On spatial dependence in stochastic frontier models with an application to European regional economic performance

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Abstract

In this paper it is argued that the diffusion of technology across regions is spatially restricted in nature. Thus, that a region's technical efficiency—its relative economic performance given its endowments—may depend more on regions nearby than on a 'global' level of technology. We estimate these spatially related technical efficiencies by using a stochastic frontier model. Specifically, our contribution is twofold. First, we introduce spatial dependence in stochastic frontier models in a relative straightforward approach. Secondly, we argue that we are able to make a distinction between technical efficiencies related to the use of a region's endowments and those related to spatial dependence: in other words, between a region's absolute and a region's relative location. Using newly constructed regional data on European regional output, labor levels and especially capital level, our results indicate that European technical efficiencies are indeed very much spatially related.

Keywords: Spatial dependence, stochastic frontiers, skew-normal distribution
JEL-classification: R1, R3

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1 Introduction

One of the most distinct features of European regions is that they differ widely in their economic performance, even when controlling for the number of inhabitants. Obviously, countries differ in terms of institutions, culture, stability, and so forth, which determine for a large part the international differences in economic performance. However, wealth and income are sometimes even more dispersed *within* countries than across countries. To illustrate this, Figure 1 show the dispersion of relative regional GDP per capital across European countries.

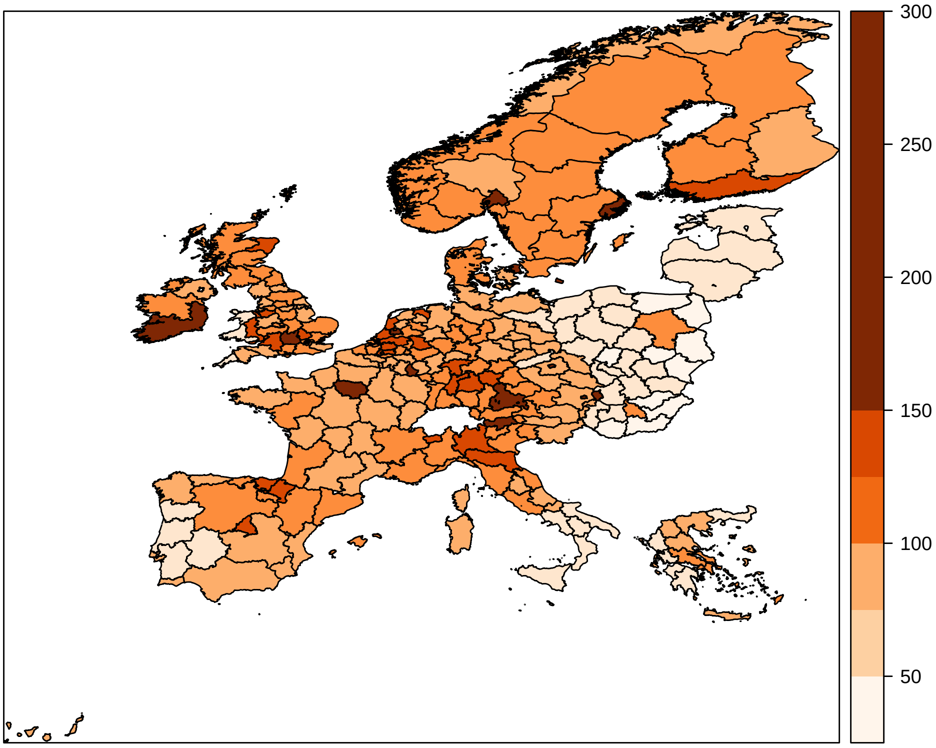

Figure 1 – GDP per capita across Europe in 2007—EU27 = 100 (source: cambridge econometrics database

Obviously, European wealth is accumulated within large metropolitan areas (most notably—the capital cities) such as Paris, London, Luxembourg, Oslo and Stockholm. However, apart from this, large differences are present as well between subnational regions. The most well-known example is the North-South division within Italy, but these differences can as well be noticed within almost all countries in Europe. Most notable examples are Spain (North-South division as well), France (with a relatively poorer central area), the UK (where the periphery seems to be much poorer), and Germany (with the former division between East and West). Thus, some regions within countries are significantly more successful than others—even when faced with similar national institutions.

