

**Aero-Elastic Optimisation based on the Adjoint Approach**

Aerodynamics Research Group

Department of Mechanical Engineering

University of Sheffield

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**Author: Anthony Stannard**

**Academic Supervisor: Prof. Ning Qin**

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# Introduction

Civil aviation is an industry that has made the world a more accessible place for everyone. People can now travel to locations that would have been impossible only a few generations ago. Climate change is caused by greenhouse gas emissions. The aviation industry is responsible for a significant proportion of global emissions. The EU commissioned report Flight Path 2050 [1] has set a series of targets for the industry to achieve by the year 2050.  
The targets set for industry are to reduce emission levels, relative to emission levels from the year 2000, by the following amounts:

* A 75% reduction in emissions per passenger kilometre
* A 65% reduction in noise emissions
* A 90% reduction of emissions relative to

It is not desirable to cut the levels of flights as this limits the opportunities for global travel to only a few people. This means major advancements in aviation technology must be achieved and implemented quickly. The aircraft industry is relentlessly pursuing more advanced designs to improve efficiency as much as possible. Aircraft optimisation is going to be key in reaching these objectives. Optimisation techniques have reached maturity in the aviation industry over the last decade and have been applied to wide range of application in aircraft design. Strategies aimed at improving the aerodynamic efficiency of an aircraft at transonic speeds have been pursued vigorously in both academia and industry. The studies conducted in [2] and [3] show the first integration of optimisations into the design process for commercial aircraft.

Optimising specific disciplines on their own has already improved aircraft efficiency. Many different authors have demonstrated various aerodynamic optimisation techniques to improve aircraft efficiency [4]–[11]. However, ignoring the strong coupling between all the disciplines that affect aircraft efficiency, will inevitably miss out on further efficiency savings that can be gained. As a wing deforms in flight due to its aerodynamic loading, the flow around it will change and therefore modify the pressure distribution, potentially drastically [12].

A clear way to reduce fuel consumption is to fly lighter aircraft. A very significant percentage of an aircraft’s weight is in its wings so minimising the weight of the wings is clearly desirable. Doing this brings complications with it, a lighter wing will typically be accompanied with lower structural stiffness. This will result in more aero-elastic deformation of the wing, leading to greater differences in the wing’s shape throughout the flight envelope. Two challenges must therefore be confronted when faced with this. First, how to determine the jig shape that will result in an optimised flight shape and second, how to ensure that off-design performance is not considerably poor.

The first challenge can be met in 2 ways. A purely aerodynamic optimisation can be performed at the design point to produce an optimum flight shape. The jig shape that will produce the desired flight shape can be determined for a given flight condition and this will produce an optimised wing shape for the design point. There are two drawbacks from this approach, one there is no guarantee that the required jig shape is feasible and second, a multi-point approach can’t be implemented \*\*\*\*\*\*\*\*\*\*\*\*\*\* CITATION NEEDED\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*. A shape which is optimised only for one point will be very poor away from the design point [13]. This poor off-design performance can be somewhat mitigated by a multi-point approach. The second approach is to perform a coupled aero-elastic optimisation. The shape of the aircraft is still the design parameter but the aero-elastic effect on the jig shape is considered during the optimisation. This means the jig shape can be optimised for directly, which in turn allows the use of a multi-point approach thus going some way to alleviate poor off-design performance.

More still needs to be done to salvage performance further away from the design condition for the following reason. A drawback of a multi-point approach is that it takes away from the drag saving potential at the design point. A low-stiffness wing is going to have such different shapes across the flight envelope that this must be mitigated for through some other means. The approach of this research project to address this is to use the exciting technology known as a variable camber wing. A variable camber wing is able to change its shape in a desirable way across the flight envelope.

The challenge of making a low-stiffness wing maintain reasonable performance all the way through its flight can now be addressed successfully. The optimal variable camber configuration can be determined for a range of design flight conditions, and at each flight condition the wing would deflect its control surfaces to form the shape that would aero-elastically deform into the optimal shape for that condition. To determine how to deflect the control surfaces in the necessary way at each flight condition would require an aero-elastic optimisation with the control surface deflections being the design parameters. This optimisation would be used late on in the design process and therefore, unlike conceptual design optimisations, the highest fidelity software must be used.

# Literature Review

## 

# Fluid-Structure Interaction

The design of an aircraft’s external shape is not just an aerodynamic problem. The shape of a wing affects its aerodynamic loading. Different aerodynamic loading profiles on a wing will displace the wing into different shapes. One way of dealing with aero-elastic behaviour is to design the jig shape so it deforms into its intended design shape in cruise flight [14], [15]. A big advantage of this approach is the decoupling of the aero-elastic system into one aerodynamic problem and one structural problem. This decreases the simulation time significantly, but it isn’t guaranteed to produce a feasible jig shape or even produce the optimum shape [16, p. 9], [17]. Additionally, this approach doesn’t work for multi-point designs which are essential in ensuring that flight conditions that are slightly off-design don’t have too large a performance deficit. As wing’s become less stiff to reduce the aircraft’s weight, the aero-elastic effects are more pronounced. It is therefore important to accurately model the aero-elasticity so that the true optimum shape can be found. The way to achieve this is to simulate the coupled effects that the structural mechanics and aerodynamics have on each other.

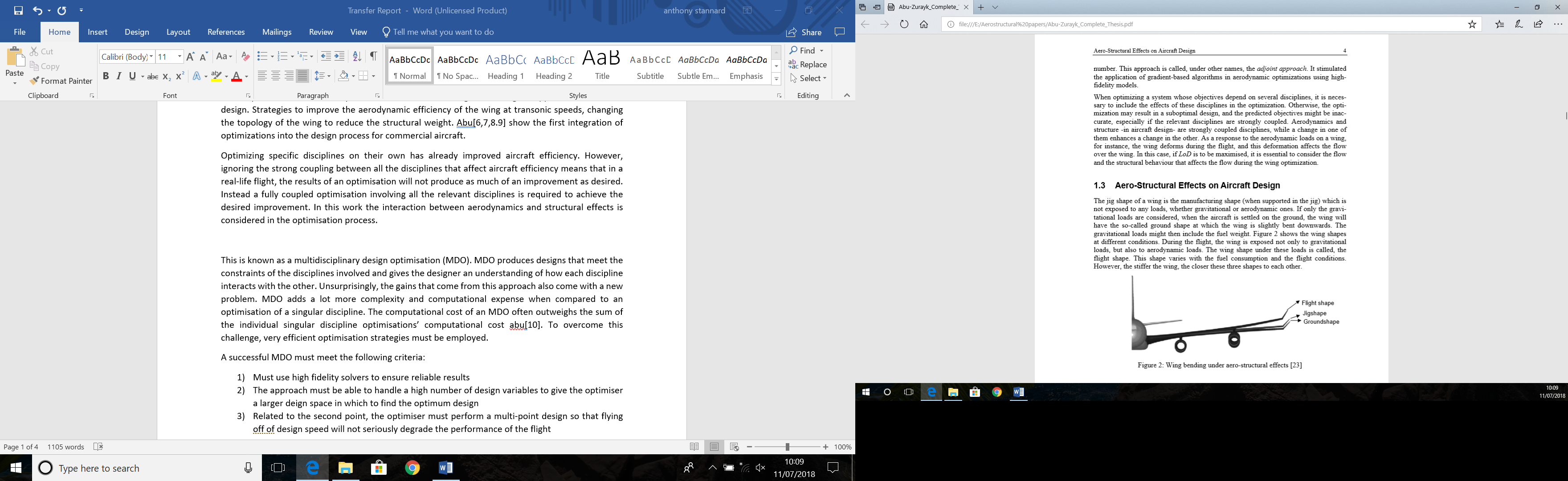


Figure 2: A visual comparison of a wing in its flight shape compared to its jig shape [18]

There are two approaches to solving a coupled aerodynamic and structural problem, either a monolithic or a partitioned approach [19]. A monolithic approach solves the aerodynamic and structural equations at the fluid-structure interface synchronously. The discretised fluid and structural models are enclosed in one system. This approach avoids any time lag between structural movement and fluid flow as the whole system is solved implicitly, including the fluid-structure interaction, which is particularly advantageous for dynamic FSI problems [20].

