

**Aero-Elastic Optimisation based on the Adjoint Approach**

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# Introduction

Civil aviation is an industry that has made the world a more accessible place for everyone. People can now travel to locations that would have been impossible only a few generations ago. Climate change is caused by greenhouse gas emissions. The aviation industry is responsible for a significant proportion of global emissions. The EU commissioned report Flight Path 2050 [1] has set a series of targets for the industry to achieve by the year 2050.  
The targets set for industry are to reduce emission levels, relative to emission levels from the year 2000, by the following amounts:

* A 75% reduction in emissions per passenger kilometre
* A 65% reduction in noise emissions
* A 90% reduction of emissions relative to

It is not desirable to cut the levels of flights as this limits the opportunities for global travel to only a few people. This means major advancements in aviation technology must be achieved and implemented quickly. The aircraft industry is relentlessly pursuing more advanced designs to improve efficiency as much as possible. Aircraft optimisation is going to be key in reaching these objectives. Optimisation techniques have reached maturity in the aviation industry over the last decade and have been applied to wide range of application in aircraft design. Strategies aimed at improving the aerodynamic efficiency of an aircraft at transonic speeds have been pursued vigorously in both academia and industry. The studies conducted in [2] and [3] show the first integration of optimisations into the design process for commercial aircraft.

Optimising specific disciplines on their own has already improved aircraft efficiency. Many different authors have demonstrated various aerodynamic optimisation techniques to improve aircraft efficiency [4]–[11]. However, ignoring the strong coupling between all the disciplines that affect aircraft efficiency, will inevitably miss out on further efficiency savings that can be gained. As a wing deforms in flight due to its aerodynamic loading, the flow around it will change and therefore modify the pressure distribution, potentially drastically [12].

A clear way to reduce fuel consumption is to fly lighter aircraft. A very significant percentage of an aircraft’s weight is in its wings so minimising the weight of the wings is clearly desirable. Doing this brings complications with it, a lighter wing will typically be accompanied with lower structural stiffness. This will result in more aero-elastic deformation of the wing, leading to greater differences in the wing’s shape throughout the flight envelope. Two challenges must therefore be confronted when faced with this. First, how to determine the jig shape that will result in an optimised flight shape and second, how to ensure that off-design performance is not considerably poor.

The first challenge can be met in 2 ways. A purely aerodynamic optimisation can be performed at the design point to produce an optimum flight shape. The jig shape that will produce the desired flight shape can be determined for a given flight condition and this will produce an optimised wing shape for the design point. There are two drawbacks from this approach, one there is no guarantee that the required jig shape is feasible and second, a multi-point approach can’t be implemented \*\*\*\*\*\*\*\*\*\*\*\*\*\* CITATION NEEDED\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*. A shape which is optimised only for one point will be very poor away from the design point [13]. This poor off-design performance can be somewhat mitigated by a multi-point approach. The second approach is to perform a coupled aero-elastic optimisation. The shape of the aircraft is still the design parameter but the aero-elastic effect on the jig shape is considered during the optimisation. This means the jig shape can be optimised for directly, which in turn allows the use of a multi-point approach thus going some way to alleviate poor off-design performance.

More still needs to be done to salvage performance further away from the design condition for the following reason. A drawback of a multi-point approach is that it takes away from the drag saving potential at the design point. A low-stiffness wing is going to have such different shapes across the flight envelope that this must be mitigated for through some other means. The approach of this research project to address this is to use the exciting technology known as a variable camber wing. A variable camber wing is able to change its shape in a desirable way across the flight envelope.

The challenge of making a low-stiffness wing maintain reasonable performance all the way through its flight can now be addressed successfully. The optimal variable camber configuration can be determined for a range of design flight conditions, and at each flight condition the wing would deflect its control surfaces to form the shape that would aero-elastically deform into the optimal shape for that condition. To determine how to deflect the control surfaces in the necessary way at each flight condition would require an aero-elastic optimisation with the control surface deflections being the design parameters. This optimisation would be used late on in the design process and therefore, unlike conceptual design optimisations, the highest fidelity software must be used.

# Literature Review

## 

# Fluid-Structure Interaction

## Proper Orthogonal Decomposition Parameterisation

## RANS Simulation in TAU

## Linear Elastic Analysis in NASTRAN

In the aerodynamic simulations, doing this had no effect on the accuracy of the results. This is because defining the displacements relative to the undeformed coordinates produced a high-quality mesh that represented the exact shape that was being modelled. However, the use of a coupling surface does have an effect on the accuracy of the aero-elastic simulation. This is because the structural analysis is always performed on an undeformed finite element mesh. Therefore, as the newer jig shapes move further away from the initial jig shape, the displacements returned from the structural solver become less accurate. However, as reported by Martins in [\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*], the error introduced by this is not significant and the simulation results are still reliable.

