

Research Tools for Social Scientists

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1 Preface

I started this online book as background material for the course Network Analysis and from the need to teach social science students (including Business Administration and econmonics students) the very basics of R and the intuition behing applying linear regression. I intend to work on this book on the fly meaning that during courses I try to see what the needs of students really are. Moreover, during the year I intend to add additional chapters, specifically about the use of stated preference modeling—and its corresponding logit estimation—and working with geographical data. Obviously, all in R. At the moment this book really has not yet passed its infant stage. Therefore, all input in the form of comments, critique and remarks are *highly* appreciated.

2 Introduction

2.1 Why do we need all this?

A natural question that arises, or at least should arise, is why bother? Why learn students in the social sciences new tools to do research. ‘Old’ tools¹ such as **Word**, **Excel** and **Statado** just as fine right? I would say yes, but up to a certain extent. If you are only interested in straightforward regression and anova techniques and collect your sample by survey research, then **Stata** and **Excel**(or even **SPSS**, the horror...) would definitely suffice. But if you would like to do more fancy and cool stuff, including creating beautiful diagrams, nice maps, simulation, network analysis, and even whole books in **html** then you need more elaborate tools.

And even though most of the best tools out there² are open source (so they are **free**, as in **free beer**) they will cost you something dearly: namely, your time. The learning curve of these tools are usually quite steep (which also means they will pay-off quickly). So, choose your battles carefully. The best tools, I would argue, have the following set of characteristics:

- They are open source. So no lock-in effects anymore with datasets (**Stata**) or versions (**Word**).
- They are scriptable. This means they can “communicate” with other programs. Usually this means that these tools revolve around **.txt** files.
- There is a large community that uses these tools. This is very handy for supplementary material and solving questions.
- They are multi-purpose, which means you can use them for various purposes. This is actually only a requirement of the data management/statistics you use.

One of the tools that meets all these requirements, and more, is **R** and the next section lays out why even social scientists should learn **R**.

2.2 Why use R and not other applications?

Ask any data scientist at the moment for the software tools most used and they will most likely answer **R** or **Python**. Of course, that should not be a valid answer (many people use **Word** as well and nobody would argue that **Word** is brilliant), but it indicates the popularity (and the community) that uses **R**.

Where 10 years ago most social scientists still used **SPSS** (and the economists **Stata**), that has now changed completely (well the economists still use **Stata** but the rest of the world moved on). And for good reasons, namely:

1. It is open source and thus free;
2. **R** is flexible and thus multi-purpose;
3. there is now a **very** large userbase; everything you can dream of (well sort of), somebody else most likely already programmed;³
4. it generates beautiful pictures, diagram, maps, and histograms (even pie diagrams);
5. relative to **Stata** or **Excel** it is fast, which is great for larger (spatial) databases.

In general, you can use **R** for statistical analysis, simulation analysis, data management, visual display of data, creating documents (and presentations), and even GIS applications. In that respect it is far more flexible than **Stata**. Last but perhaps not least, **R** is more and more used outside academia as well. Twitter, Facebook and Google use **R**, but companies as the NY times as well for interactive website diagrams.

¹Some of the tools I use are actually quite old, even so far as from the 1970s and they are still very good.

²Including **Markdown**, **LaTeX**, **Python**, **Git** and **GitHub**.

³CRAN packages give a great overview of all the official packages out there and the wide range of applications, and again they are all free!

2.3 Regression analysis: not again!

I will also spend some time on the use of regression analysis and try to explain in my own words how to use it. In general, my experience is that many students have little or no experience in regression analysis. Moreover, and perhaps even worse, they have little or no intuition for regression analysis. This sort of sucks, the more because regression is the most often statistical technique used in the social sciences (heck, in all sciences).

In my perception, students are taught the principles of statistics (typically this involves lots of things as ANOVA), where in the last course, one or two hours is spent on regression analysis. This is fine, as long as it comes back in a course as *applied* statistics or econometrics (how and when to do this stuff?) or in other applied courses. But usually this is not the case. Most courses don't care about regression analysis and the first moment you have to use it again is when writing your thesis and at that time it is a bit too late to teach the applied stuff again.

So, yeah, therefore a bit of regression analysis from my perspective. But again, you only learn this by doing and under the guidance of various teachers (and in various circumstances).

3 Basic R Usage

```
library("swirl")
library("rio")
library("ggplot2")
library("dplyr")
```

The R programming language and software environment for statistical computing is an implementation of the proprietary S programming language by Ross Ihaka and Robert Gentleman in 1992. It quickly gained in popularity (see, e.g, this Nature article from 2014 Programming tools: Adventures with R) and now has more than 8,000 user contributed packages (see as well the blog piece On the growth of CRAN packages).

3.1 Where to get it

First, you need to install R itself. You can do this by downloading this from CRAN (we choose here the server from the Netherlands). Choose your appropriate operating system, choose the **base** system, download R and install it. That's it!

The base distribution of R comes with a built-in editor, where you can write your script (more about scripts in subsection 3.3.1). This editor is however *very* basic. Therefore, it is very much advised that you download and install the Open Source editor **RStudio** as well. Again, choose your operating system and just install the latest version. The very short video (1.5 minutes) on RStudio's website gives an overview of the *basic* features (it can do quite some more stuff).

3.2 For absolute beginners

At the moment there are *many* tutorials, blogs, youtube clips, and background materials about using R on the internet. I therefore do not intend to write a complete handbook, but focus instead on what I need for my courses. Moreover, I do not intend to teach the very basic stuff. There is very good tutorial package out there called Swirl (see as well the website: <http://swirlstats.com/students.html>). I very much recommend using this package for absolute beginners. The way to do this is rather simple. First install the package by typing:

```
install.packages("swirl")
```

then start Swirl by first loading the package:

```
library("swirl")
```

And then call the function by typing:

```
swirl()
```

In the first menu choose **R Programming**. Now, there are 15 lessons. I find the first four the most useful (Basic Building Block, Workspace and Files, Sequences of Numbers, Vectors), but others are very useful as well to go through. The command `main()` by the way brings you always back to the main menu, and do not go for the credits on coursera (that is now a paid online course).

3.3 How to use it

R is truly a programming language in the sense that there is no graphical user interface (GUI) involved. You need to type your own commands. And for beginners this sort of sucks. It seems slower, you have no idea which commands to type in, and you frequently make many mistakes. However, when you start to use it more, speed of getting things done goes up (sometimes exponentially), you have a better grasp on the basic commands, and the number of mistakes go down. In contrast with programs such as **Excel** or **SPSS**, there are two big differences: (i) you use scripts and (ii) you make frequently use of packages which are essentially written by other R users.

3.3.1 Scripts

The use of scripts or program files is somewhat alien to most. Although **Stata** also makes use of so-called **do** files. You start a new script by clicking on File > New Script (R editor) or File > New File > New Script (**RStudio**). You now have a new *empty* file (which you have to save from time to time). If you fill in this file with commands, you are actually programming. The **huge** benefit of this procedure is that you record what you have done and that you can easily change something.

As an example, assume that somebody gives you a dataset with 2 variables and ask you to analyse this dataset. With **SPSS** you read this dataset in and then click on various buttons so you get some output. Now, assume further that this person actually has forgotten a variable (this happens more often than you think) and gives you a new dataset with 3 variables. Then you have to do all the clicking again (and hopefully you remembered on which buttons you actually clicked).

When you have a script file you only have to change the code in 1 or 2 places and run it all again. No sweat! So, writing up all the commands and save it for later, might cost you some time in the **beginning**, but there are **huge** time savers later on! To run a script you simply need to press the button Edit > Run all (R editor) or Code > Run Region > Run All (**RStudio**). Nobody does that however, because there are numerous Keyboard Shortcuts (I advise you to learn them, because they make you considerably faster. Actually, most Keyboard Shortcuts work in a wide variety of editors—even in **Word**.)

3.3.2 Help!

Sometimes you have found a command you would think you could use, but you do not know how, then you need to use `? operator` in front of the command. For instance, you know that the command `c()` could be useful, but how? Then type:

```
?c()
```

And the appropriate documentation will pop up.

3.3.3 Packages

R (as many other software applications nowadays) depends heavily on packages written by other parties, usually users of R. There are now many packages out there. You can find the ‘official’ ones on the CRAN website, but there are many more. Packages have to be installed (both R editor and RStudio have a separate package manager) and afterwards loaded. Say, for instance we want to use the awesome `ggplot2` package (a package to make plots look nice, actually to make more elaborate plots but anyway), then to install and load the package we give the commands:

```
install.packages("ggplot2")
library("ggplot2")
```

Now, we can use the commands from this package as if they were built-in.

Gradually, we come across some useful packages. Those we will use in a chapter will always be listed at the start of the chapter.

3.3.4 Using comments

A final word in this section about the use of comments. **Do it!**. Really, it will make your future life much easier if you have documented what you have done. You can insert comments by using the `#` operator (everything after the hashtag is a comment), so, e.g.,

```
2+2 # always wanted to know about the outcome, but were afraid to ask
```

3.4 Reading and writing data

To properly do statistics one needs data (duh!). Luckily, there are numerous ways to get data in R.

When you just have a `csv` text file (comma separated file), it is easy, you just type:

```
df <- read.csv(file="my_data.csv", header = TRUE)
```

and you read in the `data.csv` in a data frame variable called `df`. Note that the original header variables are preserved. If you would like to store your data you can do the reverse, namely:

```
write.csv(df, file = "my_data.csv")
```

Now sometimes you do not have nicely formatted `.csv` or `.txt` files, but nasty `.dta` files from Stata or `.xlsx` files from Excel. Here the package `Rio` comes very handy, being the swiss-army knife of data converters in R. Assume you have the `mtcars` dataset in various formats, then you can do

```
library("rio")

x <- import("mtcars.sav") # SPSS data file
y <- import("mtcars.xlsx") # Excel data file
z <- import("mtcars.dta") # Stata data file
```

and all dataframes `x`, `y` `z` should be identical.

3.5 Ehmmm, dataframes

I already talked a bit about dataframes, but have not yet explained what they are. Simply, it is your data. Lets construct a simple dataframe:

```
Names <- c("Thomas", "Erik", "Mark", "Eveline")
Grades <- c(5, 8, 6.5, 7)
Female <- c(FALSE, FALSE, FALSE, TRUE)

df_grades <- data.frame(Names, Grades, Female)
```

First, note a couple of things. A dataframe can consist of various data_types, in this case strings (the names)—so variable names are always in strings—, numbers (the grades) and so-called Booleans (someone is female or not). Secondly, we have named the variables. So, we now have a dataframe called `df_grades`. Great, now what? Well, we can do a couple of things. By using the command `head()` we can show the first 6 rows of this dataframe.

```
head(df_grades)
```

```
##      Names Grades Female
## 1  Thomas     5.0  FALSE
## 2    Erik     8.0  FALSE
## 3    Mark     6.5  FALSE
## 4 Eveline     7.0   TRUE
```

Because we only have four observations this actually gives our whole dataframe (you can also just type `df_grades` to get this). Using square brackets `[]` allows you to get specific information from this dataset, where the first index denotes the row and the second index denotes the column. Look at the following examples:

```
df_grades[1,2]
```

```
## [1] 5
```

```
df_grades[1,]
```

```
##      Names Grades Female
## 1 Thomas     5  FALSE
```

```
df_grades[3]
```

```
##      Female
## 1  FALSE
## 2  FALSE
## 3  FALSE
## 4   TRUE
```

```
df_grades["Names"]
```

```
##      Names
## 1  Thomas
## 2    Erik
## 3    Mark
## 4 Eveline
```

Most of the statistical stuff we will do invokes the use of dataframe and specific variables from that dataframe.