What makes regions successful? This is probably the most crucial and complex research question for both regional policymakers and regional scientists to answer. Regional policy makers would like to create policies aimed to steer regions to success, regional scientists are especially interested in the (nature of the) determinants that drive this success. At the heart, this research question deals with the absolute and relative location advantages of regions.[1]

A strongly related research question deals with the exact nature of economic performance. How to define economic performance and how to measure it is still subject to an ongoing debate. Already numerous studies have conducted with the purpose of benchmarking cities, regions and the like. Usually, these studies are not based on any behavioral or economic structure. Further, although sometimes population size is taken into account, the level of other initial endowments is usually neglected. Ideally, the latter should be taken into account in the question whether regions are successful *given* their initial situation. In the economics literature this can be reflected in the idea of stochastic frontier analysis. Here, firm performance is measured as the technical efficiency in their employment of production factors. The closer the firm's output is relative to the best possible output, the higher the firm's technical efficiency. Analogously, one can consider regions as production units (as is often done) and look at the region's technical efficiency in employing their endowments. As Figure 1 clearly shows, some regions are probably more efficient than others—even within the same country. And this should be reflected when benchmarking those regions.

The left production isoquant in Figure 2 shows how a region's technical efficiency can be measured. Denote y as the *given* maximum attainable production a region can get using the production factors x_1 and x_2. Regions A_1, A_2, A_3, B_1, B_2 and B_3 are all producing inefficiently. With the same amount of production factors x_1 and x_2 that theoretically enable them to produce y, they produce on average $\hat{y}$. The distance between Y and *haty* is then a measure for the average efficiency. More precisely, efficiency is defined as the ratio $\vec{\hat{y}}/\vec{y}$ where $\vec{y}$ is the length of the line between the origin and y. As a result, technical efficiencies must be smaller than 1.

[1]There is a large debate about the drivers of economic success, both on a national and a regional level. Some scientist favor the proposition that regions prosper because of historical events and the associated path-dependencies (e.g., Landes, 1998), while others emphasize absolute (e.g. Diamond, 1998) and relative (e.g., Fujita et al., 1999) locational advantages. Obviously, these different drivers require different policy instruments—if any at all.

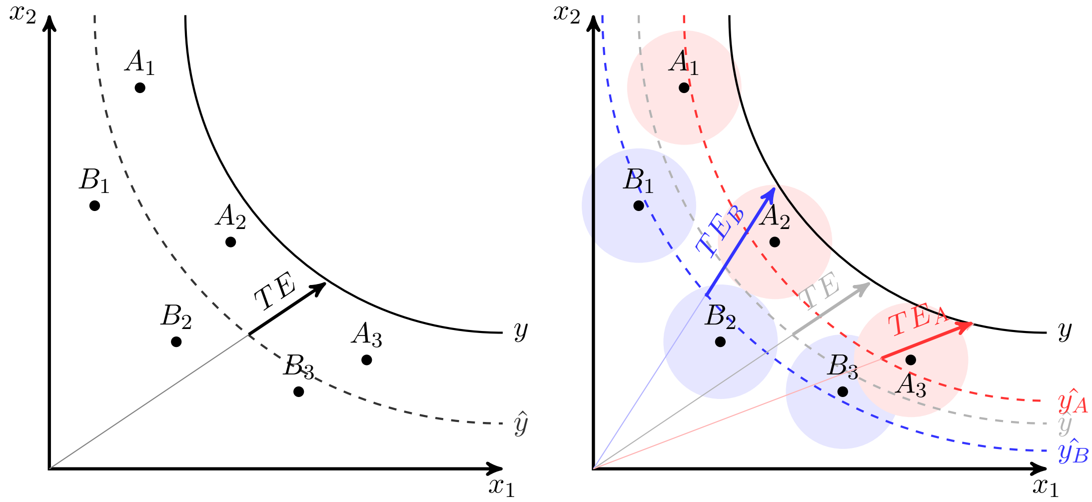

Figure 2 – Production possibility frontiers in a standard (left) and a spatial structure (right).

There is already a sizeable literature dealing with benchmarking regions using regional technical efficiencies.[2] This literature usually deals with the relative (sectoral) performance of regions and this is the approach this paper takes as well. The production factors are then usually constituted of the aggregates of various forms of labour (high skilled and low skilled) and capital (both physical and human) within a region.

Estimation of technical efficiencies may, however, be biased in the presence of spatial dependence or unobserved spatial heterogeneity amongst regions. Namely, when one assumes that only neighboring regions benefit from other regions' technological knowledge through the traditional Marshallian channels of shared customers and suppliers, shared labor pools or spillover mechanisms (or just through unobserved spatial heterogeneity), then straightforward estimation of technical efficiencies is biased. This can be been in the right production isoquant depicted in Figure 2. Assume that regions A_1, A_2 andA_3 belong to country A and regions B_1, B_2 and B_3 belong to country B, then it quite well conceivable because of spatial unobserved heterogeneity or spatial dependence that the technical efficiencies of the regions in country A and B are related. In Figure 2, region B_3 might not produce inefficient at all *given* the fact that neighbouring regions in country B produce even less efficient. This works as well the other way around. Regions very central in a network and surrounded by very efficiently producing regions, have besides a strong economic structure probably a very favourable location as well. In this context, efficiency can be related to advantages related to the *absolute* location while spatial dependence relates to the *relative* location. To control for both the inefficiency in production and the geographical location, we therefore aim to explicitly incorporate a spatial correlation structure in stochastic

[2] See, amongst others, Alvarez (2007), Brock (1999), Puig-Junoy (2001), and Puig-Junoy and Pinilla (2008).

production frontiers.

