+50 total points

1. We generally expect the variables are

+2 for completeness

(a) Discrete
(b) Continuous
(c) Discrete
(d) Discrete

+5

2. Note that

$$\Pr(P = 0) = 10k \quad \Pr(P = 1) = 16k \quad \Pr(P = 2) = 18k \quad \Pr(P = 3) = 16 \quad \Pr(P = 4) = 10k$$

The probabilities must add to 1, which means $70k = 1$ or $k = \frac{1}{70}$.

+4

(a) This probability equals

$$\Pr(P \geq 1) = 1 - \Pr(P = 0) = 1 - \frac{10}{70} = 0.857$$

−2 for not solving for k

or 85.7%.

+2

(b) This probability equals

$$\Pr(P \geq 2) = \Pr(P = 2) + \Pr(P = 3) + \Pr(P = 4) = \frac{18}{70} + \frac{16}{70} + \frac{10}{70} = 0.629$$

or 62.9%.

+6

3. The random variable M has pmf given by the table

m	0	1	2	3
$\Pr(M = m)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

So the expected value of M is given as

+4

$$E[M] = 0 \cdot \frac{1}{16} + 1 \cdot \frac{3}{16} + 2 \cdot \frac{5}{16} + 3 \cdot \frac{7}{16} = 2\frac{1}{8}$$

Note too that

$$E[M^2] = 0^2 \cdot \frac{1}{16} + 1^2 \cdot \frac{3}{16} + 2^2 \cdot \frac{5}{16} + 3^2 \cdot \frac{7}{16} = \frac{43}{8}$$

so that the standard deviation σ_M of M is given as

+2

$$\sigma_M = \sqrt{V[M]} = \sqrt{E[M^2] - E[M]^2} = \sqrt{\frac{43}{8} - \left(\frac{17}{8}\right)^2} = 0.927$$

+2 for completeness

4. The random variable X has pmf p given by the table

x	3	4	5
$p(x)$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

So the expected value $E[X]$ of X is given as

$$E[X] = 3 \cdot \frac{4}{9} + 4 \cdot \frac{2}{9} + 5 \cdot \frac{3}{9} = \frac{35}{9} = 3\frac{8}{9}$$

Note too that

$$E[X^2] = 3^2 \cdot \frac{4}{9} + 4^2 \cdot \frac{2}{9} + 5^2 \cdot \frac{3}{9} = \frac{143}{9}$$

so that the standard deviation σ_X of X is given as

$$\sigma_X = \sqrt{V[X]} = \sqrt{E[X^2] - E[X]^2} = \sqrt{\frac{143}{9} - \left(\frac{35}{9}\right)^2} = 0.875$$

+6

5. Note that X has the geometric distribution with $p = \frac{1}{150}$.

   (a) We compute

+2

$$\Pr(X > 100) = \left(1 - \frac{1}{150}\right)^{100} = 51.2\%$$

   (b) We compute

+2

$$\Pr(X \leq 200) = 1 - \Pr(X > 200) = 1 - \left(1 - \frac{1}{150}\right)^{200} = 73.8\%$$

+2

   (c) We use the formula $E[X] = \frac{1}{p} = 150$ circuit boards.

Typo, so no grading

   (d) We use the formula $\sigma_X = \sqrt{\frac{1-p}{p^2}} = 149.5$ pages.

6. Recall that

+2 for completeness

$$E[aX + b] = aE[X] + b$$

We therefore obtain

$$\begin{aligned} V[aX+b] &= E[(aX + b - E[aX+b])^2] \\ &= E[(aX + b - aE[X] - b)^2] \\ &= E[(aX - aE[X])^2] \\ &= a^2 E[(X - E[X])^2] \\ &= a^2 V[X] \end{aligned}$$

