# Constructions With Monoidal Categories

## The Clowder Project Authors

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**O1UF** This chapter contains some material on constructions with monoidal categories.

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# olug 13.1 Moduli Categories of Monoidal Structures

# 01UH 13.1.1 The Moduli Category of Monoidal Structures on a Category

Let C be a category.

Oluj Definition 13.1.1.1. The moduli category of monoidal structures on C is the category  $\mathcal{M}_{\mathbb{E}_1}(C)$  defined by

$$\mathcal{M}_{\mathbb{E}_1}(C) \stackrel{\mathrm{def}}{=} \mathsf{pt} imes_{\mathsf{Cats}} \mathsf{MonCats}, egin{pmatrix} \mathcal{M}_{\mathbb{E}_1}(C) & \longrightarrow \mathsf{MonCats} \\ & & \downarrow & & \downarrow & & \downarrow \\ & \mathsf{pt} & \stackrel{|}{\longrightarrow} \mathsf{Cats}. \end{pmatrix}$$

- 01UK Remark 13.1.1.1.2. In detail, the moduli category of monoidal structures on C is the category  $\mathcal{M}_{\mathbb{E}_1}(C)$  where:
  - Objects. The objects of  $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$  are monoidal categories  $(\mathcal{C}, \otimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}, \alpha^{\mathcal{C}}, \lambda^{\mathcal{C}}, \rho^{\mathcal{C}})$  whose underlying category is  $\mathcal{C}$ .
  - Morphisms. A morphism from  $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$  to  $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$  is a strong monoidal functor structure

$$\operatorname{id}_{\mathcal{C}}^{\otimes} \colon A \boxtimes_{\mathcal{C}} B \xrightarrow{\sim} A \otimes_{\mathcal{C}} B,$$
$$\operatorname{id}_{\mathbb{I}|\mathcal{C}}^{\otimes} \colon \mathbb{1}'_{\mathcal{C}} \xrightarrow{\sim} \mathbb{1}_{\mathcal{C}}$$

on the identity functor  $id_{\mathcal{C}} : \mathcal{C} \to \mathcal{C}$  of  $\mathcal{C}$ .

• *Identities.* For each  $M \stackrel{\text{def}}{=} (C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C) \in \text{Obj}(\mathcal{M}_{\mathbb{E}_1}(C)),$  the unit map

$$\mathbb{1}_{M,M}^{\mathcal{M}_{\mathbb{E}_1}(C)} \colon \mathrm{pt} \to \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_1}(C)}(M,M)$$

of  $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$  at M is defined by

$$\mathrm{id}_{M}^{\mathcal{M}_{\mathbb{E}_{1}}(C)} \stackrel{\mathrm{def}}{=} \left(\mathrm{id}_{\mathcal{C}}^{\otimes}, \mathrm{id}_{\mathbb{1}|\mathcal{C}}^{\otimes}\right),$$

where  $(id_C^{\otimes}, id_{1|C}^{\otimes})$  is the identity monoidal functor of C of ??.

• Composition. For each  $M, N, P \in \mathrm{Obj}(\mathcal{M}_{\mathbb{E}_{1}}(C))$ , the composition map  $\circ_{M,N,P}^{\mathcal{M}_{\mathbb{E}_{1}}(C)}$ :  $\mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(N,P) \times \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(M,N) \to \mathrm{Hom}_{\mathcal{M}_{\mathbb{E}_{1}}(C)}(M,P)$  of  $\mathcal{M}_{\mathbb{E}_{1}}(C)$  at (M,N,P) is defined by  $\left(\mathrm{id}_{C}^{\otimes,\prime},\mathrm{id}_{\mathbb{I}|C}^{\otimes,\prime}\right) \circ_{M,N,P}^{\mathcal{M}_{\mathbb{E}_{1}}(C)}\left(\mathrm{id}_{C}^{\otimes},\mathrm{id}_{\mathbb{I}|C}^{\otimes}\right) \stackrel{\mathrm{def}}{=} \left(\mathrm{id}_{C}^{\otimes,\prime}\circ\mathrm{id}_{C}^{\otimes},\mathrm{id}_{\mathbb{I}|C}^{\otimes,\prime}\circ\mathrm{id}_{\mathbb{I}|C}^{\otimes}\right).$ 

- **Q1UL** Remark 13.1.1.1.3. In particular, a morphism in  $\mathcal{M}_{\mathbb{E}_1}(C)$  from  $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$  to  $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$  satisfies the following conditions:
- 01UM 1. Naturality. For each pair  $f: A \to X$  and  $g: B \to Y$  of morphisms of C, the diagram

$$A \boxtimes_{\mathcal{C}} B \xrightarrow{f\boxtimes_{\mathcal{C}} g} X \boxtimes_{\mathcal{C}} Y$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes}} \qquad \qquad \downarrow^{\operatorname{id}_{X,Y}^{\otimes}}$$

$$A \otimes_{\mathcal{C}} B \xrightarrow{f\otimes_{\mathcal{C}} g} X \otimes_{\mathcal{C}} Y$$

commutes.