The partitioned approach is where the aerodynamic and structural equations are solved successively. The information from one discipline is exchanged with the other after it converges and this forms part of the starting conditions for the other discipline. In the static aero-elastic case, this iterative process is performed several times until the exchanged information becomes constant i.e. aero-elastic equilibrium is reached.

A major advantage of the monolithic approach is an increase in the accuracy of the result while affording larger time-steps [21]. On the other hand, the computational cost of the monolithic approach is considerably more expensive than the partitioned approach [21]. There is trade-off to be had then between computational time and accuracy. High-fidelity FSI simulations are expensive even with the partitioned approach and in an optimisation where many simulations are run, this extra time adds up considerably. The objective of this research project is to produce an optimisation tool that can directly optimise a shape while considering aerodynamic and structural effects as efficiently as possible. This is one reason why the partitioned approach was chosen. The other is the advantage of being able to use tried and tested discipline specific adjoint codes, thus substantially lowering the development overhead of the project. Therefore, this project will only consider a partitioned approach. Specifically, DLR’s partitioned approach in FlowSimulator [22] has been used as the basis for this project to be built from.

## Proper Orthogonal Decomposition Parameterisation

There are many approaches that can be taken to model the surface of an aircraft. A simple approach is to use all of the coordinates of the mesh points on the surface as the design parameters. This provides the largest design space and would allow an optimiser to return the largest performance gains. The issue with this approach is that there is no guarantee that the returned optimised shape is manufacturable [23]. Of most interest to this project is a parameterisation scheme that ensures manufacturability constraints are satisfied and one that returns a shape that is immediately useable by a designer. The motivation for this is to aid in producing a more integrated design process.

CAD engines are an important tool in geometry design due to their ability to handle complex geometries while satisfying prescribed geometric constraints. For this reason, a parameterisation scheme that uses CAD parameters in the optimisation loop has been chosen. Using CAD parameterisation in an optimisation loop provides a number of challenges. The biggest challenge is that CAD systems are typically used on the designer’s personal machine while the aerodynamic calculations are performed on high-performance computers in a remote environment, these computers often use different operating systems [24]. To maintain the benefits of CAD parameterisation, the handling of complex geometries and manufacturing constraints while removing the dependence on calls to a CAD engine, a surrogate CAD model is employed as set out by K. Bobrowski [25].

In this approach, a number of CAD design variables are created on the designer’s CAD model of the aircraft. A CAD model contains a number of curves which can be discretised to define a coordinate field. Each setting of the CAD design parameters corresponds to a new set of surface coordinates . The design space is sampled at many different settings according to Design of Experiments techniques [26] that are put together to form a snapshot matrix .

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

Proper Orthogonal Decomposition (POD) is a method which seeks to find interdependencies between the columns in a matrix. An interdependency in this context would be a basis vector of size that captures how the coordinates move across the design space. POD produces a set of optimal basis vectors that, when used in some linear combination with each other, can best approximate the surface coordinate vector across the design space. To obtain the optimal set of basis vectors or modes , an optimisation which minimises the mean square error of an orthogonal projection of the snapshots is performed [27].

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

The operator refers to the orthogonal projection of some vector, in this case the coordinate vector, onto the subspace defined by the span of the basis vectors.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

The solution to the optimisation problem in \*\*\*\*\*\*eqn 3.2\*\*\*\*\*\* is equivalent to solving an eigenvalue problem with the snapshot matrix [27]**,** [28].

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

This eigenvalue problem can be reduced from an problem to an as seen below.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

With the optimal modes found, an equation for the projection of the surface coordinates as a function of the design parameters can be written.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

From now on, the surface coordinates will be defined by the RHS of \*\*\*\*equation 3.10\*\*\*\* so the projection operator is dropped from this point on.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

All of the modes and coefficients are stored in memory. If a large number of samples have been taken, this can be quite substantial. Some modes capture the interdependencies between different design settings than others. These modes are highlighted by having a large corresponding eigenvalue. To reduce storage costs, only the most significant modes are kept.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

In order to make this parameterisation scheme useful, the parameterisation must be continuous across the design space. The CAD surface coordinates at the design parameter settings that were not sampled must be found via interpolation. Specifically, an analytic interpolation equation for the POD coefficients must be found so that the grid coordinates with any design parameters within the design space can be found. The chosen method for this task is RBF interpolation with thin plate spline as defined in equation 3.8.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (.) |

The first expression in \*\*\*\*equation 3.15\*\*\*\*\* is a first order polynomial that is added to the interpolation equation to improve the accuracy of derivative approximations [29]. It should be noted that in \*\*\*\*\*Equation 3.15 \*\*\*\*\* which is used to get a single coefficient, all the sampled design parameters are used. The prerequisite step to having the analytic equation is to calculate the constant vectors and for each mode. This is done by finding the values of the coefficients at the known design settings, so they produce the same surface coordinates. An additional condition for the RBF coefficient, as seen in \*\*\*\*equation 3.16\*\*\*\*, is also added to allow unique interpolation.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

The constants and are then obtained by introducing an interpolation matrix [30].

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |

After the linear system is solved in \*\*\*\*equation 3.21\*\*\*\* to obtain the constants for each mode, \*\*\*equation 3.13\*\*\* can be used to analytically obtain the surface coordinates across the whole design space. Thus, the need for a CAD engine in the loop has been removed whilst maintaining the benefits of a CAD parameterisation.

## RANS Simulation in TAU

\*\*\*\*\*\*\*\*\*\*Explain why field is discrete, how you get R(W(D),X(D))\*\*\*\*\*\*\*\*\*\*\*

## Linear Elastic Analysis in NASTRAN

In the aerodynamic simulations, doing this had no effect on the accuracy of the results. This is because defining the displacements relative to the undeformed coordinates produced a high-quality mesh that represented the exact shape that was being modelled. However, the use of a coupling surface does have an effect on the accuracy of the aero-elastic simulation. This is because the structural analysis is always performed on an undeformed finite element mesh. Therefore, as the newer jig shapes move further away from the initial jig shape, the displacements returned from the structural solver become less accurate. However, as reported by Martins in [\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*], the error introduced by this is not significant and the simulation results are still reliable.

\*\*\*\*\*\*\*\*Same coupling mesh can used at any point in simulation as structural model is linear elastic\*\*\*\*\*\*\*\*\*\*\*\*\*\*

## Interpolation

### CFD to CSM Interpolation

### CSM to CFD Interpolation

## RBF Mesh Deformation

|  |  |  |
| --- | --- | --- |
|  |  | (3.1) |

=================================================

|  |  |  |
| --- | --- | --- |
|  |  | (4.69) |
|  |  | (4.70) |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  | (4.71) |

The stronger non-diagonality brought in by using reduced surface points in the mesh deformation operator is not a problem for the FSI simulation. This is because the extra terms are down the columns rather than along the rows, meaning the diagonality of the FSI matrix is negligibly affected.

