Constructions With Monoidal Categories

The Clowder Project Authors

July 22, 2025

O1UF This chapter contains some material on constructions with monoidal categories.

Contents

10.1	Moduli Categories of Monoidal Structures	2
	3.1.1 The Moduli Category of Monoidal Structures on a Cate-	
gory.		2
	3.1.2 The Moduli Category of Braided Monoidal Structures	
on a	ategory	15
	3.1.3 The Moduli Category of Symmetric Monoidal Structures	
on a	ategory	15
13.2	Moduli Categories of Closed Monoidal Structures 1	15
12 2	Joduli Catogories of Refinements of Moneidal Struc-	
	Moduli Categories of Refinements of Monoidal Struc-	15
	1	15
tures	3.3.1 The Moduli Category of Braided Refinements of a Monoidal	
tures	1	
tures	3.3.1 The Moduli Category of Braided Refinements of a Monoidal	15

olug 13.1 Moduli Categories of Monoidal Structures

01UH 13.1.1 The Moduli Category of Monoidal Structures on a Category

Let C be a category.

Oluj Definition 13.1.1.1. The moduli category of monoidal structures on C is the category $\mathcal{M}_{\mathbb{E}_1}(C)$ defined by

$$\mathcal{M}_{\mathbb{E}_1}(\mathcal{C}) \stackrel{\mathrm{def}}{=} \mathsf{pt} imes_{\mathsf{Cats}} \mathsf{MonCats}, egin{array}{c} \mathcal{M}_{\mathbb{E}_1}(\mathcal{C}) & \longrightarrow \mathsf{MonCats} \\ & & & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow &$$

- 01UK Remark 13.1.1.1.2. In detail, the moduli category of monoidal structures on C is the category $\mathcal{M}_{\mathbb{E}_1}(C)$ where:
 - Objects. The objects of $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$ are monoidal categories $(\mathcal{C}, \otimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}, \alpha^{\mathcal{C}}, \lambda^{\mathcal{C}}, \rho^{\mathcal{C}})$ whose underlying category is \mathcal{C} .
 - Morphisms. A morphism from $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$ to $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$ is a strong monoidal functor structure

$$\operatorname{id}_{\mathcal{C}}^{\otimes} \colon A \boxtimes_{\mathcal{C}} B \xrightarrow{\sim} A \otimes_{\mathcal{C}} B,$$
$$\operatorname{id}_{\mathbb{I}|\mathcal{C}}^{\otimes} \colon \mathbb{1}'_{\mathcal{C}} \xrightarrow{\sim} \mathbb{1}_{\mathcal{C}}$$

on the identity functor $id_{\mathcal{C}} : \mathcal{C} \to \mathcal{C}$ of \mathcal{C} .

• *Identities.* For each $M \stackrel{\text{def}}{=} (C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C) \in \text{Obj}(\mathcal{M}_{\mathbb{E}_1}(C)),$ the unit map

$$\mathbb{1}_{M,M}^{\mathcal{M}_{\mathbb{E}_1}(C)} \colon \mathrm{pt} \to \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_1}(C)}(M,M)$$

of $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$ at M is defined by

$$\mathrm{id}_{M}^{\mathcal{M}_{\mathbb{E}_{1}}(C)} \stackrel{\mathrm{def}}{=} \left(\mathrm{id}_{\mathcal{C}}^{\otimes}, \mathrm{id}_{\mathbb{1}|\mathcal{C}}^{\otimes}\right),$$

where $\left(\mathrm{id}_{\mathcal{C}}^{\otimes},\mathrm{id}_{\mathbb{1}|\mathcal{C}}^{\otimes}\right)$ is the identity monoidal functor of \mathcal{C} of ??.

