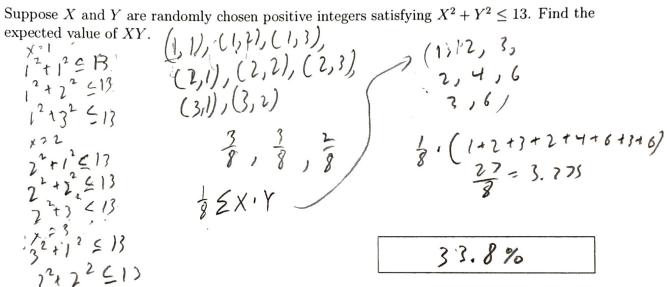# Homework 8 – due Monday 11/4

Adam A. Allan

Name (first & last): Prajnyique Ghimire

Directions: Submit your solutions in pdf format to the myCourses website under the *Homework* submodule. Please write answers in boxes and express all probabilities as percentages that are rounded to the nearest tenth of a percentage. So probability 0.1 should be entered as 10.0%.

1. Suppose X and Y are randomly chosen positive integers satisfying $X^2 + Y^2 \leq 13$. Find the expected value of XY.

$X = 1$

$1^2 + 1^2 \leq 13$

$1^2 + 2^2 \leq 13$

$1^2 + 3^2 \leq 13$

$X = 2$

$2^2 + 1^2 \leq 13$

$2^2 + 2^2 \leq 13$

$2^2 + 3^2 < 13$

$X = 3$

$3^2 + 1^2 \leq 13$

$3^2 + 2^2 \leq 13$

$(1,1), (1,2), (1,3),$
$(2,1), (2,2), (2,3),$
$(3,1), (3,2)$

$\frac{3}{8}, \frac{3}{8}, \frac{2}{8}$

$\frac{1}{8} \sum X \cdot Y$

$(1, 2, 3,$
$2, 4, 6$
$3, 6)$

$\frac{1}{8} \cdot (1+2+3+2+4+6+3+6)$

$\frac{27}{8} = 3.375$

$$\boxed{33.8\%}$$

2. Suppose a point (X, Y) chosen at random from the square defined by $0 \leq x, y \leq L$. Find the expected value of the product XY.

$$\int_0^L \int_0^L x \cdot y \frac{1}{L^2} dx \, dy$$

$$\frac{1}{L^2} \int_0^L x \cdot y \, dx$$

$$\frac{1}{L^2} \left(\frac{1 x^2}{2} \cdot yx \right]_0^L$$

$$\int_0^L \frac{1}{L^2} \left(\frac{L^2}{2} \cdot y \right) dy$$

$$\frac{1}{L^2} \left[\frac{L^2}{4} y^2 \right]_0^L \qquad \frac{1}{L^2} \left(\frac{L^4}{4} \right) = \frac{L^2}{4}$$

$$\frac{1}{L^2} \left(\frac{L^2}{4} \cdot L^2 \right)$$

$$\boxed{\frac{L^2}{4}}$$