## Trim in the Loop

### Control Surface Deployment

### Broyden Algorithm

# Adjoint Approach for Aero-Elastic Optimisation

The focus of this research was to perform high-fidelity optimisations in an efficient way. The only feasible way of performing a fast optimisation for a multi-physics high-fidelity problem is to use gradient-based optimisation, especially when there are a large number of design variables. In aerodynamic applications, often 100s of design variables are required to produce a large enough design space capable of producing a significant improvement in performance [31]. This section focuses on how to obtain the gradient after a completing an aero-elastic simulation. There are two major ways of obtaining a gradient of an objective function with respect to an aircraft’s shape. These are the finite differences (FD) approach and the adjoint method. Finite differences is of course a very old method that is very easy to implement while the adjoint method has been used in aerodynamic optimisation literature since 1988 [32]. This use of the adjoint method for aerodynamic optimisation has matured over the last few decades and it is now widely used within the commercial aviation industry [33].  
The adjoint method has the major advantage of only requiring one evaluation of the objective function. This is essential for any practical high-fidelity gradient-based optimisation. The adjoint method therefore has a negligible dependence on the number of design variables employed, allowing the optimiser to explore a large design space without adding significant computational overhead. Finite differences still have their place in aerodynamic optimisation although mostly for validation purposes.

## Finite Difference

The finite differences approach is the simplest method to obtain the gradient, it is also the simplest method to implement. Finite differences are typically implemented in one of 3 ways, two of the approaches are accurate to a 1st order approximation and these are the forward and backward difference methods. A 2nd order accurate approximation can be obtained by the central difference method. These are derived from the Taylor series and will evaluate the gradient of a one-dimensional function about a point .

|  |  |  |
| --- | --- | --- |
|  |  | (4.1) |
|  |  | (4.2) |
|  |  | (4.3) |

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* Figure showing improved accuracy of Central difference \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

An obvious issue with finite differences is selecting the step size . Mathematically, the true gradient is the value of the above expressions as the step size tends to zero. If the step size is too big then, the error of the approximation will be large. However, if they step size is too small, then subtractive cancellation errors will be large [34]. Cancellation errors occur in computing due to the limited memory numerical data types have available to them. In a CFD simulation, if the values of pressure at each node on the mesh are stored in float data types, they will have 6 digits of precision available to them. If a finite difference approximation was attempted with a step size that caused too small a perturbation in the pressure for it to be accounted for with 6 digits of precision, then the recorded approximation would be garbage information.

For complex applications, such as drag sensitivity to a shape change, a range of step sizes must be investigated to ensure a suitable step size is chosen. This will increase the computational cost of finite differences even further.

\*\*\*\*\*\*\*\*\*\*\*\* Range of Step sizes graphs \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

A suitable step size for a finite difference approximation is found when a range of step sizes are found to produce the same value. Due to the increased accuracy of central differences over the first order methods, it provides a larger range of useable step-sizes as can be seen from the graphs above. To obtain the gradient of an aerodynamic cost function with respect to design parameters through central differences will take flow solutions.

|  |  |  |
| --- | --- | --- |
|  |  | (4.4) |

|  |  |  |
| --- | --- | --- |
|  |  | (4.5) |

|  |  |  |
| --- | --- | --- |
|  |  |  |

An aerodynamic cost function calculated through a RANS simulation is explicitly sensitive to the state variables and nodal coordinates at the surface nodes. The state variables are dependent on the design variables but this relationship cannot be defined explicitly \*\*\*\*citation\*\*\*\*\* hence why RANS is used in the first place. The absence of an explicit relationship is what makes impossible to determine without finite differences.

It is clear the computational cost of finite differences is prohibitive for optimisation purposes, therefore for this research project, its use has been consigned to validation purposes.

## Flow Adjoint

To make multi-disciplinary and even aerodynamic optimisation feasible for an aircraft with more than a few design parameters, the adjoint method must be used. Two simple concepts are employed by the adjoint method to eliminate the dependency on the number of design variables. First, if you add a constant to a function, the gradient at all points will remain unchanged. Second, multiplying anything by zero will result in a product equal to zero.

|  |  |  |
| --- | --- | --- |
|  |  | (4.6) |

In equation (4.4), the cost function has been set to the value of the drag coefficient. In order to find the sensitivity of the cost-function to the design parameters efficiently, the equation will be modified by adding a constant to the right-hand-side (RHS) of (4.4) that is also a function of the design parameters.

|  |  |  |
| --- | --- | --- |
|  |  | (4.7) |
|  | *for all* | (4.8) |

This innocuous looking modification to the equation is actually very powerful. is the residual of the CFD simulation, for a sufficiently converged solution it can be considered to be a vector of zeros for all of the design space. At this point, the value of the adjoint vector is arbitrary as the term will be equal to zero for all of the design space due to being a vector of zeros. This is why the residual must be strongly converged for the adjoint method to work, the derivation depends on it. This allows the adjoint vector to be defined in a very favourable way.

|  |  |  |
| --- | --- | --- |
|  |  | (4.9) |

Both and are equal to zero so it can be seen how the sensitivity of the Lagrangian is equal to the sensitivity of the cost function. The term [[1]](#footnote-1) is equal to zero and cannot be manipulated for any gain so it is dropped from the equation. The adjoint vector will not be constant for all of the design space, instead it will meet the criteria that allows for the elimination of the state variables’ sensitivity to the design parameters . With this term eliminated, the need for multiple simulations to be run is eliminated too as every other term can be calculated from the results of one converged solution.

|  |  |  |
| --- | --- | --- |
|  |  | (4.10) |
|  |  | (4.11) |
|  |  | (4.12) |

The partial derivatives , , and can all be provided by the CFD solver TAU. The partial differentiation of an aerodynamic cost function or flow residual is a challenging task, for the TAU solver this was addressed by Dwight in [35]. Due to the complexities involved in the derivation of these terms they will not be discussed here; it is sufficient to know that they are available. The remaining unknowns are , and . As seen in the derivation, the adjoint vector is arbitrary and thanks to this, the troublesome term can be eliminated while retaining the ability to calculate the gradient.

|  |  |  |
| --- | --- | --- |
|  |  | (4.13) |

Equation (4.13) is known as the adjoint equation. The adjoint vector that satisfies (\*\*\*\*\*\*\*\*\*\*4.13\*\*\*\*\*\*\*\*\*\*\*\*) can be plugged into (4.12) and the gradient equation no longer requires the calculation of .

|  |  |  |
| --- | --- | --- |
|  |  | (4.14) |

There only unknown left to calculate in order to obtain the gradient is the mesh sensitivity to the design parameters .