The literature that combines spatial dependence and stochastic production frontiers is, although relatively recent, already very sizable. Most studies employ a parametric approach and the enumeration that follows is definitely not conclusive. One of the first parametric studies was Barrios and Ladado (2010), who uses an interative backfitting algorithm to find consistent parameter estimates although they do not allow for correlation between the technical efficiency and spatial dependence structure. Pavlyuk (2010) uses as well a parametric approach, but does not report how he estimates consistently both the spatial dependence process and technical efficiencies. Fusco and Vidoli (2013) separate out the error term in a spatial lag structure and technical efficiencies, with an application to the Italian wine sector. Kinfu and Sawhney (2015) apply a spatial stochastic frontier analysis to maternal care in India. Glass et al. (2013) decomposes productivity growth using a spatial autoregressive model, whereafter Glass et al. (2016) extends the model itself to a spatial panel setting. Finally, Jiang et al. (2017) apply a fixed effects stochastic frontier model to energy efficiency in Chinese provinces. In addition, there is a smaller literature that resorts to a Bayesian approach and simulation techniques, i.e. Schmidt et al. (2009) and Areal et al. (2010).

A specific criticism that applies to most of these (specifically parametric) studies is that they all consider a univariate error term once the spatial component is accounted for. However, as I will argue below, the error term is by definition multivariate as it is a combination of a normal and truncated normal distribution, where one of them or even both are multivariate due to the spatial correlation structure.

In contrast this study applies a different multivariate approach firmly rooted in the statistical literature. Using a relatively straightforward approach this study shows how to combine a spatial error structure with a stochastic frontier model that is (*i*) straightforward to estimate, (*ii*) able to separate spatial dependence in the error term and technical efficiencies, and (*iii*) produce consistent estimates. Moreover, I indicate how to incorporate more general spatial lag structures in stochastic frontier models. Estimation of these models, however, require simulation techniques.

The remainder of this paper is structured as follows. The next section introduces the concept of regional technical efficiencies and discusses some measurement issues. Consecutively, it treats the modeling (and its associated estimation) of technical efficiencies in two ways: a mainly econometric and a more statistical one.[3] The third section deals with the introduction of spatial

[3]Actually, both modeling approaches date back to one common source: namely, Weinstein, 1964.

dependence in stochastic production frontiers. Section 4 provides an application of spatial stochastic frontier modeling and gives an estimation of the technical efficiencies of European NUTS2 regions in 2010. The last section concludes.

2 Measuring technical inefficiencies

Since the late 70s, economists increasingly recognized that, although having access to the same set of production factors, firms do not necessarily produce the same output—and that there was consequently a need to econometrically correct for that (the seminal papers of this econometric literature are Aigner et al. (1977) and Meeusen and Broek (1977)). To explain this variation in output, it was argued that firms do not deploy production factors with the same efficiency. For example, labor might be less productive because of a lack of monitoring, which opens the possibility for various forms of shirking on the work floor. Or firms do not have access to the same technology, for whatever reason, and have therefore different output levels.

This, however, creates a problem. If most firms do not produce according to profit maximization, but systematically lower than that, then traditional production function estimates are *biased*.[4] Namely, not being able to optimize profits or costs leads to the fact that firms end up beneath an estimated ideal profit level (or above an estimated ideal costs level). Consequently, in the literature associated with stochastic production functions, the error terms are usually composed error terms: the traditional error term reflecting noise and a new error term—being strictly positive—measuring a firm's inefficiency.

In the remainder of this section I review concisely the non-spatial stochastic frontier model in subsection 2.1.[5] Thereafter, in subsection 2.2 an alternative estimation and not commonly used estimation method is introduced[6] where instead of a composed error structure we use a singular error structure in the form of a skew-normal distribution function.

[4] A similar line of reasoning could be held for minimizing cost functions. In the remainder of this chapter we do the reasoning for production functions, but note that the same theory and arguments holds for costs functions as well.

[5] See, amongst many others, Kumbhakar and Knox Lovell, 2000, Kumbhakar and Tsionas, 2006; Wang and Schmidt, 2009 and Wang and Ho, 2010 for some recent contributions to the econometric literature.

