Title

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We take J_m to be the Bessel function of order m. $m = 0$ is spherical, etc.

Consider a drum with radius a, and "tension" parameter c. Solution for a circular membrane are:

$$u_{mn}(r, \theta, t) = R(r)\Theta(\theta)T(t)$$

with

$$\begin{aligned} R(r) &= J_m(k_{mn} * r) \\ \Theta(\theta) &= \cos(m(\theta - \theta_0)) \\ T(t) &= \cos(ck_{mn}(t - t_0)) \end{aligned}$$

$k_m n$ is such that $J_m(k_m n a) = 0$. They are the normalized zeroes of the Bessel functions.

The general solution is a superposition of these:

$$u(r, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} c_{mn} u_{mn}(r, \theta, t)$$

for any coefficients c_{mn}.

When we hit the drum at position (r_0, θ_0) at time t_0, what should the coefficients be? We need to choose coefficients such that the state of the drum looks like a Dirac impulse at the hit point. We use the Hankel transform:

$$\begin{aligned} c_n &= \frac{\int\limits_0^a dr f(r) J_m(k_{mn} r) r}{\frac{a^2}{2} J_{m+1}^2(k_{mn} a)} \\ &= \frac{G J_m(\frac{\alpha_{mn}}{a} r_0)}{\frac{a^2}{2} J_{m+1}^2(k_{mn} a)} \end{aligned}$$

where we plugged in our impulse:

$$f(r) = G\delta(r - r_0)$$

for some strength factor G. See bottom of `http://www.hit.ac.il/staff/benzionS/Differential.Equations/Orthogonality_of_Bessel_functions.htm`.

We take the θ_0 and t_0 in our Θ and T functions to be where we hit the drum. These functions are maximal at these points.