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Housing market and migration revisited: a Bayesian multilevel gravity model for Dutch municipalities

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Abstract
By applying a Bayesian multilevel gravity model, this paper revisits the impact of home-ownership and social renting rates on intercity migration. Where most of the extant literatures focuses on using fixed effects for cities of origin and destination, I adopt a Bayesian multilevel approach. This approach has two main advantages. First, it allows for simultaneous estimation of city specific effects and the effects of city specific home-ownership and social renting rates on migration flows, where the impact is not necessarily symmetrical for cities of origin and destination. Second, it allows for prediction of migration flows between cities both in- and out-of-sample. The results show that home-ownership rates decrease migration flows significantly with an elasticity below -1. Municipal social renting rate has a negative impact as well, but its elasticity is close to zero. I use these estimates to predict changes in all in- and out-going migration flows in Amsterdam caused by a change in the home-ownership rate.

Keywords
Gravity model — housing market — migration — multilevel model — partial pooling — prediction

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1. Introduction

The current Dutch housing market is characterized by a low housing supply elasticity, high housing demand, and only a small segment (5%) that belongs to the private rental market (Michielsen et al., 2017). Because of the resulting large shortage of housing, many policy recommendations have been proposed, including plans for 700,000 new houses to be built in the coming decade. Unfortunately, the construction of a large amount of new dwellings is problematic in the Netherlands due to large and restrictive spatial constraints (Michielsen et al., 2019). Therefore, policy makers are also looking at changing the housing structure, e.g., by converting rent controlled housing to home-ownership properties, to tackle at least the high housing prices by enlarging the home-ownership segment.

This paper revisits the role of housing market structure as impediment for interregional migration and specifically focuses on the role of home-ownership and social renting rates. To this end, I adopt a Bayesian multi-level modelling approach which is not frequently encountered in the gravity literature[1]. Traditional gravity modelling has the disadvantage that either regional fixed effects of origins and destinations can be incorporated or the regions' characteristics when not varying over flows. This paper circumvents this disadvantage by adopting a multilevel approach with partial pooling[2], where the latter terms indicates that I do not impose fixed effects to control for origin and destination specific effects, but that I "draw" them from a distribution, hence the name partial pooling (where complete pooling states no group effects and no pooling fixed effects).

This papers adds two main elements to the literature. First, it does not only consider home-ownership but as well municipal social renting structure, which can be argued (see, e.g., Boyle, 1998; Hughes and McCormick, 1981) to have a large effect on regional mobility as well as social renting rights are usually only valid locally (within municipality) and are lost when moving residence between municipalities.

Secondly, A partial pooling approach has another advantage, namely the regional varying effects are completely probabilistic, making it feasible to predict both within and out-of-sample. In other words, with the results at hand I can predict migrations flows between existing *and* hypothetical regions. The former might be used for looking at counterfactuals; for example, the changes in in-migration for all municipalities, when one municipality changes its housing structure. The latter is useful when one wants to assess new migrations flows between one or even two new municipalities outside the sample.[3]

[1] That is, in the economic literature; a notable exception is Ranjan and Tobias (2007) in the economic trade literature. In the geographical literature this approach is more commonly adopted (see within a migration context Congdon, 2010; Congdon and Lloyd, 2012)

[2] There is a whole variety of names for these types of models, including hierarchical modeling, varying effects, mixed effects and shrinkage models. I use the more generic multilevel description as regions and flows are by definition measured at a different level (scale).

[3] See for probabilistic predictions of internation migration Azose and

There is already a large amount of literature looking into the positive and negative effects of home-ownership (see for an overview Dietz and Haurin, 2003). It is argued that home-ownership leads to, e.g., better maintenance (of the own dwelling and the neighbourhood), more savings, higher education outcomes, higher individual labor supply, and even better health. One prominent negative effect of home-ownership is that it leads to less incentives to move residence because of higher moving costs vis-à-vis renting.

And higher moving costs are detrimental to the labor market (Oswald, 1996, 1999). These is a large empirical literature (see, e.g., Munch et al., 2006, 2008; De Graaff and Van Leuvensteijn, 2013) looking at the impact of individual and aggregate home-ownership on labour market performance, where seemingly paradoxically at the aggregate level home-ownership is indeed harmful for labour market behaviour where at the individual level it is correlated with positive labour market performance.