• Composition. For each $M, N, P \in \mathrm{Obj}(\mathcal{M}_{\mathbb{E}_{1}}(C))$, the composition map $\circ_{M,N,P}^{\mathcal{M}_{\mathbb{E}_{1}}(C)}$: $\mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(N,P) \times \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(M,N) \to \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(M,P)$ of $\mathcal{M}_{\mathbb{E}_{1}}(C)$ at (M,N,P) is defined by $\left(\mathrm{id}_{C}^{\otimes,\prime},\mathrm{id}_{\mathbb{I}|C}^{\otimes,\prime}\right) \circ_{M,N,P}^{\mathcal{M}_{\mathbb{E}_{1}}(C)}\left(\mathrm{id}_{C}^{\otimes},\mathrm{id}_{\mathbb{I}|C}^{\otimes}\right) \stackrel{\mathrm{def}}{=} \left(\mathrm{id}_{C}^{\otimes,\prime}\circ\mathrm{id}_{C}^{\otimes},\mathrm{id}_{\mathbb{I}|C}^{\otimes,\prime}\circ\mathrm{id}_{\mathbb{I}|C}^{\otimes}\right).$

- **Q1UL** Remark 13.1.1.1.3. In particular, a morphism in $\mathcal{M}_{\mathbb{E}_1}(C)$ from $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$ to $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$ satisfies the following conditions:
- 01UM 1. Naturality. For each pair $f: A \to X$ and $g: B \to Y$ of morphisms of C, the diagram

$$A \boxtimes_{\mathcal{C}} B \xrightarrow{f\boxtimes_{\mathcal{C}} g} X \boxtimes_{\mathcal{C}} Y$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes}} \qquad \qquad \downarrow^{\operatorname{id}_{X,Y}^{\otimes}}$$

$$A \otimes_{\mathcal{C}} B \xrightarrow{f\otimes_{\mathcal{C}} g} X \otimes_{\mathcal{C}} Y$$

commutes.

Olum 2. Monoidality. For each $A, B, C \in \text{Obj}(C)$, the diagram

$$(A \boxtimes_{C} B) \boxtimes_{C} C$$

$$(A \otimes_{C} B) \boxtimes_{C} C$$

$$(A \otimes_{C} B) \boxtimes_{C} C$$

$$A \boxtimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A \otimes_{C} B, C}}$$

$$(A \otimes_{C} B) \otimes_{C} C$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A} \boxtimes_{C} \operatorname{id}_{B, C}^{\otimes}}$$

$$(A \otimes_{C} B) \otimes_{C} C$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A, B \otimes_{C} C}}$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

commutes.

O1UP 3. Left Monoidal Unity. For each $A \in \text{Obj}(\mathcal{C})$, the diagram

$$\mathbb{1}_{C} \boxtimes_{C} A \xrightarrow{\operatorname{id}_{\mathbb{1}'_{C}, A}^{\otimes}} \mathbb{1}_{C} \otimes_{C} A \\
\operatorname{id}_{\mathbb{1}}^{\otimes} \boxtimes_{C} \operatorname{id}_{A} \nearrow \qquad \qquad \lambda_{A}^{C}$$

$$\mathbb{1}'_{C} \boxtimes_{C} A \xrightarrow{\lambda_{A}^{C, \prime}} A$$

commutes.

01UQ 4. Right Monoidal Unity. For each $A \in \text{Obj}(\mathcal{C})$, the diagram

$$A \boxtimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{A}^{\otimes}, \mathbb{1}_{C}'} A \otimes_{C} \mathbb{1}_{C}$$

$$\operatorname{id}_{A} \boxtimes_{C} \operatorname{id}_{\mathbb{1}}^{\otimes} / \longrightarrow A \otimes_{C} \mathbb{1}_{C}$$

$$A \boxtimes_{C} \mathbb{1}_{C}' \xrightarrow{\rho_{A}^{C,'}} A$$

commutes.

OUR Proposition 13.1.1.1.4. Let C be a category.

01US 1. Extra Monoidality Conditions. Let $(id_{\mathcal{C}}^{\otimes}, id_{\mathbb{1}|\mathcal{C}}^{\otimes})$ be a morphism of $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$ from $(\mathcal{C}, \otimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}, \alpha^{\mathcal{C}}, \lambda^{\mathcal{C}}, \rho^{\mathcal{C}})$ to $(\mathcal{C}, \boxtimes_{\mathcal{C}}, \mathbb{1}'_{\mathcal{C}}, \alpha^{\mathcal{C},\prime}, \lambda^{\mathcal{C},\prime}, \rho^{\mathcal{C},\prime})$.