## Interpolation

### CFD to CSM Interpolation

### CSM to CFD Interpolation

## RBF Mesh Deformation

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## Trim in the Loop

### Control Surface Deployment

### Broyden Algorithm

# Adjoint Approach for Aero-Elastic Optimisation

The focus of this research was to perform high-fidelity optimisations in an efficient way. The only feasible way of performing a fast optimisation for a multi-physics high-fidelity problem is to use gradient-based optimisation, especially when there are a large number of design variables. In aerodynamic applications, often 100s of design variables are required to produce a large enough design space capable of producing a significant improvement in performance [14]. This section focuses on how to obtain the gradient after a completing an aero-elastic simulation. There are two major ways of obtaining a gradient of an objective function with respect to an aircraft’s shape. These are the finite differences (FD) approach and the adjoint method. Finite differences is of course a very old method that is very easy to implement while the adjoint method has been used in aerodynamic optimisation literature since 1988 [15]. This use of the adjoint method for aerodynamic optimisation has matured over the last few decades and it is now widely used within the commercial aviation industry [16].  
The adjoint method has the major advantage of only requiring one evaluation of the objective function. This is essential for any practical high-fidelity gradient-based optimisation. The adjoint method therefore has a negligible dependence on the number of design variables employed, allowing the optimiser to explore a large design space without adding significant computational overhead. Finite differences still have their place in aerodynamic optimisation although mostly for validation purposes.

## Finite Difference

The finite differences approach is the simplest method to obtain the gradient, it is also the simplest method to implement. Finite differences are typically implemented in one of 3 ways, two of the approaches are accurate to a 1st order approximation and these are the forward and backward difference methods. A 2nd order accurate approximation can be obtained by the central difference method. These are derived from the Taylor series and will evaluate the gradient of a one-dimensional function about a point .

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\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* Figure showing improved accuracy of Central difference \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

An obvious issue with finite differences is selecting the step size . Mathematically, the true gradient is the value of the above expressions as the step size tends to zero. If the step size is too big then, the error of the approximation will be large. However, if they step size is too small, then subtractive cancellation errors will be large [17]. Cancellation errors occur in computing due to the limited memory numerical data types have available to them. In a CFD simulation, if the values of pressure at each node on the mesh are stored in float data types, they will have 6 digits of precision available to them. If a finite difference approximation was attempted with a step size that caused too small a perturbation in the pressure for it to be accounted for with 6 digits of precision, then the recorded approximation would be garbage information.

For complex applications, such as drag sensitivity to a shape change, a range of step sizes must be investigated to ensure a suitable step size is chosen. This will increase the computational cost of finite differences even further.

\*\*\*\*\*\*\*\*\*\*\*\* Range of Step sizes graphs \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

A suitable step size for a finite difference approximation is found when a range of step sizes are found to produce the same value. Due to the increased accuracy of central differences over the first order methods, it provides a larger range of useable step-sizes as can be seen from the graphs above. To obtain the gradient of an aerodynamic cost function with respect to design parameters through central differences will take flow solutions.

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An aerodynamic cost function calculated through a RANS simulation is explicitly sensitive to the state variables and nodal coordinates at the surface nodes. The state variables are dependent on the design variables but this relationship cannot be defined explicitly \*\*\*\*citation\*\*\*\*\* hence why RANS is used in the first place. The absence of an explicit relationship is what makes impossible to determine without finite differences.

It is clear the computational cost of finite differences is prohibitive for optimisation purposes, therefore for this research project, its use has been consigned to validation purposes.

## Flow Adjoint

To make multi-disciplinary and even aerodynamic optimisation feasible for an aircraft with more than a few design parameters, the adjoint method must be used. Two simple concepts are employed by the adjoint method to eliminate the dependency on the number of design variables. First, if you add a constant to a function, the gradient at all points will remain unchanged. Second, multiplying anything by zero will result in a product equal to zero.

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In equation (4.4), the cost function has been set to the value of the drag coefficient. In order to find the sensitivity of the cost-function to the design parameters efficiently, the equation will be modified by adding a constant to the right-hand-side (RHS) of (4.4) that is also a function of the design parameters.

