Sets

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0000 This chapter (will eventually) contain material on axiomatic set theory, as well as a couple other things.

Contents

0001 3.1 Sets and Functions

0002 3.1.1 Functions

0003 **Definition 3.1.1.1.1 ▶ Functions**

A **function** is a functional and total relation.

0004 **Notation 3.1.1.1.2 ▶ Additional Notation for Functions**

Throughout this work, we will sometimes denote a function $f\colon X\to Y$ by

$$f\overset{\mathrm{def}}{=}[\![x\mapsto f(x)]\!].$$

01Y2 1. For example, given a function

$$\Phi\colon \mathrm{Hom}_{\mathsf{Sets}}(X,Y)\to K$$

taking values on a set of functions such as $\mathrm{Hom}_{\mathsf{Sets}}(X,Y)$, we will sometimes also write

$$\Phi(f)\overset{\mathrm{def}}{=}\Phi([\![x\mapsto f(x)]\!]).$$

01Y3 2. This notational choice is based on the lambda notation

$$f\overset{\mathrm{def}}{=}(\lambda x.\ f(x)),$$

but uses a "$\mapsto$" symbol for better spacing and double brackets instead of either:

01Y4 (a) Square brackets $[x\mapsto f(x)]$;

01Y5 (b) Parentheses $(x\mapsto f(x))$;

hoping to improve readability when dealing with e.g.:

01Y6 (a) Equivalence classes, cf.:

01Y7 i. $[\![[x]\mapsto f([x])]\!]$

01Y8 ii. $[[x]\mapsto f([x])]$

01Y9 iii. $(\lambda[x].\ f([x]))$

01YA (b) Function evaluations, cf.:

01YB i. $\Phi([\![x\mapsto f(x)]\!])$

01YC ii. $\Phi((x\mapsto f(x)))$

01YD iii. $\Phi((\lambda x.\ f(x)))$

01YE 3. We will also sometimes write $-$, $-_1$, $-_2$, etc. for the arguments of a function. Some examples include:

01YF (a) Writing $f(-_1)$ for a function $f\colon A\to B$.

01YG (b) Writing $f(-_1, -_2)$ for a function $f\colon A \times B \to C$.

01YH (c) Given a function $f\colon A \times B \to C$, writing

$$f(a, -)\colon B \to C$$

for the function $[\![b \mapsto f(a, b)]\!]$.

01YJ (d) Denoting a composition of the form

$$A \times B \xrightarrow{\phi \times \mathrm{id}_B} A' \times B \xrightarrow{f} C$$

by $f(\phi(-_1), -_2)$.

01YK 4. Finally, given a function $f\colon A \to B$, we will sometimes write

$$\mathrm{ev}_a(f) \stackrel{\mathrm{def}}{=} f(a)$$

for the value of f at some $a \in A$.

For an example of the above notations being used in practice, see the proof of the adjunction

$$(A \times - \dashv \mathrm{Hom}_{\mathsf{Sets}}(A, -))\colon\quad \begin{tikzcd} \mathsf{Sets} \arrow[r, bend left, "A \times -"] \arrow[r, phantom, "\perp"] & \mathsf{Sets}, \arrow[l, bend left, "\mathrm{Hom}_{\mathsf{Sets}}(A,-)"] \end{tikzcd}$$

stated in Constructions With Sets, Item 2 of Proposition 4.1.3.1.4.

0005 3.2 The Enrichment of Sets in Classical Truth Values

0006 3.2.1 (-2)-Categories

0007 **Definition 3.2.1.1.1 ▶ (-2)-Categories**

A (-2)-**category** is the "necessarily true" truth value.[1,2,3]

[1]Thus, there is only one (-2)-category.

[2]A $(-n)$-category for $n = 3, 4, \ldots$ is also the "necessarily true" truth value, coinciding with a (-2)-category.

[3]For motivation, see [BS10, p. 13].

0008 3.2.2 (-1)-Categories

0009 **Definition 3.2.2.1.1 ▶ (-1)-Categories**

A (-1)-**category** is a classical truth value.