This difference between individual and aggregate level is explained by sorting. Home-owners are indeed less mobile than private renters because of higher fixed and sunk moving costs which has a negative *aggregate* effect on labour market performance. However, home-owners are different from renters as they do *individually* better on the labour market (due to individual unobserved heterogeneity). So home-owners in countries with high home-ownership rates perform worse on the labour market vis-à-vis home-owners in countries with low home-ownership rates; but they still perform better than private renters. For social renters, the effect is different from private renters. On the individual level they are less mobile than renters at the free market as well, but their labor market performance is also worse than private renters (Hughes and McCormick, 1981; De Graaff et al., 2009).

That housing market structure has a sizeable effect on migration decisions is empirically well-established, especially at the micro-level, where it is widely accepted that home-ownership has a negative effect on regional mobility (Dietz and Haurin, 2003; Dohmen, 2005). For example, Palomares-Linares and van Ham (2018) find that home-ownership has a very strong immobility effect on internal migration in Spain during the period 2001–2011.

In the literature, less attention has been given to interregional migration on the aggregate level with respect to the housing market as a specific barrier.[4] For the UK, Congdon (2010) found within a multilevel gravity model that social rented housing had little effect on the attractivity of a region, although it had a small positive effect on preventing people from moving residence. For the Canadian case, Amirault et al. (2016) looked at the impact of home-ownership on migration flows within a gravity model using a Poisson pseudo maximum likelihood estimator and found an elasticity around -1.

Raftery, 2015.

[4] See Cushing and Poot (2004) for a historical overview of common themes within migration research.

The anticipate the results of this paper, I find strong negative effects of home-ownership rates on inter-municipal migration flows. Further, social renting rates also affect migration flows negatively, but the effect is far less pronounced than for home-ownership and overlaps zero to a large extent. A possible interpretation of this finding is that those who sort into social renting are by definition less mobile than those who sort into home-ownership (this argument is put forward by Boyle, 1998, as well). Finally, I show that a 0.1 decrease in the rate of home-ownership in Amsterdam leads to in increase of x in-migration y out-migration and significant changes as well in the neighbouring municipaties.

This paper reads as follows. The next section describes the data and focuses especially on the distribution of regional migration flows and regional labour market structure. Section 4 describes the modelling approach, where starting from traditional gravity model and using the descriptives of the migration flows, a Bayesian multilevel gravity model is constructed. Section 5 gives both the model results and interprets them by providing as well predictions within and out-of-sample. The last section concludes.

2. Data

I use inter-municipal migration flows measured in individuals between all of the the 393 Dutch municipalities in 2015. There is no information available on within municipality residential migration. So, I have 393 regional characteristics (or doubled when accounting for both regions of origin and destination) and 154,056 aggregate migration flows ($393 \times 393 - 393$).

Figure 1 shows the distribution of migrant flows within my sample. The left panel deals with migrant flows below 20, the right panel with migrant flows of 20 and larger. Two main observations can be made.

First, there is strong but consistent decay in migration flow size in both panels, which points to a persistent underlying pattern. However, the right 'tail' in this distribution is rather thick.[5] Thus, there are still observations quite far right in the distribution. Indeed, the sample mean is about 10, while the sample variance is around 40, leading to a strong presence of *overdispersion* (unconditional on other explanatory variables). Secondly, two thirds of the dataset consists of zero observations. Although they do seem to be genuine observations and not caused by another process (this will be checked later), they do need to be taken specifically into account.

Zeven explanatory variables are added to the model. First, to account for spatial distance decay between origin i and destination j, distance between all municipalities are calculated as Eucledian distance between centroids (dist_{ij}). Secondly, as municipality mass we use population size for both city of origin and city of destination (so pop_i and pop_j). Finally, for housing market structure we use variables indicating percentage of homeownership (home_i and home_j and percentage of

[5] The largest migration flows are between the municipalities of Amsterdam and Amstelveen and amount to about 3,500 migrants.

