ECE 450 Exam 3

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Problem Statement

For this exam, a filter with the following specifications and lowest order must be found. The requirements are as follows:

$$\omega_p \geq 400\frac{rad}{s} \quad \omega_s \leq 200\frac{rad}{s}$$
$$|H_p| \geq 0.9 \quad |H_s| \leq 0.2$$

Filter Selection

To help choose which filter should be used, we eliminated Chebychev I immediately, as it is non monotonic in the passband. The options left are butterworth and Chebychev II. Using the programs to find the minimum order required, the lowest orders found were:

$$Butterworth = 5$$

$$Chebychev\ II = 3$$

Given these results, the Chebychev II filter was chosen.

Low Pass Base

To start the filter design, a low pass filter with corner frequency of 1 rad/s is made. The design starts with finding ε, α, and the major and minor axes of the root ellipse.

$$\varepsilon = \sqrt{\frac{H_s^2}{1-H_s^2}} = 0.100$$

$$\alpha = \frac{1}{\varepsilon} + \sqrt{1+\frac{1}{\varepsilon^2}} = 19.95$$
$$a = \frac{1}{2}\left(\alpha^{\frac{1}{n}} - \alpha^{-\frac{1}{n}}\right) = 1.17$$
$$b = \frac{1}{2}\left(\alpha^{\frac{1}{n}} + \alpha^{-\frac{1}{n}}\right) = 1.54$$

The next step is to find the poles. There will be a single pole on the axis and a double pole at 180° - 60°. The poles are:

$$s_1 = a$$

$$s_{2,3} = acos(\theta_i) + bjsin(\theta_i)$$

As this is the inverse Chebychev filter, the poles need to be inverted:

$$q_1 = \frac{1}{a}$$

$$q_{2,3} = \frac{1}{acos(\theta_i) + bjsin(\theta_i)}$$

For the numerator, the zeros are needed. As there is only one set of poles not on the axis, we need only find the one:

$$\omega_k = sec\left(\frac{\pi}{6}\right) = 1.15$$

Once the code has run its course, we are left with a transfer function polynomial of:

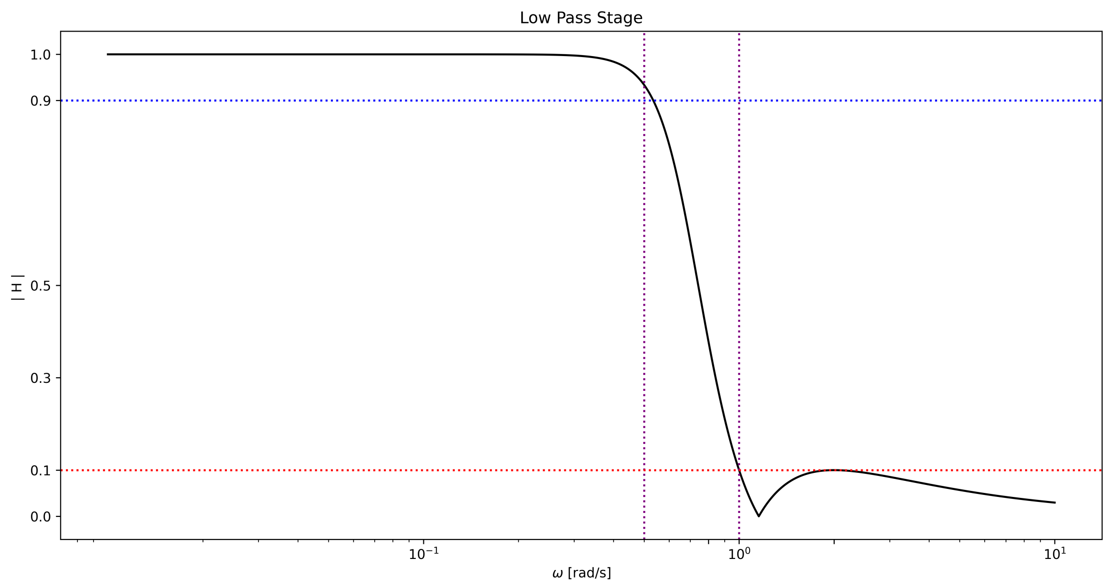
$$H(s) = K\frac{s^2 + 1.15^2}{s^3 + 0.3s^2 - 2.7 * 10^{-16}s + .4}$$

K is found when s = 0:

$$K = \frac{0.4}{0.15^2} = 0.35$$

When we graph this function, we get:

Figure 1: Low Pass Base Centered at 1 rad/sec



High Pass Conversion

To convert the graph to a high pass filter, we replace each s with $\frac{1}{s}$ and that yields:

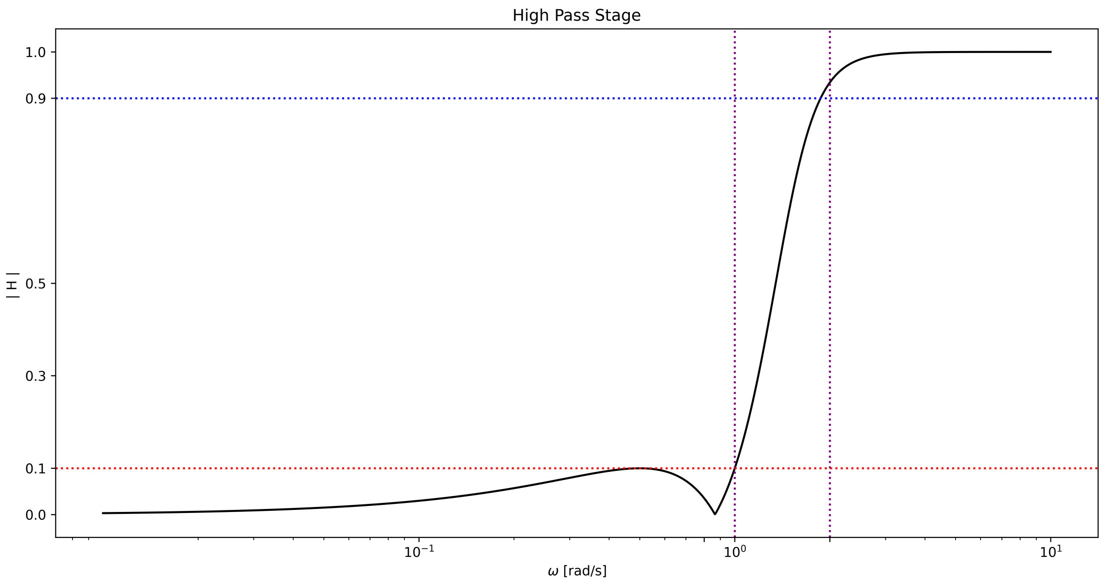
$$H(0) = \left(\frac{\left(\frac{1}{s}\right)^2 + 1.15^2}{\left(\frac{1}{s}\right)^3 + 0.3\left(\frac{1}{s}\right)^2 - 2.7 * 10^{-16}\left(\frac{1}{s}\right) + 0.4} \right) = K \frac{s + 1.15^2 s^3}{0.4s^3 - 2.7 * 10^{-16} s^2 + 0.3s + 1}$$

Normalizing that polynomial yields assuming K = 1 to facilitate the passband going to 1:

$$\frac{s^3 + 0.75s}{s^3 - 6.9 * 10^{-16} s^2 + 0.75s + 2.48}$$

If we graph this we can see that it is a flip of the low pass filter around the ω_s value at 1 rad/sec:

Figure 2: High Pass Filter Centered at 1 rad/sec



Frequency Scale

Now that the high pass filter has been found, simply scaling the filter by 200 rad/s will shift the corner frequency to our desired criterion:

$$H(\frac{s}{\omega}) = \frac{s^3 + 30000s}{s^3 - 1.38 * 10^{-13} s^2 + 30000s + 20 * 10^6}$$

Graphing this will show how the function retains its shape and has moved to meet the requirements of the problem statement: If we graph this we can see that it is a flip of the low pass filter around the ω_s value at 1 rad/sec:

Figure 3: High Pass Filter Centered at 200 rad/sec

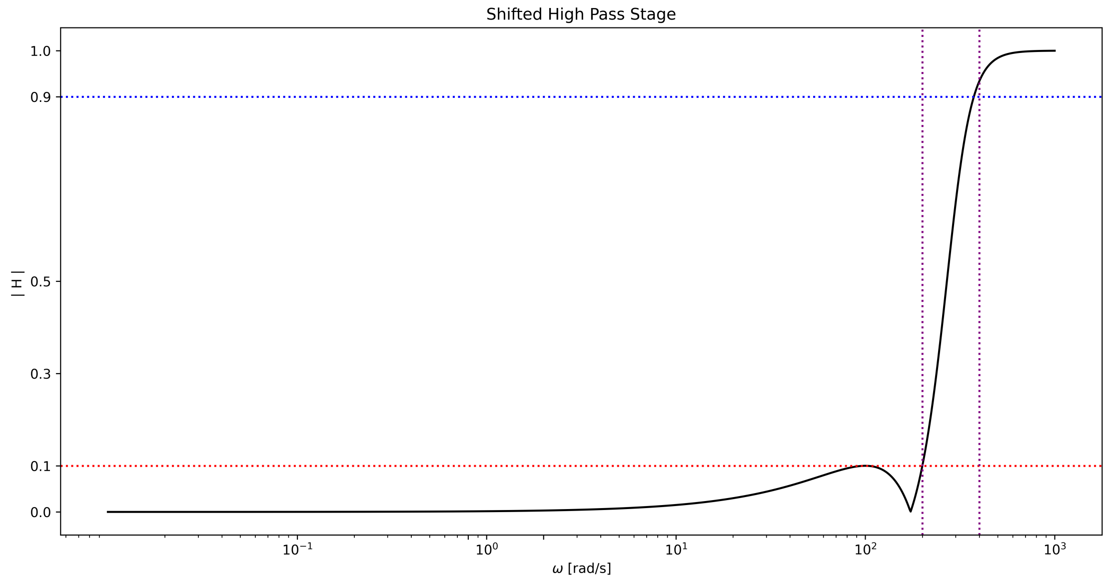
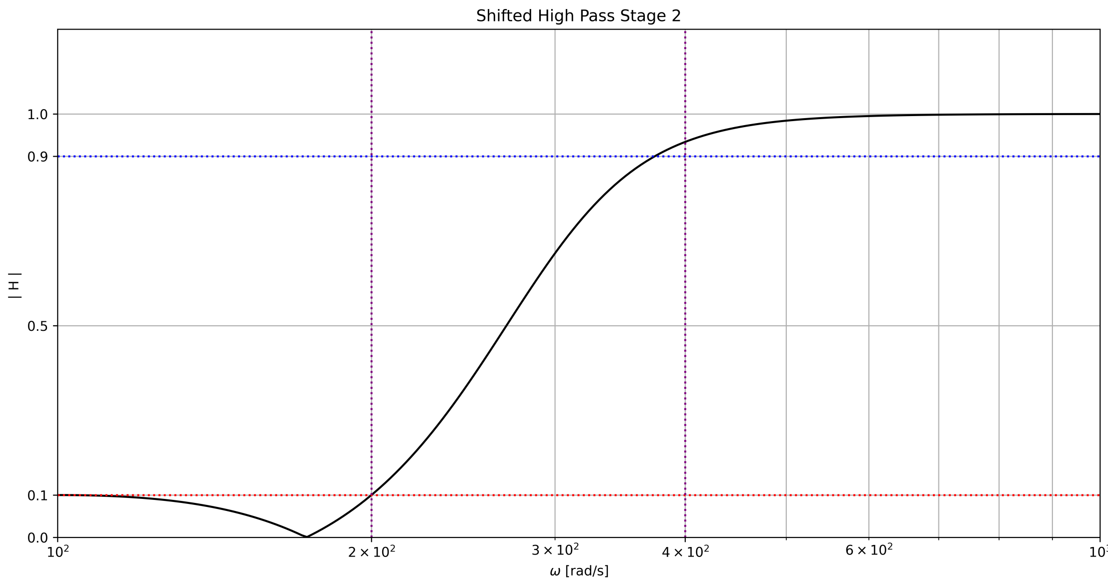


Figure 4: High Pass Filter Zoomed into Area of Interest



To verify the filter, time domain graphs were creating showing what happens at 200 rad/sec and 400 rad/sec (The blue lines represent the outputs):

Figure 5: Filter Output at 200 rad/sec

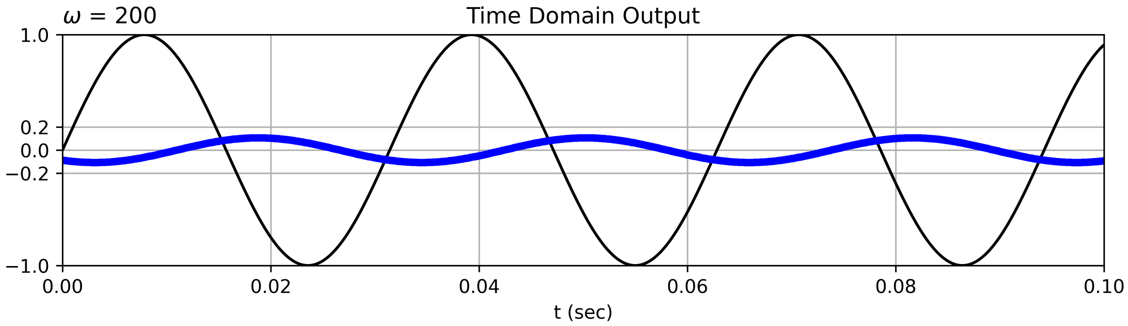
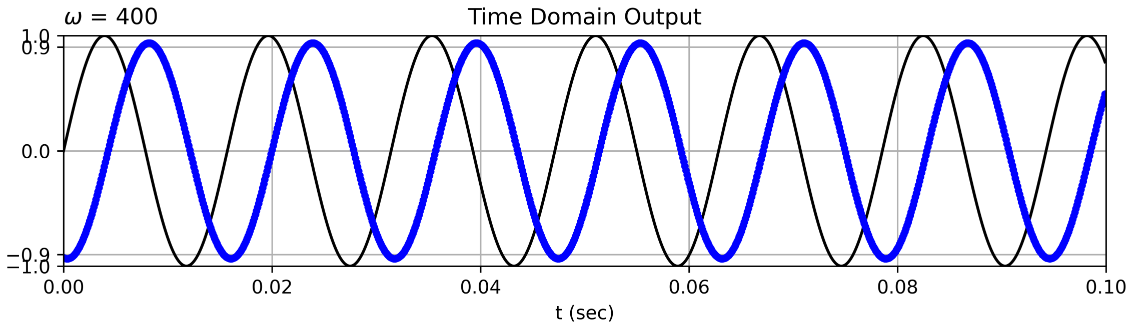


Figure 6: Filter output at 400 rad/sec



Conclusion

The filter made was much more complicated to create and simulate than a butterworth version, but the chosen filter was able to do the same with a two degree lower filter. The graphs of the magnitude give important verification that the filters were designed correctly at each stage.