000A **Remark 3.2.2.1.2 ▶ Motivation for (-1)-Categories**

[1](-1)-categories should be thought of as being "categories enriched in (-2)-categories", having a collection of objects and, for each pair of objects, a Hom-object $\mathrm{Hom}(x, y)$ that is a (-2)-category (i.e. trivial). As a result, a (-1)-category $\mathcal{C}$ is either:[2]

01D2 1. *Empty*, having no objects.

01D3 2. *Contractible*, having a collection of objects $\{a, b, c, \ldots\}$, but with $\mathrm{Hom}_{\mathcal{C}}(a, b)$ being a (-2)-category (i.e. trivial) for all $a, b \in \mathrm{Obj}(\mathcal{C})$, forcing all objects of $\mathcal{C}$ to be uniquely isomorphic to each other.

Thus there are only two (-1)-categories up to equivalence:

01D4 1. The (-1)-category false (the empty one);

01D5 2. The (-1)-category true (the contractible one).

[1]For more motivation, see [BS10, p. 13].

[2]See [BS10, pp. 33–34].

000B

The **poset of truth values**[1] is the poset $(\{\mathsf{true}, \mathsf{false}\}, \preceq)$ consisting of:

- *The Underlying Set.* The set $\{\mathsf{true}, \mathsf{false}\}$ whose elements are the truth values true and false.
- *The Partial Order.* The partial order

$$\preceq\colon \{\mathsf{true}, \mathsf{false}\} \times \{\mathsf{true}, \mathsf{false}\} \to \{\mathsf{true}, \mathsf{false}\}$$

on $\{\mathsf{true}, \mathsf{false}\}$ defined by[2]

$$\begin{aligned} \mathsf{false} \preceq \mathsf{false} &\overset{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{false} &\overset{\text{def}}{=} \mathsf{false}, \\ \mathsf{false} \preceq \mathsf{true} &\overset{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{true} &\overset{\text{def}}{=} \mathsf{true}. \end{aligned}$$

[1] *Further Terminology:* Also called the **poset of** (-1)**-categories**.
[2] This partial order coincides with logical implication.

000C **Notation 3.2.2.1.4 ▶ Further Notation for The Poset of Truth Values**

We also write $\{\mathsf{t}, \mathsf{f}\}$ for the poset $\{\mathsf{true}, \mathsf{false}\}$.

000D **Proposition 3.2.2.1.5 ▶ Cartesian Closedness of the Poset of Truth Values**

The poset of truth values $\{\mathsf{t}, \mathsf{f}\}$ is Cartesian closed with product given by[1]

$$\begin{aligned} \mathsf{t} \times \mathsf{t} &= \mathsf{t}, & \mathsf{f} \times \mathsf{t} &= \mathsf{f}, \\ \mathsf{t} \times \mathsf{f} &= \mathsf{f}, & \mathsf{f} \times \mathsf{f} &= \mathsf{f}, \end{aligned}$$

$\times$	t	f
t	t	f
f	f	f

and internal Hom $\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$ given by the partial order of $\{\mathsf{t}, \mathsf{f}\}$, i.e. by

$$\begin{aligned} \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t}) &= \mathsf{t}, & \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t}) &= \mathsf{t}, \\ \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f}) &= \mathsf{f}, & \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f}) &= \mathsf{t}, \end{aligned}$$

$\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$	t	f
t	t	t
f	t	f

.

[1] Note that $\times$ coincides with the "and" operator, while $\mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}$ coincides with the logical implication operator.