How to change variables or generate new variables in such a dataframe. Well, remember you can always access elements from an object by the `$`-sign. So, let's suppose we want to add a new variable to our dataframe where we point to the `Grades` variable. We can do this as follows:

```
df_grades$Grades_revised <- df_grades$Grades + 1
```

Now, the grades dataframe looks like:

```
head(df_grades)
```

```
##      Names Grades Female Grades_revised
## 1  Thomas    5.0  FALSE             6.0
## 2   Erik    8.0  FALSE             9.0
## 3   Mark    6.5  FALSE             7.5
## 4 Eveline    7.0   TRUE             8.0
```

We can even do this in a more simple way, by invoking the most brilliant `dplyr` package. Say, we want to create a log variable of the “Grades” variable and then a new variable where add +1. No idea why, but we want it. Then

```
df_grades <- mutate(df_grades,
  log_grades = log(Grades),
  log_grades_revised = log_grades + 1)
```

Does the trick. Note that I only have to invoke `mutate` once. `dplyr` can be used for filtering, selecting, reshaping, sorting and a whole lot of other stuff. A very handy cheatsheet can be found [here](#).

Now, the grades dataframe looks like:

```
head(df_grades)
```

```
##      Names Grades Female Grades_revised log_grades log_grades_revised
## 1  Thomas    5.0  FALSE             6.0    1.609438         2.609438
## 2   Erik    8.0  FALSE             9.0    2.079442         3.079442
## 3   Mark    6.5  FALSE             7.5    1.871802         2.871802
## 4 Eveline    7.0   TRUE             8.0    1.945910         2.945910
```

3.6 Regression modeling

Before we start laying-out how to do regression modeling in R, we first need data. And for this purpose we will simulate our data by the following commands:

```
x <- runif(100, min = 0, max = 1) # create 100 uniformly distributed numbers in interval (0,1)
y <- 2 + 6*x + rnorm(100, mean = 0, sd = 1) # rnorm stands for the normal distribution
df <- data.frame(y,x) # Strictly not necessary but for the sake of the exposition
```

So in fact we have now created the following model:

$$y_i = 2 + 6x_i + \epsilon_i,$$

where ϵ_i is standard normally distributed (as I will explain later in Section 4.1 this is for convenience but **not** absolutely needed to do linear regression).

checking this with

```
head(df)
```

```
##      y      x
## 1 1.813045 0.31684671
## 2 7.813838 0.73022612
## 3 2.355888 0.09872103
## 4 5.653408 0.29544488
## 5 3.707419 0.23773422
## 6 3.178336 0.27828697
```

indeed shows the first 6 combination of x and y . If we now perform a linear regression, then we expect that the estimated intercept should be very close to 2 and the estimated slope parameter should be very close to 6. In R we have the command `lm()` (from linear model) to do this as follows:

```
linear_model <- lm(y~x, data = df)
```

We now have performed a regression of y onto x , using the data `df` and save the result in a variable called `linear_model`. First note the first part of the `lm(),` expression. `y~x` means that y is the left-hand side variable and x the right-hand side variable (if we have more variables, the formula becomes something like `asy~x+u+v+w+z`). The second part of the `lm(),` expression denotes the specific dataframe to be used. Namely, in R you can have multiple dataframes so you have to specify which one is to be used.

Right, but now what? Well, we have now a variable called `linear_model`, just typing in `linear_model` only gets you the real basic results:

```
linear_model
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Coefficients:
## (Intercept)          x
##      2.044      5.549
```

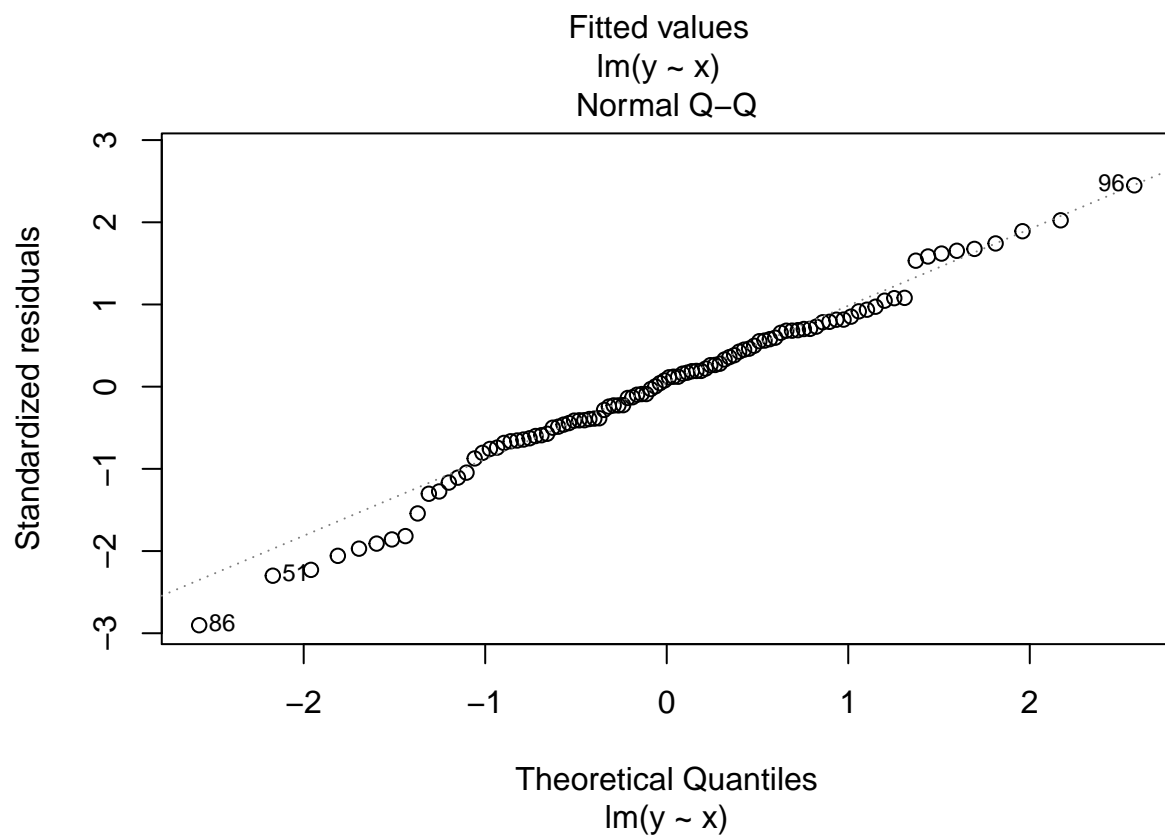
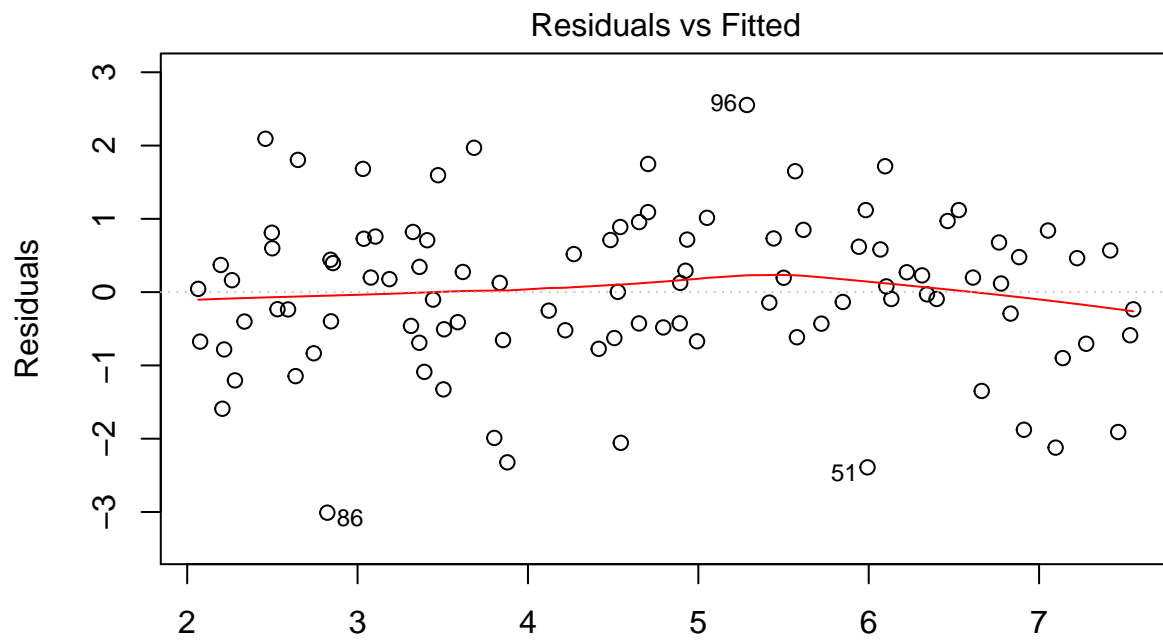
But, you want more, right? Standard errors, t-statistics, R-squares, the whole lot. For this, you need the `summary()` command, which gives you the following outcome

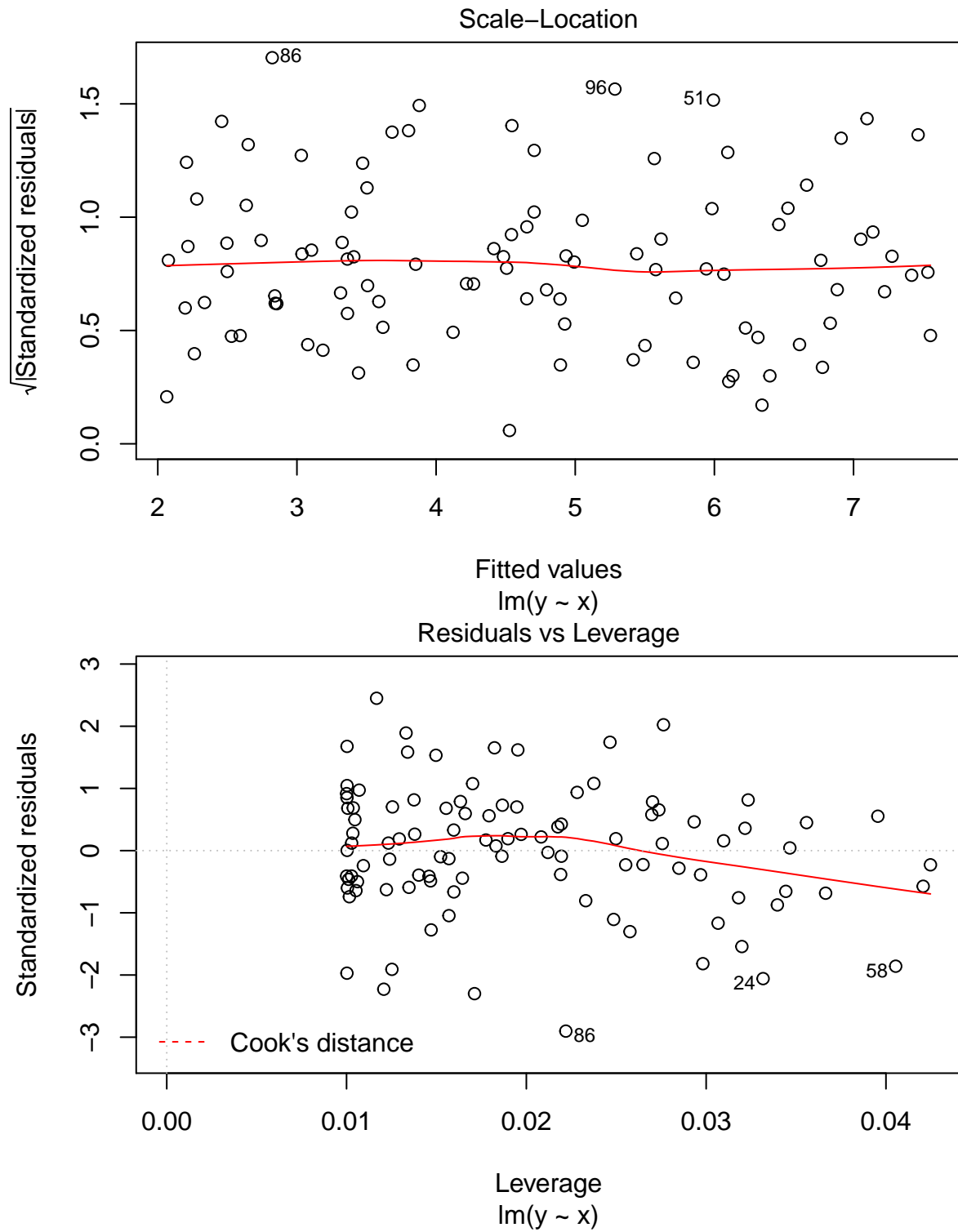
```
summary(linear_model)
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.00912 -0.59496  0.09804  0.70891  2.55356
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.0442     0.1963   10.41  <2e-16 ***
## x             5.5490     0.3575   15.52  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.049 on 98 degrees of freedom
## Multiple R-squared:  0.7108, Adjusted R-squared:  0.7079
## F-statistic: 240.9 on 1 and 98 DF,  p-value: < 2.2e-16
```

