

Types of Morphisms in Bicategories

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Ø19H In this chapter, we study special kinds of morphisms in bicategories:

1. *Monomorphisms and Epimorphisms in Bicategories* (*Sections 14.1 and 14.2*). There is a large number of different notions capturing the idea of a “monomorphism” or of an “epimorphism” in a bicategory.

Arguably, the notion that best captures these concepts is that of a *pseudomononic morphism* (*Definition 14.1.10.1.1*) and of a *pseudoepic morphism* (*Definition 14.2.10.1.1*), although the other notions introduced in *Sections 14.1 and 14.2* are also interesting on their own.

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019J **14.1 Monomorphisms in Bicategories**

019K **14.1.1 Representably Faithful Morphisms**

Let C be a bicategory.

019L **Definition 14.1.1.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably faithful**¹ if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is faithful.

019M **Remark 14.1.1.1.2.** In detail, f is representably faithful if, for all diagrams in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \downarrow \downarrow \downarrow \beta \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if we have

$$\text{id}_f \star \alpha = \text{id}_f \star \beta,$$

then $\alpha = \beta$.

019N **Example 14.1.1.1.3.** Here are some examples of representably faithful morphisms.

- 019P 1. *Representably Faithful Morphisms in Cats_2 .* The representably faithful morphisms in Cats_2 are precisely the faithful functors; see [Categories, Item 2](#) of [Definition 11.6.1.1.2](#).
- 019Q 2. *Representably Faithful Morphisms in \mathbf{Rel} .* Every morphism of \mathbf{Rel} is representably faithful; see [Relations, Item 1](#) of [Definition 8.5.11.1.1](#).

¹*Further Terminology:* Also called simply a **faithful morphism**, based on [Item 1](#) of

019R 14.1.2 Representably Full Morphisms

Let C be a bicategory.

019S **Definition 14.1.2.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably full**² if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is full.

019T **Remark 14.1.2.1.2.** In detail, f is representably full if, for each $X \in \text{Obj}(C)$ and each 2-morphism

$$\beta: f \circ \phi \Rightarrow f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of C , there exists a 2-morphism

$$\alpha: \phi \Rightarrow \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

of C such that we have an equality

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B = X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \text{id}_f \star \alpha.$$

019U **Example 14.1.2.1.3.** Here are some examples of representably full morphisms.

019V 1. *Representably Full Morphisms in Cats_2 .* The representably full morphisms in Cats_2 are precisely the full functors; see **Categories, ??** of **Definition 11.6.2.1.2**.

019W 2. *Representably Full Morphisms in \mathbf{Rel} .* The representably full morphisms in \mathbf{Rel} are characterised in **Relations, Item 2** of **Definition 8.5.11.1.1**.

Definition 14.1.1.1.3.

²*Further Terminology:* Also called simply a **full morphism**, based on **Item 1** of

019X 14.1.3 Representably Fully Faithful Morphisms

Let C be a bicategory.

019Y **Definition 14.1.3.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably fully faithful**³ if the following equivalent conditions are satisfied:

019Z 1. The 1-morphism f is representably faithful (**Definition 14.1.1.1.1**) and representably full (**Definition 14.1.2.1.1**).

01A0 2. For each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is fully faithful.

01A1 **Remark 14.1.3.1.2.** In detail, f is representably fully faithful if the conditions in **Definition 14.1.1.1.2** and **Definition 14.1.2.1.2** hold:

1. For all diagrams in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if we have

$$\text{id}_f \star \alpha = \text{id}_f \star \beta,$$

then $\alpha = \beta$.

2. For each $X \in \text{Obj}(C)$ and each 2-morphism

$$\beta: f \circ \phi \Rightarrow f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of C , there exists a 2-morphism

$$\alpha: \phi \Rightarrow \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

Definition 14.1.2.1.3.

³*Further Terminology:* Also called simply a **fully faithful morphism**, based on **Item 1** of **Definition 14.1.3.1.3**.

of C such that we have an equality

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B = X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \text{id}_f \star \alpha.$$

01A2 Example 14.1.3.1.3. Here are some examples of representably fully faithful morphisms.