PROOF 3.2.2.1.6 ▶ PROOF OF PROPOSITION 3.2.2.1.5

Existence of Products

We claim that the products $\mathsf{t}\times\mathsf{t}$, $\mathsf{t}\times\mathsf{f}$, $\mathsf{f}\times\mathsf{t}$, and $\mathsf{f}\times\mathsf{f}$ satisfy the universal property of the product in $\{\mathsf{t},\mathsf{f}\}$. Indeed, suppose we have diagrams of the form

$$
\begin{tikzcd}
& P_1 \arrow[dl, bend right, "p^1_1"'] \arrow[dr, bend left, "p^1_2"] & \\
\mathsf{t} & \mathsf{t}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_2 \arrow[dl, bend right, "p^2_1"'] \arrow[dr, bend left, "p^2_2"] & \\
\mathsf{t} & \mathsf{t}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_3 \arrow[dl, bend right, "p^3_1"'] \arrow[dr, bend left, "p^3_2"] & \\
\mathsf{f} & \mathsf{f}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_4 \arrow[dl, bend right, "p^4_1"'] \arrow[dr, bend left, "p^4_2"] & \\
\mathsf{f} & \mathsf{f}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f}
\end{tikzcd}
$$

where the pr_1 and pr_2 morphisms are the only possible ones (since $\{\mathsf{t},\mathsf{f}\}$ is posetal). We claim that there are unique morphisms making the diagrams

$$
\begin{tikzcd}
& P_1 \arrow[dl, bend right, "p^1_1"'] \arrow[dr, bend left, "p^1_2"] \arrow[d, dashed, "\exists!"] & \\
\mathsf{t} & \mathsf{t}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_2 \arrow[dl, bend right, "p^2_1"'] \arrow[dr, bend left, "p^2_2"] \arrow[d, dashed, "\exists!"] & \\
\mathsf{t} & \mathsf{t}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_3 \arrow[dl, bend right, "p^3_1"'] \arrow[dr, bend left, "p^3_2"] \arrow[d, dashed, "\exists!"] & \\
\mathsf{f} & \mathsf{f}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t}
\end{tikzcd}
\quad
\begin{tikzcd}
& P_4 \arrow[dl, bend right, "p^4_1"'] \arrow[dr, bend left, "p^4_2"] \arrow[d, dashed, "\exists!"] & \\
\mathsf{f} & \mathsf{f}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f}
\end{tikzcd}
$$

commute. Indeed:

01YL 1. If $P_1=\mathsf{t}$, then $p^1_1=p^1_2=\mathrm{id}_{\mathsf{t}}$, so there's a unique morphism from P_1 to t making the diagram commute, namely id_{t}.

01YM 2. If $P_1=\mathsf{f}$, then $p^1_1=p^1_2$ are given by the unique morphism from f to t, so there's a unique morphism from P_1 to t making the diagram commute, namely the unique morphism from f to t.

01YN 3. If $P_2=\mathsf{t}$, then there is no morphism p^2_2.

01YP 4. If $P_2=\mathsf{f}$, then p^2_1 is the unique morphism from f to t while $p^2_2=\mathrm{id}_{\mathsf{f}}$, so there's a unique morphism from P_2 to f making the diagram commute, namely id_{f}.

01YQ 5. The proof for P_3 is similar to the one for P_2.

01YR 6. If $P_4=\mathsf{t}$, then there is no morphism p^4_1 or p^4_2.

01YS
7. If $P_4 = \mathsf{f}$, then $p_1^4 = p_2^4 = \mathrm{id}_{\mathsf{f}}$, so there's a unique morphism from P_4 to f making the diagram commute, namely id_{f}.

This finishes the existence of products part of the proof.

Cartesian Closedness

We claim there's a bijection

$$\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(A \times B, C) \cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(A, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(B, C)\Big),$$

natural in $A, B, C \in \{\mathsf{t}, \mathsf{f}\}$. Indeed:

- For $(A, B, C) = (\mathsf{t}, \mathsf{t}, \mathsf{t})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{t}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\\ &= \{\mathrm{id}_{\mathsf{true}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{t})\Big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{t}, \mathsf{f})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{t}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\\ &= \emptyset\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\Big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{f}, \mathsf{t})$, we have
$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{f}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{pt}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\Big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{t}, \mathsf{f}, \mathsf{f})$, we have
$$\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t} \times \mathsf{f}, \mathsf{f}) \cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})$$

$$
\begin{aligned}
&\cong \{\mathrm{id}_{\mathsf{false}}\}\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{t}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\Big).
\end{aligned}
$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{t}, \mathsf{t})$, we have

