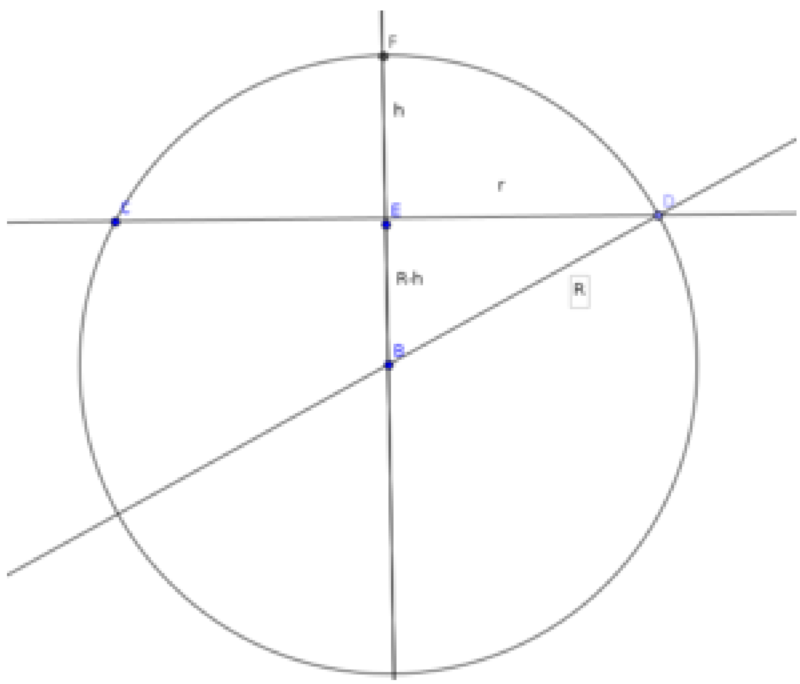
18.01 Problem Set Solutions
Problem Set 2

Niranjan Krishna
realniranjankrishna@gmail.com

September 10, 2020

Link to Problem Set : Problem Set 2b

1. In the figure h is the height of the spherical cap, while R is the radius of the sphere and r is the radius of the disc



   (a) Therefore

$$(R-h)^2 + r^2 = R^2$$
$$(R-h)^2 = R^2 - r^2$$
$$R - h = \sqrt{R^2 - r^2}$$

$$h = R - \sqrt{R^2 - r^2}$$

Using the aforementioned formula we can find the area

$$2\pi Rh = 2\pi R(R - \sqrt{R^2 - r^2})$$

(b)

$$2\pi R(R - \sqrt{R^2 - r^2}) = 2\pi R(R - \sqrt{R^2(1 - \frac{r^2}{R^2}})$$

$$= 2\pi R(R - R\sqrt{1 - (\frac{r}{R})^2})$$

$$= 2\pi R^2(1 - \sqrt{1 - (\frac{r}{R})^2})$$

For finding the linear approximation we need to select a suitable x that is close to 0. Lucky for us we have $\frac{r^2}{R^2}$. When $\frac{r}{R}$ is small $\frac{r}{R^2}$ is close to 0. So $x = -\frac{r^2}{R^2}$
Therfore

$$1 + r_c x = 1 + \frac{1}{2}\frac{-r^2}{R^2} = 1 - \frac{r^2}{2R^2}$$

So the Area becomes

$$2\pi R^2(\frac{r^2}{2R^2}). = \pi r^2$$

In a similar way the quadratic approximation can also be found. THe quadratic approximation formula is

$$1 + r_c x + \frac{r_c(r_c - 1)}{2}x^2 = 1 + \frac{1}{2}\frac{-r^2}{R^2} + \frac{\frac{1}{2}(1 - \frac{1}{2})}{2}\frac{-r^2}{R^2} = \pi r^2 + \pi\frac{r^4}{4R^2}$$

(c) i. One dimple removes πr^2 area from the ball. Area

$$= 4\pi R^2 - 100\pi r^2$$

ii. One dimple removes $\pi r^2 + \pi\frac{r^4}{4R^2}$ area from the ball. Area

$$= 4\pi R^2 - 100\pi r^2 + 25\pi\frac{r^4}{R^2}$$

iii. The actual calculation is left as an exercise to the readers

(d) If we use linear approximation we can say that

$$4\pi R^2 \geq n\pi r^2$$

$$= n \leq \frac{4R^2}{r^2}$$

(e) If we use quadratic approximation we can say that

$$4\pi R^2 \geq n(\pi r^2 + \pi\frac{r^4}{4R^2})$$

$$= n \leq \frac{4R^2}{r^2 + \frac{r^4}{4R^2}}$$

Calculation is left to readers

2. Drawing left to readers