Sets

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This chapter (will eventually) contain material on axiomatic set theory, as well as a couple other things.

Contents

3.1 Sets and Functions

3.1.1 Functions

Definition 3.1.1.1.1. A **function** is a functional and total relation.

Notation 3.1.1.1.2. Throughout this work, we will sometimes denote a function $f\colon X \to Y$ by

$$f \stackrel{\text{def}}{=} [\![x \mapsto f(x)]\!].$$

1. For example, given a function

$$\Phi\colon \mathrm{Hom}_{\mathsf{Sets}}(X, Y) \to K$$

taking values on a set of functions such as $\mathrm{Hom}_{\mathsf{Sets}}(X, Y)$, we will sometimes also write

$$\Phi(f) \overset{\text{def}}{=} \Phi(\llbracket x \mapsto f(x) \rrbracket).$$

2. This notational choice is based on the lambda notation

$$f \overset{\text{def}}{=} (\lambda x. f(x)),$$

but uses a "$\mapsto$" symbol for better spacing and double brackets instead of either:

   (a) Square brackets $[x \mapsto f(x)]$;
   (b) Parentheses $(x \mapsto f(x))$;

hoping to improve readability when dealing with e.g.:

   (a) Equivalence classes, cf.:
      i. $\llbracket [x] \mapsto f([x]) \rrbracket$
      ii. $\big[[x] \mapsto f([x])\big]$
      iii. $(\lambda [x]. f([x]))$
   (b) Function evaluations, cf.:
      i. $\Phi(\llbracket x \mapsto f(x) \rrbracket)$
      ii. $\Phi((x \mapsto f(x)))$
      iii. $\Phi((\lambda x. f(x)))$

3. We will also sometimes write $-$, $-_1$, $-_2$, etc. for the arguments of a function. Some examples include:

   (a) Writing $f(-_1)$ for a function $f: A \to B$.
   (b) Writing $f(-_1, -_2)$ for a function $f: A \times B \to C$.
   (c) Given a function $f: A \times B \to C$, writing

$$f(a, -): B \to C$$

   for the function $\llbracket b \mapsto f(a, b) \rrbracket$.

(d) Denoting a composition of the form

$$A \times B \xrightarrow{\phi \times \mathrm{id}_B} A' \times B \xrightarrow{f} C$$

by $f(\phi(-_1), -_2)$.

4. Finally, given a function $f : A \longrightarrow B$, we will sometimes write

$$\mathrm{ev}_a(f) \stackrel{\mathrm{def}}{=} f(a)$$

for the value of f at some $a \in A$.

For an example of the above notations being used in practice, see the proof of the adjunction

$$(A \times - \dashv \mathrm{Hom}_{\mathsf{Sets}}(A, -)) \colon \quad \begin{tikzcd} \mathsf{Sets} \arrow[r, bend left, "A\times-"] \arrow[r, phantom, "\perp"] & \mathsf{Sets}, \arrow[l, bend left, "\mathrm{Hom}_{\mathsf{Sets}}(A,-)"] \end{tikzcd}$$

stated in Constructions With Sets, Item 2 of Definition 4.1.3.1.3.

3.2 The Enrichment of Sets in Classical Truth Values

3.2.1 (-2)-Categories

Definition 3.2.1.1.1. A (-2)-**category** is the "necessarily true" truth value.[1,2,3]

3.2.2 (-1)-Categories

Definition 3.2.2.1.1. A (-1)-**category** is a classical truth value.

Remark 3.2.2.1.2. [4](-1)-categories should be thought of as being "categories enriched in (-2)-categories", having a collection of objects and, for each pair of objects, a Hom-object $\mathrm{Hom}(x, y)$ that is a (-2)-category (i.e. trivial).

As a result, a (-1)-category C is either:[5]

[1]Thus, there is only one (-2)-category.

[2]A $(-n)$-category for $n = 3, 4, \ldots$ is also the "necessarily true" truth value, coinciding with a (-2)-category.

[3]For motivation, see [BS10, p. 13].

[4]For more motivation, see [BS10, p. 13].

[5]See [BS10, pp. 33–34].

1. *Empty*, having no objects.
2. *Contractible*, having a collection of objects $\{a, b, c, \dots\}$, but with $\mathrm{Hom}_{C}(a, b)$ being a (-2)-category (i.e. trivial) for all $a, b \in \mathrm{Obj}(C)$, forcing all objects of C to be uniquely isomorphic to each other.