$$
\begin{aligned}
\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{t},\mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t})\\
&\cong \mathrm{pt}\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t})\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t},\mathsf{t})\Big).
\end{aligned}
$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{t}, \mathsf{f})$, we have

$$
\begin{aligned}
\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{t},\mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\\
&\cong \{\mathrm{id}_{\mathsf{false}}\}\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t},\mathsf{f})\Big).
\end{aligned}
$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{t})$, we have

$$
\begin{aligned}
\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{f},\mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t})\\
&\cong \mathrm{pt}\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t})\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{t})\Big).
\end{aligned}
$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{f})$, we have

$$
\begin{aligned}
\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{f},\mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\\
&= \{\mathrm{id}_{\mathsf{false}}\}\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\\
&\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\Big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f},\mathsf{f})\Big).
\end{aligned}
$$

Since $\{\mathsf{t},\mathsf{f}\}$ is posetal, naturality is automatic (Categories, Item 4 of Proposition 11.2.7.1.2).

000E
3.2.3 0-Categories

000F

Definition 3.2.3.1.1 ▶ 0-Categories

A 0-**category** is a poset.[1]

[1] *Motivation:* A 0-category is precisely a category enriched in the poset of (-1)-categories.

000G

Definition 3.2.3.1.2 ▶ 0-Groupoids

A 0-**groupoid** is a 0-category in which every morphism is invertible.[1]

[1] That is, a *set*.

000H
3.2.4 Tables of Analogies Between Set Theory and Category Theory

Here we record some analogies between notions in set theory and category theory. The analogies relating to presheaves relate equally well to copresheaves, as the opposite X^{op} of a set X is just X again.

01D6

Remark 3.2.4.1.1 ▶ Basic Analogies Between Set Theory and Category Theory

The basic analogies between set theory and category theory are summarised in the following table:

Set Theory	Category Theory
Enrichment in $\{\mathsf{true}, \mathsf{false}\}$	Enrichment in Sets
Set X	Category $\mathcal{C}$
Element $x \in X$	Object $X \in \mathrm{Obj}(\mathcal{C})$
Function $f \colon X \to Y$	Functor $F \colon \mathcal{C} \to \mathcal{D}$
Function $X \to \{\mathsf{true}, \mathsf{false}\}$	Copresheaf $\mathcal{C} \to \mathsf{Sets}$
Function $X \to \{\mathsf{true}, \mathsf{false}\}$	Presheaf $\mathcal{C}^{\mathsf{op}} \to \mathsf{Sets}$

01D7 **Remark 3.2.4.1.2 ▶ Analogies Between Set Theory and Category Theory: Powersets and Categories of Presheaves**

The category of presheaves $\mathsf{PSh}(\mathcal{C})$ and the category of copresheaves $\mathsf{CoPSh}(\mathcal{C})$ on a category $\mathcal{C}$ are the 1-categorical counterparts to the powerset $\mathcal{P}(X)$ of subsets of a set X. The further analogies built upon this are summarised in the following table:

Set Theory	Category Theory
Powerset $\mathcal{P}(X)$	Presheaf category $\mathsf{PSh}(\mathcal{C})$
Characteristic function $\chi_{\{x\}}\colon X \to \{\mathsf{t},\mathsf{f}\}$	Representable presheaf $h_X\colon \mathcal{C}^{\mathsf{op}} \hookrightarrow \mathsf{Sets}$
Characteristic embedding $\chi_{(-)}\colon X \hookrightarrow \mathcal{P}(X)$	Yoneda embedding $よ\colon \mathcal{C}^{\mathsf{op}} \hookrightarrow \mathsf{PSh}(\mathcal{C})$
Characteristic relation $\chi_X(-_1,-_2)\colon X \times X \to \{\mathsf{t},\mathsf{f}\}$	Hom profunctor $\mathrm{Hom}_{\mathcal{C}}(-_1,-_2)\colon \mathcal{C}^{\mathsf{op}} \times \mathcal{C} \to \mathsf{Sets}$
The Yoneda lemma for sets $\mathrm{Hom}_{\mathcal{P}(X)}(\chi_x,\chi_U) = \chi_U(x)$	The Yoneda lemma for categories $\mathrm{Nat}(h_X,\mathcal{F}) \cong \mathcal{F}(X)$
The characteristic embedding is fully faithful, $\mathrm{Hom}_{\mathcal{P}(X)}(\chi_x,\chi_y) = \chi_X(x,y)$	The Yoneda embedding is fully faithful, $\mathrm{Nat}(h_X,h_Y) \cong \mathrm{Hom}_{\mathcal{C}}(X,Y)$
Subsets are unions of their elements $U = \bigcup_{x\in U}\{x\}$ or $\chi_U = \operatorname*{colim}_{\chi_x\in\mathcal{P}(U)}(\chi_x)$	Presheaves are colimits of representables, $\mathcal{F} \cong \operatorname*{colim}_{h_X\in\int_{\mathcal{C}}\mathcal{F}}(h_X)$