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This innocuous looking modification to the equation is actually very powerful. is the residual of the CFD simulation, for a sufficiently converged solution it can be considered to be a vector of zeros for all of the design space. At this point, the value of the adjoint vector is arbitrary as the term will be equal to zero for all of the design space due to being a vector of zeros. This is why the residual must be strongly converged for the adjoint method to work, the derivation depends on it. This allows the adjoint vector to be defined in a very favourable way.

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Both and are equal to zero so it can be seen how the sensitivity of the Lagrangian is equal to the sensitivity of the cost function. The term [[1]](#footnote-1) is equal to zero and cannot be manipulated for any gain so it is dropped from the equation. The adjoint vector will not be constant for all of the design space, instead it will meet the criteria that allows for the elimination of the state variables’ sensitivity to the design parameters . With this term eliminated, the need for multiple simulations to be run is eliminated too as every other term can be calculated from the results of one converged solution.

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The partial derivatives , , and can all be provided by the CFD solver TAU. The partial differentiation of an aerodynamic cost function or flow residual is a challenging task, for the TAU solver this was addressed by Dwight in [18]. Due to the complexities involved in the derivation of these terms they will not be discussed here; it is sufficient to know that they are available. The remaining unknowns are , and . As seen in the derivation, the adjoint vector is arbitrary and thanks to this, the troublesome term can be eliminated while retaining the ability to calculate the gradient.

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Equation (4.13) is known as the adjoint equation. The adjoint vector that satisfies (\*\*\*\*\*\*\*\*\*\*4.13\*\*\*\*\*\*\*\*\*\*\*\*) can be plugged into (4.12) and the gradient equation no longer requires the calculation of .

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There only unknown left to calculate in order to obtain the gradient is the mesh sensitivity to the design parameters .

## Mesh Adjoint

The mesh sensitivity can of course be calculated by finite differences. This isn’t a ridiculous prospect, as calculating mesh deformations is significantly less time consuming that a full RANS simulation.

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Obtaining the mesh sensitivity this way through (\*\*\*\*\*\*\*\*\*\*4.15\*\*\*\*\*\*\*\*\*\*\*\*) gets the final unknown term and the gradient can be calculated. However, as mesh sizes get larger, the mesh deformation will still take a significant time to compute. To further improve the efficiency of the gradient calculation, a mesh adjoint can be applied using the same principles as described in the previous flow adjoint section. The mesh adjoint was first used in [19], the addition of a mesh residual provides another mesh sensitivity term which can then be used to remove the mesh sensitivity from the gradient equation. This requires the mesh adjoint vector to meet a criterion that is shown below.

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The mesh residual is defined by the mesh deformation process, I.E the relationship between the surface mesh and the volume mesh.

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The relationship is obviously determined by the choice of mesh deformation strategy. An explicit mesh deformation strategy, one where the volume mesh points can be determined by a simple matrix vector product such as Delaunay Graph Mapping, will have the residual defined in (\*\*\*\*\*\*4.18\*\*\*\*\*). An implicit mesh deformation strategy, one which requires the solution of a large linear system such as linear elasticity, will have the residual defined in (\*\*\*\*\*\*\*\*\*\*\*\*4.19\*\*\*\*\*\*\*\*\*\*\*\*\*).

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It can be seen that the addition of a mesh adjoint had no effect on the flow adjoint equation so that can be carried out in exactly the same way as before to obtain the flow adjoint vector . This leaves the gradient equation in the following form:

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Now the mesh adjoint vector can be chosen so that the mesh sensitivity is multiplied with a vector of zeros, thus eliminating it from the gradient equation.

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Equation (\*\*\*\*\*4.23\*\*\*\*\*) highlights an advantage of an explicit mesh deformation scheme over and implicit one. For an explicit scheme, the jacobian is the identity matrix making the calculation of the mesh adjoint vector an addition problem. An implicit scheme would require solving a large linear system that will naturally take a significant time to do.

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The jacobian is the identity matrix multiplied by minus 1. It does have to be calculated for explicit schemes, however. In this work, the mesh deformation scheme employed is an explicit version of RBF mesh deformation. The mesh adjoint terms for the RBF mesh deformation scheme are explored in section 4.3.1.

The only unknown left in the gradient equation is the surface mesh sensitivity to the design variables. This is much simpler than the volume mesh sensitivity and it often has an analytic relationship. An analytic relationship means there is no need for any finite differences and can be computed very quickly. The relationship between the time taken to calculate the gradient and the number of design variables is now completely negligible.