[6] That is, for econometricians, not for statisticians. Dominguez-Molina et al. (2003), Gupta et al. (2004) and Dominguez-Molina et al. (2007) already applied multivariate skew-normal distributions within a stochastic production frontier modeling framework—although only theoretically and not empirically.

2.1 Stochastic production frontiers

To start with, assume for simplicity that the production, y_i, of firm i ($i \in \{1, \ldots, N\}$) can be modeled as a Cobb-Douglas production function, thus:[7]

$$y_i = f(X_i; \beta')TEi, \tag{1}$$

where X_i; is the vector of production factors, β are the parameters of the Cobb-Douglas production function and TE_i denotes the so-called firm specific technical efficiency. Thus, TE_i is a distance measure of the firm to the (maximum) production of the best production firm there is – *within* the whole sample of firms. As a consequence TE_i must be smaller or equal to one for each firm i. Aigner et al. (1977) and Meeusen and Broek (1977) already specified (1) by assuming that $TE = \exp(-u_i)$, where u_i represents a stochastic variable. If we now assume a loglinear specification, then we may assume the following econometric specification:

$$\ln(y_i) = \ln(X_i)\beta - u_i + v_i, \tag{2}$$

with u_i being a stochastic variable as well, where $u \sim N(0, \sigma_u^2)$ and $v \sim N(0, \sigma_v^2)$ and with the explicit condition that $u_i > 0$.

For likelihood purposes one usually considers the composite stochastic variable $\epsilon = v - u$. Further, usually both u and v are conveniently considered independent. This enables us to find the marginal density of ϵ, namely:

$$\begin{aligned} f(\epsilon) &= \int_0^\infty f(u, \epsilon) du \\ &= \frac{2}{\sigma}\phi\left(\frac{\epsilon}{\sigma}\right)\Phi\left(-\frac{\epsilon\lambda}{\sigma}\right) \end{aligned} \tag{3}$$

where $\sigma_\epsilon = \sqrt{\sigma_u^2 + \sigma_v^2}$ and $\lambda = \frac{\sigma_u}{\sigma_v}$. Note that the marginal distribution of ϵ is a conditional distribution of u and v and that in the estimation u and v are intertwined by this conditional nature.[8] Note further that an estimate of the technical efficiency can now be obtained by finding the distribution of $f(u|\epsilon)$.

[7]For practical purposes such a production function is in its simplest form denoted as:

$$Y = AL^\alpha K^{1-\alpha}$$

where L stands for labor, K for capital and A for the level of technology (also known as: labor augmenting technology).

[8]A different way of denoting this is to observe that we interested in the probability $\pi(v|u' > 0)$ where u' is now a normal normal variable.

Obviously, estimation of this model with ordinary least squares regression creates a bias because of the simultaneous appearance of the two stochastic variables with one being truncated at zero. The traditional estimation procedure uses a likelihood procedure based on the density in eqn. (3). However, introducing a more complex error structure in specification (2) is rather cumbersome and not very intuitive. The next subsection proposes however an alternative specification and corresponding estimation procedure, which is in our opinion more insightful and more straightforward to adapt.

2.2 A skew-normal approach

The stochastic error structure, ϵ, in specification (2) actually dates back to Weinstein (1964), and can be rewritten in its most simple form as the sum of a normal and and a truncated normal distributed variable:

$$\epsilon = \delta|\mu| + \sqrt{1-\delta^2}\nu, \tag{4}$$

where μ and ν are independent $N(0,1)$ variables, and $\delta \in (-1,1)$. Here, the stochastic variable ϵ is generated by means of convolution.

A different genesis of ϵ can be realised by conditioning:

$$\epsilon = (\nu|\mu > 0), \tag{5}$$

where (μ,ν) is distributed as a bivariate normal random variable with correlation δ. From here, it is quite straightforward to show that both geneses (4) and (5) of ϵ lead to the same density function:

$$\epsilon Z \sim SN(\alpha) = 2\phi(x)\Phi(\alpha x), \tag{6}$$

which is called the skew-normal density function.[9] The parameter α in density (6) is a skewness parameter and determines the shape of the density function.

Density (6) is shown in Figure (3) for some values of the parameter α. When α is positive, the density is skewed to the right and when it is negative it is skewed to the left. When α is zero, the density becomes a standard normal density function and if $\alpha \to \infty(-\infty)$ then the density converges to the half-normal density; $2\phi(x)$ for $z \geq 0(\leq 0)$.

[9] The seminal paper in this field is by Azzalini, 1985, and is still probably the most influentious author in this field. Other relevant references with respect to the skew-normal distribution are, among others, Azzalini and DallaValle (1996), Azzalini and Capitanio (1999), Azzalini (2005), Arellano-Valle and Azzalini (2006) and Arellano-Valle and Azzalini (2008).