01D8 **Remark 3.2.4.1.3 ▶ Analogies Between Set Theory and Category Theory: Categories of Elements**

We summarise the analogies between un/straightening in set theory and category theory in the following table:

Set Theory	Category Theory
Assignment $U \mapsto \chi_U$	Assignment $\mathcal{F} \mapsto \int_C \mathcal{F}$
Un/straightening isomorphism $\mathcal{P}(X) \cong \mathsf{Sets}(X, \{\mathsf{t}, \mathsf{f}\})$	Un/straightening equivalence $\mathsf{DFib}(C) \overset{\text{eq.}}{\cong} \mathsf{PSh}(C)$

01D9 **Remark 3.2.4.1.4 ▶ Analogies Between Set Theory and Category Theory: Functions Between Powersets and Functors Between Presheaf Categories**

We summarise the analogies between functions $\mathcal{P}(X) \to \mathcal{P}(Y)$ and functors $\mathsf{PSh}(C) \to \mathsf{PSh}(\mathcal{D})$ in the following table:

Set Theory	Category Theory
Direct image function $f_!: \mathcal{P}(X) \to \mathcal{P}(Y)$	Left Kan extension functor $F_!: \mathsf{PSh}(C) \to \mathsf{PSh}(\mathcal{D})$
Inverse image function $f^{-1}: \mathcal{P}(Y) \to \mathcal{P}(X)$	Precomposition functor $F^*: \mathsf{PSh}(\mathcal{D}) \to \mathsf{PSh}(C)$
Codirect image function $f_*: \mathcal{P}(X) \to \mathcal{P}(Y)$	Right Kan extension functor $F_*: \mathsf{PSh}(C) \to \mathsf{PSh}(\mathcal{D})$

01DA **Remark 3.2.4.1.5 ▶ Analogies Between Set Theory and Category Theory: Relations and Profunctors**

We summarise the analogies between functions, relations and profunctors in the following table:

Set Theory	Category Theory
Relation $R: X \times Y \to \{\mathsf{t}, \mathsf{f}\}$	Profunctor $\mathfrak{p}: \mathcal{D}^{\mathsf{op}} \times C \to \mathsf{Sets}$
Relation $R: X \to \mathcal{P}(Y)$	Profunctor $\mathfrak{p}: C \to \mathsf{PSh}(\mathcal{D})$
Relation as a cocontinuous morphism of posets $R: (\mathcal{P}(X), \subset) \to (\mathcal{P}(Y), \subset)$	Profunctor as a colimit-preserving functor $\mathfrak{p}: \mathsf{PSh}(C) \to \mathsf{PSh}(\mathcal{D})$

Appendices

A Other Chapters

References

[BS10] John C. Baez and Michael Shulman. "Lectures on n-Categories and Cohomology". In: *Towards higher categories*. Vol. 152. IMA Vol. Math. Appl. Springer, New York, 2010, pp. 1–68. DOI: 10.1007/978-1-4419-1524-5_1. URL: https://doi.org/10.1007/978-1-4419-1524-5_1 (cit. on p. 4).