Figure 1. Histogram of migrant flows. Left panel shows the histogram of small migrant flows ($0 \leq N < 20$) and the right panel shows the histogram of large migrant flows ($N \geq 20$). Note the different scale of the y-axes.

social renting (soc_i and soc_j), again in both cities of origin and destination. Social renting in the Netherlands includes all kinds of rent controlled housing but typically involves local housing corporations offering housing to lower income households, where eligibility is based on (local) waiting lists. Both social renting and homeownership are assumed to impede regional mobility in the Netherlands as argued in De Graaff et al. (2009).

Figure 2 shows the distribution of social renting and homeownership across Dutch municipalities in 2015. Clearly, both home-ownership and social housing are prevalent across Dutch municipalities, with an average of 25% of social housing and around 60% of homeownership. Moreover, it is worthwhile to note that social renting is especially prevalent in the larger cities with a correlation of 0.4 between city size and social renting (e.g., Amsterdam has about a 40% social renting rate). Also, some smaller dutch municipalities do not exhibit any social renting. Homeownership and city size correlate negatively (-0.51). Finally, there is a large negative correlation between social renting and homeownership (-0.84) across municipalities.

3. Modeling framework

3.1 The traditional gravity model

A workhorse model to study aggregate migration flows has throughout time been the gravity model (see Anderson (2011) for a generic survey of the use of gravity models and Poot et al. (2016) for an overview of migration applications). I therefore start by adopting the basic gravity model specification pioneered by Tinbergen (1962), so:

$$\text{migrants}_{ij} = \text{pop}_i^{\beta_1} \text{pop}_j^{\beta_2} \text{dist}_{ij}^{\gamma} \tag{1}$$

Note, that in model (1) the variable dist_{ij} may represent in general all sorts of frictions, not only physical distance.

Importantly, Anderson and Van Wincoop (2003) argued that origin and destination specific variables should be incorporated to take into account multilateral resistance terms. Most often, this is done by log-linearising model (1)[6] and incorporating fixed effects for origins and destinations, as follows:

$$\log(\text{migrants}_{ij}) = o_i + d_j + \gamma \log(\text{dist}_{ij}) \tag{2}$$

Note that now all origin and destination specific variables are absorbed by the fixed effects o_i and d_j and that only variables affecting the frictions (dist_{ij}) can be incorporated.[7][8] Figure 3 denotes the problem schematically. Typically, one wants to model migration flows between i and j, whilst taken into account both the regional specific effects (o_i and d_j) and the regional variables ($\mathbf{X}_i$ and $\mathbf{X}_j$) one is interested in, such as housing market, population structures and cultural variables.

Moreover, equation (2 is typically estimated with regression type of models, which is often very cumbersome given the large amount of zeros migrants flows. 'Quick and dirty' remedies as adding a small amount to the flow variable or removing all zeros have been proven to seriously bias the parameters (Linders and De Groot, 2006; Burger et al., 2009).

Therefore, I next allow for a different strategy, where I would like to tackle simultaneously the two disadvantes of

[6] In our case, note that zeros are present in our social renting variable. We therefore add a small number to this variables (0.0001). Doing this only on the *right-hand side* does only marginally affect our results.

[7] If there is another variable dimension—say, repeated observations over time—then this might be circumvented. However, this requires enough variation in the data as time-invariant variables can still not be taken into account.

[8] An often applied strategy is to use differences between origin and destination specific variables. Take for example Δh_{ij} as the difference in home-ownership rates between i and j. A disadvantage of this approach is that the difference between 10% and 20% home-ownership rates and the difference between 80% and 90% home-ownership rates would be valued as the same.

Figure 2. Histogram of social housing (left) and homeownership (right) percentages in Dutch municipalities 2015

Figure 3. Decomposition of variables impacting migration flows from i to j $(\{i, j\} \in \{1, \ldots, R\})$

above: incorporating both city varying effects and city specific variables and modelling the distribution of migrants flows as they are displayed in Figure 1—even when being zero.