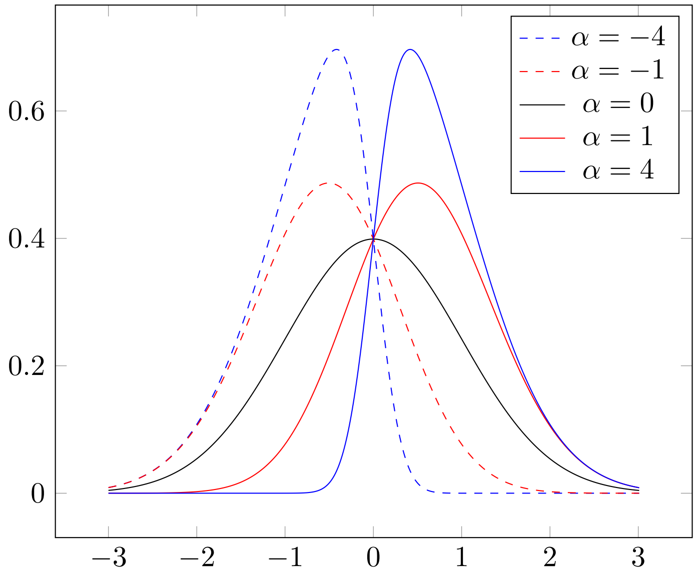

Figure 3 – Several examples of the skew-normal distribution function with varying α.

Apart from its relation with the normal distribution we will use for our application the following property: if $\epsilon \sim SN(\alpha)$ and $\ln(y) = \ln(X\beta) + \sigma\epsilon$, then the affine transformation $\ln(y) \sim SN(\ln(X\beta), \sigma^2, \alpha)$ holds, which can be expressed as:

$$\epsilon \sim 2\phi((\ln y) - \ln(X\beta); \sigma^2)\Phi(\alpha(\ln(y) - \ln(X\beta))). \tag{7}$$

Note that in this case $\ln X\beta$, σ^2 and α can be seen as a location parameter, a scale parameter and a skewness parameter, respectively.

The direct relation between specification (2) and (7) can be seen as well through stating $\ln(y_i) - \ln(X_i)\beta = \pi(v|u > 0) = \epsilon$, where

$$\epsilon = \begin{pmatrix} u \\ v \end{pmatrix} \sim N(0, \Omega^*), \qquad \Omega^* = \begin{pmatrix} 1 & \delta' \\ \delta' & \sigma^2 \end{pmatrix}, \tag{8}$$

and where $\alpha = \delta/\sqrt{1-\delta^2}$, $\delta = \sigma_u$, and $\sqrt{1-\delta^2}\sigma_\epsilon = \sigma_v$. The latter equality signifies the intrinsic relation between u and v which is implicit in specification (2). Note that specification (2) only holds when $\delta < 0$. We do not explicitly impose this condition on the model, but choose to leave this as an empirical test.

Unfortunately, estimation of the density in (7) is a bit involved.[10] When using maximum

[10]This basically is due to the fact that a direct estimation of the parameters is problematic around $\alpha = 0$. We therefore have to resolve to an alternative parametrization, the so-called centred parametrization (**ARELLANOVALLE2008**)

likelihood, the log-likelihood can be denoted as:

$$\ell\ell = -\frac{n}{2}\ln(\pi) + n\ln\left(\frac{\sigma_\epsilon}{\sigma}\right) - \frac{1}{2}\sum_i \epsilon_i^2 + \sum_i \ln\left((2\Phi(\alpha\epsilon_i))\right); \tag{9}$$

where

$$\begin{aligned}\epsilon_i &= \left(\sqrt{\frac{2}{\pi}}\delta\right) + \frac{\sigma_\epsilon}{\sigma}\left(\ln(y_i) - \ln(X_i)\beta\right) \\ \sigma_\epsilon &= \sqrt{1 - \left(\sqrt{\frac{2}{\pi}}\delta\right)^2}\end{aligned} \tag{10}$$

Skew-normal distributions are not much used in econometrics, but for our purpose they will do very nicely.[11] They allow us to use a single error term instead of a composite one, which has some benefits (such as clarity) when working with multivariate distributions. Moreover, the interpretation of the parameters seems as well more intuitive (using scale, location and skewness parameters). A disadvantage of using skew-normal distributions is the need to use a reparametrization of the parameters in order to estimate them properly.

The next subsection introduces a multivariate variant of this distribution function and applies it to spatial autocorrelation in both the endogeneous variable and the residuals.

