

## Raw amplitude

$$\frac{e^2}{16\pi^4} 1 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) \gamma^\mu (m + (k_{\sigma_1} + p_{\sigma_1}) \gamma^{\sigma_1}) (m + p_{\sigma_1} \gamma^{\sigma_2}) \gamma^\nu}{(-m^2 + (k_\eta + p_\eta) (k^\eta + p^\eta)) (-m^2 + p_\eta p^\eta)} \right) \quad (1)$$

## Simplified numerator

$$\begin{aligned} \frac{e^2}{16\pi^4} 1 \int \frac{d^d p}{(2\pi)^4} \left( \frac{1}{(-m^2 + (k_\eta + p_\eta) (k^\eta + p^\eta)) (-m^2 + p_\eta p^\eta)} \right. & (\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \\ & + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu) \left. \right) \end{aligned} \quad (2)$$

## Feynman parameterization

Here, we perform the following expansion:

$$\frac{1}{A_1} \cdots \frac{1}{A_n} = (n-1)! \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \cdots \int_0^{1-z_1-\cdots-z_{n-1}} dz_n \frac{1}{(z_1 A_1 + \cdots + z_n A_n)^n}$$

We use this form because a single denominator raised to a power can be simplified with the Golden Integral.

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu + \epsilon_\nu(k) \bar{\epsilon}_\mu(k) p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (3)$$

## Expanded numerator

We split the numerator into additive terms, to process individually. The following is a list of such terms:

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (4)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (5)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (6)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (7)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (8)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (9)$$

## Golden Integral

We resolve internal momentas with this transformation:

$$\int \frac{d^d q}{(2\pi)^d} \frac{(q^2)^a}{(q^2 + D)^b} = i \frac{\Gamma(b - a - \frac{1}{2}d) \Gamma(a + \frac{1}{2}d)}{(4\pi)^{d/2} \Gamma(b) \Gamma(\frac{1}{2}d)} D^{-(b-a-d/2)}$$

After this section, all internal momenta should disappear. We will now resolve each term in a queue. Each term may produce additional terms, which are pushed to the back of the queue and resolved later.

## Evaluating internal momenta in this term (5 terms left)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (10)$$

Integrating over  $p$   
Completing the square

$$A = z_1 + z_2 \quad (11)$$

$$B = 2z_1 k^\eta \quad (12)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (13)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta}^\eta \right)^2} \right) \quad (14)$$

**Expanding numerator into 1 term(s)**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta}^\eta \right)^2} \right) \quad (15)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta}^\eta \right)^2} \right) \quad (16)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5}} \right) \quad (17)$$

**Evaluating internal momenta in this term (5 terms left)**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (18)$$

Integrating over  $p$   
 Completing the square

$$A = z_1 + z_2 \quad (19)$$

$$B = 2z_1 k^\eta \quad (20)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (21)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (22)$$

**Expanding numerator into 1 term(s)**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (23)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{0.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (24)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{0.5}} \right) \quad (25)$$

## Evaluating internal momenta in this term (5 terms left)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (26)$$

Integrating over  $p$

Completing the square

$$A = z_1 + z_2 \quad (27)$$

$$B = 2z_1 k^\eta \quad (28)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (29)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m \left( z_1 (z_1 + z_2)^{0.5} k_{\sigma_1} - 2 (z_1 + z_2) q_{1\sigma_1} \right) \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{2 (z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (30)$$

## Expanding numerator into 3 term(s)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (31)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_2 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (32)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{2 (z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (33)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (34)$$

Found 1 q-vector terms in the numerator.  
Term vanishes due to Ward identity

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_2 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (35)$$

Found 1 q-vector terms in the numerator.  
Term vanishes due to Ward identity

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{2 (z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (36)$$

Found 0 q-vector terms in the numerator.  
Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu}{2 (z_1 + z_2)^{1.5}} \right) \quad (37)$$

