

Raw amplitude

$$\frac{e^2}{16\pi^4} g_{\mu\nu} \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{\bar{u}(p) \gamma^\mu (m + (-k_{\sigma_2} + p_{\sigma_2}) \gamma^{\sigma_2}) \gamma^\nu u(p)}{(-m^2 + (-k_\eta + p_\eta) (-k^\eta + p^\eta)) (-t + k_\eta k^\eta)^2} \quad (1)$$

Simplified numerator

$$\frac{e^2}{16\pi^4} g_{\mu\nu} \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p) - k_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) + p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(-m^2 + (-k_\eta + p_\eta) (-k^\eta + p^\eta)) (-t + k_\eta k^\eta)^2} \quad (2)$$

Feynman parameterization

Here, we perform the following expansion:

$$\frac{1}{A_1} \cdots \frac{1}{A_n} = (n-1)! \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \cdots \int_0^{1-z_1-\cdots-z_{n-1}} dz_n \frac{1}{(z_1 A_1 + \cdots + z_n A_n)^n}$$

We use this form because a single denominator raised to a power can be simplified with the Golden Integral.

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p) - k_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) + p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(-m^2 z_3 - t z_1 - t z_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (3)$$

Expanded numerator

We split the numerator into additive terms, to process individually. The following is a list of such terms:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(-m^2 z_3 - t z_1 - t z_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (4)$$

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(-m^2 z_3 - t z_1 - t z_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (5)$$

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} - \frac{k_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(-m^2 z_3 - tz_1 - tz_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (6)$$

Golden Integral

We resolve internal momentas with this transformation:

$$\int \frac{d^d q}{(2\pi)^d} \frac{(q^2)^a}{(q^2 + D)^b} = i \frac{\Gamma(b - a - \frac{1}{2}d) \Gamma(a + \frac{1}{2}d)}{(4\pi)^{d/2} \Gamma(b) \Gamma(\frac{1}{2}d)} D^{-(b-a-d/2)}$$

After this section, all internal momenta should disappear. We will now resolve each term in a queue. Each term may produce additional terms, which are pushed to the back of the queue and resolved later.

Evaluating internal momenta in this term (2 terms left)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(-m^2 z_3 - tz_1 - tz_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (7)$$

Integrating over k

Completing the square

$$A = z_1 + z_2 + z_3 \quad (8)$$

$$B = -2z_3 p^\eta \quad (9)$$

$$C = -m^2 z_3 - tz_1 - tz_2 + z_3 p_\eta p^\eta \quad (10)$$

After $k \rightarrow q_1$ substitutions

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1 + 4z_2 + 4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1^\eta \right)^3} \quad (11)$$

Expanding numerator into 1 term(s)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{m\bar{u}(p)\gamma^\mu\gamma^\nu u(p)}{(z_1+z_2+z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (12)$$

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{m\bar{u}(p)\gamma^\mu\gamma^\nu u(p)}{(z_1+z_2+z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (13)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{m\bar{u}(p)\gamma^\mu\gamma^\nu u(p)}{(z_1+z_2+z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (14)$$

Evaluating internal momenta in this term (2 terms left)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{p_{\sigma_2} \bar{u}(p)\gamma^\mu\gamma^{\sigma_2}\gamma^\nu u(p)}{(-m^2 z_3 - tz_1 - tz_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (15)$$

Integrating over k

Completing the square

$$A = z_1 + z_2 + z_3 \quad (16)$$

$$B = -2z_3 p^\eta \quad (17)$$

$$C = -m^2 z_3 - tz_1 - tz_2 + z_3 p_\eta p^\eta \quad (18)$$

After $k \rightarrow q_1$ substitutions

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1^\eta \right)^3} \quad (19)$$

Expanding numerator into 1 term(s)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1^\eta \right)^3} \quad (20)$$

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1^\eta \right)^3} \quad (21)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (22)$$

Evaluating internal momenta in this term (2 terms left)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d k}{(2\pi)^4} \frac{k_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(-m^2 z_3 - tz_1 - tz_2 + z_1 k_\eta k^\eta + z_2 k_\eta k^\eta + z_3 k_\eta k^\eta - z_3 k_\eta p^\eta - z_3 k^\eta p_\eta + z_3 p_\eta p^\eta)^3} \quad (23)$$