### Mesh Sensitivity for the RBF Mesh Deformation Method

As seen in section 3.5, the mesh deformation scheme employed in this project is an explicit RBF mesh deformation [20]. As this is an explicit scheme, it enables the gradient to be calculated faster. Two terms need to be found, the sensitivity of the mesh to the surface and the sensitivity of the surface to the design parameters . In this implementation, the CFD surface is approximated by a surrogate CAD model that is defined by the design parameters. This means is defined analytically as seen in section (\*\*\*\*4.3.2). The sensitivity of the volume mesh to the surrogate CAD surface is derived in this section.

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The interpolation coefficients and are different for but the RBF function is the same for each direction.

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The volume points and base points used in the RBF function are the undeformed points defined by the first set of design parameters. When the design parameters are updated, the displacements required to move the undeformed base points to their new position is calculated, this vector is .

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The displacements are from the undeformed base points rather than the previous design iteration, this makes the mesh adjoint easier to implement as the RBF function and scaling factor aren’t dependent on the design variables. If the RBF function were dependent on the design variables, then a matrix of the size would need to be calculated during each gradient equation. The enormous term can be omitted with no loss of mesh quality simply by deforming from the undeformed condition.

This means the only terms in (\*\*\*\*\*\*\*\*\*\*\*4.25\*\*\*\*\*\*\*\*\*\*\*) dependent on the design parameters are and . These are implicitly dependent on the design parameters through the known deformation of the surrogate base points . The result in (\*\*\*\*\*\*\*\*\*4.28\*\*\*\*\*\*\*\*) demonstrates that only the interpolation coefficient’s sensitivity to the displacements need to be found.

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|  |  | (.27) |

The distance from the surface is obviously for all base points. It can be seen from (\*\*\*\*\*\*\*\*\*4.26\*\*\*\*\*\*\*\*\*\*\*) that the value of will be 1 for all of the base points, hence why has disappeared from the interpolation equation. To aid in the derivation of the mesh adjoint, (\*\*\*\*\*\*\*4.27\*\*\*\*\*\*\*\*\*) is put in a matrix format.

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|  |  | (.29) |

The sensitivity of the interpolation coefficients to the surface displacements is the inverse of the matrix in (\*\*\*\*\*\*\*\*\*\*4.29\*\*\*\*\*\*\*\*\*). The first four rows in (\*\*\*\*\*\*\*\*\*4.29\*\*\*\*\*\*\*\*) set conditions to ensure that the interpolation gives a unique result. From a linear algebra perspective, for the matrix to give a unique solution, the matrix must be a square matrix in order to have an inverse.

The sensitivity of the deformed volume mesh points to the interpolation coefficients are found in a similar way.

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(\*\*\*\*\*\*\*Equation 4.30\*\*\*\*\*\*\*) shows that the only the sensitivity of the mesh coordinate displacements to the interpolation coefficients need to be found. It is also another way of revealing that this implementation of RBF mesh deformation is an explicit mesh deformation technique. This term can be seen by rewriting (\*\*\*\*\*\*\*equation 4.25\*\*\*\*\*\*\*) in matrix form.

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In practical applications, the matrix in (\*\*\*\*equation 4.31\*\*\*\*) is too large to be stored. It must be calculated on the fly each iteration which is unfortunate as it doesn’t change from one iteration to another. The smaller matrix in (\*\*\*\*\*equation 4.29\*\*\*\*) can be stored however, as it has been a deliberate choice to keep the number of base points below 5000. When this condition is enforced, the matrix can be inverted in a number of seconds as seen in (\*\*\*\*\*\*\*\*\*\*figure \*\*\*\*\*\*\*\*\*).

=====================================================================

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* Figure of lengths of time to invert matrices from 1000 – 8000\*\*\*\*\*\*\*\*\*\*\*\*\*

=====================================================================

A literature review of mesh deformation strategies concluded that this approach was the best in terms of efficiency, robustness and mesh quality [21]. As seen in this section, it also makes the calculation of the mesh adjoint easier which is another advantage. The only remaining term is the sensitivity of the base points to the design parameters .

### Surface Sensitivity of the Surrogate CAD Model

The parameterisation model used in this research project is the CAD-based surrogate model as defined in the AIAA journal paper [20] by Kamil Bobrowsi. A description of this technique and the motivation for using it have been provided in (\*\*\*\*\*\*PODShaper section in FSI\*\*\*\*\*\*). This section derives the sensitivity of the surface to the design parameters .