- 01A3** 1. *Representably Fully Faithful Morphisms in \mathbf{Cats}_2 .* The representably fully faithful morphisms in \mathbf{Cats}_2 are precisely the fully faithful functors; see [Categories, Item 6](#) of [Definition 11.6.3.1.2](#).
- 01A4** 2. *Representably Fully Faithful Morphisms in \mathbf{Rel} .* The representably fully faithful morphisms of \mathbf{Rel} coincide ([Relations, Item 3](#) of [Definition 8.5.11.1.1](#)) with the representably full morphisms in \mathbf{Rel} , which are characterised in [Relations, Item 2](#) of [Definition 8.5.11.1.1](#).

01A5 14.1.4 Morphisms Representably Faithful on Cores

Let C be a bicategory.

01A6 Definition 14.1.4.1.1. A 1-morphism $f: A \rightarrow B$ of C is **representably faithful on cores** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Core}(\text{Hom}_C(X, A)) \rightarrow \text{Core}(\text{Hom}_C(X, B))$$

given by postcomposition by f is faithful.

01A7 Remark 14.1.4.1.2. In detail, f is representably faithful on cores if, for all diagrams in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if α and β are 2-isomorphisms and we have

$$\text{id}_f \star \alpha = \text{id}_f \star \beta,$$

then $\alpha = \beta$.

01A8 14.1.5 Morphisms Representably Full on Cores

Let C be a bicategory.

01A9 **Definition 14.1.5.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably full on cores** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Core}(\text{Hom}_C(X, A)) \rightarrow \text{Core}(\text{Hom}_C(X, B))$$

given by postcomposition by f is full.

01AA **Remark 14.1.5.1.2.** In detail, f is representably full on cores if, for each $X \in \text{Obj}(C)$ and each 2-isomorphism

$$\beta: f \circ \phi \xRightarrow{\sim} f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of C , there exists a 2-isomorphism

$$\alpha: \phi \xRightarrow{\sim} \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

of C such that we have an equality

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B = X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \text{id}_f \star \alpha.$$

01AB 14.1.6 Morphisms Representably Fully Faithful on Cores

Let C be a bicategory.

01AC **Definition 14.1.6.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably fully faithful on cores** if the following equivalent conditions are satisfied:

- 01AD 1. The 1-morphism f is representably faithful on cores ([Definition 14.1.5.1.1](#)) and representably full on cores ([Definition 14.1.4.1.1](#)).

01AE 2. For each $X \in \text{Obj}(C)$, the functor

$$f_* : \text{Core}(\text{Hom}_C(X, A)) \rightarrow \text{Core}(\text{Hom}_C(X, B))$$

given by postcomposition by f is fully faithful.

01AF **Remark 14.1.6.1.2.** In detail, f is representably fully faithful on cores if the conditions in [Definition 14.1.4.1.2](#) and [Definition 14.1.5.1.2](#) hold:

1. For all diagrams in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if α and β are 2-isomorphisms and we have

$$\text{id}_f \star \alpha = \text{id}_f \star \beta,$$

then $\alpha = \beta$.

2. For each $X \in \text{Obj}(C)$ and each 2-isomorphism

$$\beta : f \circ \phi \xrightarrow{\sim} f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of C , there exists a 2-isomorphism

$$\alpha : \phi \xrightarrow{\sim} \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

of C such that we have an equality

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B = X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \text{id}_f \star \alpha.$$

01AG **14.1.7 Representably Essentially Injective Morphisms**

Let C be a bicategory.

01AH **Definition 14.1.7.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably essentially injective** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is essentially injective.

01AJ **Remark 14.1.7.1.2.** In detail, f is representably essentially injective if, for each pair of morphisms $\phi, \psi: X \rightrightarrows A$ of C , the following condition is satisfied:

(\star) If $f \circ \phi \cong f \circ \psi$, then $\phi \cong \psi$.

01AK **14.1.8 Representably Conservative Morphisms**

Let C be a bicategory.

01AL **Definition 14.1.8.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **representably conservative** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is conservative.

01AM **Remark 14.1.8.1.2.** In detail, f is representably conservative if, for each pair of morphisms $\phi, \psi: X \rightrightarrows A$ and each 2-morphism

$$\alpha: \phi \Rightarrow \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

of C , if the 2-morphism

$$\text{id}_f \star \alpha: f \circ \phi \Rightarrow f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \parallel \\ \text{id}_f \star \alpha \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

is a 2-isomorphism, then so is α .