Thus there are only two (-1)-categories up to equivalence:

1. The (-1)-category false (the empty one);
2. The (-1)-category true (the contractible one).

Definition 3.2.2.1.3. The **poset of truth values**[6] is the poset $(\{\mathsf{true}, \mathsf{false}\}, \preceq)$ consisting of:

- *The Underlying Set.* The set $\{\mathsf{true}, \mathsf{false}\}$ whose elements are the truth values true and false.
- *The Partial Order.* The partial order

  $$\preceq\colon \{\mathsf{true}, \mathsf{false}\} \times \{\mathsf{true}, \mathsf{false}\} \to \{\mathsf{true}, \mathsf{false}\}$$

  on $\{\mathsf{true}, \mathsf{false}\}$ defined by[7]

  $$\begin{aligned} \mathsf{false} \preceq \mathsf{false} &\overset{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{false} &\overset{\text{def}}{=} \mathsf{false}, \\ \mathsf{false} \preceq \mathsf{true} &\overset{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{true} &\overset{\text{def}}{=} \mathsf{true}. \end{aligned}$$

Notation 3.2.2.1.4. We also write $\{\mathsf{t}, \mathsf{f}\}$ for the poset $\{\mathsf{true}, \mathsf{false}\}$.

Proposition 3.2.2.1.5. The poset of truth values $\{\mathsf{t}, \mathsf{f}\}$ is Cartesian closed with product given by[8]

$$\begin{aligned} \mathsf{t} \times \mathsf{t} &= \mathsf{t}, & \mathsf{f} \times \mathsf{t} &= \mathsf{f}, \\ \mathsf{t} \times \mathsf{f} &= \mathsf{f}, & \mathsf{f} \times \mathsf{f} &= \mathsf{f}, \end{aligned}$$

$\times$	t	f
t	t	f
f	f	f

[6] *Further Terminology:* Also called the **poset of (-1)-categories**.
[7] This partial order coincides with logical implication.
[8] Note that $\times$ coincides with the "and" operator, while $\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$ coincides with the logical

and internal Hom $\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$ given by the partial order of $\{\mathsf{t},\mathsf{f}\}$, i.e. by

$$\begin{aligned}\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t},\mathsf{t}) &= \mathsf{t}, & \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t}) &= \mathsf{t},\\ \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t},\mathsf{f}) &= \mathsf{f}, & \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f}) &= \mathsf{t},\end{aligned}$$

$\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$	t	f
t	t	t
f	t	f

.

Proof. Existence of Products: We claim that the products $\mathsf{t}\times\mathsf{t}$, $\mathsf{t}\times\mathsf{f}$, $\mathsf{f}\times\mathsf{t}$, and $\mathsf{f}\times\mathsf{f}$ satisfy the universal property of the product in $\{\mathsf{t},\mathsf{f}\}$. Indeed, suppose we have diagrams of the form

$$\begin{tikzcd} & P_1 \arrow[dl, bend right, "p^1_1"'] \arrow[dr, bend left, "p^1_2"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_2 \arrow[dl, bend right, "p^2_1"'] \arrow[dr, bend left, "p^2_2"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd} \quad \begin{tikzcd} & P_3 \arrow[dl, bend right, "p^3_1"'] \arrow[dr, bend left, "p^3_2"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_4 \arrow[dl, bend right, "p^4_1"'] \arrow[dr, bend left, "p^4_2"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd}$$

where the pr_1 and pr_2 morphisms are the only possible ones (since $\{\mathsf{t},\mathsf{f}\}$ is posetal). We claim that there are unique morphisms making the diagrams

$$\begin{tikzcd} & P_1 \arrow[dl, bend right, "p^1_1"'] \arrow[dr, bend left, "p^1_2"] \arrow[d, dashed, "\exists!"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_2 \arrow[dl, bend right, "p^2_1"'] \arrow[dr, bend left, "p^2_2"] \arrow[d, dashed, "\exists!"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd} \quad \begin{tikzcd} & P_3 \arrow[dl, bend right, "p^3_1"'] \arrow[dr, bend left, "p^3_2"] \arrow[d, dashed, "\exists!"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_4 \arrow[dl, bend right, "p^4_1"'] \arrow[dr, bend left, "p^4_2"] \arrow[d, dashed, "\exists!"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd}$$

commute. Indeed:

1. If $P_1 = \mathsf{t}$, then $p^1_1 = p^1_2 = \mathrm{id}_{\mathsf{t}}$, so there's a unique morphism from P_1 to t making the diagram commute, namely id_{t}.