7. We compute

+2 for completeness

$$
\begin{aligned}
V[X_1 + X_2] &= E[(X_1 + X_2 - E[X_1 + X_2])^2] \\
&= E[(X_1 - E[X_1] + X_2 - E[X_2])^2] \\
&= E[(X_1 - E[X_1])^2 + (X_2 - E[X_2])^2 + 2(X_1 - E[X_1])(X_2 - E[X_2])] \\
&= E[(X_1 - E[X_1])^2] + E[(X_2 - E[X_2])^2] + 2E[(X_1 - E[X_1])(X_2 - E[X_2])] \\
&= V[X_1] + V[X_2] + 2E[X_1 - E[X_1]]E[X_2 - E[X_2]] \\
&= V[X_1] + V[X_2] + 2(E[X_1] - E[X_1])(E[X_2] - E[X_2]) \\
&= V[X_1] + V[X_2]
\end{aligned}
$$

+8

8. We say that a "success" corresponds to having the disease. So the chance p of a success equals $\frac{1}{5000} = 0.0002$. We now use the binomial distribution with $n = 15000$ many trials.

   (a) The chance that at least someone in the town has the disease equals

   +2

   $$1 - \binom{n}{0} p^0 (1-p)^{n-0} = 0.950$$

   or 95.0%.

   (b) The probability that at most 2 people in the town have the disease equals

   +2

   $$\binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1} + \binom{n}{2} p^2 (1-p)^{n-2} = 0.423$$

   or 42.3%.

   (c) The expected number of people in the town who have the disease equals

   +2

   $$np = 3$$

   (d) The variance for the number of people in the town with the disease equals

   +2

   $$np(1-p) = 2.9994$$

9. We say a "success" is selecting a fish entrée. The probability p of success is given as

+2 for completeness

$$p = \frac{8}{8 + 12 + 10} = \frac{4}{15}$$

The probability of at most two successes from six trials equals

$$\binom{6}{0}\left(\frac{4}{15}\right)^0\left(1 - \frac{4}{15}\right)^{6-0} + \binom{6}{1}\left(\frac{4}{15}\right)^1\left(1 - \frac{4}{15}\right)^{6-1} + \binom{6}{2}\left(\frac{4}{15}\right)^2\left(1 - \frac{4}{15}\right)^{6-2} = 0.803$$

or 80.3%.

+4

10. Let X denote the number of components that have failed. We want to compute the conditional probability $\Pr(X \geq 2|X \geq 1)$. We use the definition of conditional probability and the binomial distribution to get

$$\begin{aligned}
\Pr(X \geq 2|X \geq 1) &= \frac{\Pr((X \geq 2)\&(X \geq 1))}{\Pr(X \geq 1)} \\
&= \frac{\Pr(X \geq 2)}{\Pr(X \geq 1)} \\
&= \frac{1 - \Pr(X = 0) - \Pr(X = 1)}{1 - \Pr(X = 0)} \\
&= \frac{1 - \binom{10}{0}(0.2)^0(1-0.2)^{10-0} - \binom{10}{1}(0.2)^1(1-0.2)^{10-1}}{1 - \binom{10}{0}(0.2)^0(1-0.2)^{10-0}} \\
&= \frac{1 - 0.26844 - 0.10737}{1 - 0.10737} \\
&= 0.699
\end{aligned}$$

or 69.9%.

+8

11. We have to adjust the value of λ for the different time intervals. To do this we can always set up a proportion.

+2

(a) We have $\frac{\lambda}{5} = \frac{48}{60}$ so that $\lambda = 4$. Then

$$\Pr(X \leq 2) = \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} = 0.238$$

or 23.8%.

+2

(b) There will be an average of 4 callers waiting by that time.

(c) The probability equals

+2

$$\Pr(X = 0) = \frac{e^{-4}4^0}{0!} = 0.018$$

or 1.8%.

+2

(d) We have $\frac{\lambda}{3} = \frac{48}{60}$ so that $\lambda = 2.4$. Then

$$\Pr(X = 0) = \frac{e^{-2.4}2.4^0}{0!} = 0.091$$

or 9.1%. One must have sympathy for this poor agent!

+2 for completeness

12. In the calculator we set the "average rate of success" as 400 and the "Poisson random variable (x)" as either 425 or 375. We hit **Calculate** and then read the output.

(a) 89.8%

(b) 90.0%