

$$Q_1 \quad f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2} \times \sqrt{2\pi}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Likelihood function

$$L(\theta_1, \theta_2) = \prod_{i=1}^n f(x_i, \theta_1, \theta_2) = \theta_2^{-n/2} (2\pi)^{-n/2} e^{-\frac{1}{2\theta_2} \sum (x_i - \theta_1)^2}$$

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log \theta_2 - \frac{n}{2} \log 2\pi - \frac{\sum (x_i - \theta_1)^2}{2\theta_2}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = \frac{-2 \sum (x_i - \theta_1)(-1)}{2\theta_2} = 0$$

$$\sum x_i - n\theta_1 = 0$$

$$\hat{\theta}_1 = \hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = -\frac{n}{2\theta_2} - \frac{\sum (x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$-n\theta_2 + \sum (x_i - \theta_1)^2 = 0$$

$$\hat{\theta}_2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\theta_1 = \frac{\sum x_i}{n}$$

$$\theta_2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

Q2 $P(y|\theta) = \text{Bin}(y, m, \theta)$

$$= {}^m C_y \times \theta^y [1-\theta]^{m-y}$$

Log Likelihood Function = $\log \theta(y|\theta)$

$$ll(\theta) = \log {}^m C_y + y \log \theta + (m-y) \log(1-\theta)$$

Diffn. w.r.t. θ

$$\frac{\partial ll(\theta)}{\partial \theta} = \frac{y}{\theta} - \frac{(m-y)}{1-\theta} = 0$$

$$\frac{y}{\hat{\theta}} = \frac{(m-y)}{1-\hat{\theta}}$$

$$(m-y) \hat{\theta} = y(1-\hat{\theta})$$

$$m\hat{\theta} - y\hat{\theta} = y - y\hat{\theta}$$

$$m\hat{\theta} = y$$

$$\hat{\theta} = \frac{y}{m}$$