01AN 14.1.9 Strict Monomorphisms

Let C be a bicategory.

01AP Definition 14.1.9.1.1. A 1-morphism $f: A \rightarrow B$ of C is a **strict monomorphism** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is injective on objects, i.e. its action on objects

$$f_*: \text{Obj}(\text{Hom}_C(X, A)) \rightarrow \text{Obj}(\text{Hom}_C(X, B))$$

is injective.

01AQ Remark 14.1.9.1.2. In detail, f is a strict monomorphism in C if, for each diagram in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if $f \circ \phi = f \circ \psi$, then $\phi = \psi$.

01AR Example 14.1.9.1.3. Here are some examples of strict monomorphisms.

01AS 1. *Strict Monomorphisms in \mathbf{Cats}_2 .* The strict monomorphisms in \mathbf{Cats}_2 are precisely the functors which are injective on objects and injective on morphisms; see [Categories, Item 1](#) of [Definition 11.7.2.1.2](#).

01AT 2. *Strict Monomorphisms in \mathbf{Rel} .* The strict monomorphisms in \mathbf{Rel} are characterised in [Relations, Definition 8.5.10.1.1](#).

01AU 14.1.10 Pseudomonic Morphisms

Let C be a bicategory.

01AV Definition 14.1.10.1.1. A 1-morphism $f: A \rightarrow B$ of C is **pseudomonic** if, for each $X \in \text{Obj}(C)$, the functor

$$f_*: \text{Hom}_C(X, A) \rightarrow \text{Hom}_C(X, B)$$

given by postcomposition by f is pseudomonic.

01AW Remark 14.1.10.1.2. In detail, a 1-morphism $f: A \rightarrow B$ of C is pseudomonic if it satisfies the following conditions:

- 01AX 1. For all diagrams in C of the form

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B,$$

if we have

$$\mathrm{id}_f \star \alpha = \mathrm{id}_f \star \beta,$$

then $\alpha = \beta$.

- 01AY 2. For each $X \in \mathrm{Obj}(C)$ and each 2-isomorphism

$$\beta: f \circ \phi \xrightarrow{\sim} f \circ \psi, \quad X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of C , there exists a 2-isomorphism

$$\alpha: \phi \xrightarrow{\sim} \psi, \quad X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A$$

of C such that we have an equality

$$X \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} A \xrightarrow{f} B = X \begin{array}{c} \xrightarrow{f \circ \phi} \\ \beta \Downarrow \\ \xrightarrow{f \circ \psi} \end{array} B$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \mathrm{id}_f \star \alpha.$$

01AZ **Proposition 14.1.10.1.3.** Let $f: A \rightarrow B$ be a 1-morphism of C .

- 01B0 1. *Characterisations.* The following conditions are equivalent:

01B1 (a) The morphism f is pseudomonadic.

01B2 (b) The morphism f is representably full on cores and representably faithful.

01B3 (c) We have an isocomma square of the form

$$A \overset{\text{eq.}}{\cong} A \overset{\leftrightarrow}{\times}_B A, \quad \begin{array}{ccc} A & \xrightarrow{\text{id}_A} & A \\ \text{id}_A \downarrow & \nearrow \text{dashed} & \downarrow F \\ A & \xrightarrow{F} & B \end{array}$$

in C up to equivalence.

01B4 2. *Interaction With Cotensors.* If C has cotensors with $\mathbb{1}$, then the following conditions are equivalent:

- (a) The morphism f is pseudomonic.
- (b) We have an isocomma square of the form

$$A \overset{\text{eq.}}{\cong} A \overset{\leftrightarrow}{\times}_{\mathbb{1} \pitchfork F} B, \quad \begin{array}{ccc} A & \hookrightarrow & \mathbb{1} \pitchfork A \\ F \downarrow & \nearrow \text{dashed} & \downarrow \mathbb{1} \pitchfork F \\ B & \hookrightarrow & \mathbb{1} \pitchfork B \end{array}$$

in C up to equivalence.

Proof. *Item 1, Characterisations:* Omitted.

Item 2, Interaction With Cotensors: Omitted. □

01B5 14.2 Epimorphisms in Bicategories

01B6 14.2.1 Corepresentably Faithful Morphisms

Let C be a bicategory.