**Olum** 2. Monoidality. For each  $A, B, C \in \text{Obj}(\mathcal{C})$ , the diagram

$$(A \boxtimes_{C} B) \boxtimes_{C} C$$

$$(A \otimes_{C} B) \boxtimes_{C} C$$

$$(A \otimes_{C} B) \boxtimes_{C} C$$

$$A \boxtimes_{C} (B \boxtimes_{C} C)$$

$$id^{\otimes}_{A \otimes_{C} B, C}$$

$$(A \otimes_{C} B) \otimes_{C} C$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

$$id_{A} \boxtimes_{C} id^{\otimes}_{B, C}$$

$$(A \otimes_{C} B) \otimes_{C} C$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

$$id^{\otimes}_{A, B, C}$$

$$A \boxtimes_{C} (B \otimes_{C} C)$$

commutes.

**O1UP** 3. Left Monoidal Unity. For each  $A \in \text{Obj}(\mathcal{C})$ , the diagram

$$\mathbb{1}_{C} \boxtimes_{C} A \xrightarrow{\operatorname{id}_{\mathbb{1}'_{C}, A}^{\otimes}} \mathbb{1}_{C} \otimes_{C} A \\
\mathbb{1}'_{C} \boxtimes_{C} A \xrightarrow{\lambda_{A}^{C, \prime}} A$$

commutes.

**01UQ** 4. Right Monoidal Unity. For each  $A \in \text{Obj}(\mathcal{C})$ , the diagram

$$A \boxtimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{A,\mathbb{1}_{C}'}^{\otimes}} A \otimes_{C} \mathbb{1}_{C}$$

$$\operatorname{id}_{A} \boxtimes_{C} \operatorname{id}_{\mathbb{1}}^{\otimes} / \longrightarrow A$$

$$A \boxtimes_{C} \mathbb{1}_{C}' \xrightarrow{\rho_{A}^{C,'}} A$$

commutes.

**Olum** Proposition 13.1.1.1.4. Let C be a category.

01US 1. Extra Monoidality Conditions. Let  $(id_C^{\otimes}, id_{1|C}^{\otimes})$  be a morphism of  $\mathcal{M}_{\mathbb{E}_1}(C)$  from  $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$  to  $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$ .

**01UT** (a) The diagram

$$(A \boxtimes_{C} B) \boxtimes_{C} C \xrightarrow{\operatorname{id}_{A,B}^{\otimes} \boxtimes_{C} \operatorname{id}_{C}} (A \otimes_{C} B) \boxtimes_{C} C$$

$$\operatorname{id}_{A\boxtimes_{C}B,C}^{\otimes} \downarrow \qquad \qquad \downarrow \operatorname{id}_{A\otimes_{C}B,C}^{\otimes}$$

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} \operatorname{id}_{C}} (A \otimes_{C} B) \otimes_{C} C$$

commutes.

**01UU** (b) The diagram

$$A \boxtimes_{C} (B \boxtimes_{C} C) \xrightarrow{\operatorname{id}_{A} \boxtimes_{C} \operatorname{id}_{B,C}^{\otimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

$$\operatorname{id}_{A,B\boxtimes_{C} C}^{\otimes} \downarrow \qquad \qquad \downarrow \operatorname{id}_{A,B\otimes_{C} C}^{\otimes}$$

$$A \otimes_{C} (B \boxtimes_{C} C) \xrightarrow{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}} A \otimes_{C} (B \otimes_{C} C)$$

commutes.

01WB 2. Extra Monoidal Unity Constraints. Let  $(id_{\mathcal{C}}^{\otimes}, id_{1|\mathcal{C}}^{\otimes})$  be a morphism of  $\mathcal{M}_{\mathbb{E}_1}(\mathcal{C})$  from  $(\mathcal{C}, \otimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}, \alpha^{\mathcal{C}}, \lambda^{\mathcal{C}}, \rho^{\mathcal{C}})$  to  $(\mathcal{C}, \boxtimes_{\mathcal{C}}, \mathbb{1}_{\mathcal{C}}', \alpha^{\mathcal{C},'}, \lambda^{\mathcal{C},'}, \rho^{\mathcal{C},'})$ .