3 Spatial stochastic production frontiers

The extension of density (7) to a multivariate setting is rather straightforward and has already been documented extensively in, amongst others, Azzalini and Capitanio (1999) Azzalini (2005) and **ARELLANOVALLE2006**. In this paper, we only concisely review the statistical framework needed for our application. To start with, we represent the underlying stochasting structure via conditioning. Assume that the k-dimensional variable ϵ is composited as $U + V$ where U has size 1 and V has size k, respectively, such that:

$$\epsilon \sim \begin{pmatrix} U \\ V \end{pmatrix} \sim N_{1+k}\left(0, \Omega^*\right), \qquad \Omega^* = \begin{pmatrix} 1 & \delta' \\ \delta & \Omega \end{pmatrix} \tag{11}$$

[11] Interestingly, the statistics literature mentions as a possible application of skew-normal distribution function the area of stochastic production frontier models. However, this has yet not permeated in the econometrics literature—although there is a nice R package that is able to deal with various forms of skew-normal and skew-t distribution functions, see http://pbil.univ-lyon1.fr/library/sn/html/sn.html. For more information about the skew-normal distribution function, see http://azzalini.stat.unipd.it/SN.

where Ω^* is a positive definite correlation matrix and – for our purposes – $\delta < 0$, then

$$\epsilon = (V|U > 0) \sim SN(0, \Omega, \alpha), \qquad \alpha = (1 - \delta' \Omega \delta)^{-1/2} \Omega^{-1} \delta. \tag{12}$$

Moreover, the affine transformation $\ln(y) = \ln(X\beta) + \sigma\epsilon$ leads to the following log-likelihood:

$$\ell\ell = -\frac{1}{2} n \ln|\Omega| - \frac{1}{2} n \mathrm{tr}(\Omega^{-1} V) + \varsigma_0 \left(\frac{\alpha}{\sigma} (\ln(y) - \ln(X)\beta) \right), \tag{13}$$

where

$$\begin{aligned} V &= n^{-1} (\ln(y) - \ln(X)\beta)(\ln(y) - \ln(X)\beta)', \\ \varsigma_0(x) &= \ln(2\Phi(x)). \end{aligned}$$

Fortunately, opposite to the univariate case, a reparametrization of the parameters does not seem necessary for the multivariate case.

Introducing spatial dependence is now rather straightforward (**ANSELIN1988A**). Define W as a spatial weight matrix, then the spatial autoregressive (SAR) frontier model can be expressed as:

$$\ln(y) = \rho W \ln(y) + \ln(X)\beta + \epsilon. \tag{14}$$

Model (14) can also be expressed as: $\epsilon \sim SN((\ln(y) - \rho W \ln(y) - \ln(X)\beta), \sigma(I - \rho W), \alpha)$. For further use, we refer to this model as a SAR frontier model.

Correspondingly, a stochastic frontier model containing spatial dependence in the residuals (also known as a SEM model in the nomenclature of LeSage and Pace (2009)) can be denote as:

$$\ln(y) = \ln(X)\beta + (I - \lambda W)^{-1}\epsilon, \tag{15}$$

with the density function $\epsilon \sim SN((\ln(y) - \ln(X)\beta), \sigma(I - \lambda W), \alpha)$. We refer to this model as a SEM frontier model.

Given the stochastic structure in (11), both the SAR and SEM frontier models can now be readily estimated by the use of maximum likelihood methods—as long as the number of observations is not too large. Note that this specification allows us to separate spatial autocorrelation and technical efficiency effects.[12]

As mentioned above, finding technical (in)efficiencies resolves in finding the expectation of the

[12] A more complex specification where technical inefficiencies themselves are spatially dependent as well would be

distribution of $u|\epsilon$. This distribution can be denoted as:

$$u|\epsilon \sim N^c\left(\left(D'\Sigma^{-1}D+I\right)^{-1}D'\Sigma^{-1}e,\left(D'\Sigma^{-1}D+I\right)^{-1}\right) \tag{16}$$

where N^c indicates a normal distribution truncated at 0, e a vector of realised residuals, D is a diagonal matrix with δ's on the diagonal and Σ equals $\sqrt{(I-D^2)}\Omega$. The expectation can now be readily derived.

The next section offers an empirical application by looking at regional technical efficiency of various economic sectors in European NUTS-2 regions.

4 The efficiency of European regions

In this section we apply the concept of spatial stochastic frontiers to European NUTS-2 regional production functions. The next subsection first describes concisely the data, the subsequent subsection gives the estimation results, and the last subsection provides a discussion to put the results into perspective.