01UT (a) The diagram

$$(A \boxtimes_{C} B) \boxtimes_{C} C \xrightarrow{\operatorname{id}_{A,B}^{\otimes} \boxtimes_{C} \operatorname{id}_{C}} (A \otimes_{C} B) \boxtimes_{C} C$$

$$\operatorname{id}_{A\boxtimes_{C}B,C}^{\otimes} \downarrow \qquad \qquad \downarrow \operatorname{id}_{A\otimes_{C}B,C}^{\otimes}$$

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} \operatorname{id}_{C}} (A \otimes_{C} B) \otimes_{C} C$$

commutes.

01UU (b) The diagram

$$A \boxtimes_{C} (B \boxtimes_{C} C) \xrightarrow{\operatorname{id}_{A} \boxtimes_{C} \operatorname{id}_{B,C}^{\otimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

$$\operatorname{id}_{A,B\boxtimes_{C} C}^{\otimes} \downarrow \qquad \qquad \downarrow \operatorname{id}_{A,B\otimes_{C} C}^{\otimes}$$

$$A \otimes_{C} (B \boxtimes_{C} C) \xrightarrow{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}} A \otimes_{C} (B \otimes_{C} C)$$

commutes.

01WB 2. Extra Monoidal Unity Constraints. Let $\left(\operatorname{id}_{\mathcal{C}}^{\otimes}, \operatorname{id}_{\mathbb{1}|\mathcal{C}}^{\otimes}\right)$ be a morphism of $\mathcal{M}_{\mathbb{E}_{1}}(\mathcal{C})$ from $\left(\mathcal{C}, \otimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}, \alpha^{\mathcal{C}}, \lambda^{\mathcal{C}}, \rho^{\mathcal{C}}\right)$ to $\left(\mathcal{C}, \boxtimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}', \alpha^{\mathcal{C},'}, \lambda^{\mathcal{C},'}, \rho^{\mathcal{C},'}\right)$.

01WC (a) The diagram

commutes.

01WD (b) The diagram

commutes.

01WE (c) The diagram

commutes.

01WF (d) The diagram

commutes.

01UV 3. Mixed Associators. Let $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$ and $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$ be monoidal structures on C and let

$$\operatorname{id}_{-1,-2}^{\otimes} : -_1 \boxtimes_{\mathcal{C}} -_2 \to -_1 \otimes_{\mathcal{C}} -_2$$

be a natural transformation.

01UW (a) If there exists a natural transformation

$$\alpha_{A,B,C}^{\otimes} \colon (A \otimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \to A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \otimes_{C} B) \boxtimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\otimes}} A \otimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A \otimes_{C} B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\mathcal{C},\prime}} A \boxtimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes} \boxtimes_{\mathcal{C}} \operatorname{id}_{\mathcal{C}}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B} \boxtimes_{\mathcal{C}} C}$$

$$(A \otimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\otimes}} A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

commute, then the natural transformation id $^{\otimes}$ satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

01UX (b) If there exists a natural transformation

$$\alpha_{ABC}^{\boxtimes}: (A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \to A \boxtimes_{\mathcal{C}} (B \otimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} \operatorname{id}_{C}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} C}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{C} B) \boxtimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C,\prime}} A \boxtimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A\boxtimes_{C}B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A}\boxtimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

commute, then the natural transformation id^{\otimes} satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

01UY (c) If there exists a natural transformation

$$\alpha_{A,B,C}^{\boxtimes,\otimes} \colon (A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \to A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes,\otimes}} A \otimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A,B} \otimes_{C} \operatorname{id}_{C}} \qquad \qquad \downarrow^{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\mathcal{C},\prime}} A \boxtimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

$$\downarrow^{\operatorname{id}_{A\boxtimes_{\mathcal{C}} B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B\boxtimes_{\mathcal{C}} C}^{\otimes}}$$

$$(A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes,\otimes}} A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

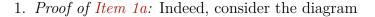
commute, then the natural transformation id^{\otimes} satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