3.2 A Bayesian multilevel gravity model

Firstly, as regional migrants flows are discrete and relatively rare give the size of the population, the most appropriate way to go forward is to model number of migrants with a Poisson type of model. However, given that the sampling variance is four times the sampling mean of the migration flows (although not conditional on the covariates), we likely need to correct for overdispersion of heteroskedasticity (Silva and Tenreyro, 2006, states that heteroskedasticity (rather than the presence of too many zeros) is responsible for the main source of bias.). An often used distribution to account for overdispersion is the gamma-poisson model (also known as the negative binomial model). So, we use that for our outcome variable.

To account for the multiplicative nature of the theoretical model as in (1), I adopt a log-link for the expectation variable in the Poisson model.

Finally, to adopt both region effects and variables I adopt a multilevel model with partial pooling. This entails that our regional varying effects (the formerly fixed effecs) are now drawn from a, in this case Normal, distribution, where the parameters of this distribution are estimated as well (in the literature they are known as well as hyper-parameters). Intuitively, this entails that regions are partially pooled indicating that information between regions is shared. This is very attractive, as fixed effects assume no pooling. In that case, the model only learns from the information contained in that specific region whereas with partial pooling it is ensured that outliers (very high or low effects) are effectively *shrunk* towards the mean. Indeed, this is a further extension of that best feature of linear regression: regression towards the mean.

The total model looks now as follows:

$$\text{Migrants}_{ij} \sim \text{GammaPoisson}(\lambda_{ij}, \tau) \tag{3a}$$

$$\begin{aligned}\log(\lambda_{ij}) = &\alpha + o_{\text{mun}[i]} + d_{\text{mun}[j]} + \\ &\beta_1 \log(\text{pop}_i) + \beta_2 \log(\text{pop}_j) + \\ &\beta_3 \log(\text{home}_i) + \beta_4 \log(\text{home}_j) + \\ &\beta_5 \log(\text{soc}_i) + \beta_6 \log(\text{soc}_j) + \\ &\beta_7 \log(\text{dist}_{ij})\end{aligned} \tag{3b}$$

$$o_{\text{mun}} \sim \text{Normal}(\alpha_o, \sigma_o) \tag{3c}$$

$$d_{\text{mun}} \sim \text{Normal}(\alpha_d, \sigma_d) \tag{3d}$$

$$\beta_1, \ldots, \beta_7 \sim \text{Normal}(0, 2) \tag{3e}$$

$$\alpha_o, \alpha_d \sim \text{Normal}(0, 2) \tag{3f}$$

$$\sigma_o, \sigma_d \sim \text{HalfCauchy}(0, 1) \tag{3g}$$

$$\tau \sim \text{Gamma}(0.01, 0.01) \tag{3h}$$

The first line (3a) models the outcome variable, being the number of migrants, using a Poisson distribution (with parameter λ_{ij}) allowing for overdispersion by using an additional parameter τ. The linear part of the model is given by (3b) and states that the poisson outcome variable is on a log-scale and that most variables are on a log-scale as well, allowing for direct comparison of the parameters being elasticities. Equations (3c) and (3d) constitute the multilevel part, where parameters σ_o and σ_d measure the amount of pooling. If they tend to zero, then the data exhibits complete pooling. If they become very large (go to infinity) there is no pooling (thus fixed effects). All the other parameters are priors (chosen such that they are rather conservative but given the amount of data they are of little influence).

4. Results

4.1 Parameter estimates

Model (3) is estimated by using the *No U-Turn Sampler* (NUTS) from the Stan application.[9] NUTS is a relatively recent developed Hamiltonian Monte Carlo (a specific form of Markov Chain Monte Carlo simulation) method, able to draw samples efficiently from large multilevel models (Hoffman and Gelman, 2014). Parameter estimates and probability intervals of the main parameters (so not the region specific effects: there are 786 of them) are given in Table 1. Perhaps more insightful, they are as well graphically depicted in Figure 4.

Obviously, housing structure indeed impedes regional mobility, but that it is primarily home-ownership rates and not social renting rates that have a negative effect. The home-ownership elasticities are slightly larger in absolute size than what Amirault et al. (2016) reported. Furthermore, if anything, estimations for parameters σ_o and σ_d point to more pooling

Figure 4. Forest plot of parameter means and 95% probability intervals (group specific origin and destination estimates are not presented)

[9] See https://mc-stan.org/. As interface to Stan (see for an overview article of Stan Carpenter et al., 2017) I used the R-package (Bürkner, 2017) `brms`.