Interestingly, our parameters are close to 2 and 6, but not that close (which happens with a limited amount of observations, notice as well the relatively large standard errors). So, the command `summary()` gives all the needed statistical output, but what about regression diagnostics. For this you can ask for a plot of the variable `linear_model`


```
plot(linear_model)
```



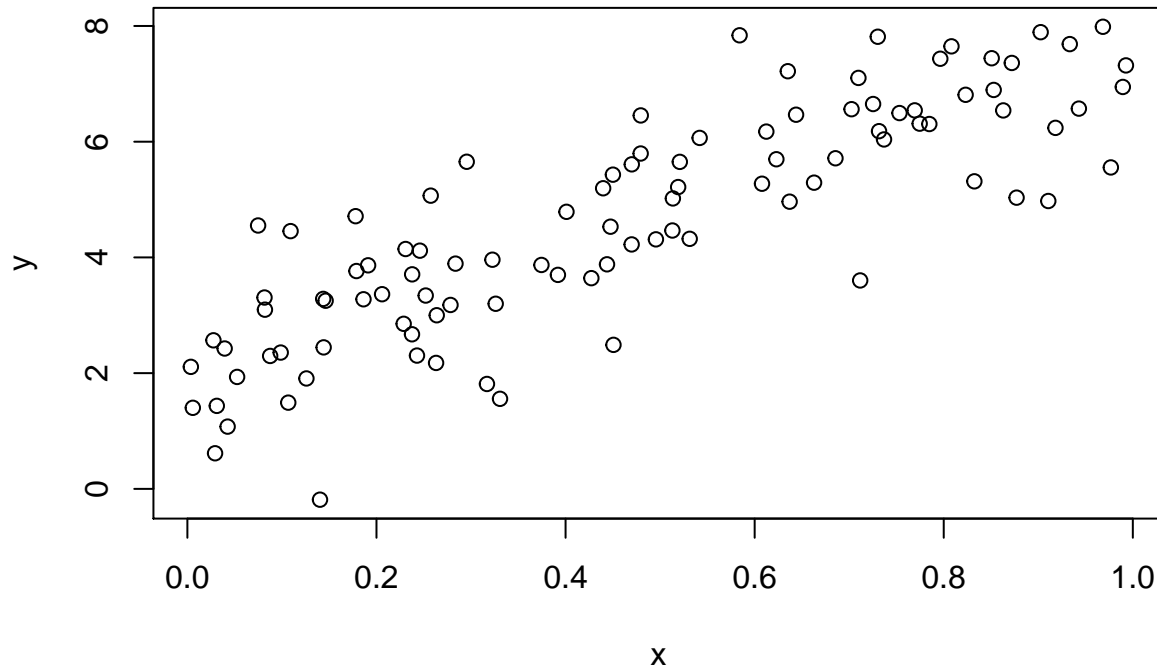


And that is basically it, at least for the basics. There is much more to say about the `lm()` command, but that is for later.

3.7 Making plots

Where R truly shines is in making plots, diagrams, histograms, etcetera. The first thing with data you want to do is to make a scatterplot. With our x and y data this can be easily done by:

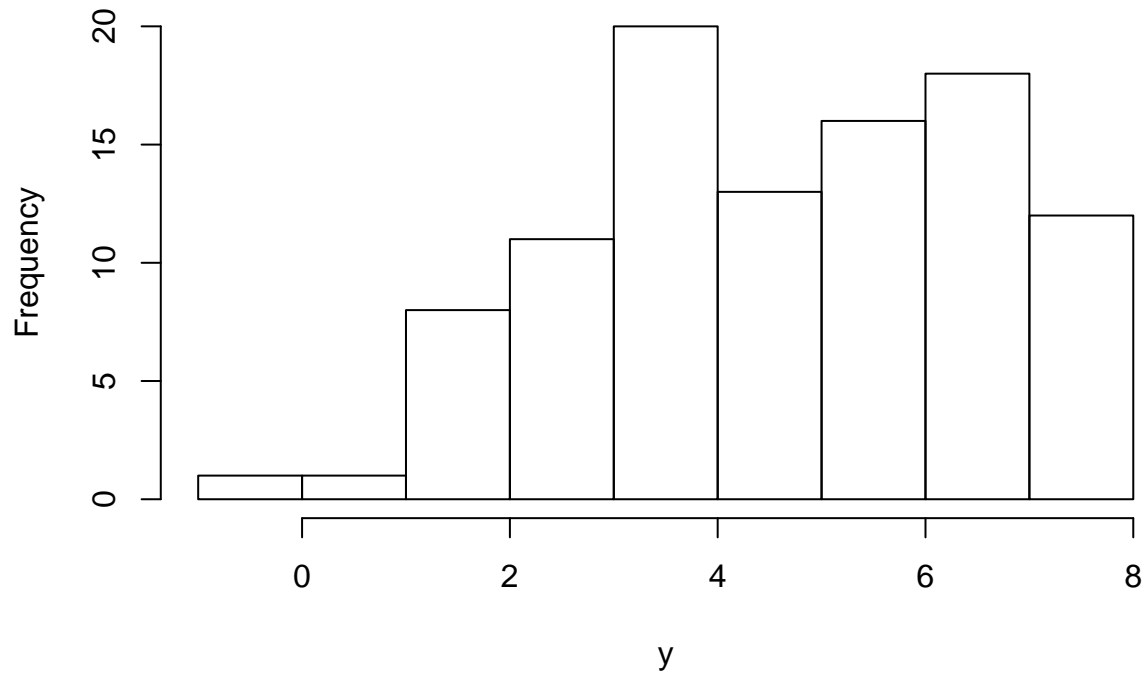
```
plot(x,y)
```



If you would like to create a histogram, just use `hist()` as

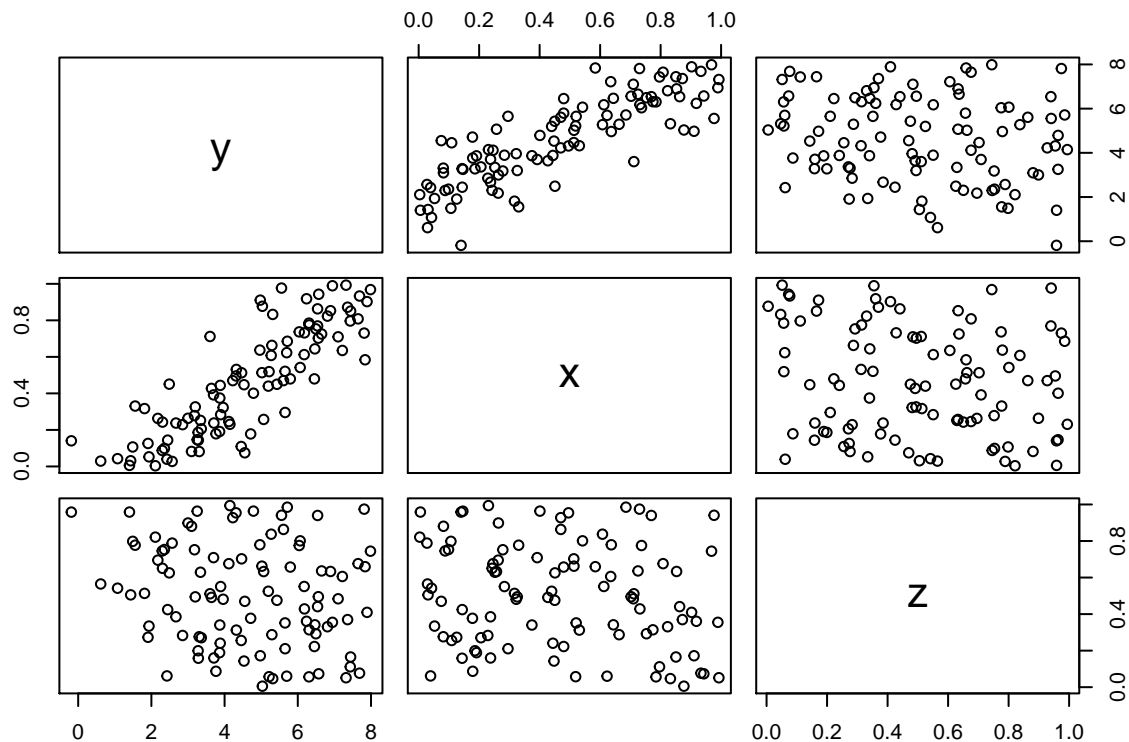
```
hist(y)
```

Histogram of y



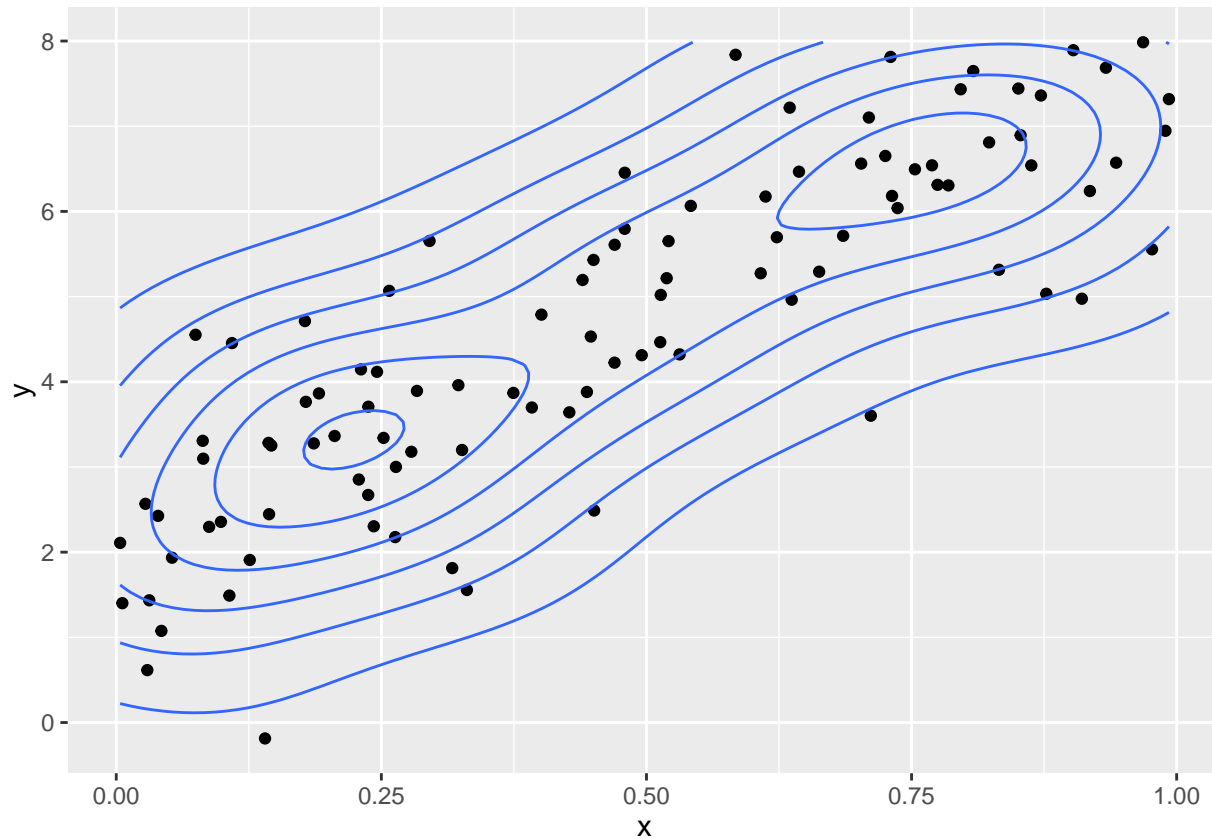
However, to go one step further you also make a plot of a dataframe. for our `df` dataframe this is not very insightful, but let's add another variable z uncorrelated with x and y and then plot the dataframe (the `$` indicate that z is a variable in dataframe `df`).

```
df$z <- runif(100, 0,1)
plot(df)
```



And luckily, this plot confirms what we expect. x are correlated y by construction and z is not correlated with either x and y . These are all the so-called baseline plots. They are great (and already highly customizable), but lately there has been a new kid on the block called `ggplot2`. It goes to far to explain the details of `ggplot2` (gg here stands for the grammar of graphics), but suffice to say that `ggplot2` works with building blocks, so that every piece of the figure that you want (or can think of) can be constructed. Just as an example, let's redo our scatterplot but now using `ggplot2` and say we want to add some density lines from our observations (just because we can). This can be done in the following way

```
ggplot(df, aes(x,y))+geom_point() + geom_density2d()
```



3.8 Recap

For the absolute beginner R is huge and daunting. You need to learn by taking small steps and by practicing (a lot). Do not aim immediately at big and complex projects but start small and at the basics. You will then learn that you quickly make progress and at a certain time even become efficient than when using mousing driven tools (clicking) as `Excel` or `SPSS`. In later chapters I dive in to some more detailed topics, and hopefully the material provides you with a background solid enough to understand and work with those topics using R.

4 Digression: Linear Regression and how to apply it

```
library("stargazer")
```

In the social sciences (in fact, in all sciences) linear regressions (also called OLS or ordinary least squares) or one of its relatives is the most *used* empirical research tool there is. Why? It is relatively simple,

computationally fast, easy to interpret and relies on a relatively weak set of assumptions. Unfortunately, the assumptions needed to be able to apply linear regression, are often not well understood: both in the social sciences and beyond. Moreover, students in the social sciences typically get little guidance in how to apply linear regression in practice and how to interpret the results. Note that this does not have anything to do with the specific software students use, but more with that fact that regression techniques in a wide number of situations (courses) are just not given (apart from statistical or research methods courses). However, given the fact that data becomes increasingly more available, knowing when to use regression techniques, how to apply them and especially how to interpret and assess them is now becoming an issue of paramount importance. Or, perhaps more compelling, you need them to write your thesis.

In this chapter, I will first focus in section 4.1 on the *essential* theory behind regression analysis. I really keep it to the bare minimum. But if you understand these basics, I would argue that you understand more than 75% of the theoretical background (the rest is just nitty-gritty). Section 4.2 will focus on applications of linear regression, specification issues (how many variables) and how to interpret the results.

4.1 Theoretical background

This subsection first deals with the model (what are you trying to explain), then about the three critical assumptions of (ordinary) least squares, discusses subsequently typical situations when these assumptions are violated, and finishes with a discussion about less important stuff (on which, alas, quite some attention is given in bachelor courses).

Before we start, I would like to make one important remark. In general, statistical models can be used for (i) finding **statistical** relations, finding (ii) **causal** relations and for (iii) **predicting**. All three uses require the same assumptions, but have different focuses. In statistics, generally the focus is on finding statistical relations, such as whether the Dutch population is *on average* taller than the German population. In economics the focus is very much on finding causal relations, so the need for explanatory power is not very large. Models that do not explain much (where the R^2 's are low, say < 0.2) are just as good as models that explain quite a lot, as long as the least squares assumptions hold. In transportation science in general (and other disciplines that deals with making large models) predictions and thus explanatory power is key. Here it is now very important that you perfectly understand what causes what as long as out-of-sample predicting is good (say for predicting future commuting flows).

I usually have finding causal relations in mind when talking about least squares (already difficult enough), but note again that the same least squares assumptions should hold when you want to predict or want to find statistical relations.

4.1.1 The model

Assume we are interested in the effect of the weight of a car on the fuel efficiency of the car (measures in miles per gallon). We state the following univariate regression model:

$$y_i = \alpha + \beta x_i + \epsilon_i,$$

where y denotes the fuel efficiency of the car, x denotes the weight of the car and subscript i stands for the i -th observation. α and β are the parameters of the model and they are **unknown** so have to be **estimated**. ϵ is a so-called error term and denotes all the variation that is not captured by our two variables (α and β) and our weight variable x .

The following observations are rather important:

1. What is on the left hand side of the $=$ sign is what is to be explained (in this case miles per gallon). What is on the right hand side is what we use to explain y . In this case x , the weight of the car.
2. The parameters α and β constitute a **linear** relation between x and y
3. The parameter α is the constant and in the univariate setting denotes where the linear relation crosses the y -axis.

4. β gives the impact of x on y . Because it is a linear relation, the effect is simple. One unit change of x is associated with a β change of y . In general, we can say that β is equal to the marginal effect ($\frac{\partial y}{\partial x}$). Moreover, in a univariate setting β denotes the slope of the relation between x and y .
5. The regression error term ϵ gives all variation that is not captured by our model, so $y_i - (\alpha + \beta x_i) = \epsilon_i$. In this case, weight of the car most likely does not capture all variation in miles per gallon, so quite some variation is left in ϵ . Something else is captured as well by ϵ and that is the measurement error of y . So, if we have not measured miles per gallon precisely enough then that variation is captured as well by ϵ .

Now, let's assume that we want to incorporate another variable (say the number of cylinders c), because we think that that variable is very important in explaining miles per gallon. Then we get the following *multivariate* regression model:

$$y_i = \alpha + \beta_1 x_i + \beta_2 c_i + \epsilon_i,$$

where we now have two variables on the right hand side (x_i and c_i) and three parameters (α , β_1 and β_2). In effect nothing changes with the intuition behind the model. Except for the interpretation of the parameter β_1 (and thus also β_2). Parameter β_1 now measures the impact of x on y *holding c constant*. So, multivariate regression models is nothing more (and less) than controlling for other factors. And we see later why that is very important.

4.1.2 The least squares assumptions

We are interested in the effect of the weight of the car on fuel efficiency and, therefore, our **estimate** of β_1 should be very close to the **true** β_1 , especially when we have a large number of observations. Regression is great and utterly brilliant in finding this estimate, as long as the following three least squares assumptions hold:

1. There are no large outliers
2. All left hand side (in this case y) and right side variables (in this case x and c) are *i.i.d.*
3. For the error term the following must hold: $E[\epsilon|X = x] = 0$.

The first one is easy to understand. OLS is very sensitive to large outliers in the dataset. It is therefore always good to look for outliers and think whether they are *real* observations or perhaps typo's (in Excel or something). Do not throw outliers immediately away but check whether they are correct.

The second is relatively easy to understand as well (but not that much in practice). *i.i.d.* in this case stands for independently and identically distributed. This basically means that the observations in your dataset are independent from each other: in other words, the observations should have been correctly *sampled*.

The third looks the most horrible, and, to be quite honest, is so—both in theory as in reality. This is also the assumption that is least well understood. And especially in assessing whether regression output is correct (is your estimate *really* close to β_1) this assumption is crucial.

4.1.3 Possible violations and how to spot them

A violation of the first assumption is usually easy to spot. There is a very strange outlier. But this also marks the importance of analysing *descriptive statistics*, including means, maximums and minimums, scatterplots and histograms.

Whether your data is not *i.i.d.* can come because of a couple of reasons. The most straightforward is getting your data via snowballing (asking your friends and families using facebook to fill in a questionnaire and to ask their friends and families to do so as well). Usually, this means that you have a very specific sample and that the estimate you get is not close to the true value for the whole population. Observations might also be dependent upon each other, because of unobserved factors. In our case, it might be that a type of cars (American) are less fuel efficient than other cars (European).

Another typical violation of the *i.i.d.* assumption is in time-series, where what happened in the previous period might have a large effect on the current period.

In general, however, violations of the *i.i.d.* property are not that devastating for your model as long as you are only interested in finding the true β_1 : namely, it usually only affects the precision of your estimate (the standard error) and not the estimate itself. When this assumption is violated, we therefore say that the estimate is **inefficient**. When you want to predict however, this assumption is crucial, as you would like your estimate to be *as precise* as possible.

When the third assumption is violated, we say that our estimate is **biased**, in other words **wrong**: our parameter estimate does not come close to the true parameter. And this happens more often than not. So, what does $E[\epsilon|X = x] = 0$ actually mean. Loosely speaking, the parameters on the right hand side (in our case x and c) and the error term (ϵ) are not allowed to be correlated. There are several ways that this might happen, from which the following are in our case the most relevant:

- 1) Reverse causality: x impacts y , but y might impact x as well. This is the classical chicken and egg problem. Do firm perform better because of good management, or do good managers go to the better performing firms?
- 2) Unobserved heterogeneity bias: there are factors that are not in the model but influence both x and y . For example, if american cars have both an influence on fuel efficiency and weight of the cars then the estimate that we find is not close to the true value of β_1 .
- 3) Misspecification: we assume that our model is linear, but it actually is not. Then, again, our estimate that we find is not close to the true value of β_1 .

There are other sources of violations, but in this case, these are the most important ones. Reverse causality is usually hard to correct for, but unobserved heterogeneity bias is luckily easier. Namely, we add *relevant* control variables (as we did with c). In this case we can *minimize* possible unobserved heterogeneity bias. Misspecifications are in general as well relative easy to correct for. From our descriptive statistics and scatterplot we usually can infer the relation between x and y and control for possible nonlinearities by using (natural) logarithms and squared terms.