01B7 **Definition 14.2.1.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **corepresentably faithful** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is faithful.

01B8 **Remark 14.2.1.1.2.** In detail, f is corepresentably faithful if, for all diagrams in C of the form

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \downarrow \parallel \downarrow \beta \\ \xrightarrow{\psi} \end{array} X,$$

if we have

$$\alpha \star \text{id}_f = \beta \star \text{id}_f,$$

then $\alpha = \beta$.

01B9 Example 14.2.1.1.3. Here are some examples of corepresentably faithful morphisms.

01BA 1. *Corepresentably Faithful Morphisms in \mathbf{Cats}_2 .* The corepresentably faithful morphisms in \mathbf{Cats}_2 are characterised in **Categories, Item 5** of **Definition 11.6.1.1.2**.

01BB 2. *Corepresentably Faithful Morphisms in \mathbf{Rel} .* Every morphism of \mathbf{Rel} is corepresentably faithful; see **Relations, Item 1** of **Definition 8.5.13.1.1**.

01BC 14.2.2 Corepresentably Full Morphisms

Let C be a bicategory.

01BD Definition 14.2.2.1.1. A 1-morphism $f: A \rightarrow B$ of C is **corepresentably full** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is full.

01BE Remark 14.2.2.1.2. In detail, f is corepresentably full if, for each $X \in \text{Obj}(C)$ and each 2-morphism

$$\beta: \phi \circ f \Rightarrow \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of C , there exists a 2-morphism

$$\alpha: \phi \Rightarrow \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C such that we have an equality

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X = A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \alpha \star \text{id}_f.$$

01BF **Example 14.2.2.1.3.** Here are some examples of corepresentably full morphisms.

01BG 1. *Corepresentably Full Morphisms in \mathbf{Cats}_2 .* The corepresentably full morphisms in \mathbf{Cats}_2 are characterised in [Categories, Item 7](#) of [Definition 11.6.2.1.2](#).

01BH 2. *Corepresentably Full Morphisms in \mathbf{Rel} .* The corepresentably full morphisms in \mathbf{Rel} are characterised in [Relations, Item 2](#) of [Definition 8.5.13.1.1](#).

01BJ 14.2.3 Corepresentably Fully Faithful Morphisms

Let C be a bicategory.

01BK **Definition 14.2.3.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **corepresentably fully faithful**⁴ if the following equivalent conditions are satisfied:

01BL 1. The 1-morphism f is corepresentably full ([Definition 14.2.2.1.1](#)) and corepresentably faithful ([Definition 14.2.1.1.1](#)).

01BM 2. For each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is fully faithful.

01BN **Remark 14.2.3.1.2.** In detail, f is corepresentably fully faithful if the conditions in [Definition 14.2.1.1.2](#) and [Definition 14.2.2.1.2](#) hold:

1. For all diagrams in C of the form

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \downarrow \downarrow \downarrow \beta \\ \xrightarrow{\psi} \end{array} X,$$

if we have

$$\alpha \star \text{id}_f = \beta \star \text{id}_f,$$

then $\alpha = \beta$.

⁴*Further Terminology:* Corepresentably fully faithful morphisms have also been called **lax epimorphisms** in the literature (e.g. in [\[Adá+01\]](#)), though we will always use the name “corepresentably fully faithful morphism” instead in this work.

2. For each $X \in \text{Obj}(C)$ and each 2-morphism

$$\beta: \phi \circ f \Rightarrow \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of C , there exists a 2-morphism

$$\alpha: \phi \Rightarrow \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C such that we have an equality

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X = A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \alpha \star \text{id}_f.$$

01BP Example 14.2.3.1.3. Here are some examples of corepresentably fully faithful morphisms.

01BQ 1. *Corepresentably Fully Faithful Morphisms in Cats_2 .* The fully faithful epimorphisms in Cats_2 are characterised in [Categories, Item 10](#) of [Definition 11.6.3.1.2](#).

01BR 2. *Corepresentably Fully Faithful Morphisms in \mathbf{Rel} .* The corepresentably fully faithful morphisms of \mathbf{Rel} coincide ([Relations, Item 3](#) of [Definition 8.5.13.1.1](#)) with the corepresentably full morphisms in \mathbf{Rel} , which are characterised in [Relations, Item 2](#) of [Definition 8.5.13.1.1](#).