## Mesh Adjoint

The mesh sensitivity can of course be calculated by finite differences. This isn’t a ridiculous prospect, as calculating mesh deformations is significantly less time consuming that a full RANS simulation.

|  |  |  |
| --- | --- | --- |
|  |  | (4.15) |
|  |  |  |

Obtaining the mesh sensitivity this way through (\*\*\*\*\*\*\*\*\*\*4.15\*\*\*\*\*\*\*\*\*\*\*\*) gets the final unknown term and the gradient can be calculated. However, as mesh sizes get larger, the mesh deformation will still take a significant time to compute. To further improve the efficiency of the gradient calculation, a mesh adjoint can be applied using the same principles as described in the previous flow adjoint section. The mesh adjoint was first used in [36], the addition of a mesh residual provides another mesh sensitivity term which can then be used to remove the mesh sensitivity from the gradient equation. This requires the mesh adjoint vector to meet a criterion that is shown below.

|  |  |  |
| --- | --- | --- |
|  |  | (4.16) |
|  |  | (4.17) |

The mesh residual is defined by the mesh deformation process, I.E the relationship between the surface mesh and the volume mesh.

|  |  |  |
| --- | --- | --- |
|  |  | (4.18) |
|  |  |  |
|  |  | (4.19) |

The relationship is obviously determined by the choice of mesh deformation strategy. An explicit mesh deformation strategy, one where the volume mesh points can be determined by a simple matrix vector product such as Delaunay Graph Mapping, will have the residual defined in (\*\*\*\*\*\*4.18\*\*\*\*\*). An implicit mesh deformation strategy, one which requires the solution of a large linear system such as linear elasticity, will have the residual defined in (\*\*\*\*\*\*\*\*\*\*\*\*4.19\*\*\*\*\*\*\*\*\*\*\*\*\*).

|  |  |  |
| --- | --- | --- |
|  |  | (4.20) |
|  |  |  |
|  |  | (4.21) |
|  |  |  |

It can be seen that the addition of a mesh adjoint had no effect on the flow adjoint equation so that can be carried out in exactly the same way as before to obtain the flow adjoint vector . This leaves the gradient equation in the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (4.22) |

Now the mesh adjoint vector can be chosen so that the mesh sensitivity is multiplied with a vector of zeros, thus eliminating it from the gradient equation.

|  |  |  |
| --- | --- | --- |
|  |  | (4.23) |

Equation (\*\*\*\*\*4.23\*\*\*\*\*) highlights an advantage of an explicit mesh deformation scheme over and implicit one. For an explicit scheme, the jacobian is the identity matrix making the calculation of the mesh adjoint vector an addition problem. An implicit scheme would require solving a large linear system that will naturally take a significant time to do.

|  |  |  |
| --- | --- | --- |
|  |  | (4.24) |

The jacobian is the identity matrix multiplied by minus 1. It does have to be calculated for explicit schemes, however. In this work, the mesh deformation scheme employed is an explicit version of RBF mesh deformation. The mesh adjoint terms for the RBF mesh deformation scheme are explored in section 4.3.1.

The only unknown left in the gradient equation is the surface mesh sensitivity to the design variables. This is much simpler than the volume mesh sensitivity and it often has an analytic relationship. An analytic relationship means there is no need for any finite differences and can be computed very quickly. The relationship between the time taken to calculate the gradient and the number of design variables is now completely negligible.

### Mesh Sensitivity for the RBF Mesh Deformation Method

As seen in section 3.5, the mesh deformation scheme employed in this project is an explicit RBF mesh deformation [25]. As this is an explicit scheme, it enables the gradient to be calculated faster. Two terms need to be found, the sensitivity of the mesh to the surface and the sensitivity of the surface to the design parameters . In this implementation, the CFD surface is approximated by a surrogate CAD model that is defined by the design parameters. This means is defined analytically as seen in section (\*\*\*\*4.3.2). The sensitivity of the volume mesh to the surrogate CAD surface is derived in this section.

|  |  |  |
| --- | --- | --- |
|  |  | (4.25) |

The interpolation coefficients and are different for but the RBF function is the same for each direction.

|  |  |  |
| --- | --- | --- |
|  |  | (4.26) |

The volume points and base points used in the RBF function are the undeformed points defined by the first set of design parameters. When the design parameters are updated, the displacements required to move the undeformed base points to their new position is calculated, this vector is .

|  |  |  |
| --- | --- | --- |
|  |  | (4.27) |
|  |  | (4.28) |

The displacements are from the undeformed base points rather than the previous design iteration, this makes the mesh adjoint easier to implement as the RBF function and scaling factor aren’t dependent on the design variables. If the RBF function were dependent on the design variables, then a matrix of the size would need to be calculated during each gradient equation. The enormous term can be omitted with no loss of mesh quality simply by deforming from the undeformed condition.

This means the only terms in (\*\*\*\*\*\*\*\*\*\*\*4.25\*\*\*\*\*\*\*\*\*\*\*) dependent on the design parameters are and . These are implicitly dependent on the design parameters through the known deformation of the surrogate base points . The result in (\*\*\*\*\*\*\*\*\*4.28\*\*\*\*\*\*\*\*) demonstrates that only the interpolation coefficient’s sensitivity to the displacements need to be found.

|  |  |  |
| --- | --- | --- |
|  |  | (4.29) |

The distance from the surface is obviously for all base points. It can be seen from (\*\*\*\*\*\*\*\*\*4.26\*\*\*\*\*\*\*\*\*\*\*) that the value of will be 1 for all of the base points, hence why has disappeared from the interpolation equation. To aid in the derivation of the mesh adjoint, (\*\*\*\*\*\*\*4.27\*\*\*\*\*\*\*\*\*) is put in a matrix format.

|  |  |  |
| --- | --- | --- |
|  |  | (4.30) |

The sensitivity of the interpolation coefficients to the surface displacements is the inverse of the matrix in (\*\*\*\*\*\*\*\*\*\*4.29\*\*\*\*\*\*\*\*\*). The first four rows in (\*\*\*\*\*\*\*\*\*4.29\*\*\*\*\*\*\*\*) set conditions to ensure that the interpolation gives a unique result. From a linear algebra perspective, for the matrix to give a unique solution, the matrix must be a square matrix in order to have an inverse.

The sensitivity of the deformed volume mesh points to the interpolation coefficients are found in a similar way.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

(\*\*\*\*\*\*\*Equation 4.30\*\*\*\*\*\*\*) shows that the only the sensitivity of the mesh coordinate displacements to the interpolation coefficients need to be found. It is also another way of revealing that this implementation of RBF mesh deformation is an explicit mesh deformation technique. This term can be seen by rewriting (\*\*\*\*\*\*\*equation 4.25\*\*\*\*\*\*\*) in matrix form.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

In practical applications, the matrix in (\*\*\*\*equation 4.31\*\*\*\*) is too large to be stored. It must be calculated on the fly each iteration which is unfortunate as it doesn’t change from one iteration to another. The smaller matrix in (\*\*\*\*\*equation 4.29\*\*\*\*) can be stored however, as it has been a deliberate choice to keep the number of base points below 5000. When this condition is enforced, the matrix can be inverted in a number of seconds as seen in (\*\*\*\*\*\*\*\*\*\*figure \*\*\*\*\*\*\*\*\*).

=====================================================================

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* Figure of lengths of time to invert matrices from 1000 – 8000\*\*\*\*\*\*\*\*\*\*\*\*\*

=====================================================================

A literature review of mesh deformation strategies concluded that this approach was the best in terms of efficiency, robustness and mesh quality [37]. As seen in this section, it also makes the calculation of the mesh adjoint easier which is another advantage. The only remaining term is the sensitivity of the base points to the design parameters .