## Evaluating internal momenta in this term (5 terms left)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m p_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (38)$$

Integrating over  $p$

Completing the square

$$A = z_1 + z_2 \quad (39)$$

$$B = 2z_1 k^\eta \quad (40)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (41)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m \left( z_1 (z_1 + z_2)^{0.5} k_{\sigma_1} - 2 (z_1 + z_2) q_{1\sigma_1} \right) \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (42)$$

## Expanding numerator into 3 term(s)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (43)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_2 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (44)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (45)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (46)$$

Found 1 q-vector terms in the numerator.  
Term vanishes due to Ward identity

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_2 q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (47)$$

Found 1 q-vector terms in the numerator.  
Term vanishes due to Ward identity

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (48)$$

Found 0 q-vector terms in the numerator.  
Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{1.5}} \right) \quad (49)$$



## Evaluating internal momenta in this term (5 terms left)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (50)$$

Integrating over  $p$

Completing the square

$$A = z_1 + z_2 \quad (51)$$

$$B = 2z_1 k^\eta \quad (52)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (53)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) \left( z_1 (z_1 + z_2)^{0.5} k_{\sigma_1} - 2 (z_1 + z_2) q_{1\sigma_1} \right)^2 \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{4 (z_1 + z_2)^{3.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (54)$$

## Expanding numerator into 3 term(s)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (55)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{4 (z_1 + z_2)^{2.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (56)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (57)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (58)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{1.5}} \right) \quad (59)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{4(z_1 + z_2)^{2.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (60)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{4(z_1 + z_2)^{2.5}} \right) \quad (61)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_{1\eta} \right)^2} \right) \quad (62)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

## Evaluating internal momenta in this term (6 terms left)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d p}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} p_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(-m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta + z_1 k_\eta p^\eta + z_1 k^\eta p_\eta + z_1 p_\eta p^\eta + z_2 p_\eta p^\eta)^2} \right) \quad (63)$$

Integrating over  $p$

Completing the square

$$A = z_1 + z_2 \quad (64)$$

$$B = 2z_1 k^\eta \quad (65)$$

$$C = -m^2 z_1 - m^2 z_2 + z_1 k_\eta k^\eta \quad (66)$$

After  $p \rightarrow q_1$  substitutions

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) \left( z_1 (z_1 + z_2)^{0.5} k_{\sigma_1} - 2 (z_1 + z_2) q_{1\sigma_1} \right) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (67)$$

## Expanding numerator into 3 term(s)

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{2 (z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (68)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (69)$$

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_2 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1+4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1^\eta \right)^2} \right) \quad (70)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{2(z_1 + z_2)^{1.5} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1{}^\eta \right)^2} \right) \quad (71)$$

Found 0 q-vector terms in the numerator.

Apply golden integral

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( - \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{2(z_1 + z_2)^{1.5}} \right) \quad (72)$$

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1{}^\eta \right)^2} \right) \quad (73)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

**Integrating this term:**

$$\frac{e^2}{16\pi^4} 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \int \frac{d^d q_1}{(2\pi)^4} \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_2 k_{\sigma_1} q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu}{(z_1 + z_2)^{2.0} \left( -m^2 z_1 - m^2 z_2 - \frac{z_1^2 k_\eta k^\eta}{4z_1 + 4z_2} + z_1 k_\eta k^\eta + q_{1\eta} q_1{}^\eta \right)^2} \right) \quad (74)$$

Found 1 q-vector terms in the numerator.