As a sidenote, natural logarithms are the ones most used for various reasons not discussed here. If we take the logarithm of both sides then for our univariate regression we get:

$$\ln(y_i) = \alpha + \beta \ln(x_i) + \epsilon_i,$$

and all the aforementioned rules and assumptions still apply. But there is something peculiar to this regression. Namely, if we are interested in the marginal effect ($\frac{\partial y}{\partial x}$), we get the following:

$$\frac{\partial y}{\partial x} = \frac{\partial e^{\ln(y_i)}}{\partial x} = \frac{\partial e^{(\alpha + \beta \ln(x_i) + \epsilon_i)}}{\partial x} = \frac{\beta}{x} e^{(\alpha + \beta \ln(x_i) + \epsilon_i)} = \frac{\beta}{x} y,$$

In other words:

$$\beta = \frac{\partial y}{\partial x} \frac{x}{y}$$

which is simply the **elasticity** between x and y . So, if there are logarithms on both the left and right hand side then the parameters (the β 's) denotes elasticities.

4.1.4 Normality, heteroskedasticity and multicollinearity

Until now, we have not discussed the concepts normality, heteroskedasticity and multicollinearity. That is simply because they are not that relevant (as long as we have enough observations, typically above 40). The validity of OLS hinges just upon the three assumptions mentioned above (and they are already difficult enough). In fact, if the three assumptions are satisfied, then, as an **outcome**, the parameters (β) are normally distributed. It goes to far to explain why (the theorems required for this are deeply fundamental to statistics), but in any case, normality is **not** a core OLS assumption. It would be nice if both y and x are normally distributed because then the standard errors are minimized, but again, whether the estimate of β you find is correct or not (biased or unbiased) does not depend on normality **assumptions**.

Heteroskedasticity (in other words your standard errors are not constant) as well leads to quite some confusion. In general heteroskedasticity only leads to inefficient estimates (so only affects the standard errors). Nothing more, nothing less. And there are corrections for that (robust standard errors in **Stata** and similar procedures in **R**), so that nobody needs to care anymore about heteroskedasticity.

Finally, there is multicollinearity. And this come in two flavours: perfect and imperfect multicollinearity. Perfect multicollinearity occurs, e.g., when your model contains two identical variables. Then, OLS can not decide which one to use and usually one of the variables is dropped, or your computer program gives an error (*computer says no*).

Imperfect multicollinearity occurs when two variables are highly (but not perfectly) correlated. This occurs less often than one may think. Variables that are highly correlated (say *age* and *age*²) can be perfectly incorporated in a model. Only when the correlation becomes very high (say above 95% or even higher) then something strange happens: the standard errors get very large. Why? That is because of the definition mentioned above. Parameter β_1 measures the impact of *x* on *y* *holding c constant*. But if *x* and *c* are very highly correlated and you control for *c*, then not much variation is left over for *x*. So, *c* actually removes the variation *within* the variable *x*. This always happens, and there is a trade-off between adding more variables and leaving enough variation (note that there is always correlation between variables), but usually it all goes fine. Good judgement and sound thinking typically helps more than strange statistics (say VIF?).

4.2 Applications of linear regression

This section gives an application of regression analysis. Assume we are still interested in the effect of the weight of a car on the fuel efficiency of the car (measures in miles per gallon). We have found a dataset (internal in **R**), so the first thing we have to do is look at the descriptives of the dataset.

4.2.1 Descriptives

The build-in dataset **mtcars** has, besides several other variables, information on weight of a car in (1000 lbs or in about 450 kilos) and miles per gallon. With the following command **head()** we can look at the first 6 observations.

```
head(mtcars)
```

| ## | mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|----------------------|------|-----|------|-----|------|-------|-------|----|----|------|------|
| ## Mazda RX4 | 21.0 | 6 | 160 | 110 | 3.90 | 2.620 | 16.46 | 0 | 1 | 4 | 4 |
| ## Mazda RX4 Wag | 21.0 | 6 | 160 | 110 | 3.90 | 2.875 | 17.02 | 0 | 1 | 4 | 4 |
| ## Datsun 710 | 22.8 | 4 | 108 | 93 | 3.85 | 2.320 | 18.61 | 1 | 1 | 4 | 1 |
| ## Hornet 4 Drive | 21.4 | 6 | 258 | 110 | 3.08 | 3.215 | 19.44 | 1 | 0 | 3 | 1 |
| ## Hornet Sportabout | 18.7 | 8 | 360 | 175 | 3.15 | 3.440 | 17.02 | 0 | 0 | 3 | 2 |
| ## Valiant | 18.1 | 6 | 225 | 105 | 2.76 | 3.460 | 20.22 | 1 | 0 | 3 | 1 |

Again, we are interested in the relation weight of a car (the variable **wt**) and miles per gallon (the variable **mpg**). Note, that in this case **mpg** is the first column and **wt** is the sixth column. (The column with car names above is not a real variable, but are the row names). Recall that the command **c()** combines stuff, so we can look at the summary statistics of the variables we are only interested in by:

```
summary(mtcars[,c(1,6)])
```

| ## | mpg | wt |
|-------------|--------|---------------|
| ## Min. | :10.40 | Min. :1.513 |
| ## 1st Qu.: | 15.43 | 1st Qu.:2.581 |
| ## Median : | 19.20 | Median :3.325 |
| ## Mean : | 20.09 | Mean :3.217 |
| ## 3rd Qu.: | 22.80 | 3rd Qu.:3.610 |
| ## Max. | :33.90 | Max. :5.424 |

There does not seem to be anything out of the ordinary here, but to be sure, we construct a scatterplot between weight of the car and miles per gallon.

```
plot(mpg~wt, mtcars)
```

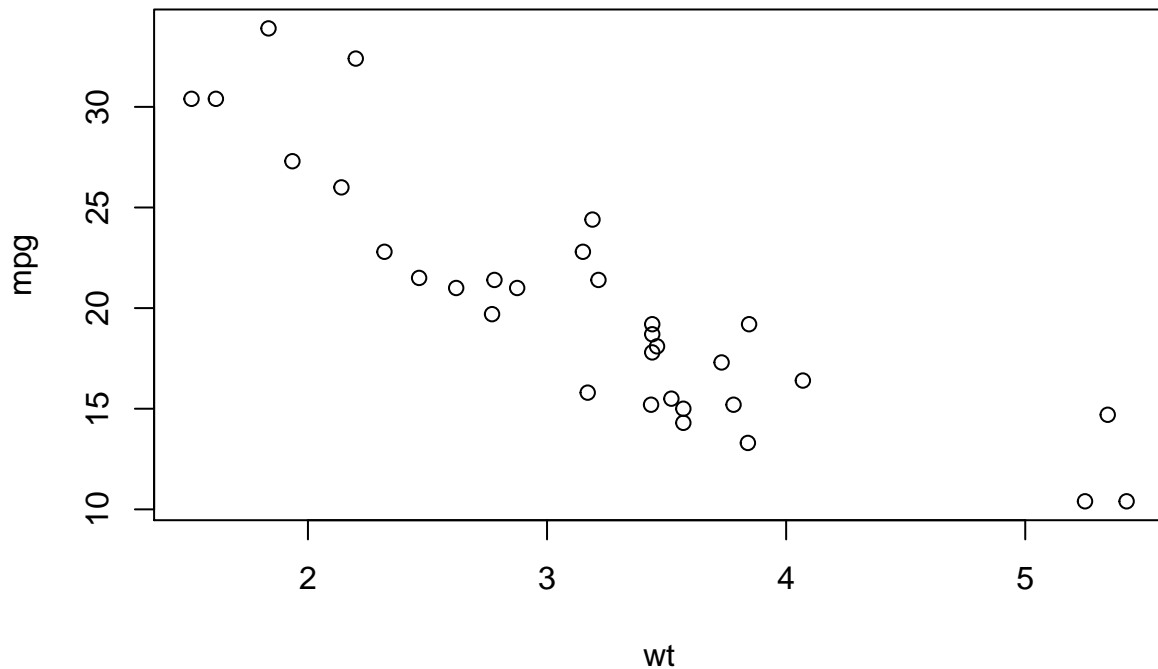


Figure 1: A scatterplot between miles per gallon and car weight.

So, there do not seem to be many outliers here. Moreover, as we would expect, there seems to be a downward sloping relation between weight of the car and miles per gallon (hopefully you agree, that this makes sense). To **quantify** this relation, the next subsection will perform a least squares estimation.

4.2.2 Baseline model

So, if we are only interested in the relation between weight of a car (the variable `wt`) and miles per gallon, we can easily perform the following regression (recall again that the command `lm()` performs a least squares estimation and that `<-` denotes an assignment to a variable:

```
baselinemodel <- lm(mpg~wt, mtcars)
```

```
summary(baselinemodel)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.046 on 30 degrees of freedom
## Multiple R-squared:  0.7528, Adjusted R-squared:  0.7446
## F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

At this point it is good to stop and see what we have got. We have the formula call (we know this one), some stuff about the residuals, stuff about the coefficients (most important for us), and some diagnostics (including the notorious R^2). We zoom in on the results about the coefficients. We have an estimation of the intercept (our α of above) and of `wt` (our β of above). For both, we have as well a standard error, a t -value, a probability and a bunch of stars. What do they all mean again? We focus on `wt` here (typically we are less interested in the constant or intercept).

The estimate is easy; that is β or the marginal effect of x on y . So, increasing x with 1 (or with a 1000 lbs) decreases the miles per gallon with 5.3445 (which is quite lot).

The standard error denote the *precision* of the estimate. As a rule of thumb: you know with 95% certainty that the true value of β lies within the interval $[\text{estimate} - 2 \times \text{standard error}, \text{estimate} + 2 \times \text{standard error}]$. The standard error is very important and is used to **test** possible values of the parameter. One of the most important tests is whether $\beta = 0$. Why? Because, if $\beta = 0$ then the variable `wt` does nothing on `mpg`. This specific test is always denoted by the t -value, its associated probability and the corresponding star thingies. In this case, the t -value is high in an absolute sense, so it is *very* improbable that the estimate could be zero (something like a probability of 0.0000000001 which is small indeed), and the stars neatly indicate that this probability (of $\beta = 0$ is smaller than 0.001.

One nice trick is to plot the regression line in the scatterplot above. One can do so by the command `abline()` or:

```
plot(mpg~wt, mtcars)
abline(lm(mpg~wt, mtcars))
```

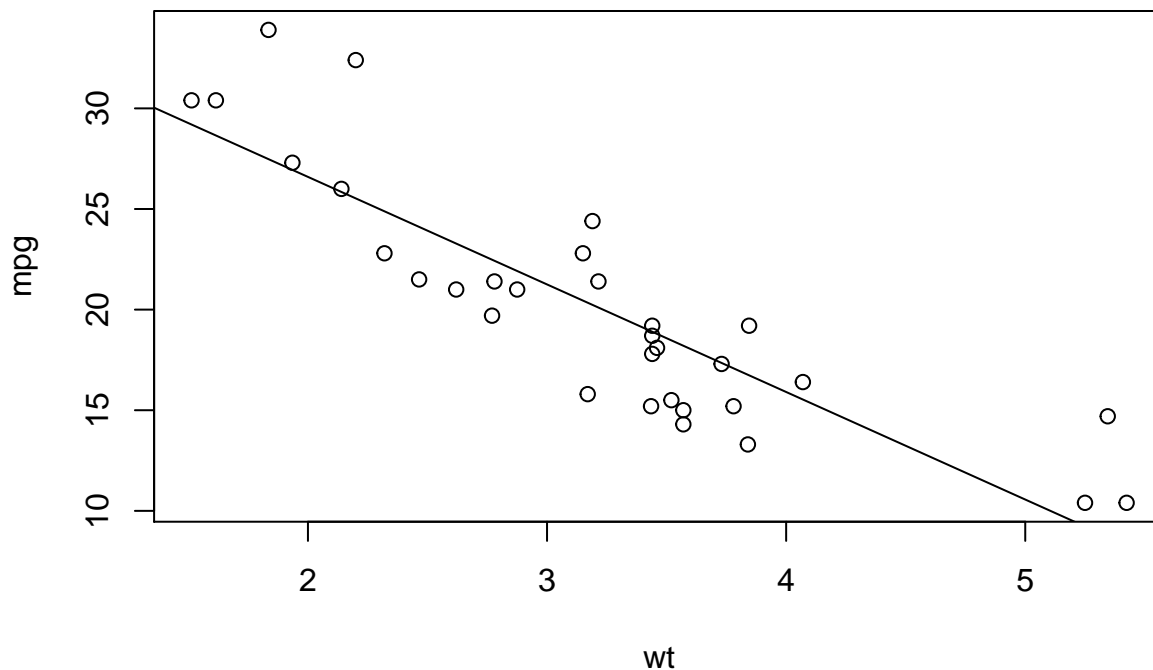


Figure 2: A scatterplot between miles per gallon and car weight with regression line.

4.2.3 Specifiction issues

I can imagine that you are not very satisfied yet with the analysis. First of all, the relation between `mpg` and `wt` might be nonlinear and, secondly, you would like to include additional variables. First we look at the possible nonlinearity in the regression relation (note that we can use the regression formula as before, but that we can specify logarithmic relations by `log()`):

```
logmodel <- lm(log(mpg)~log(wt), mtcars)
summary(logmodel)

##
## Call:
## lm(formula = log(mpg) ~ log(wt), data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.18141 -0.10681 -0.02125  0.08109  0.26930
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.90181    0.08790   44.39 < 2e-16 ***
## log(wt)       -0.84182    0.07549  -11.15 3.41e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1334 on 30 degrees of freedom
## Multiple R-squared:  0.8056, Adjusted R-squared:  0.7992
## F-statistic: 124.4 on 1 and 30 DF,  p-value: 3.406e-12
```

We have now the same type of output as before, but I would like to focus on two things here. First, the R^2 has gone up and that is what you typically get in the social sciences. Logarithmically transformed variables usually **fit** better. Secondly, the interpretation of the estimate now differs. It has become an elasticity with size -0.84 , which is quite high again. If the car doubles in weights, the fuel efficiency of the car goes down by 84%!

Secondly, you might want to include other variables, such as being a foreing car (opposite to a car from USA) (`vs`), the number of cylinders (`cyl`), the gross horsepower (`hp`) and how quick the car does over a quarter of a mile (`qsec`). Again, we are not interested in these additional variables or whether they crank up the R^2 . The only thing we are interested is in whether the coefficient of `log(wt)` changes.

```
extendedmodel <- lm(log(mpg)~log(wt)+vs+cyl + hp + qsec, mtcars)
summary(extendedmodel)

##
## Call:
## lm(formula = log(mpg) ~ log(wt) + vs + cyl + hp + qsec, data = mtcars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.201746 -0.065242 -0.009506  0.079809  0.205166
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.5280554  0.4610947   7.651 4.04e-08 ***
## log(wt)       -0.6514583  0.1509786  -4.315 0.000205 ***
## vs            -0.0149215  0.0863277  -0.173 0.864111
## cyl           -0.0160253  0.0314486  -0.510 0.614651
```

```
## hp          -0.0007670  0.0006861  -1.118  0.273786
## qsec         0.0212017  0.0267153   0.794  0.434603
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1121 on 26 degrees of freedom
## Multiple R-squared:  0.8811, Adjusted R-squared:  0.8582
## F-statistic: 38.53 on 5 and 26 DF,  p-value: 3.227e-11
```

Clearly, the coefficient of `log(wt)` changes with the inclusion of additional variables. So there was unobserved heterogeneity bias in our baseline model (and most likely there still is in our extended model), even though the additional variables are not significant.

4.2.4 Reporting

Hopefully you agree that the regression output of above looks **horrible** and that you do not need all these statistics and stuff. Therefore, you need to construct your own table for presentation format. The minimum what should be in these tables are the coefficient names, the estimates, the standard errors (and the start would be nice as well), the R^2 and the number of observations used.

Unfortunately, creating tables of your own is a pain in the ... Luckily, within R (and by the way in programs as *Stata* as well) you can do this automatically! In R there are several packages one can use. I prefer the package *Stargazer* and after using this package we get the following outcome for our logarithmic specification:⁴

```
stargazer(logmodel, extendedmodel, header=FALSE, type = "html", omit.stat = c("rsq", "f"))
```

Dependent variable:

log(mpg)

(1)

(2)

log(wt)

-0.842***

-0.651***

(0.075)

(0.151)

vs

-0.015

(0.086)

cyl

-0.016

(0.031)

hp

-0.001

(0.001)

qsec

⁴To get nice tables directly into Word, you need something else (as you can imagine, because Word is not scriptable). With *Stargazer* should give the command as such `stargazer(logmodel, extendedmodel, out = "table1.txt", omit.stat = c("rsq", "f"))` and there after you can read in and edit the table `table1.txt` in Word.

```

0.021
(0.027)
Constant
3.902***
3.528***
(0.088)
(0.461)
Observations
32
32
Adjusted R2
0.799
0.858
Residual Std. Error
0.133 (df = 30)
0.112 (df = 26)
Note:
p<0.1; p<0.05; p<0.01

```

Note that we now display both specifications, the first the univariate case and the second the multivariate case. This is now typically done in social science research. You start with a baseline and then add variables to check whether the variable of interest remains robust.

4.2.5 Internal validation

So, after done all modeling exercices you are not done! Now, it is time to validate your results internally. Can you be confident of your results? Or is the parameter that you have found still suspect of possible biases. Here again come the three least squares assumptions:

1. No large outliers: we have checked that with our descriptive statistics and our scatterplot and it seems that assumption is met.
2. Both miles per gallon and car weight should be *i.i.d.*. We can reasonably assume that the sampling has been correctly and that this is the case.
3. The following condition should hold: $E[\epsilon|X = x] = 0$. This one is dubious. Probably there is no reverse causality (difficult to image that miles per gallon would influence car weight), but most likely there are other factors that impact car weight and miles per gallon which are omitted (where is the car build, what is the car brand, etcetera.). One possible strategy here is to add additional variables until the parameter of interest remains robust (does not change anymore).

Note that the number of observations in this case is only 32, so whether our standard errors are automatically normally distributed is highly questionable, but for the sake of the exposition (and the fact that small sample statistics are very complex) we leave it just with this observation.

5 Network analysis with R

```
library("igraph")
library("igraphdata")
```

5.1 Introduction

Work in progress⁵

5.2 Creating networks

There are various ways to create a network: one can it oneself, one can create a structured network, and one can read a network from file or the internet. (The latter is typically done with very large and dynamics network data.)

5.2.1 Creating network yourself

First of all, I will show how to create a network “by hand”. We first start with an undirected network. Suppose we have a network with four nodes (named 1,2, 3 and, you guessed it, 4). Then we can create a graph called `g_4` by:

```
g_4 <- graph (edges = c(1,2,2,3,3,4, 4,1), n = 4, directed = FALSE)
```

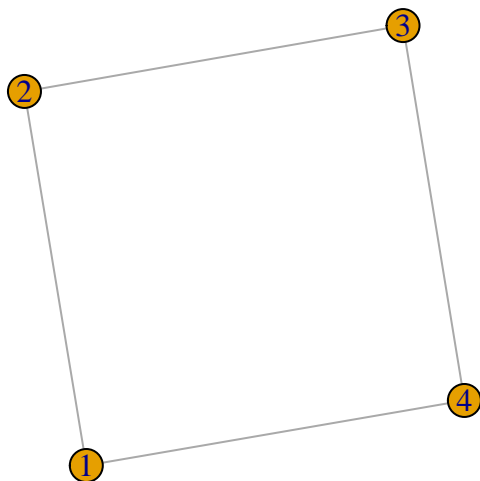
Let us have a look at `g_4`

```
g_4
```

```
## IGRAPH U--- 4 4 --
## + edges:
## [1] 1--2 2--3 3--4 1--4
```

So, `g_4` is an igraph sort of object which has four edges 1--2 2--3 3--4 1--4. Note that these coincide with `c(1,2,2,3,3,4, 4,1)`, so an edge is always formed by two components (they do not have to be numbers). Now plotting `g_4` leads to:

```
plot(g_4)
```



⁵Obviously I want to provide a self-contained manual, however, there are as well some pretty good and more extensive tutorials on internet: e.g., the one over here.

The connectivity matrix of the network can be directly given by:

```
g_4[]
```

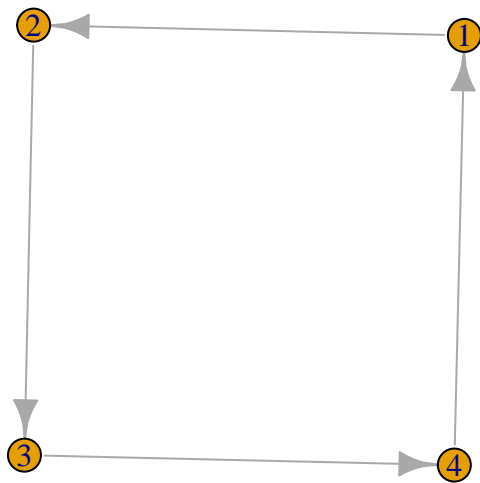
```
## 4 x 4 sparse Matrix of class "dgCMatrix"
##
## [1,] . 1 . 1
## [2,] 1 . 1 .
## [3,] . 1 . 1
## [4,] 1 . 1 .
```

Now, for more directed graphs we need to set `directed = TRUE`.

```
g_4 <- graph(edges = c(1,2,2,3,3,4, 4,1), n = 4, directed = TRUE)
```

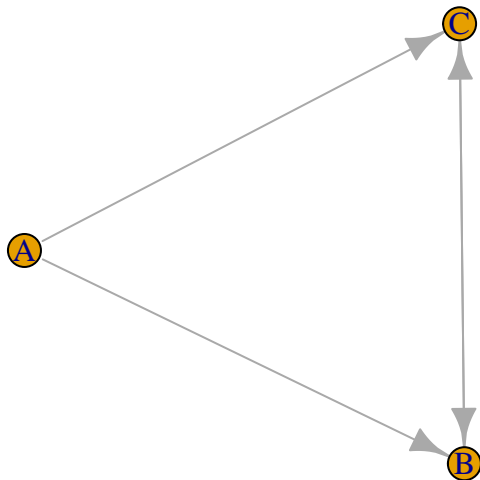
with as plot

```
plot(g_4)
```



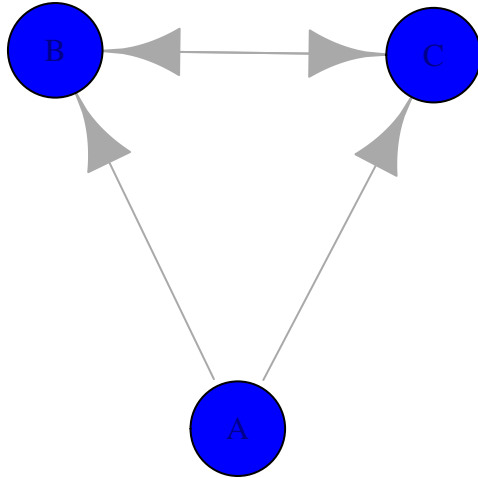
Graphs may also be generated by using `-`, `++` and `+-+`, in combination with the command `graph_from_literal`. For example:

```
g_directed <- graph_from_literal(A-->B, B +-+ C, C+-A)
plot(g_directed)
```



Finally, we can influence the color and sizes (amongst others) of all the elements of the network, so:


```
plot(g_directed, edge.arrow.size=2, vertex.size = 50, vertex.color = "blue" )
```

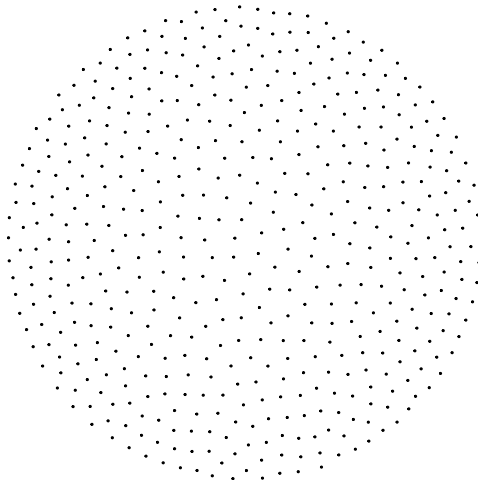


However, this way of creating of networks is not very efficient. We therefore offer some other ways as well, starting with created directly (well-structured networks).

5.2.2 Structured networks

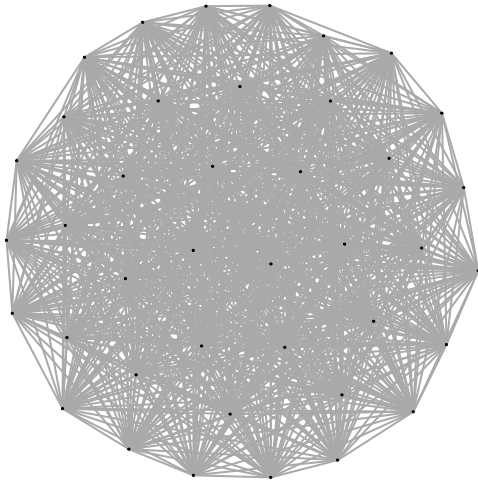
With the package `igraph` it is rather straightforward to create structured networks. Let's start with an empty network. We therefore use the command `make_empty_graph(500)` in order to create 500 nodes and no links.

```
g_empty <- make_empty_graph(500)
plot(g_empty, vertex.label= NA, edge.arrow.size=0.02, vertex.size = 0.5)
```



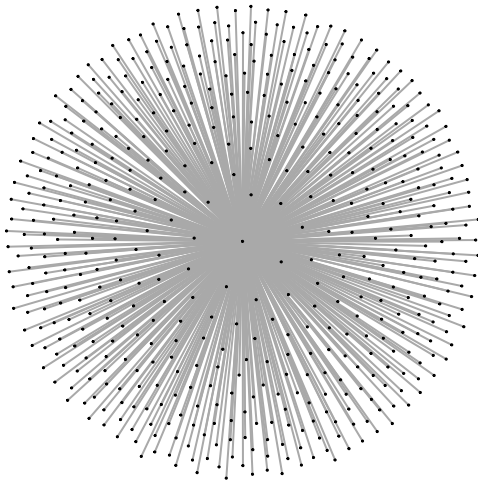
The other extreme is obviously the fully connected network. With the command `make_full_graph(40)` we can create a fully connected network between 40 nodes. (Please do not do 500 nodes; it takes a long time to create that network (with $500 \times 500 - 500$ links)).

```
g_full <- make_full_graph(40)
plot(g_full, vertex.label= NA, edge.arrow.size=0.02, vertex.size = 0.5)
```



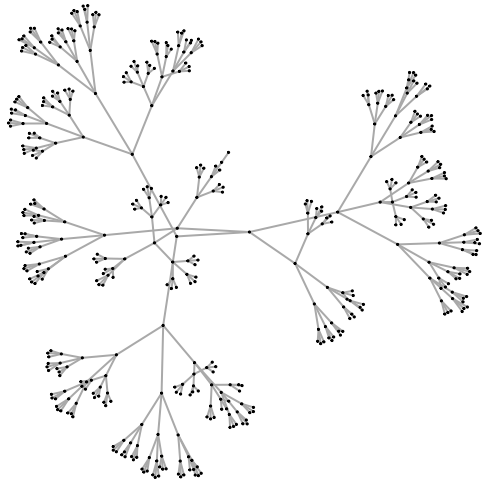
The `make_star(500)` command create a star network (or a hub-and-spoke network) with 500 nodes.

```
g_star <- make_star(500)
plot(g_star, vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



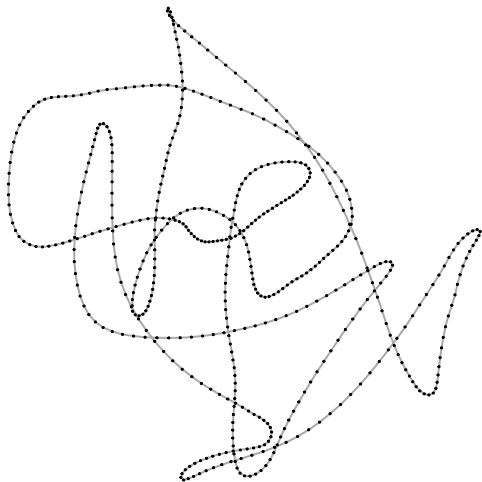
With the `make_tree(500, children = 3, mode="undirected"` command, we create an undirected hierarchical tree with each time tree children

```
g_tree <- make_tree(500, children = 3, mode = "undirected")
plot(g_tree, vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



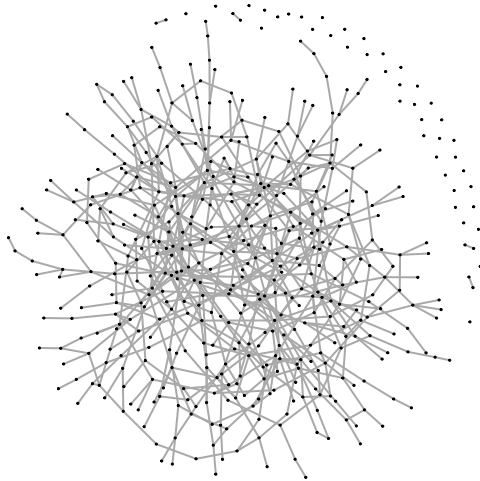
And the `graph.ring(500)` creates a ring (or line) structure where each node is connected to two other nodes in a sequential way

```
g_ring <- graph.ring(500)
plot(g_ring, vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



For the real-world network we can use the following commands. First, the command `erdos.renyi.game(500, 0.005)` creates a random network, where each node has a probability of 0.005 to be connected to each other node.

```
plot(erdos.renyi.game(500, 0.005), vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



For the small-world network, we need to rewire the ring network. We can do by the command `rewire` as follows:

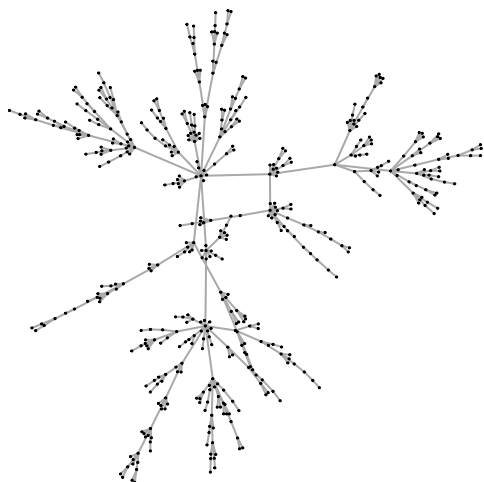
```
plot(rewire(g_ring,each_edge(prob = 0.5)), vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



Note that the probability of rewiring is now 0.5.

Finally, for a power-law we need to invoke an algorithm. In this case the `barabasi.game`, where we now create a power-law type of network with the following underlying formula $P(k) = k^{-0.5}$.

```
g_power_law <- barabasi.game(500, 0.5)
plot(g_power_law, vertex.label= NA, edge.arrow.size=0.02,vertex.size = 0.5)
```



Note the tree-like structure.

5.2.3 Reading in networks

There are various sources for networks on the internet. One of them is nexus. With the following command I import a network from this website:

```
g_florence <- nexus_get("padgett")
```

which contains business and marriage ties (PADGB) in 14th century Florence and looks like this

```
plot(g_florence$PADGB)
```

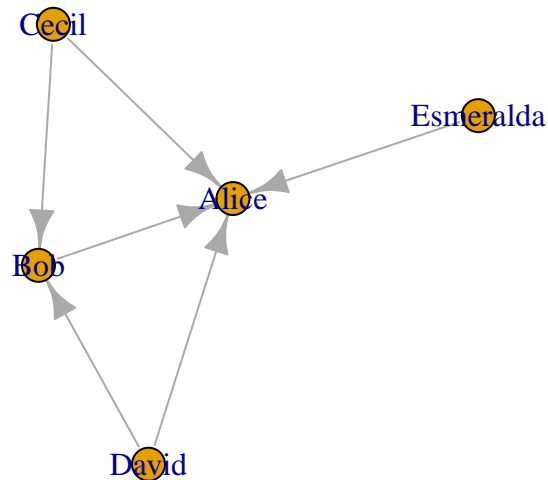


Another good but smaller source is the `igraphdata` package which actually contains a network of american domestic flights.

Another possibility is to read in the data from a `.csv` file. Something like (I took and modified the example from <http://stackoverflow.com/questions/23687806/creating-a-network-graph-using-igraph-in-r>):

```
# Make up data
relations <- data.frame(from=c("Bob", "Cecil", "Cecil", "David", "David", "Esmeralda"),
                        to=c("Alice", "Bob", "Alice", "Alice", "Bob", "Alice"))
# Alternatively, you could read in the data from a similar CSV file as follows:
# relations <- read.csv("relations.csv")
```

```
# Load (DIRECTED) graph from data frame
g_relations <- graph.data.frame(relations, directed=TRUE)
plot(g_relations)
```



Note that this only requires a from and to columns in a .csv file, which resembles trade, commuting, migration and transport networks (in a gravity-model type of way). The only thing is that we need to convert this data in a igraph structure with `graph.data.frame`.

5.3 Network characteristics

So, it is fine that you have a network, but what about the network characteristics. Well these are easily given once you have a network.

For density, or the completeness of the network, one can look at

```
edge_density(g_florence$PADGB)
```

```
## [1] 0.1666667
```

Which give the proportion of the present edges from all possible edges in the network.