4.1 Data & specification

NUTS-2 (Nomenclature of Units for Territorial Statistics) is a geocode standard for referencing the subdivisions of European countries for statistical purposes, where the addition 2 stands for the geographical level of more or less provinces. We use the European regional database by Cambridge Econometrics: a database containing detailed sectoral information about regional gross value added, regional provision of labour, and regional investment. The data we use stems from 2010, although previous years are available as well and the economic sectors that they comprise are: agriculture, energy and manufacturing, construction, market services and non-market services. The countries we include in our estimation can be seen in Figure **??** and are basically all EU25 countries except for Romania and Bulgaria.

created by rewriting (11) as:

$$\epsilon \sim \begin{pmatrix} U \\ V \end{pmatrix} \sim N_{d+k}\left(0,\Omega^*\right), \qquad \Omega^*=\begin{pmatrix} \Phi & \Delta' \\ \Delta & \Omega \end{pmatrix}.$$

Such a stochastic structure, however, involves multivariate truncated normal distributions, which with large dimensions of Φ becomes infeasible to estimate with maximum likelihood procedures and need simulation techniques such as Monte Carlo Markov Chain procedures (**DOMINGUEZMOLINA2007**).

To define our spatial weight matrix W, we use a k-nearest neighbour algoritm with $k = 4$, where the k-nearest neighbours get a weight of 1. The weights of all other neighbours are set at 0. Finally, we row-standardize W.

These data allow us to estimate the following Cobb-Douglas function:

$$\ln(Y_{rs}) = \beta_0 + \ln(L_{rs})\beta_1 + \ln(K_{rs})\beta_2 + \epsilon_{rs}, \tag{17}$$

where Y is gross value added, L the number of workers multiplied by the average hours worked per week, K is the amount of capital, r is the region, s the sector, and ϵ an error term that can be distributed normally or (multivariate) skew-normally.

Unfortunately, there is no precise data on capital stock for NUTS-2 levels. Therefore, we use an estimation of the capital stock based on the available investment data. Capital stock decreases by depreciation and increases by investments. By assuming that capital stock depreciates with 4% per year and is increased with each year's investments, it is possible to generate an approximation of the regional capital stock on the long term.

The next subsection provides the results for various sectors and specifications of the production function of (17).

4.2 Results

We start our estimation by pooling all sectors. Table 1 gives the results for various econometrics specifications. We start with the OLS estimation. Unfortunately, the factor rewards (or output elasticities) of K and L are not conform theory, although not significantly different from constant returns to scale. A frontier analysis does not alter those strange results, although the likelihood improves significantly. Finally, allowing for spatial dependence (whether that would be a SEM or a SAR frontier model) does not change the estimates of the factor rewards, considerably. However, it is clear that a SEM frontier model performs best in terms of log-likelihood. Moreover, a λ of almost 0.5 indicates significant spatial dependence in the error terms, which should be reflected in the estimations of the regional technical efficiencies.

Figure **??** shows the difference between the non-spatial technical efficiencies and the efficiencies, which are generated by a SEM frontier model. Clearly, the introduction of spatial dependence ensures that regions in the center become less efficient and regions in the periphery become

Table 1 – Estimation results for all sectors combined ($N = 256$)

Variable	**OLS**	**Frontier model**	**SAR frontier model**	**SEM frontier model**
Const.	-1.07**	-0.82**	-1.44**	-0.57**
ln K	1.10**	1.08**	1.03**	1.02**
ln L	-0.05	-0.04	-0.00	0.01
σ	0.21**	0.30**	0.29**	0.26**
α		-1.96**	-2.01**	-1.91**
ρ			0.15**	
λ				0.49**
Log-l.	0.12	0.49	0.51	0.60

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

more efficient (the distribution of technical efficiencies over regions becomes more homogeneous). In other words, regions in the periphery produce technically inefficient but less so when taking their geographical location into account. Thus, where regions are located matters just as their economic structure.

Because the estimates off all sectors combined are still somewhat implausible, we turn to sectoral estimations in Table 2, where we specify for each sector a SEM frontier model.[13]

Table 2 – Estimation of SEM frontier models for separate sectors ($N = 256$)

Variable	**Agriculture**	**Energy and manufacturing**	**Construction**	**Market services**	**Non-market services**
Const.	0.51**	0.10	0.90**	-1.08**	0.29
ln K	0.48**	0.82**	0.34**	0.79**	0.35**
ln L	0.41**	0.17**	0.64**	0.27**	0.67**
σ	0.65**	0.36**	0.60**	0.26**	0.49**
α	-3.26**	-1.37**	-3.44**	-1.17**	-3.45**
λ	0.19*	0.66**	0.51**	0.48**	0.55**
Log-l.	-0.16	0.17	-0.10	0.48	0.09

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

The estimated factors rewards in Table 2 are much more in line with theory. Note that they all point to constant returns to scale. Energy and manufacturing seems to be the most capital

[13] Checks with other specifications show that the SEM frontier model always performs best in terms of log-likelihood.

intensive sector and non-market services the most labour intensive sector. The most skewed sectors, with α's around -3.5, and thus the sectors with the largest differences in regional efficiency are construction and non-market services. Spatial dependence is in allmost all sectors – except agriculture – rather high with λ's between a half and a third.