Term vanishes due to Ward identity

**Evaluating internal momenta in this term (6 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5}} \right) \quad (75)$$

**Evaluating internal momenta in this term (5 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1}}{(z_1 + z_2)^{0.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (76)$$

**Evaluating internal momenta in this term (4 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (77)$$

**Evaluating internal momenta in this term (3 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu \right) \quad (78)$$

**Evaluating internal momenta in this term (2 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu}{(z_1 + z_2)^{1.5}} \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (79)$$

**Evaluating internal momenta in this term (1 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1}}{4(z_1 + z_2)^{2.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (80)$$

**Evaluating internal momenta in this term (0 terms left)**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (81)$$

**Final amplitudes after momenta integration**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5}} \right) \quad (82)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1}}{(z_1 + z_2)^{0.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (83)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (84)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu \right) \quad (85)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu}{(z_1 + z_2)^{1.5}} \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (86)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1}}{4(z_1 + z_2)^{2.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (87)$$

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (88)$$

## Integrating cutoffs

Here we integrate all  $t$ -variables, which represent the upper and lower cutoffs.

### Integrating this term

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m^2 \gamma^\mu \gamma^\nu}{(z_1 + z_2)^{0.5}} \right) \quad (89)$$

Denominator only

$$1 \quad (90)$$

### Integrating this term

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m k_{\sigma_1}}{(z_1 + z_2)^{0.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (91)$$

Denominator only

$$1 \quad (92)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \quad (93)$$

Denominator only

$$1 \quad (94)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) m z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu \right) \quad (95)$$

Denominator only

$$1 \quad (96)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu}{(z_1 + z_2)^{1.5}} \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (97)$$

Denominator only

$$1 \quad (98)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1}}{4(z_1 + z_2)^{2.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (99)$$

Denominator only

$$1 \quad (100)$$



**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2}e^21\int\limits_0^1dz_1\int\limits_0^{-z_1+1}dz_2\left(-\frac{\epsilon_\nu(k)\bar{\epsilon}_\mu(k)z_1k_{\sigma_1}}{2\left(z_1+z_2\right)^{1.5}}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu\right)$$

(101)

Denominator only

1

(102)

**Integrating  $z$ -variables**

Here we integrate all  $z$ -variables, the Feynman parameters.

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2}e^21\int\limits_0^1dz_1\int\limits_0^{-z_1+1}dz_2\left(\frac{\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m^2\gamma^\mu\gamma^\nu}{\left(z_1+z_2\right)^{0.5}}\right)$$

(103)

Integrating wrt  $z_2$ ...

$$-2\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m^2\sqrt{z_1}\gamma^\mu\gamma^\nu+2\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m^2\gamma^\mu\gamma^\nu$$

(104)

Integrating wrt  $z_1$ ...

$$\frac{2m^2}{3}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)\gamma^\mu\gamma^\nu$$

(105)

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2}e^21\int\limits_0^1dz_1\int\limits_0^{-z_1+1}dz_2\left(\frac{\epsilon_\nu(k)\bar{\epsilon}_\mu(k)mk_{\sigma_1}}{\left(z_1+z_2\right)^{0.5}}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu\right)$$

(106)

Integrating wrt  $z_2 \dots$

$$-2\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m\sqrt{z_1}k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu + 2\epsilon_\nu(k)\bar{\epsilon}_\mu(k)mk_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu \quad (107)$$

Integrating wrt  $z_1 \dots$

$$\frac{2m}{3}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu \quad (108)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2}e^2\int_0^1dz_1\int_0^{-z_1+1}dz_2\left(-\frac{\epsilon_\nu(k)\bar{\epsilon}_\mu(k)mz_1k_{\sigma_1}}{2(z_1+z_2)^{1.5}}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu\right) \quad (109)$$

Integrating wrt  $z_2 \dots$

$$-\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m\sqrt{z_1}k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu + \epsilon_\nu(k)\bar{\epsilon}_\mu(k)mz_1k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu \quad (110)$$

Integrating wrt  $z_1 \dots$

$$-\frac{m}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu \quad (111)$$

**Integrating this term**

$$\frac{1249999896.56489i}{\pi^2}e^2\int_0^1dz_1\int_0^{-z_1+1}dz_2\left(-\frac{\epsilon_\nu(k)\bar{\epsilon}_\mu(k)mz_1k_{\sigma_1}}{2(z_1+z_2)^{1.5}}\gamma^\mu\gamma^{\sigma_2}\gamma^\nu\right) \quad (112)$$