The diameter of a network can be retrieved as:

```
diameter(g_florence$PADGB)
```

```
## [1] 5
```

somewhat more interesting is given by the degree function which gives the number of connections. For our power-law network this amounts to

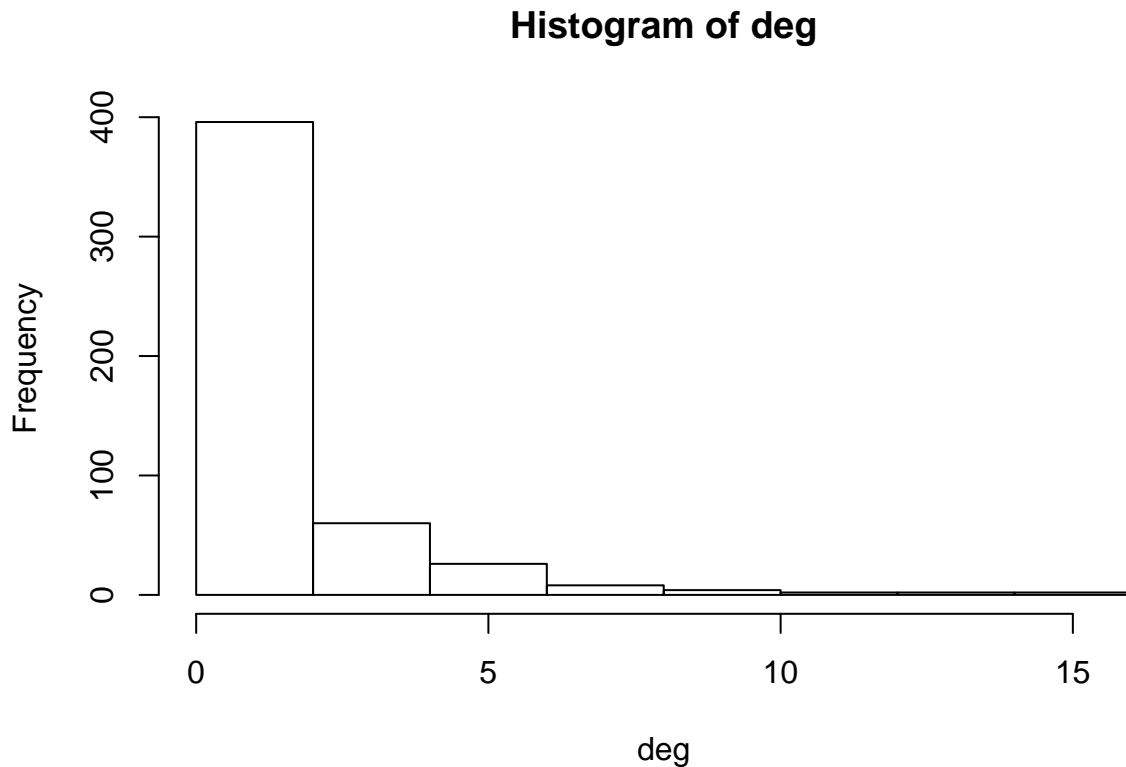
```
deg <- degree(g_power_law)
deg
```

```
## [1] 15 11 14 8 7 10 16 6 5 1 11 6 9 7 9 6 13 6 4 3 3 4 2
## [24] 2 4 4 2 1 10 1 5 1 5 1 7 1 8 2 1 6 4 3 5 1 2 3
## [47] 7 4 6 3 5 1 1 4 4 3 6 5 1 1 2 1 8 2 2 1 1 1 3
## [70] 1 4 2 3 1 3 5 2 2 2 2 2 2 4 1 1 1 3 1 2 1 4 3
## [93] 1 7 1 2 5 1 6 3 2 3 5 4 2 5 1 1 1 3 2 2 1 1 3
## [116] 1 1 3 3 1 5 1 2 5 5 1 4 4 2 2 2 2 1 4 1 5 3 1
## [139] 3 1 2 4 5 1 1 1 2 3 1 3 1 1 1 1 1 1 4 2 1 2 1
## [162] 1 1 2 3 1 1 1 3 1 2 1 2 2 1 1 3 3 1 3 1 1 2 6
## [185] 4 1 1 1 1 1 4 2 1 3 3 6 2 6 3 1 2 2 1 2 2 1 1
```

```
## [208] 1 1 1 1 1 2 1 1 1 1 2 2 3 1 1 1 1 1 1 1 1 2
## [231] 1 1 1 1 3 2 2 1 1 1 3 1 2 2 1 2 1 1 1 1 1 1
## [254] 2 2 1 1 1 3 2 1 2 1 2 1 4 1 1 1 1 1 1 1 1 2
## [277] 1 1 1 1 1 1 1 2 1 1 1 1 2 3 1 1 1 1 1 1 3 4
## [300] 1 1 1 1 1 1 1 1 2 1 1 3 1 2 1 2 1 1 2 1 1 1
## [323] 1 1 1 1 1 1 2 1 1 1 1 1 2 1 2 1 2 1 1 2 1 3
## [346] 2 1 1 1 1 1 1 1 2 2 1 1 1 1 1 1 1 1 1 1 2 1
## [369] 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [392] 1 1 1 1 1 1 1 1 1 2 3 3 1 1 1 1 1 1 2 1 1 1
## [415] 2 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 2 1 1 1
## [438] 1 1 1 1 1 1 2 1 1 1 1 1 1 1 2 1 1 1 1 1 1 2
## [461] 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [484] 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

which is not that insightful, but a histogram is:

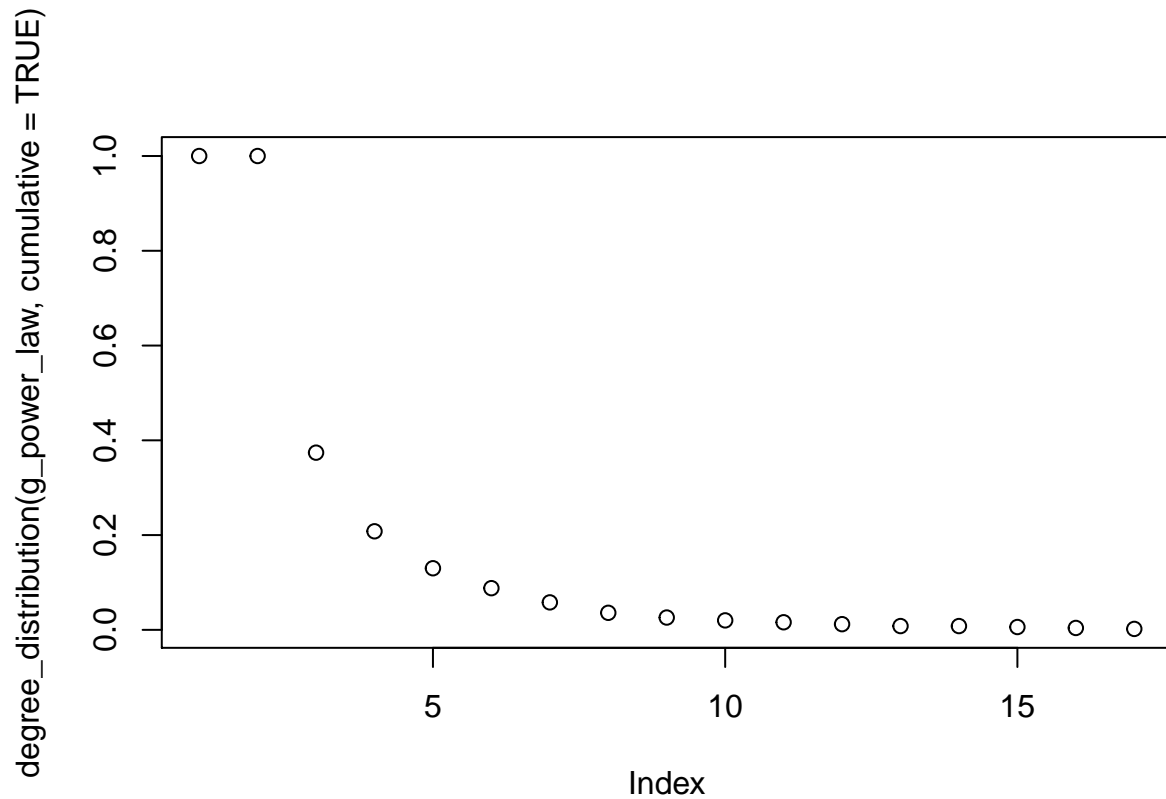
```
hist(deg)
```



Which is typical for a power-law.

The `degree_distribution` is insightful here as well

```
plot(degree_distribution(g_power_law, cumulative = TRUE))
```



Finally, we may be interested in the shortest path. This can easily be retrieved using distances (with underlying Dijkstra's algorithm) as follows for the Florence network:

```
distances(g_florence$PADGB)
```

```
##          ACCIAIUOL ALBIZZI BARBADORI BISCHERI CASTELLAN GINORI GUADAGNI
## ACCIAIUOL          0         2         2         4         3         3         3
## ALBIZZI           2         0         2         2         3         1         1
## BARBADORI         2         2         0         3         1         3         3
## BISCHERI          4         2         3         0         2         3         1
## CASTELLAN         3         3         1         2         0         4         3
## GINORI            3         1         3         3         4         0         2
## GUADAGNI          3         1         3         1         3         2         0
## LAMBERTES         4         2         4         2         4         3         1
## MEDICI            1         1         1         3         2         2         2
## PAZZI             3         3         3         5         4         4         4
## PERUZZI           4         3         2         1         1         4         2
## PUCCI             Inf        Inf        Inf        Inf        Inf        Inf        Inf
## RIDOLFI           2         2         2         2         2         3         2
## SALVIATI          2         2         2         4         3         3         3
## STROZZI           3         3         2         1         1         4         2
## TORNABUON        2         2         2         2         3         3         1
##          LAMBERTES MEDICI PAZZI PERUZZI PUCCI RIDOLFI SALVIATI STROZZI
## ACCIAIUOL         4         1         3         4     Inf         2         2         3
## ALBIZZI           2         1         3         3     Inf         2         2         3
## BARBADORI         4         1         3         2     Inf         2         2         2
## BISCHERI          2         3         5         1     Inf         2         4         1
## CASTELLAN         4         2         4         1     Inf         2         3         1
## GINORI            3         2         4         4     Inf         3         3         4
```


| | | | | | | | | |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|
| ## GUADAGNI | 1 | 2 | 4 | 2 | Inf | 2 | 3 | 2 |
| ## LAMBERTES | 0 | 3 | 5 | 3 | Inf | 3 | 4 | 3 |
| ## MEDICI | 3 | 0 | 2 | 3 | Inf | 1 | 1 | 2 |
| ## PAZZI | 5 | 2 | 0 | 5 | Inf | 3 | 1 | 4 |
| ## PERUZZI | 3 | 3 | 5 | 0 | Inf | 2 | 4 | 1 |
| ## PUCCI | Inf | Inf | Inf | Inf | 0 | Inf | Inf | Inf |
| ## RIDOLFI | 3 | 1 | 3 | 2 | Inf | 0 | 2 | 1 |
| ## SALVIATI | 4 | 1 | 1 | 4 | Inf | 2 | 0 | 3 |
| ## STROZZI | 3 | 2 | 4 | 1 | Inf | 1 | 3 | 0 |
| ## TORNABUON | 2 | 1 | 3 | 3 | Inf | 1 | 2 | 2 |
| ## TORNABUON | | | | | | | | |
| ## ACCIAIUOL | 2 | | | | | | | |
| ## ALBIZZI | 2 | | | | | | | |
| ## BARBADORI | 2 | | | | | | | |
| ## BISCHERI | 2 | | | | | | | |
| ## CASTELLAN | 3 | | | | | | | |
| ## GINORI | 3 | | | | | | | |
| ## GUADAGNI | 1 | | | | | | | |
| ## LAMBERTES | 2 | | | | | | | |
| ## MEDICI | 1 | | | | | | | |
| ## PAZZI | 3 | | | | | | | |
| ## PERUZZI | 3 | | | | | | | |
| ## PUCCI | Inf | | | | | | | |
| ## RIDOLFI | 1 | | | | | | | |
| ## SALVIATI | 2 | | | | | | | |
| ## STROZZI | 2 | | | | | | | |
| ## TORNABUON | 0 | | | | | | | |

Note that the Pucci family is not in the network and therefore the distances are set to infinity.

6 In conclusion

To be written