2. If $P_1 = \mathsf{f}$, then $p^1_1 = p^1_2$ are given by the unique morphism from f to t, so there's a unique morphism from P_1 to t making the diagram commute, namely the unique morphism from f to t.

3. If $P_2 = \mathsf{t}$, then there is no morphism p^2_2.

4. If $P_2 = \mathsf{f}$, then p^2_1 is the unique morphism from f to t while $p^2_2 = \mathrm{id}_{\mathsf{f}}$, so there's a unique morphism from P_2 to f making the diagram commute, namely id_{f}.

5. The proof for P_3 is similar to the one for P_2.

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implication operator.

6. If $P_4 = \mathsf{t}$, then there is no morphism p_1^4 or p_2^4.
7. If $P_4 = \mathsf{f}$, then $p_1^4 = p_2^4 = \mathrm{id}_{\mathsf{f}}$, so there's a unique morphism from P_4 to f making the diagram commute, namely id_{f}.

This finishes the existence of products part of the proof.
Cartesian Closedness: We claim there's a bijection

$$\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(A \times B, C) \cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(A, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(B, C)\big),$$

natural in $A, B, C \in \{\mathsf{t}, \mathsf{f}\}$. Indeed:

- For $(A, B, C) = (\mathsf{t}, \mathsf{t}, \mathsf{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{t}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\\ &= \{\mathrm{id}_{\mathsf{true}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{t}, \mathsf{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{t}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\\ &= \emptyset\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{f}, \mathsf{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{f}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{pt}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{f}, \mathsf{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{f}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \{\mathrm{id}_{\mathsf{false}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{t}, \mathsf{t})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f} \times \mathsf{t}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{pt}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\big).\end{aligned}$$
- For $(A, B, C) = (\mathsf{f}, \mathsf{t}, \mathsf{f})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f} \times \mathsf{t}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \{\mathrm{id}_{\mathsf{false}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\big).\end{aligned}$$
- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{t})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f} \times \mathsf{f}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{pt}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\big).\end{aligned}$$
- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{f})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f} \times \mathsf{f}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &= \{\mathrm{id}_{\mathsf{false}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\big).\end{aligned}$$

Since $\{\mathsf{t}, \mathsf{f}\}$ is posetal, naturality is automatic (Categories, Item 4 of Definition 11.2.7.1.2). □

3.2.3 0-Categories

Definition 3.2.3.1.1. A 0-**category** is a poset.[9]

Definition 3.2.3.1.2. A 0-**groupoid** is a 0-category in which every morphism is invertible.[10]

[9] *Motivation:* A 0-category is precisely a category enriched in the poset of (-1)-categories.
[10] That is, a *set*.

3.2.4 Tables of Analogies Between Set Theory and Category Theory

Here we record some analogies between notions in set theory and category theory. The analogies relating to presheaves relate equally well to copresheaves, as the opposite X^{op} of a set X is just X again.

Remark 3.2.4.1.1. The basic analogies between set theory and category theory are summarised in the following table:

Set Theory	Category Theory
Enrichment in $\{\mathsf{true}, \mathsf{false}\}$	Enrichment in Sets
Set X	Category $\mathcal{C}$
Element $x \in X$	Object $X \in \mathrm{Obj}(\mathcal{C})$
Function $f\colon X \to Y$	Functor $F\colon \mathcal{C} \to \mathcal{D}$
Function $X \to \{\mathsf{true}, \mathsf{false}\}$	Copresheaf $\mathcal{C} \to \mathsf{Sets}$
Function $X \to \{\mathsf{true}, \mathsf{false}\}$	Presheaf $\mathcal{C}^{\mathrm{op}} \to \mathsf{Sets}$

Remark 3.2.4.1.2. The category of presheaves $\mathsf{PSh}(\mathcal{C})$ and the category of copresheaves $\mathsf{CoPSh}(\mathcal{C})$ on a category $\mathcal{C}$ are the 1-categorical counterparts to the powerset $\mathcal{P}(X)$ of subsets of a set X. The further analogies built upon this are summarised in the following table:

Set Theory	Category Theory
Powerset $\mathcal{P}(X)$	Presheaf category $\mathsf{PSh}(\mathcal{C})$
Characteristic function $\chi_{\{x\}}\colon X \to \{\mathsf{t},\mathsf{f}\}$	Representable presheaf $h_X\colon \mathcal{C}^{\mathsf{op}} \to \mathsf{Sets}$
Characteristic embedding $\chi_{(-)}\colon X \to \mathcal{P}(X)$	Yoneda embedding $よ\colon \mathcal{C}^{\mathsf{op}} \to \mathsf{PSh}(\mathcal{C})$
Characteristic relation $\chi_X(-_1,-_2)\colon X\times X \to \{\mathsf{t},\mathsf{f}\}$	Hom profunctor $\mathrm{Hom}_{\mathcal{C}}(-_1,-_2)\colon \mathcal{C}^{\mathsf{op}}\times\mathcal{C} \to \mathsf{Sets}$
The Yoneda lemma for sets $\mathrm{Hom}_{\mathcal{P}(X)}(\chi_x,\chi_U) = \chi_U(x)$	The Yoneda lemma for categories $\mathrm{Nat}(h_X,\mathcal{F}) \cong \mathcal{F}(X)$
The characteristic embedding is fully faithful, $\mathrm{Hom}_{\mathcal{P}(X)}(\chi_x,\chi_y) = \chi_X(x,y)$	The Yoneda embedding is fully faithful, $\mathrm{Nat}(h_X,h_Y) \cong \mathrm{Hom}_{\mathcal{C}}(X,Y)$
Subsets are unions of their elements $U = \bigcup_{x\in U}\{x\}$ or $\chi_U = \operatorname*{colim}_{\chi_x\in\mathcal{P}(U)}(\chi_x)$	Presheaves are colimits of representables, $\mathcal{F} \cong \operatorname*{colim}_{h_X\in\int_{\mathcal{C}}\mathcal{F}}(h_X)$

Remark 3.2.4.1.3. We summarise the analogies between un/straightening in set theory and category theory in the following table:

Set Theory	Category Theory
Assignment $U \mapsto \chi_U$	Assignment $\mathcal{F} \mapsto \int_{\mathcal{C}}\mathcal{F}$
Un/straightening isomorphism $\mathcal{P}(X) \cong \mathsf{Sets}(X,\{\mathsf{t},\mathsf{f}\})$	Un/straightening equivalence $\mathsf{DFib}(\mathcal{C}) \overset{\text{eq.}}{\cong} \mathsf{PSh}(\mathcal{C})$

Remark 3.2.4.1.4. We summarise the analogies between functions $\mathcal{P}(X) \to \mathcal{P}(Y)$ and functors $\mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$ in the following table:

Set Theory	Category Theory
Direct image function $f_!\colon \mathcal{P}(X) \to \mathcal{P}(Y)$	Left Kan extension functor $F_!\colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$
Inverse image function $f^{-1}\colon \mathcal{P}(Y) \to \mathcal{P}(X)$	Precomposition functor $F^*\colon \mathsf{PSh}(\mathcal{D}) \to \mathsf{PSh}(\mathcal{C})$
Codirect image function $f_*\colon \mathcal{P}(X) \to \mathcal{P}(Y)$	Right Kan extension functor $F_*\colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$

Remark 3.2.4.1.5. We summarise the analogies between functions, relations and profunctors in the following table:

Set Theory	Category Theory
Relation $R\colon X \times Y \to \{\mathsf{t}, \mathsf{f}\}$	Profunctor $\mathfrak{p}\colon \mathcal{D}^{\mathsf{op}} \times \mathcal{C} \to \mathsf{Sets}$
Relation $R\colon X \to \mathcal{P}(Y)$	Profunctor $\mathfrak{p}\colon \mathcal{C} \to \mathsf{PSh}(\mathcal{D})$
Relation as a cocontinuous morphism of posets $R\colon (\mathcal{P}(X), \subset) \to (\mathcal{P}(Y), \subset)$	Profunctor as a colimit-preserving functor $\mathfrak{p}\colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$

Appendices

A Other Chapters

References

[BS10] John C. Baez and Michael Shulman. "Lectures on n-Categories and Cohomology". In: *Towards higher categories*. Vol. 152. IMA Vol. Math. Appl. Springer, New York, 2010, pp. 1–68. DOI: 10.1007/978-1-4419-1524-5_1. URL: https://doi.org/10.1007/978-1-4419-1524-5_1 (cit. on p. 3).