01WC (a) The diagram

$$\mathbb{1}_{C} \otimes_{C} \mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}_{C} \boxtimes_{C} \mathbb{1}'_{C}$$

$$\downarrow^{\lambda_{\mathbb{1}'_{C}}^{C}} \qquad \qquad \downarrow^{\rho_{\mathbb{1}_{C}}^{C,'}}$$

$$\mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}}^{\otimes}} \mathbb{1}_{C}$$

commutes.

**01WD** (b) The diagram

commutes.

01WE (c) The diagram

commutes.

01WF (d) The diagram

commutes.

**01UV** 3. Mixed Associators. Let  $(C, \otimes_C, \mathbb{1}_C, \alpha^C, \lambda^C, \rho^C)$  and  $(C, \boxtimes_C, \mathbb{1}'_C, \alpha^{C,\prime}, \lambda^{C,\prime}, \rho^{C,\prime})$  be monoidal structures on C and let

$$\operatorname{id}_{-1,-2}^{\otimes} : -_1 \boxtimes_{\mathcal{C}} -_2 \to -_1 \otimes_{\mathcal{C}} -_2$$

be a natural transformation.

**01UW** (a) If there exists a natural transformation

$$\alpha_{A,B,C}^{\otimes} \colon (A \otimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \to A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \otimes_{C} B) \boxtimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\otimes}} A \otimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A \otimes_{C} B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\mathcal{C},\prime}} A \boxtimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes} \boxtimes_{\mathcal{C}} \operatorname{id}_{\mathcal{C}}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B} \boxtimes_{\mathcal{C}} C}$$

$$(A \otimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\otimes}} A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

commute, then the natural transformation id  $^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

**01UX** (b) If there exists a natural transformation

$$\alpha_{ABC}^{\boxtimes}: (A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \to A \boxtimes_{\mathcal{C}} (B \otimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} \operatorname{id}_{C}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B}^{\otimes} \otimes_{C} C}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{C} B) \boxtimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C,\prime}} A \boxtimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A\boxtimes_{C}B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A}\boxtimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes}} A \boxtimes_{C} (B \otimes_{C} C)$$

commute, then the natural transformation  $id^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

**01UY** (c) If there exists a natural transformation

$$\alpha_{A,B,C}^{\boxtimes,\otimes} \colon (A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \to A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

making the diagrams

$$(A \boxtimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes,\otimes}} A \otimes_{C} (B \boxtimes_{C} C)$$

$$\downarrow^{\operatorname{id}_{A,B} \otimes_{C} \operatorname{id}_{C}} \qquad \qquad \downarrow^{\operatorname{id}_{A} \otimes_{C} \operatorname{id}_{B,C}^{\otimes}}$$

$$(A \otimes_{C} B) \otimes_{C} C \xrightarrow{\alpha_{A,B,C}^{C}} A \otimes_{C} (B \otimes_{C} C)$$

and

$$(A \boxtimes_{\mathcal{C}} B) \boxtimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\mathcal{C},\prime}} A \boxtimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

$$\downarrow^{\operatorname{id}_{A\boxtimes_{\mathcal{C}} B,C}} \qquad \qquad \downarrow^{\operatorname{id}_{A,B\boxtimes_{\mathcal{C}} C}^{\otimes}}$$

$$(A \boxtimes_{\mathcal{C}} B) \otimes_{\mathcal{C}} C \xrightarrow{\alpha_{A,B,C}^{\boxtimes,\otimes}} A \otimes_{\mathcal{C}} (B \boxtimes_{\mathcal{C}} C)$$

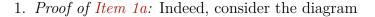
commute, then the natural transformation  $id^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