Integrating wrt  $z_2 \dots$

$$-\epsilon_\nu(k)\bar{\epsilon}_\mu(k)m\sqrt{z_1}k_{\sigma_1}\gamma^\mu\gamma^{\sigma_2}\gamma^\nu + \epsilon_\nu(k)\bar{\epsilon}_\mu(k)mz_1k_{\sigma_1}\gamma^\mu\gamma^{\sigma_2}\gamma^\nu \quad (113)$$

Integrating wrt  $z_1 \dots$

$$-\frac{m}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_2}\gamma^\nu \quad (114)$$

## Integrating this term

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu}{(z_1 + z_2)^{1.5}} \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (115)$$

Integrating wrt  $z_2 \dots$

$$-2\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu + \frac{2q_{1\sigma_1}}{\sqrt{z_1}} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \quad (116)$$

Integrating wrt  $z_1 \dots$

$$2\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \quad (117)$$

## Integrating this term

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( \frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1}}{4(z_1 + z_2)^{2.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (118)$$

Integrating wrt  $z_2 \dots$

$$\frac{1}{6} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) \sqrt{z_1} k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu - \frac{1}{6} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1^2 k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \quad (119)$$

Integrating wrt  $z_1 \dots$

$$\frac{1}{18} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \quad (120)$$

## Integrating this term

$$\frac{1249999896.56489i}{\pi^2} e^2 1 \int_0^1 dz_1 \int_0^{-z_1+1} dz_2 \left( -\frac{\epsilon_\nu(k) \bar{\epsilon}_\mu(k) z_1 k_{\sigma_1}}{2(z_1 + z_2)^{1.5}} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \quad (121)$$

Integrating wrt  $z_2 \dots$

$$-\epsilon_\nu(k)\bar{\epsilon}_\mu(k)\sqrt{z_1}k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu + \epsilon_\nu(k)\bar{\epsilon}_\mu(k)z_1k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu \quad (122)$$

Integrating wrt  $z_1 \dots$

$$-\frac{1}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu \quad (123)$$

## Final amplitudes after $z$ -variable integration

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(\frac{2m^2}{3}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)\gamma^\mu\gamma^\nu\right) \quad (124)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(\frac{2m}{3}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu\right) \quad (125)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(-\frac{m}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^\nu\right) \quad (126)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(-\frac{m}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_2}\gamma^\nu\right) \quad (127)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(2\epsilon_\nu(k)\bar{\epsilon}_\mu(k)q_{1\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu\right) \quad (128)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(\frac{1}{18}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu\right) \quad (129)$$

$$\frac{1249999896.56489i}{\pi^2}e^2 1\left(-\frac{1}{6}\epsilon_\nu(k)\bar{\epsilon}_\mu(k)k_{\sigma_1}\gamma^\mu\gamma^{\sigma_1}\gamma^{\sigma_2}\gamma^\nu\right) \quad (130)$$

# Evaluating spins and gamma matrices

TODO. Here's the sum for now:

$$\begin{aligned} & \frac{1249999896.56489i}{\pi^2} e^2 1 \left( \frac{2m^2}{3} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) \gamma^\mu \gamma^\nu \right) + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( \frac{2m}{3} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( -\frac{m}{6} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^\nu \right) \\ & + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( -\frac{m}{6} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_2} \gamma^\nu \right) + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( 2\epsilon_\nu(k) \bar{\epsilon}_\mu(k) q_{1\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \\ & + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( \frac{1}{18} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) + \frac{1249999896.56489i}{\pi^2} e^2 1 \left( -\frac{1}{6} \epsilon_\nu(k) \bar{\epsilon}_\mu(k) k_{\sigma_1} \gamma^\mu \gamma^{\sigma_1} \gamma^{\sigma_2} \gamma^\nu \right) \end{aligned} \quad (131)$$