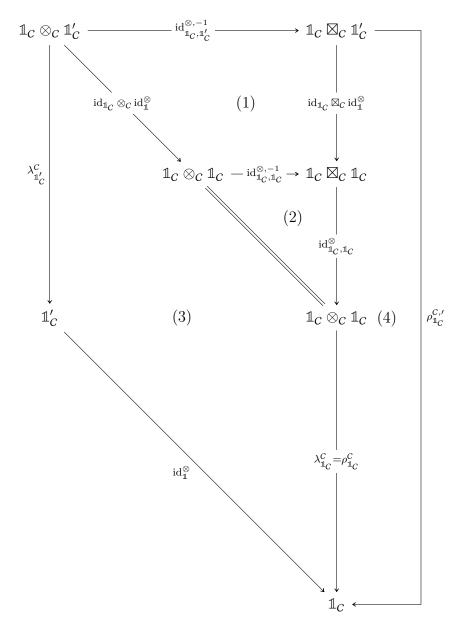
Proof. Item 1, Extra Monoidality Conditions: We claim that Items 1a and 1b are indeed true:

- 1. Proof of Item 1a: This follows from the naturality of id^{\otimes} with respect to the morphisms $id_{A,B}^{\otimes}$ and id_{C} .
- 2. Proof of Item 1b: This follows from the naturality of id^{\otimes} with respect to the morphisms id_A and $id_{B,C}^{\otimes}$.

This finishes the proof.

Item 2, Extra Monoidal Unity Constraints: We claim that *Items 2a* and *2b* are indeed true:

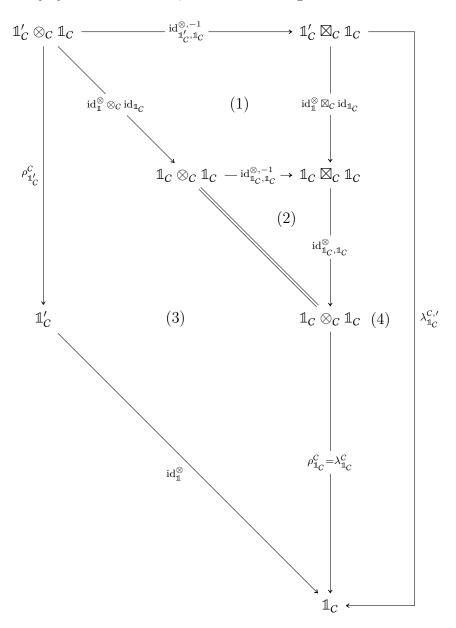




whose boundary diagram is the diagram whose commutativity we wish to prove. Since:

- Subdiagram (1) commutes by the naturality of $\mathrm{id}_{\mathcal{C}}^{\otimes,-1};$
- Subdiagram (2) commutes trivially;

- Subdiagram (3) commutes by the naturality of λ^C , where the equality $\rho_{\mathbb{1}_C}^C = \lambda_{\mathbb{1}_C}^C$ comes from ??;
- Subdiagram (4) commutes by the right monoidal unity of $(id_C, id_C^{\otimes}, id_{C|1}^{\otimes})$; so does the boundary diagram, and we are done.
- 2. Proof of Item 1b: Indeed, consider the diagram



whose boundary diagram is the diagram whose commutativity we wish to prove. Since:

- Subdiagram (1) commutes by the naturality of $id_C^{\otimes,-1}$;
- Subdiagram (2) commutes trivially;
- Subdiagram (3) commutes by the naturality of ρ^{C} , where the equality $\rho_{\mathbb{1}_{C}}^{C} = \lambda_{\mathbb{1}_{C}}^{C}$ comes from ??;
- Subdiagram (4) commutes by the left monoidal unity of $(id_C, id_C^{\otimes}, id_{C|1}^{\otimes})$; so does the boundary diagram, and we are done.
- 3. Proof of Item 2c: Indeed, consider the diagram

Since:

- The boundary diagram commutes trivially;
- Subdiagram (1) commutes by Item 1b;

it follows that the diagram

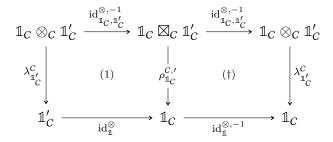
$$\mathbb{1}'_{C} \otimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}'_{C} \boxtimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes}} \mathbb{1}'_{C} \otimes_{C} \mathbb{1}_{C}$$

$$\downarrow^{C,'}_{\mathbb{1}'_{C}} \qquad \qquad (\dagger) \qquad \qquad \downarrow^{\rho^{C}_{\mathbb{1}'_{C}}}_{\mathbb{1}'_{C}}$$

$$\mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}}^{\otimes},-1} \qquad \mathbb{1}'_{C}$$

commutes. But since $\mathrm{id}_{\mathbb{1}_C,\mathbb{1}_C'}^{\otimes,-1}$ is an isomorphism, it follows that the diagram (†) also commutes, and we are done.