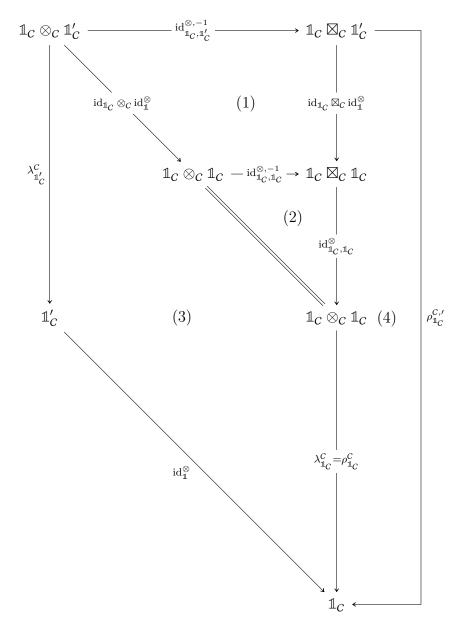
*Proof.* Item 1, Extra Monoidality Conditions: We claim that Items 1a and 1b are indeed true:

- 1. Proof of Item 1a: This follows from the naturality of  $id^{\otimes}$  with respect to the morphisms  $id_{A,B}^{\otimes}$  and  $id_{C}$ .
- 2. Proof of Item 1b: This follows from the naturality of  $id^{\otimes}$  with respect to the morphisms  $id_A$  and  $id_{B,C}^{\otimes}$ .

This finishes the proof.

*Item 2, Extra Monoidal Unity Constraints*: We claim that *Items 2a* and *2b* are indeed true:

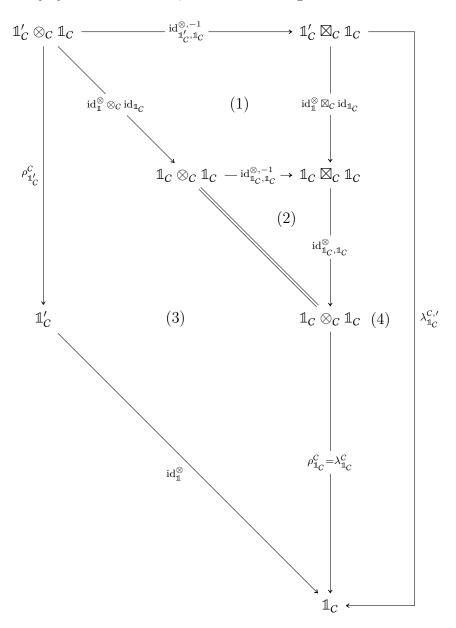




whose boundary diagram is the diagram whose commutativity we wish to prove. Since:

- Subdiagram (1) commutes by the naturality of  $\mathrm{id}_{\mathcal{C}}^{\otimes,-1};$
- Subdiagram (2) commutes trivially;

- Subdiagram (3) commutes by the naturality of  $\lambda^C$ , where the equality  $\rho_{\mathbb{1}_C}^C = \lambda_{\mathbb{1}_C}^C$  comes from ??;
- Subdiagram (4) commutes by the right monoidal unity of  $(id_C, id_C^{\otimes}, id_{C|1}^{\otimes})$ ; so does the boundary diagram, and we are done.
- 2. Proof of Item 1b: Indeed, consider the diagram



whose boundary diagram is the diagram whose commutativity we wish to prove. Since:

- Subdiagram (1) commutes by the naturality of  $id_C^{\otimes,-1}$ ;
- Subdiagram (2) commutes trivially;
- Subdiagram (3) commutes by the naturality of  $\rho^{C}$ , where the equality  $\rho_{\mathbb{1}_{C}}^{C} = \lambda_{\mathbb{1}_{C}}^{C}$  comes from ??;
- Subdiagram (4) commutes by the left monoidal unity of  $(id_C, id_C^{\otimes}, id_{C|1}^{\otimes})$ ; so does the boundary diagram, and we are done.
- 3. Proof of Item 2c: Indeed, consider the diagram

Since:

- The boundary diagram commutes trivially;
- Subdiagram (1) commutes by Item 1b;

it follows that the diagram

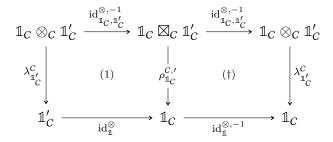
$$\mathbb{1}'_{C} \otimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}'_{C} \boxtimes_{C} \mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes}} \mathbb{1}'_{C} \otimes_{C} \mathbb{1}_{C}$$

$$\downarrow^{C,'}_{\mathbb{1}'_{C}} \qquad \qquad (\dagger) \qquad \qquad \downarrow^{\rho^{C}_{\mathbb{1}'_{C}}}_{\mathbb{1}'_{C}}$$

$$\mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}}^{\otimes},-1} \qquad \mathbb{1}'_{C}$$

commutes. But since  $\mathrm{id}_{\mathbb{1}_C,\mathbb{1}_C'}^{\otimes,-1}$  is an isomorphism, it follows that the diagram (†) also commutes, and we are done.