Figure **??** shows the maps of the estimated regional technical efficiencies provided by the estimates in Table 2. Obviously, there are large differences in the regional distribution, but on average regions in the centre of Europe seem to have larger technical efficiencies than regions in the periphery. Moreover, agriculture has on average a much larger regional technical efficiency tham, e.g., market services, where regional differences are much more pronounced.

The next subsection aims to put these results into a somewhat wider perspective.

4.3 Discussion

Introducing spatial dependence decreases the dispersion in regional technical efficiencies. In other words, it removes the extremes; the most inefficient regions become somewhat more efficient and the most efficient regions become slightly less efficient. Obviously, space matters as well in this case.

However, according to Figure **??** the impact of spatial dependence does not seem to be that large given the still large dispersion in technical efficiencies. This is probably caused by the quite restrictve Cobb-Douglas production function we use and the associated data.

To see this, Table 3 gives the most efficient regions for each sector given our parameter estimates. Although the general pattern seems to be fine, technical efficiency estimates for individual regions do not always seem to be appropriate, given the prevalence of, e.g., Greek regions in Table 3. To be more specific, there are two general issues here that need to be adressed:

Spatial heterogeneity It might very well be that there is still a large amount of regional variety present in the data. This might be taken into account by fixed effects or additional regional variables. Using fixed effects is not straightforward in a stochastic frontier model, however. Moreover, using both regional fixed effects and spatial dependence might cause an identification problem (**ANSELIN2002A**).

Misspecification of the production function Another issue is the misspecification of the pro-

Table 3 – Five most efficient regions in each sector

Agriculture	*Energy and manufacturing*
Algarve	Southern and Easter Ireland
Castilla y Leon	iudad Autónoma de Ceuta
Notio Aigaio	Ciudad Autónoma de Melilla
Berlin	Outer London
Malta	Groningen
Construction	*Market services*
Voreio Aigaio	Attiki
Luxembourg	Luxembourg
North Eastern Scotland	Dytiki Makedonia
Southern and Easter Ireland	Bremen
Dresden	Stockholm
Non-market services	
Norra Mellansverige	
Luxembourg	
Friesland	
Hovedstadsregio	
Cheshire	

duction function. A Cobb-Douglas production function is very restrictive and one might feel more at ease with, e.g., a translog production function. Moreover, one rather important production factor is missing: namely, human capital. It is known that the estimates could be seriously biased (**MANKIW1992**) when omitting human capital.

Probably, the most fruitful approach would be to incorporate additional variables (especially human capital). Unfortunately, our dataset does not contain such variables. However, we can use a recently devised regional input-output matrix, not to add more production factors, but to decompose our gross value added variable (Y), namely :

$$Y = p(V^T - U) = \beta_0 K^{\beta_1} L^{\beta_2}, \tag{18}$$

where p is a price vector and V and U are make and use tables, respectively. This allows us to add more regional and sectoral variation in the estimation and would solve at least partly for both the spatial heterogeneity and the misspecification of the production function. Note however that using an input-output framework with its fixed shares only allow for a Cobb-Douglas production function.

5 In conclusion

The main aim of this paper was to introduce spatial dependence in stochastic production frontier analysis. We did this by using a skew-normal distribution function approach, which we argue is s (*i*) straightforward to use, (*ii*) able to separate spatial dependence and technical efficiencies, and (*iii*) produce consistent estimates. Moreover, we argue that the results can be interpreted in the view of relative and absolute geographic location. Obviously, there is more to this because technical efficiencies may be spatially dependend as well (for instance, there are spillovers in the adoption of new technology that improve the efficiency or there are specific institutions, such as former guilds or unions, that prohibit the adoption of new technologies are spatially concentrated). However, when looking at the efficiency of regions, taking into account spatial dependence – whether in the inefficiency part or not – strongly improves the estimates of the tecnical efficiencies; in other words, they become less biased.

When looking at European regions, taking spatial dependence into account controls more or less for the core-periphery pattern in Europe. Thus, regions in the periphery do not produce that inefficiently only because of their economic structure, but partly as well because of their location and the related diminished access to knowledge and information. Obviously, our estimations are a bit restrictive regarding the data and specification we use. Therefore, for immediate further research, we need to see whether we can use the information from a regional input-output table and mold that into a production function form, so that we at least remove some of the spatial heterogeneity and perhaps as well part of our misspecification issues.

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