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The constants and are calculated by satisfying that the surrogate model produces the exact displacements taken from the samples. This means conditions must be met, these are shown in (\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*section 3.1\*\*\*\*\*\*\*\*\*\*).

The POD modes are a reduced set of constant eigenvectors resulting from the snapshot matrix described in (\*\*\*\*\*\*PODShaper section in FSI\*\*\*\*\*\*). As shown in (\*\*\*\*\*\*\*4.32\*\*\*\*\*\*), only the POD coefficients are functions of the design parameters.

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All the values in (\*\*\*\*4.37\*\*\*\*) have been derived so the surface sensitivity can be calculated. The gradient as defined in (\*\*\*\*\*4.24\*\*\*\*) can now be found.

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All the terms on the RHS of (\*\*\*\*\*4.24\*\*\*\*) can be calculated analytically once the flow adjoint vector has been computed. The computational cost of is similar to the cost of calculating the displacements from a new set of design parameters. This is only weakly dependent on the number of design variables [20] making this mesh adjoint implementation very efficient.

## Aero-Elastic Adjoint Derivation

This purpose of this research project has been to enable fast aerodynamic shape optimisation while considering the structural effects of the aerodynamic loading. Up to this point, only aerodynamics has been considered when deriving the gradient. In an aero-elastic simulation, the mesh coordinates are no longer at a constant position throughout the simulation.

The mesh coordinates are a function of 2 things. First, are the base points which are defined at the start of the simulation. Second, are the structural displacements that change throughout the simulation. It is useful for the purpose the gradient derivation to define to meshes. The jig mesh , that is independent of the structural displacements, and the converged mesh that is a function of the structural displacements

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It is important to be clear on the exact relationship that the displaced mesh has with the displacements as different implementations can result in the aero-elastic adjoint equation being much harder to solve.

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All the displacement values are defined relative to the undeformed coordinates that are defined in the first optimisation iteration. As mentioned in (\*\*\*\*section 4.3.1\*\*\*\*) this is important as it means matrix variables aren’t dependent on the design variables, thus preventing the need to compute three-dimensional matrices when obtaining the gradient.

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is related to the aerodynamic surface displacements through the mesh deformation procedure. Then the aerodynamic surface displacements are related to the structural displacements through the interpolation procedure which is discussed in the next section. The structural displacements are the state variables of the structural solver and their sensitivity to the design variables can’t be found analytically. Therefore, to enable fast gradient-based optimisation, the Lagrangian must be extended to include a term that can eliminate . Just like with the flow adjoint and mesh adjoint, the residual of the structural solver is added to the Lagrangian.

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To reduce clutter in the derivation, the mesh adjoint terms will be omitted.

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A significant change in the gradient equation can be seen in (\*\*\*\*4.51\*\*\*\*), the cost function and aerodynamic residual are now dependent on the converged displaced mesh and not directly on the jig mesh. along with the structural force vector are the variables through which the aerodynamic and structural solvers are coupled. In order to tackle this gradient, each term must be understood and derived. To start this process, each term is expanded so anything which hasn’t yet been derived can be.

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|  |  | (4.56) |
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|  |  | (4.57) |
|  |  | (4.58) |

Substituting (\*\*\*\*\*4.52 to 4.57\*\*\*\*) into (\*\*\*\*4.58\*\*\*\*) produces the gradient equation that the aero-elastic adjoint equations can be formed from. As the displaced mesh has simply been defined as the addition of the jig mesh coordinates and the structural displacements, the jacobian is simply the identity matrix.

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, , and are found in the same way as they were in the regular aerodynamic adjoint equations. is new, but it can also be calculated within the flow solver TAU (\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*insert citation of this term being calculated in TAU\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*).

## Interpolation Derivatives

### CFD to CSM Interpolation

### CSM to CFD Interpolation

## Non-Consistent Mesh Deformation

## Solving the Coupled System

## Trim Correction

### Constant Lift Correction

The gradient equation that was found in equation (\*\*\*\*\*\*\*\*4.18\*\*\*\*\*\*\*\*\*) is true only for the case of a simulation at a constant angle of attack. Typically, an aerodynamic shape optimisation is performed at a design point or multiple design points at specified lift coefficients.

### Constant Pitching Moment Correction

## Validation

# Variable Camber Continuous Trailing Edge

## Theoretical Design

## Parameterisation

# Multipoint Optimisation of the XRF1 at Cruise Condition

# VCCTEF Optimisation of the XRF1 at Off-Design Conditions

# Conclusion

# References

1. The following vector identity for vectors and should be noted.  
    [↑](#footnote-ref-1)