### Surface Sensitivity of the Surrogate CAD Model

The parameterisation model used in this research project is the CAD-based surrogate model as defined in the AIAA journal paper [25] by Kamil Bobrowsi. A description of this technique and the motivation for using it have been provided in (\*\*\*\*\*\*PODShaper section in FSI\*\*\*\*\*\*). This section derives the sensitivity of the surface to the design parameters .

|  |  |  |
| --- | --- | --- |
|  |  | (4.34) |
|  |  | (4.35) |

|  |  |  |
| --- | --- | --- |
|  |  | (4.36) |

The constants and are calculated by satisfying that the surrogate model produces the exact displacements taken from the samples. This means conditions must be met, these are shown in (\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*section 3.1\*\*\*\*\*\*\*\*\*\*).

The POD modes are a reduced set of constant eigenvectors resulting from the snapshot matrix described in (\*\*\*\*\*\*PODShaper section in FSI\*\*\*\*\*\*). As shown in (\*\*\*\*\*\*\*4.32\*\*\*\*\*\*), only the POD coefficients are functions of the design parameters.

|  |  |  |
| --- | --- | --- |
|  |  | (4.37) |
|  |  |  |
|  |  | (4.38) |
|  |  |  |
|  |  | (4.39) |
|  |  |  |
|  |  | (4.40) |

All the values in (\*\*\*\*4.37\*\*\*\*) have been derived so the surface sensitivity can be calculated. The gradient as defined in (\*\*\*\*\*4.24\*\*\*\*) can now be found.

|  |  |  |
| --- | --- | --- |
|  |  | (4.41) |

All the terms on the RHS of (\*\*\*\*\*4.24\*\*\*\*) can be calculated analytically once the flow adjoint vector has been computed. The computational cost of is similar to the cost of calculating the displacements from a new set of design parameters. This is only weakly dependent on the number of design variables [25] making this mesh adjoint implementation very efficient.

## Aero-Elastic Adjoint Derivation

This purpose of this research project has been to enable fast aerodynamic shape optimisation while considering the structural effects of the aerodynamic loading. Up to this point, only aerodynamics has been considered when deriving the gradient. In an aero-elastic simulation, the mesh coordinates are no longer at a constant position throughout the simulation.

The mesh coordinates are a function of 2 things. First, are the base points which are defined at the start of the simulation. Second, are the structural displacements that change throughout the simulation. It is useful for the purpose the gradient derivation to define to meshes. The jig mesh , that is independent of the structural displacements, and the converged mesh that is a function of the structural displacements

|  |  |  |
| --- | --- | --- |
|  |  | (4.42) |
|  |  | (4.43) |

It is important to be clear on the exact relationship that the displaced mesh has with the displacements as different implementations can result in the aero-elastic adjoint equation being much harder to solve.

|  |  |  |
| --- | --- | --- |
|  |  | (4.44) |
|  |  | (4.45) |

All the displacement values are defined relative to the undeformed coordinates that are defined in the first optimisation iteration. As mentioned in (\*\*\*\*section 4.3.1\*\*\*\*) this is important as it means matrix variables aren’t dependent on the design variables, thus preventing the need to compute three-dimensional matrices when obtaining the gradient.

|  |  |  |
| --- | --- | --- |
|  |  | (4.46) |
|  |  | (4.47) |
|  |  | (4.48) |

is related to the aerodynamic surface displacements through the mesh deformation procedure. Then the aerodynamic surface displacements are related to the structural displacements through the interpolation procedure which is discussed in the next section. The structural displacements are the state variables of the structural solver and their sensitivity to the design variables can’t be found analytically. Therefore, to enable fast gradient-based optimisation, the Lagrangian must be extended to include a term that can eliminate . Just like with the flow adjoint and mesh adjoint, the residual of the structural solver is added to the Lagrangian.

|  |  |  |
| --- | --- | --- |
|  |  | (4.49) |
|  |  | (4.50) |

To reduce clutter in the derivation, the mesh adjoint terms will be omitted.

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | (4.51) |
|  | |  | (4.52) |
|  |  | | (4.53) |

A significant change in the gradient equation can be seen in (\*\*\*\*4.51\*\*\*\*), the cost function and aerodynamic residual are now dependent on the converged displaced mesh and not directly on the jig mesh. along with the structural force vector are the variables through which the aerodynamic and structural solvers are coupled. In order to tackle this gradient, each term must be understood and derived. To start this process, each term is expanded so anything which hasn’t yet been derived can be.

|  |  |  |
| --- | --- | --- |
|  |  | (4.54) |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  | (4.55) |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  | (4.56) |
|  |  |  |
|  |  | (4.57) |

|  |  |  |
| --- | --- | --- |
|  |  | (4.58) |
|  |  |  |
|  |  | (4.59) |
|  |  | (4.60) |

Substituting (\*\*\*\*\*4.54 to 4.59\*\*\*\*) into (\*\*\*\*4.60\*\*\*\*) produces the gradient equation that the aero-elastic adjoint equations can be formed from. As the displaced mesh has simply been defined as the addition of the jig mesh coordinates and the structural displacements, the jacobian is simply the identity matrix.

|  |  |  |
| --- | --- | --- |
|  |  | (4.61) |

, , and are found in the same way as they were in the regular aerodynamic adjoint equations. and are new, but they can also be calculated within the flow solver TAU [38]. This leaves the following terms; , and .

is very similar to the term that was calculated in section 4.3.1. It uses exactly the same interpolation method, RBF mesh deformation with reduced surface points. However, there is one subtle difference to do with the surface points. The surface points used in the jig shape calculation aren’t actually on the CFD mesh, they are points defined on the surrogate CAD model surface. Whereas, in the interpolation of the structural displacements to the volume mesh, there is an intermediary step of calculating all the displacements on the surface of the actual CFD mesh. A subset of these displacements is then interpolated to the volume mesh. This means the interpolation coefficients used in the mesh deformation of the structural displacements are different from those used in the jig shape calculation. However, as will be explained in (\*\*\*section 4.5\*\*\*), the mesh deformation term used in the coupled-adjoint is not consistent with the actual mesh deformation used in the simulation.

## Non-Consistent Mesh Deformation

In the derivation of the gradient, the sensitivity of the volume displacements to the surface displacements is assumed to be the mesh deformation operation used in the fluid-structure interaction simulation. This ensures the calculated gradient is exact. A convergence issue has been found with this approach when it is combined with a mesh deformation strategy that only uses some of the surface points.

|  |  |  |
| --- | --- | --- |
|  |  | (4.62) |

In the FSI simulation, a subset of displacements on the surface which adequately define the displacement profile are interpolated to the volume mesh. Displacements over a wing don’t require as high a fidelity as aerodynamic terms in order to be sufficiently captured. In (\*\*\*\*equation 4.62\*\*\*) the vector that undergoes the transpose of the mesh deformation operation is a force derived sensitivity. This poses a problem for the convergence of the coupled adjoint if the mesh deformation operation uses a subset of the surface points.