4. Proof of Item 2d: Indeed, consider the diagram



Since:

- The boundary diagram commutes trivially;
- Subdiagram (1) commutes by Item 1a;

it follows that the diagram

$$\mathbb{1}_{C} \otimes_{C} \mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}_{C} \boxtimes_{C} \mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}_{C} \otimes_{C} \mathbb{1}'_{C}$$

$$\downarrow \qquad \qquad \downarrow^{\lambda_{\mathbb{1}'_{C}}^{C}} \downarrow \qquad \downarrow^{\lambda_{\mathbb{1}'_{C}}^{C}}$$

$$\mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}}^{\otimes,-1}} \mathbb{1}_{C}$$

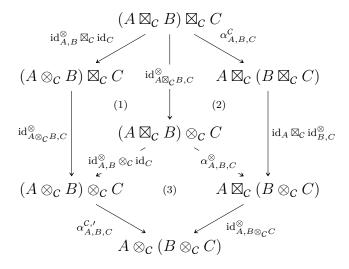
commutes. But since $id_{1}^{\otimes,-1}$ is an isomorphism, it follows that the diagram (†) also commutes, and we are done.

This finishes the proof.

Item 3, Mixed Associators: We claim that Items 3a to 3c are indeed true:

01UZ 1. Proof of Item 3a: We may partition the monoidality diagram for id^{\otimes}



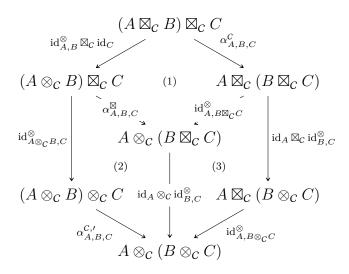


Since:

- Subdiagram (1) commutes by Item 1a of Item 1.
- Subdiagram (2) commutes by assumption.
- Subdiagram (3) commutes by assumption.

it follows that the boundary diagram also commutes, i.e. id^{\otimes} satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

01V0 2. Proof of Item 3b: We may partition the monoidality diagram for id^{\otimes} of Item 2 of Definition 13.1.1.1.3 as follows:

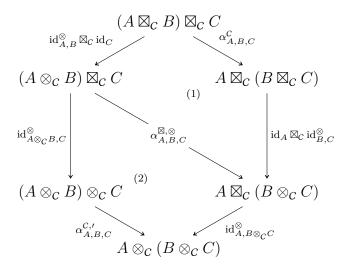


Since:

- Subdiagram (1) commutes by assumption.
- Subdiagram (2) commutes by assumption.
- Subdiagram (3) commutes by Item 1b of Item 1.

it follows that the boundary diagram also commutes, i.e. id^{\otimes} satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

3. Proof of Item 3c: We may partition the monoidality diagram for id^{\otimes} of Item 2 of Definition 13.1.1.1.3 as follows:



Since subdiagrams (1) and (2) commute by assumption, it follows that the boundary diagram also commutes, i.e. id^{\otimes} satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

This finishes the proof.

- 01V2 13.1.2 The Moduli Category of Braided Monoidal Structures on a Category
- 01V3 13.1.3 The Moduli Category of Symmetric Monoidal Structures on a Category
- olva 13.2 Moduli Categories of Closed Monoidal Structures
- olv 13.3 Moduli Categories of Refinements of Monoidal Structures
- 01V6 13.3.1 The Moduli Category of Braided Refinements of a Monoidal Structure

Appendices

A Other Chapters

T	•	•	•
Pre	lım	ına	ries

- 1. Introduction
- 2. A Guide to the Literature

Sets

- 3. Sets
- 4. Constructions With Sets
- 5. Monoidal Structures on the Category of Sets
- 6. Pointed Sets
- 7. Tensor Products of Pointed Sets

Relations

- 8. Relations
- 9. Constructions With Relations
- 10. Conditions on Relations

Categories

- 11. Categories
- 12. Presheaves and the Yoneda Lemma

Monoidal Categories

13. Constructions With Monoidal Categories

Bicategories

Extra Part

14. Types of Morphisms in Bicategories

15. Notes