Integrating over k

Completing the square

$$A = z_1 + z_2 + z_3 \quad (24)$$

$$B = -2z_3 p^\eta \quad (25)$$

$$C = -m^2 z_3 - tz_1 - tz_2 + z_3 p_\eta p^\eta \quad (26)$$

After $k \rightarrow q_1$ substitutions

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{\left(\frac{1}{2} z_3 (z_1 + z_2 + z_3)^{0.5} p_{\sigma_2} + (z_1 + z_2 + z_3) q_{1\sigma_2}\right) \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta\right)^3} \quad (27)$$

Expanding numerator into 4 term(s)

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_1 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta\right)^3} \quad (28)$$

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_2 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta\right)^3} \quad (29)$$

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_3 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta\right)^3} \quad (30)$$

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta\right)^3} \quad (31)$$

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_1 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - t z_1 - t z_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (32)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_2 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - t z_1 - t z_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (33)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_3 q_{1\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{2.0} \left(-m^2 z_3 - t z_1 - t z_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (34)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

Integrating this term:

$$\frac{e^2}{8\pi^4} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \int \frac{d^d q_1}{(2\pi)^4} - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - t z_1 - t z_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta + q_{1\eta} q_1{}^\eta \right)^3} \quad (35)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (36)$$

Evaluating internal momenta in this term (2 terms left)

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (37)$$

Evaluating internal momenta in this term (1 terms left)

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (38)$$

Evaluating internal momenta in this term (0 terms left)

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (39)$$

Final amplitudes after momenta integration

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (40)$$

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (41)$$

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (42)$$

Integrating cutoffs

Here we integrate all t -variables, which represent the upper and lower cutoffs.

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{m \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (43)$$

Denominator only

$$\frac{1}{-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta} \quad (44)$$

Integrating wrt t ...

$$\begin{aligned} & \frac{1}{-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\Lambda^2 \left(-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3 \right) - 4m^2 z_1 z_3 - 4m^2 z_2 z_3 - 4m^2 z_3^2 + 4z_1 z_3 p_\eta p^\eta + 4z_2 z_3 p_\eta p^\eta + 3z_3^2 p_\eta p^\eta \right) \\ & - \frac{1}{-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\lambda^2 \left(-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3 \right) - 4m^2 z_1 z_3 - 4m^2 z_2 z_3 - 4m^2 z_3^2 + 4z_1 z_3 p_\eta p^\eta + 4z_2 z_3 p_\eta p^\eta + 3z_3^2 p_\eta p^\eta \right) \end{aligned} \quad (45)$$

Keeping only highest order term...

$$-\frac{2 \log(\Lambda)}{z_1 + z_2} \quad (46)$$

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt \frac{p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2 + z_3)^{0.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (47)$$

Denominator only

$$\frac{1}{-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta} \quad (48)$$

Integrating wrt t ...

$$\begin{aligned} & \frac{1}{-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\Lambda^2 \left(-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3 \right) - 4m^2 z_1 z_3 - 4m^2 z_2 z_3 - 4m^2 z_3^2 + 4z_1 z_3 p_\eta p^\eta + 4z_2 z_3 p_\eta p^\eta + 3z_3^2 p_\eta p^\eta \right) \\ & - \frac{1}{-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\lambda^2 \left(-4z_1^2 - 8z_1 z_2 - 4z_1 z_3 - 4z_2^2 - 4z_2 z_3 \right) - 4m^2 z_1 z_3 - 4m^2 z_2 z_3 - 4m^2 z_3^2 + 4z_1 z_3 p_\eta p^\eta + 4z_2 z_3 p_\eta p^\eta + 3z_3^2 p_\eta p^\eta \right) \end{aligned} \quad (49)$$

Keeping only highest order term...

$$-\frac{2 \log(\Lambda)}{z_1 + z_2} \quad (50)$$

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \int_{\lambda^2}^{\Lambda^2} dt - \frac{z_3 p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{2(z_1 + z_2 + z_3)^{1.5} \left(-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta \right)} \quad (51)$$

Denominator only

$$\frac{1}{-m^2 z_3 - tz_1 - tz_2 - \frac{z_3^2 p_\eta p^\eta}{4z_1+4z_2+4z_3} + z_3 p_\eta p^\eta} \quad (52)$$