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Convergence of RBF with reduced surface points \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

The problem from a linear algebra perspective is that the coupled adjoint matrix becomes less diagonally dominant with each omitted surface point. In this project the coupled adjoint is solved using a lagged Gauss-Seidel approach which is only guaranteed to converge if the matrix is strictly diagonally dominant [39]. A matrix is diagonally dominant if the absolute value of all the diagonal elements are greater than the sum of the absolute values of the non-diagonal elements in their respective rows.

|  |  |  |
| --- | --- | --- |
|  |  | (4.63) |

This was not a problem for the FSI simulation for the reason discussed in \*\*\*\*\*section 3.5\*\*\*\*\*, the extra terms that occurred in the matrix were down the columns rather than along the rows. It therefore had a negligible effect on the diagonality of the FSI matrix. The coupled-adjoint matrix is effectively the transpose of the FSI matrix thus making the extra terms a problem.

|  |  |  |
| --- | --- | --- |
|  |  | (4.64) |

The structural force vector is the one defined from the structural residual meaning it is only a function of through the aerodynamic force. It is not equal to the stiffness matrix **.**

|  |  |  |
| --- | --- | --- |
|  |  | (4.65) |

In fact, is a small term that captures the effect of a change in the CFD surface normals on the aerodynamic force and therefore the interpolated structural force. This means the matrix effectively still has the same diagonality as the stiffness matrix. The mesh deformation operator is only prevalent in the bottom block row of (\*\*\*\*4.64\*\*\*\*).

|  |  |  |
| --- | --- | --- |
|  |  | (4.66) |

To make the analysis of the mesh deformation clearer, a simplification is made about the interpolation in order to isolate the mesh deformation effects. The simplification is to assume that the CSM and CFD surface meshes align perfectly thus making the force and displacement transfer a simple one for one transfer between corresponding nodes.

|  |  |  |
| --- | --- | --- |
|  |  | (4.67) |
|  |  |  |
|  |  | (4.68) |

All the terms in the matrix are off-diagonal in the coupled-adjoint matrix. This means for any given row in , the larger the sum of the row’s absolute values, the less likely the coupled-adjoint is to converge. When using a mesh deformation operation with reduced points, some rows in become all zeros while others become denser. Specifically, the rows related to the subset of surface points used in the mesh deformation become denser.

This reduces the diagonality of the coupled-adjoint matrix but the matrix has strong diagonality. Therefore, the reduction in diagonality only becomes a problem if many elements in are large. For the element in to be large, the volume node must be close to the surface node. Similarly, the sensitivity of the flow residual vector to the grid coordinates in its stencil must be large too. The term is complex even for a 2D Euler case [40]. However, some things can be asserted qualitatively. The value will be larger when the air speed is higher simply because the change in flux over the affected control volume face will be larger thus changing the residual by more. Secondly, the presence of a shock switches on the scalar dissipation in TAU which uses an unstructured generalisation of the JST scheme [41], [42]. The scalar dissipation is also a function of the grid points meaning that the presence of shock waves on the surface are a source of non-diagonality in the coupled-adjoint matrix.

\*\*\*\*Comparison of gradients with RBF at flow condition without shockwaves\*\*\*\*

To significantly increase the chance of convergence for the coupled-adjoint, a different mesh deformation strategy is assumed to have been used. The assumed mesh deformation strategy is one that uses all the surface points, thus removing all the extra off-diagonal terms.

For the assumed mesh deformation strategy to be an adequate substitution, it must produce similar mesh deformations to those produced by the RBF method. As stated in the previous paragraph, it must use all the surface points and it should also be fast. Two mesh deformation strategies were considered due to their use of all the surface points and fast calculation times. These strategies were Delaunay Graph Mapping \*\*\*\*\*DGM citation\*\*\*\*\* and inverse distance weighting \*\*\*\*IDW citation\*\*\*\*.

### Delaunay Graph Mapping

Delaunay Graph Mapping starts by subdividing the mesh via Delaunay triangulation. Delaunay triangulation is a method of connecting a cloud of points in a unique way by ensuring all joined points meet the Delaunay criteria. This criteria states that “The circumcircle of each triangle must not include other points outside of those constructing the triangle” [43]. This concept is extended to 3D by creating tetrahedra elements and ensuring the elements’ circumsphere doesn’t include points other than those in the element.

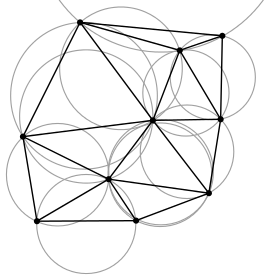


Figure \*\*\*\*\*\*\*\*\*\*: No vertex is within the circumcircle of another triangle [44].

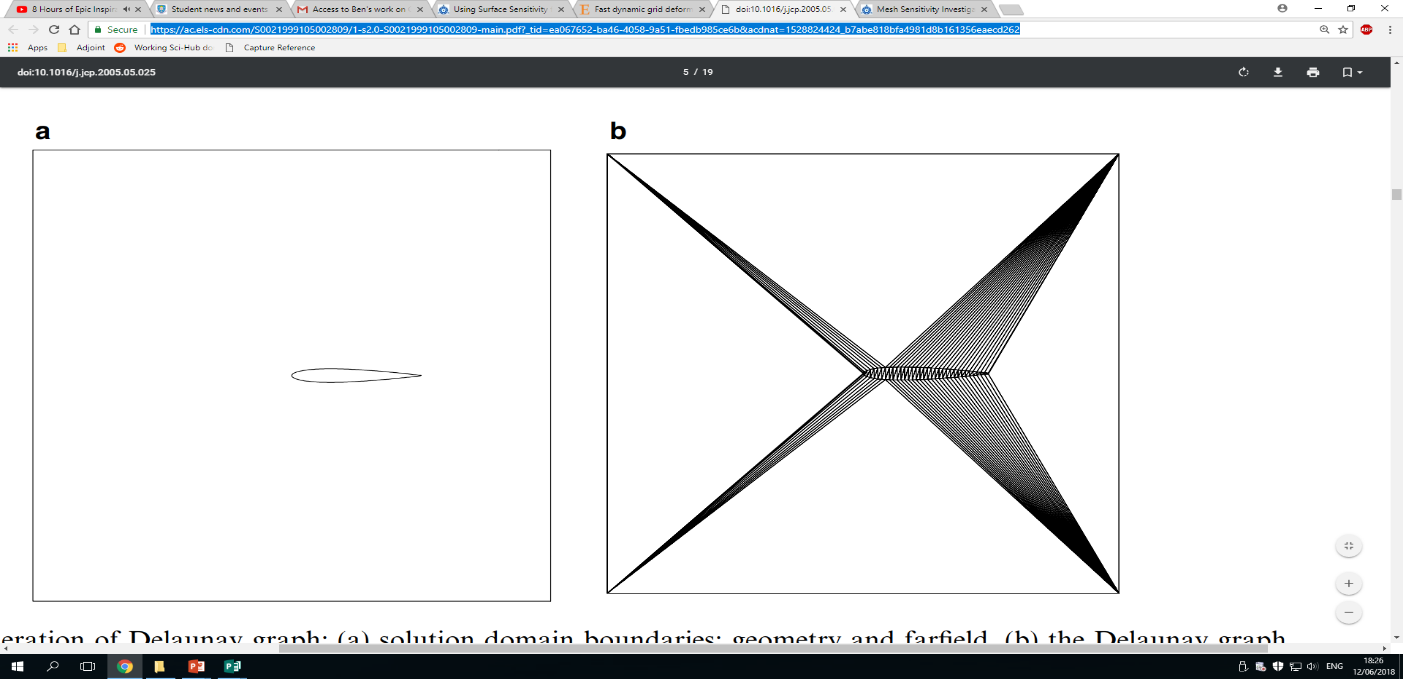


Figure 11: A Delaunay triangle subdivision of a mesh where only the corner nodes of the farfield were used [45].