4. Proof of Item 2d: Indeed, consider the diagram



Since:

- The boundary diagram commutes trivially;
- Subdiagram (1) commutes by Item 1a;

it follows that the diagram

$$\mathbb{1}_{C} \otimes_{C} \mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}_{C} \boxtimes_{C} \mathbb{1}'_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}_{C},\mathbb{1}'_{C}}^{\otimes,-1}} \mathbb{1}_{C} \otimes_{C} \mathbb{1}'_{C}$$

$$\downarrow \qquad \qquad \downarrow^{\lambda_{\mathbb{1}'_{C}}^{C}} \downarrow \qquad \downarrow^{\lambda_{\mathbb{1}'_{C}}^{C}}$$

$$\mathbb{1}_{C} \xrightarrow{\operatorname{id}_{\mathbb{1}}^{\otimes,-1}} \mathbb{1}_{C}$$

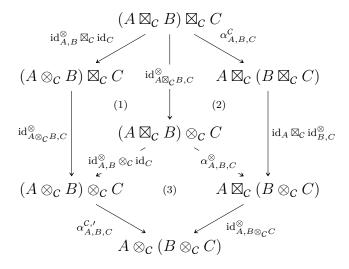
commutes. But since  $id_{1}^{\otimes,-1}$  is an isomorphism, it follows that the diagram (†) also commutes, and we are done.

This finishes the proof.

Item 3, Mixed Associators: We claim that Items 3a to 3c are indeed true:

**01UZ** 1. Proof of Item 3a: We may partition the monoidality diagram for  $id^{\otimes}$ 



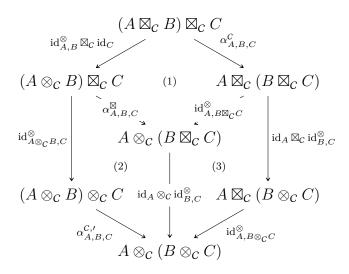


Since:

- Subdiagram (1) commutes by Item 1a of Item 1.
- Subdiagram (2) commutes by assumption.
- Subdiagram (3) commutes by assumption.

it follows that the boundary diagram also commutes, i.e.  $id^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

01V0 2. Proof of Item 3b: We may partition the monoidality diagram for  $id^{\otimes}$  of Item 2 of Definition 13.1.1.1.3 as follows:

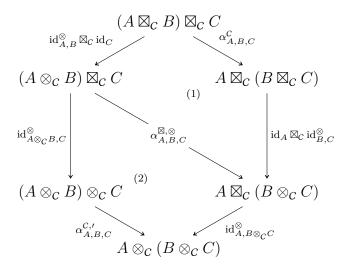


Since:

- Subdiagram (1) commutes by assumption.
- Subdiagram (2) commutes by assumption.
- Subdiagram (3) commutes by Item 1b of Item 1.

it follows that the boundary diagram also commutes, i.e.  $id^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

3. Proof of Item 3c: We may partition the monoidality diagram for  $id^{\otimes}$  of Item 2 of Definition 13.1.1.1.3 as follows:



Since subdiagrams (1) and (2) commute by assumption, it follows that the boundary diagram also commutes, i.e.  $id^{\otimes}$  satisfies the monoidality condition of Item 2 of Definition 13.1.1.1.3.

This finishes the proof.

- 01V2 13.1.2 The Moduli Category of Braided Monoidal Structures on a Category
- 01V3 13.1.3 The Moduli Category of Symmetric Monoidal Structures on a Category
- olva 13.2 Moduli Categories of Closed Monoidal Structures
- olv 13.3 Moduli Categories of Refinements of Monoidal Structures
- 01V6 13.3.1 The Moduli Category of Braided Refinements of a Monoidal Structure

## Appendices

## A Other Chapters

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Pre	lım	ına	ries

- 1. Introduction
- 2. A Guide to the Literature

#### Sets

- 3. Sets
- 4. Constructions With Sets
- 5. Monoidal Structures on the Category of Sets
- 6. Pointed Sets
- 7. Tensor Products of Pointed Sets

#### Relations

- 8. Relations
- 9. Constructions With Relations
- 10. Conditions on Relations

#### Categories

- 11. Categories
- 12. Presheaves and the Yoneda Lemma

#### **Monoidal Categories**

13. Constructions With Monoidal Categories

## Bicategories

### Extra Part

14. Types of Morphisms in Bicategories

15. Notes