18.01 Problem Set Solutions
Problem Set 2

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Link to Problem Set : Problem Set 2a

1. (Use a graphing calculator)

2. (a) Here we have to use the chain rule.

$$t = x^4, u = tan(t), v = u^3$$

Therfore

$$\frac{d}{dx}tan^3(x^4) = \frac{dv}{du} \cdot \frac{du}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dv}{du} = \frac{d}{du}u^3 = 3 \cdot u^2 = 3 \cdot tan(x^4)^2 = 3 \cdot tan(x^4)^2$$

$$\frac{du}{dt} = \frac{d}{dt}tan(t) = sec^2(t) = sec^2(x^4)$$

$$\frac{du}{dx} = \frac{d}{dx}x^4 = 4x^3$$

Multiplying all of them together we get

$$\frac{d}{dx}tan^3(x^4) = 3 \cdot tan(x^4)^2 \cdot sec^2(x^4) \cdot 4x^3$$

$$= 12x^3(\cdot tan(x^4) \cdot sec(x^4))^2$$

(b)

$$\frac{d}{dy}\sin^2(y)\cos^2(y) = \frac{d}{dy}\sin^2(y) \cdot \cos^2(y) + \frac{d}{dy}\cos^2(y) \cdot \sin^2(y)$$

$$= 2 \cdot \sin(y) \cdot \cos(y) \cdot \cos^2(y) + 2 \cdot \cos(y) \cdot -\sin(y) \cdot \sin^2 y$$

$$= 2\sin(y)\cos^3(y) - 2\cos(y)\sin^3(y)$$

$$= 2\sin(y)\cos(y)(\cos^2(y) - \sin^2(y))$$

3. (a)

$$y' = u' \cdot v + v' \cdot u$$

$$y'' = (u' \cdot v)' + (v' \cdot u)'$$
$$y'' = u'' \cdot v + u' \cdot v' + u' \cdot v' + v'' \cdot u'$$
$$y'' = u''v + 2u'v' + v''u'$$

(b)

$$y'' = u''v + 2u'v' + v''u'$$

$$\begin{aligned} y''' &= (u'''v + v'u'') + 2(u''v' + v''u') + (v'''u + u'v'') \\ &= u'''v + 3u''v' + 3v''u' + v'''u \end{aligned}$$

4. (a)

$$y = \cos^{-1}(x)$$

$$\cos(y) = x$$
$$\frac{d}{dx}\cos(y) = \frac{d}{dx}x$$
$$-\sin(y)\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{-\sin(y)}$$

$\cos(y)$ represents a triangle with adjacent side x and hypotnuse 1. So the opposite side will be $\sqrt{1-x^2}$ So $\sin(y)$ in this triangle will be $\frac{\sqrt{1-x^2}}{1}$. Therefore $\frac{dy}{dx}$ will be

$$\frac{dy}{dx} = \frac{1}{-\sin(y)} = \frac{1}{-\sqrt{1-x^2}}$$

(b) Assuming $0 < \theta < \frac{\pi}{2}$. Let

$$\cos(a) = x$$

By the identity $\cos(x) = \sin(90 - x)$

$$\sin(90 - a) = x$$

Since

$$\cos^{-1}(x) = a$$
$$\frac{d}{dx}\cos^{-1}(x) = \frac{d}{dx}a$$

Since

$$\sin^{-1}(x) = 90 - a$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{d}{dx}(90 - a) = \frac{d}{dx} - a = -\frac{d}{dx}a$$

Adding them together we get

$$\frac{d}{dx}\cos^{-1}(x) + \frac{d}{dx}\sin^{-1}(x) = \frac{d}{dx}a + -\frac{d}{dx}a = 0$$

5. (From the textbook)

6.

$$y = (u_1 \cdot u_2 ... \cdot u_n)$$

$$\ln y = \ln(u_1 \cdot u_2 ... \cdot u_n)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\ln(u_1 \cdot u_2 ... \cdot u_n)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\ln(u_1 \cdot u_2 ... \cdot u_n)$$

$$\frac{dy}{dx} = y \cdot \frac{d}{dx}\ln(u_1 \cdot u_2 ... \cdot u_n)$$

$$\frac{dy}{dx} = (u_1 \cdot u_2 ... \cdot u_n) \cdot \frac{d}{dx}\ln(u_1 \cdot u_2 ... \cdot u_n)$$

By $\ln ab = \ln a + \ln b$

$$\frac{dy}{dx} = (u_1 \cdot u_2 ... \cdot u_n) \cdot \frac{d}{dx}(\ln u_1 + \ln u_2 ... + \ln u_n)$$

$$\frac{dy}{dx} = (u_1 \cdot u_2 ... \cdot {}_n) \cdot (\frac{u_1'}{u_1} + \frac{u_2'}{u_2} ... + \frac{u_n'}{u_n})$$

$$\frac{dy}{dx} = (u_1 \cdot u_2 ... \cdot {}_n) \cdot (\frac{u_1'}{u_1} + \frac{u_2'}{u_2} ... + \frac{u_n'}{u_n})$$

$$\frac{dy}{dx} = u_1' \cdot (u_2 \cdot u_3 ... \cdot u_n) + u_2' \cdot (u_1 \cdot u_3 ... \cdot u_n) + u_n' \cdot (u_1 \cdot u_2 ... \cdot u_{n-1})$$