The Delaunay triangulation is conducted on all the surface points and some farfield points so that all mesh points are enclosed within a subdomain. With each mesh node within a Delaunay element, an explicit relationship between the node and the nodes that make up the element can be made. Figure \*\*\*\*\*\*12\*\*\*\*\*\* shows a mesh node within its Delaunay element .

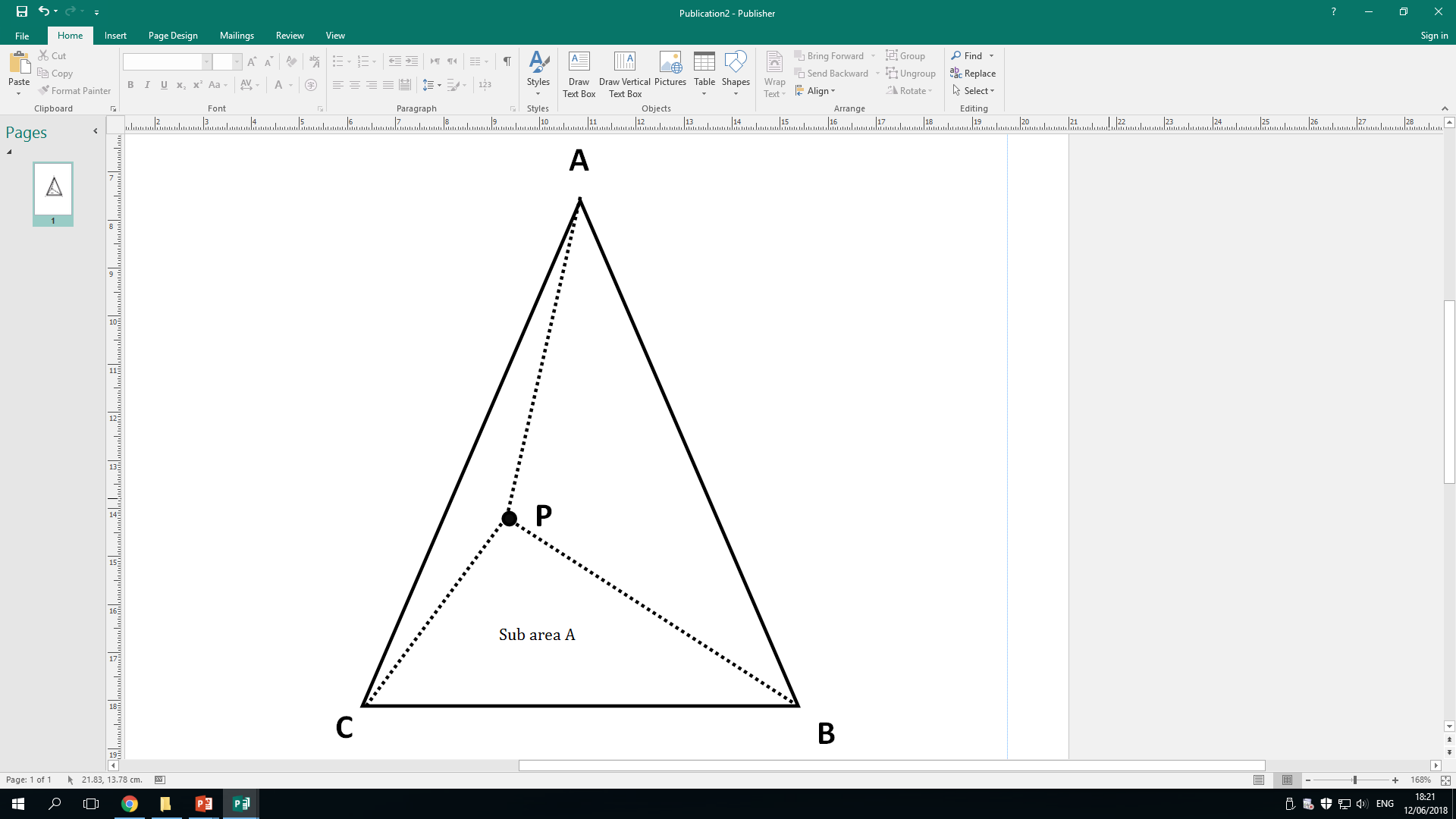


Figure 12: A mesh node within its Delaunay element.

In 2D the mesh node is related to the Delaunay element vertices via area ratios, in 3D it is related via volume ratios. The area ratios are the ratio of the sub-area and the total element area .

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

Using \*\*\*\*figure 12\*\*\*\* as a reference, sub-area refers to the triangle . and can be replaced by the notation , and respectively. Now the mesh location can be specified in terms of the Delaunay vertex locations.

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| --- | --- | --- |
|  |  | (.) |

To have the best chance of converging the coupled adjoint, every node on the surface must be designated as a Delaunay vertex. If the method were used to propagate a surface deformation to the volume mesh, the Delaunay vertices would be updated directly and the new volume mesh points would be calculated using equation \*\*\*4.70\*\*\*. However, this method is not intended to be used for a volume mesh deformation as the RBF method works very well. Instead, this method is used as a substitute during the coupled adjoint when performing a transpose mesh deformation operation. In matrix form this method is sparse, only 4 non-zero values occur per row meaning it can be stored in memory making it fast and easy to implement.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

is made up of both surface nodes and farfield nodes so only a section of the Delaunay deformation matrix is needed.

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|  |  | (.) |
|  |  |  |
|  |  | (.) |

The matrix can be plugged directly into the coupled-adjoint equation. As is shown in \*\*\*figure 14\*\*\* below, the coupled-adjoint now converges.

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Comparison of coupled-adjoint RBF and Delaunay convergence\*\*\*\*\*\*\*\*\*

**Simple Wing Optimisation with Euler Equations**

This method was used in an aero-elastic coupled-adjoint optimisation of a poorly designed wing with good effect. A linear elastic structural model was used along with the Euler equations to capture the aerodynamics. The optimisation problem was defined as below:

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*LANN-wing optimisation Mach 0.73\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*.

This wing was not well optimised beforehand so lots of drag-reduction gains could be obtained.

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Image of shock wave before\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

The figure above shows a large shock wave along half the span at the leading edge which is causing massive wave-drag. The optimiser simply reduced this and thus reduced the drag enormously.

### Inverse Distance Weighting

The convergence problem was solved easily by using the Delaunay method and it was even used effectively in an optimisation of a poorly designed wing. However, due to the large optimisation gains to be had, the gradient didn’t have to be too accurate to be able to produce good results.

\*\*\*\*\*\*\*\*\*\*\*\*\*\* Convergence of RBF and Delaunay at Mach 0.5\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

The convergence of the Delaunay method was obtained by improving the diagonality of the coupled-adjoint matrix but in doing so made the values of the coupled-adjoint matrix significantly different. To maintain the benefit of using the RBF mesh deformation with reduced points and still be able to obtain a very accurate gradient, the inverse distance weighting method was implemented. It deforms the mesh in a similar way to RBF in that each surface point effects every volume point. The deformation is also similar in that the mesh movement is explicitly dependant on the radial distance from the surface points. It has also been shown to have a similar accuracy to RBF mesh deformation [46].

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

An advantage of IDW is that it doesn’t need to be solved iteratively meaning just like the Delaunay method, it can be substituted right into the coupled-adjoint equation. One disadvantage it has when compared to the Delaunay method is that it can’t be stored explicitly. It has rows and columns in its interpolation matrix which for high fidelity full aircraft meshes will be too large to store. In \*\*\*\*equation 4.75\*\*\*\* the exponent value of 3 was selected as it produced comparable results with aero-elastic simulations which used the RBF approach.