01BS 14.2.4 Morphisms Corepresentably Faithful on Cores

Let C be a bicategory.

01BT Definition 14.2.4.1.1. A 1-morphism $f: A \rightarrow B$ of C is **corepresentably faithful on cores** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Core}(\text{Hom}_C(B, X)) \rightarrow \text{Core}(\text{Hom}_C(A, X))$$

given by precomposition by f is faithful.

01BU Remark 14.2.4.1.2. In detail, f is corepresentably faithful on cores if, for all diagrams in C of the form

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} X,$$

if α and β are 2-isomorphisms and we have

$$\alpha \star \text{id}_f = \beta \star \text{id}_f,$$

then $\alpha = \beta$.

01BV 14.2.5 Morphisms Corepresentably Full on Cores

Let C be a bicategory.

01BW Definition 14.2.5.1.1. A 1-morphism $f: A \rightarrow B$ of C is **corepresentably full on cores** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Core}(\text{Hom}_C(B, X)) \rightarrow \text{Core}(\text{Hom}_C(A, X))$$

given by precomposition by f is full.

01BX Remark 14.2.5.1.2. In detail, f is corepresentably full on cores if, for each $X \in \text{Obj}(C)$ and each 2-isomorphism

$$\beta: \phi \circ f \xRightarrow{\sim} \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of C , there exists a 2-isomorphism

$$\alpha: \phi \xRightarrow{\sim} \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C such that we have an equality

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X = A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \alpha \star \text{id}_f.$$

01BY 14.2.6 Morphisms Corepresentably Fully Faithful on Cores

Let C be a bicategory.

01BZ **Definition 14.2.6.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **corepresentably fully faithful on cores** if the following equivalent conditions are satisfied:

01C0 1. The 1-morphism f is corepresentably full on cores (Definition 14.2.5.1.1) and corepresentably faithful on cores (Definition 14.2.1.1.1).

01C1 2. For each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Core}(\text{Hom}_C(B, X)) \rightarrow \text{Core}(\text{Hom}_C(A, X))$$

given by precomposition by f is fully faithful.

01C2 **Remark 14.2.6.1.2.** In detail, f is corepresentably fully faithful on cores if the conditions in Definition 14.2.4.1.2 and Definition 14.2.5.1.2 hold:

1. For all diagrams in C of the form

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} X,$$

if α and β are 2-isomorphisms and we have

$$\alpha \star \text{id}_f = \beta \star \text{id}_f,$$

then $\alpha = \beta$.

2. For each $X \in \text{Obj}(C)$ and each 2-isomorphism

$$\beta: \phi \circ f \xRightarrow{\sim} \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of C , there exists a 2-isomorphism

$$\alpha: \phi \xRightarrow{\sim} \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C such that we have an equality

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X = A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \alpha \star \text{id}_f.$$

01C3 14.2.7 Corepresentably Essentially Injective Morphisms

Let C be a bicategory.

01C4 **Definition 14.2.7.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **corepresentably essentially injective** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is essentially injective.

01C5 **Remark 14.2.7.1.2.** In detail, f is corepresentably essentially injective if, for each pair of morphisms $\phi, \psi: B \rightrightarrows X$ of C , the following condition is satisfied:

(\star) If $\phi \circ f \cong \psi \circ f$, then $\phi \cong \psi$.

01C6 14.2.8 Corepresentably Conservative Morphisms

Let C be a bicategory.

01C7 **Definition 14.2.8.1.1.** A 1-morphism $f: A \rightarrow B$ of C is **corepresentably conservative** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is conservative.

01C8 **Remark 14.2.8.1.2.** In detail, f is corepresentably conservative if, for each pair of morphisms $\phi, \psi: B \rightrightarrows X$ and each 2-morphism

$$\alpha: \phi \xRightarrow{\sim} \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C , if the 2-morphism

$$\alpha \star \text{id}_f: \phi \circ f \xRightarrow{\sim} \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \parallel \\ \alpha \star \text{id}_f \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

is a 2-isomorphism, then so is α .

01C9 14.2.9 Strict Epimorphisms

Let C be a bicategory.