Integrating wrt $t...$

$$\begin{aligned} & \frac{1}{-4z_1^2 - 8z_1z_2 - 4z_1z_3 - 4z_2^2 - 4z_2z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\Lambda^2 \left(-4z_1^2 - 8z_1z_2 - 4z_1z_3 - 4z_2^2 - 4z_2z_3 \right) - 4m^2z_1z_3 - 4m^2z_2z_3 - 4m^2z_3^2 + 4z_1z_3p_\eta p^\eta + 4z_2z_3p_\eta p^\eta + 3z_3^2p_\eta p^\eta \right) \\ & - \frac{1}{-4z_1^2 - 8z_1z_2 - 4z_1z_3 - 4z_2^2 - 4z_2z_3} (4z_1 + 4z_2 \\ & + 4z_3) \log \left(\lambda^2 \left(-4z_1^2 - 8z_1z_2 - 4z_1z_3 - 4z_2^2 - 4z_2z_3 \right) - 4m^2z_1z_3 - 4m^2z_2z_3 - 4m^2z_3^2 + 4z_1z_3p_\eta p^\eta + 4z_2z_3p_\eta p^\eta + 3z_3^2p_\eta p^\eta \right) \end{aligned} \quad (53)$$

Keeping only highest order term...

$$-\frac{2 \log (\Lambda)}{z_1+z_2} \quad (54)$$

Integrating z -variables

Here we integrate all z -variables, the Feynman parameters.

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 - \frac{2m \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{(z_1+z_2)(z_1+z_2+z_3)^{0.5}} \quad (55)$$

Integrating wrt $z_3...$

$$-\frac{4m \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{z_1+z_2} + \frac{4m \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p)}{\sqrt{z_1+z_2}} \quad (56)$$

Integrating wrt $z_2...$

$$-8m\sqrt{z_1} \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p) + 4m \log (\Lambda) \log (z_1) \bar{u}(p) \gamma^\mu \gamma^\nu u(p) + 8m \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p) \quad (57)$$

Integrating wrt $z_1...$

$$-\frac{4m}{3} \log (\Lambda) \bar{u}(p) \gamma^\mu \gamma^\nu u(p) \quad (58)$$

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 - \frac{2 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2)(z_1 + z_2 + z_3)^{0.5}} \quad (59)$$

Integrating wrt $z_3 \dots$

$$-\frac{4 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu}{z_1 + z_2} \gamma^{\sigma_2} \gamma^\nu u(p) + \frac{4 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu}{\sqrt{z_1 + z_2}} \gamma^{\sigma_2} \gamma^\nu u(p) \quad (60)$$

Integrating wrt $z_2 \dots$

$$-8\sqrt{z_1} \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) + 4 \log(\Lambda) \log(z_1) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) + 8 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) \quad (61)$$

Integrating wrt $z_1 \dots$

$$-\frac{4}{3} \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) \quad (62)$$

Integrating this term

$$\frac{ie^2}{16\pi^2} g_{\mu\nu} \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int_0^{-z_1-z_2+1} dz_3 \frac{z_3 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p)}{(z_1 + z_2)(z_1 + z_2 + z_3)^{1.5}} \quad (63)$$

Integrating wrt $z_3 \dots$

$$-\frac{\log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu}{z_1 + z_2} \left(\frac{4z_1}{\sqrt{z_1 + z_2}} + \frac{4z_2}{\sqrt{z_1 + z_2}} \right) \gamma^{\sigma_2} \gamma^\nu u(p) + \frac{\log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu}{z_1 + z_2} (2z_1 + 2z_2 + 2) \gamma^{\sigma_2} \gamma^\nu u(p) \quad (64)$$

Integrating wrt $z_2 \dots$

$$8\sqrt{z_1} \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) + (-2z_1 + 2) \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) - 2 \log(\Lambda) \log(z_1) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) - 8 \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) \quad (65)$$

Integrating wrt $z_1 \dots$

$$\frac{1}{3} \log(\Lambda) p_{\sigma_2} \bar{u}(p) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu u(p) \quad (66)$$

Final amplitudes after z -variable integration

$$\frac{ie^2}{16\pi^2}g_{\mu\nu} - \frac{4m}{3}\log(\Lambda)\bar{u}(p)\gamma^\mu\gamma^\nu u(p) \quad (67)$$

$$\frac{ie^2}{16\pi^2}g_{\mu\nu} - \frac{4}{3}\log(\Lambda)p_{\sigma_2}\bar{u}(p)\gamma^\mu\gamma^{\sigma_2}\gamma^\nu u(p) \quad (68)$$

$$\frac{ie^2}{16\pi^2}g_{\mu\nu} \frac{1}{3}\log(\Lambda)p_{\sigma_2}\bar{u}(p)\gamma^\mu\gamma^{\sigma_2}\gamma^\nu u(p) \quad (69)$$