\*\*\*\*\*\*\*\*\*\*\*Mesh comparisons with different exponents, maybe use Alistairs\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

No floating point exponents were examined as it would have made the interpolation significantly more computationally expensive. The question then arises, why not just use IDW in both the FSI and coupled-adjoint procedures. The simple reason is the RBF procedure is known to be robust \*\*\*\*\*\*\*\*\*\*\*\*Add citation\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* so it could be trusted to maintain a good mesh quality throughout the optimisation.

\*\*\*\*\*\*\*\*\*\*\*Convergence of IDW\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

It can be seen that just like the Delaunay coupled-adjoint, the IDW approach converges too.

\*\*\*\*\*\*\*\*\*Comparison of RBF and IDW gradient at Mach 0.5\*\*\*\*\*\*\*\*\*\*\*\*

\*\*\*\*\*\*\*Figure 132\*\*\*\*\* demonstrates the major benefit IDW has over Delaunay in this context, it more closely matches the gradient produced by RBF approach. As it can more accurately produce the gradient, it will be able to squeeze out more gains than the Delaunay approach as the optimizer approaches the local minimum.

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*LANN-wing optimisation IDW Delaunay comparison\*\*\*\*\*\*\*\*\*\*\*

In the case of the LANN-wing, both approaches significantly improve the aerodynamic performance of the wing. The IDW approach however is able to keep decreasing the drag for a few more iterations than the Delaunay approach.

## Interpolation Derivatives

The two remaining terms that haven’t been discussed, and , are known as cross-derivatives. They represent the sensitivity between vectors from two separate disciplines. In this case, one vector is from aerodynamics and the other is from structural mechanics. They are used to interpolate the information from one discipline to the other in the FSI simulation.

It would be possible to use entirely separate interpolation processes in each direction. For instance, a simple nearest neighbour interpolation could be used to calculate the forces on the structural mesh. While RBF interpolation could be used to calculate the displacements on the aerodynamic mesh. However, there is an implementation benefit to having the interpolation process in one direction be the transpose of the process in the other.

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| --- | --- | --- |
|  |  | (.) |
|  |  |  |
|  |  | (.) |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

The implementation benefit being that no new code needs to be written for the coupled-adjoint when performing the transpose interpolation process. This isn’t ground-breaking but it makes life easier for the developer.

The interpolation technique that is used in both directions then must be suitable for both. As described earlier in \*\*\*section 3.\*\*\*\*\*, RBF with thin plate spline is the best approach. The approach employed allows all the structural coupling surface nodes to be used which increases the convergence properties of the coupled-adjoint matrix. The reason for this improved convergence is the same as the reason for the improved convergence that comes from using a mesh deformation strategy that uses all the aerodynamic surface points.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

Some rows in the off-diagonal matrix become all zero while the rest become more significant.  
\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Maybe include some maths showing the change in interpolation coefficients results in more significant elements in the rows where the values are used when there are omitted elements. Also show that this isn’t the case for the fuselage where it always will be zero as it’s a separate interpolation family and all its values are fixed.\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

## Solving the Coupled System

This project solved the coupled-adjoint equations in a similar to way in which the FSI equations were solved. Rather than create a single linear system of equations to be solved, a lagged iteration scheme is used. The major benefit of this approach is that the discipline specific solvers can be reused with only minor modifications.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

In the case of the flow-adjoint, the right-hand side of the equation must be modified by the interpolated structural term. The modification is a simple addition. In the case of the structural adjoint, the gravity switch needs to be turned off. This is because the equation in \*\*4.81\*\* has no reference to the gravity force like the structural problem in the FSI simulation has. If the structural coupling surface wasn’t constant throughout the optimization, then the gravitational forces sensitivity to the design parameters would have to be accounted for. The stiffness matrix for a linear-elastic simulation is symmetric, meaning the structural solver can be used directly to obtain the structural adjoint vector by supplying the RHS of \*\*\*4.83\*\*\* as the force.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  |  |
|  |  | (.) |

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

1. The solution is initialised for
2. Equation (\*\*\*4.82\*\*\*) is solved to find
3. Equation (\*\*\*4.83\*\*\*) is solved to find
4. Equation (\*\*\*4.83\*\*\*) is solved to find the gradient evaluation.
5. Steps 2, 3 and 4 are cycled through until the gradient converges.

The convergence criteria selected in this project is that the difference between the L2 norm of the gradient of the last iteration must be under \*\*\*\*5%\*\*\*\* of the previous iteration’s L2 norm. The jacobians and as seen in \*\*\*4.84\*\*\* are exactly the same in this aero-elastic adjoint as they would be in a regular aerodynamic adjoint evaluation. This is because the mesh residual is independent of the structural displacements.

## Trim Correction

The gradient equation that was found in equation (\*\*\*\*\*\*\*\*4.18\*\*\*\*\*\*\*\*\*) is true only for the case of a simulation at a constant angle of attack. The FSI simulations in this research project have been trimmed in the loop. As discussed in \*\*\*section 3.4\*\*\*, this means there are 2 implicit trim parameters which are adjusted during the FSI simulation to ensure that the final result occurs at trim. Trim means that the target lift-coefficient is reached while the pitching moment is zero. The trim parameters are the deflection angle of the horizontal tail and the angle of attack . The true dependencies of the terms in the drag gradient are defined below:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |

These extra terms obviously make a difference to value of the gradient.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

A useful observation can be made about \*\*\*eqn 4.88\*\*\*, the term that multiplies is equal to zero. Also, as and are 1-dimensional, the terms that multiply it must also be 1-dimensional. The values and are already known as they have been calculated while finding . The unknown terms in the brackets are , , and . The terms , and are available from the TAU solver so they do not need to be derived. The sensitivity of the mesh to the horizontal tail deflection angle does need to be derived.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

The only new term here is .

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Deflection angle derivation\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*.

Now all the partial derivative terms have been found, a way to deal with the trim parameters sensitivity to the design parameters is also needed. This can be done without resorting to finite differences. Instead, the fact that the gradients of both the pitching moment and lift coefficient are zero is taken advantage of.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

A similar gradient equation can be formed for the pitching moment.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

By performing the coupled-adjoint for both the lift gradient and pitching moment gradient at constant alpha to obtain and , a simultaneous equation can be setup to obtain and .

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |

Every term in the true drag gradient is now known and the gradient can be found. It is worth reiterating at this point the benefit of trimming within the FSI loop. This procedure allows the use of an unconstrained optimisation algorithm. This enables the local minimum to be found faster by the optimiser \*\*\*\*\*citation to Andrei’s paper\*\*\*\*\* and it ensures that each design returned by the optimiser is feasible. This ties in with the use of the surrogate CAD model parameterisation, no volume constraints need to be applied as that is done within the CAD engine while maintaining feasible design shapes. This optimisation tool can be more directly integrated into the design process due to these features, making the added complexity of the optimisation process worth it.

## Validation

# Variable Camber Continuous Trailing Edge

## Theoretical Design

## Parameterisation

# Multipoint Optimisation of the XRF1 at Cruise Condition

# VCCTEF Optimisation of the XRF1 at Off-Design Conditions

# Conclusion

# References

1. The following vector identity for vectors and should be noted.  
    [↑](#footnote-ref-1)