01CA Definition 14.2.9.1.1. A 1-morphism $f: A \rightarrow B$ is a **strict epimorphism in C** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is injective on objects, i.e. its action on objects

$$f_*: \text{Obj}(\text{Hom}_C(B, X)) \rightarrow \text{Obj}(\text{Hom}_C(A, X))$$

is injective.

01CB Remark 14.2.9.1.2. In detail, f is a strict epimorphism if, for each diagram in C of the form

$$A \xrightarrow{f} B \begin{matrix} \xrightarrow{\phi} \\ \xrightarrow{\psi} \end{matrix} X,$$

if $\phi \circ f = \psi \circ f$, then $\phi = \psi$.

01CC Example 14.2.9.1.3. Here are some examples of strict epimorphisms.

01CD 1. *Strict Epimorphisms in Cats_2 .* The strict epimorphisms in Cats_2 are characterised in [Categories, Item 1](#) of [Definition 11.7.3.1.2](#).

01CE 2. *Strict Epimorphisms in \mathbf{Rel} .* The strict epimorphisms in \mathbf{Rel} are characterised in [Relations, Definition 8.5.12.1.1](#).

01CF 14.2.10 Pseudoepic Morphisms

Let C be a bicategory.

01CG Definition 14.2.10.1.1. A 1-morphism $f: A \rightarrow B$ of C is **pseudoepic** if, for each $X \in \text{Obj}(C)$, the functor

$$f^*: \text{Hom}_C(B, X) \rightarrow \text{Hom}_C(A, X)$$

given by precomposition by f is pseudomonc.

01CH Remark 14.2.10.1.2. In detail, a 1-morphism $f: A \rightarrow B$ of C is pseudoepic if it satisfies the following conditions:

01CJ 1. For all diagrams in C of the form

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \beta \\ \xrightarrow{\psi} \end{array} X,$$

if we have

$$\alpha \star \text{id}_f = \beta \star \text{id}_f,$$

then $\alpha = \beta$.

01CK 2. For each $X \in \text{Obj}(C)$ and each 2-isomorphism

$$\beta: \phi \circ f \xRightarrow{\sim} \psi \circ f, \quad A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of C , there exists a 2-isomorphism

$$\alpha: \phi \xRightarrow{\sim} \psi, \quad B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X$$

of C such that we have an equality

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{\phi} \\ \alpha \Downarrow \\ \xrightarrow{\psi} \end{array} X = A \begin{array}{c} \xrightarrow{\phi \circ f} \\ \beta \Downarrow \\ \xrightarrow{\psi \circ f} \end{array} X$$

of pasting diagrams in C , i.e. such that we have

$$\beta = \alpha \star \text{id}_f.$$

01CL **Proposition 14.2.10.1.3.** Let $f: A \rightarrow B$ be a 1-morphism of C .

01CM 1. *Characterisations.* The following conditions are equivalent:

01CN (a) The morphism f is pseudoepic.

01CP (b) The morphism f is corepresentably full on cores and corepresentably faithful.

01CQ

(c) We have an isococcomma square of the form

$$B \stackrel{\text{eq.}}{\cong} B \coprod_A B, \quad \begin{array}{ccc} B & \xleftarrow{\text{id}_B} & B \\ \text{id}_B \uparrow & \nearrow & \uparrow F \\ B & \xleftarrow{F} & A \end{array}$$

in \mathcal{C} up to equivalence.*Proof.* **Item 1**, Characterisations: Omitted. □

Appendices

A Other Chapters

Preliminaries

1. **Introduction**
2. **A Guide to the Literature**

Sets

3. **Sets**
4. **Constructions With Sets**
5. **Monoidal Structures on the Category of Sets**
6. **Pointed Sets**
7. **Tensor Products of Pointed Sets**

Relations

8. **Relations**
9. **Constructions With Relations**

10. **Conditions on Relations**

Categories

11. **Categories**
12. **Presheaves and the Yoneda Lemma**

Monoidal Categories

13. **Constructions With Monoidal Categories**

Bicategories

14. **Types of Morphisms in Bicategories**

Extra Part

15. **Notes**

References

- [Adá+01] Jiří Adámek, Robert El Bashir, Manuela Sobral, and Jiří Velebil. “On Functors Which Are Lax Epimorphisms”. In: *Theory Appl. Categ.* 8 (2001), pp. 509–521. issn: 1201-561X (cit. on p. 13).