Exam 4 Name:

MT 3312: Advanced Math for Engineers I

You may use your formula sheet on this exam and a device to write and erase with, but no other items may be used. No calculators are allowed either.

Answers without supporting work will not be accepted. Partial credit will be given only for steps that are correct.

1. Let S be the set consisting of all complex numbers z that satisfy $1 < |z - 3 + 4i| < 3$.

   (a) (10 points) Sketch the set S in the complex plane.

   (b) (5 points) Is the set S a domain? Explain your answer. An answer without an explanation will receive no credit.

2. Let $f(z) = e^z$.

(a) (10 points) Write $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$, assuming that $z = x + iy$.

(b) (10 points) Show that the functions $u(x, y)$ and $v(x, y)$ that you found in part (a) satisfy the Cauchy-Riemann equations.

3. Let $z = 2 - 2i$.

(a) (10 points) Write z in polar form.

(b) (10 points) Find $(2 - 2i)^5$. Write your answer in $x + iy$ form.

4. (15 points) Find all solutions to the complex equation $z^3 + 3i = 0$. Write your answers in $x + iy$ form.

5. (10 points) Evaluate the following limit, if it exists.

$$\lim_{z \to 1+i} \frac{z^2 - 2i}{z^2 - 2z + 2}$$

6. (10 points) Find $(1-i)^{3i}$. Simplify as much as possible.

7. Find the derivatives of the following complex functions. You do not need to simplify once you have fully completed taking the derivative.

   (a) (5 points) $f(z) = \tan\left(z^5 + \dfrac{1}{z^3}\right) - \sin^{-1}(z^4) + \operatorname{Ln}\left(\cos(z) - 6z^2\right)$

   (b) (5 points) $f(z) = \left(2z^2 - z\right)^{1/4} + \dfrac{4e^{z^3}}{\sec(z^2)} + i^i$