Table 1. Parameter estimates with 95% probability intervals (group specific origin and destination estimates are not presented)

Parameter	mean	sd	2.5%	97.5%
Intercept	−0.74	0.04	−0.82	−0.66
$\log(\text{pop}_i)$	0.89	0.03	0.83	0.96
$\log(\text{pop}_j)$	0.88	0.04	0.79	0.97
$\log(\text{home}_i)$	−1.48	0.19	−1.86	−1.10
$\log(\text{home}_j)$	−1.27	0.25	−1.75	−0.78
$\log(\text{soc}_i)$	−0.04	0.04	−0.11	0.03
$\log(\text{soc}_j)$	−0.06	0.03	−0.12	−0.01
$\log(\text{dist}_{ij})$	−1.96	0.01	−1.97	−1.95
σ_o	0.45	0.02	0.42	0.49
σ_j	0.61	0.02	0.57	0.66
τ	1.22	0.01	1.20	1.24

than less, so fixed effects in this case could lead to substantial overfitting.

The other variables are conform previous literature. The elasticities of population for both regions of origin and destination are around 0.9 and the distance-decay parameter is −1.96. The latter is significantly larger in absolute value that typical parameters reported by regression type of models, but is more in less line with parameters estimated by negative binomial or poisson type of models.

For each municipalities the models estimates as well an origin specific effect (o_i) and a destination specific effect (d_j). These are depicted in maps 5 and 6 and can be interpreted as having relatively less or more migration *relative* to population size and housing structure.

Figure 5. Region specific origin effects

Figure 5 first shows the region specific origin effects. Clearly, there is a relatively larger push factor in the peripheral Dutch municipalities. These are actually regions that loose population. The areas that loose relatively few people are to be found in the Dutch core region (the "Randstad") including the largest Dutch cities: Amsterdam, Rotterdam, The Hague and Utrecht.

Figure 6. Region specific destination effects

Figure 6 shows the region specific destination effects. Interestingly, these seem to mirror to a certain extent the region specific origin effects. Again, relatively most people seem to move to the peripheral municipalities—albeit to a lesser extent than the push factor. That means that controlling for housing and population, relatively few people move to the largest cities in the Netherlands. This is due to the scarcity of available housing in that period (partly due to the housing structure) and the fact that this model does not reflect price differences. The "Randstad area" in the Netherlands is very popular, but because of housing shortage this is mainly reflected in higher prices and an increasingly tighter housing market. Housing is still available in the least attractive regions of the Netherlands and this is where most dynamics in terms of moving residence take place. This indicates as well that the region specific destination effects should not be interpreted as attractivity effects as, e.g., in Congdon (2010).

Figures 5 and 6 indicates as well that the region specific origin and destination effects are most likely highly correlated. Moreover, given the facts that nearby municipalities often show similar values, ideally both types of effects should be modelled as being spatially autocorrelated.

4.2 Model predictions

To assess to what extent the model predictions are in line with the observed data, Figure 7 shows the histogram of both observed and predicted flows side by side. Eyeballing the evidence, it seems that the distribution of predicted flows comes very close to the distribution of the observed mgiration flows. The distribution of predicted migration flows seems somewhat flatter though, as it predicts around 12,500 less

Figure 7. Histogram of observed and predicted migrant flows. Left panel shows the histogram of small migrant flows ($0 \leq N < 20$) and the right panel shows the histogram of large migrant flows ($N \geq 20$). Note the different scale of the y-axes. Predicted migrant flows above 4,020 were removed for reasons of clarity and comparability with the observed migrant flows.

zeros and more or less the same amount more ones. What is not shown in the histogram (and if, it would be very difficult to discern) is that the model predicts a couple of large outliers—flows as large as 100,000 migrants.

Figure 8. Changes in migration flows when percentage home-ownership in Amsterdam increases by 10%

5. In conclusion

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