

**Step 1**

$$\begin{array}{ccccc}
 & S^0 \wedge S^0 & \xrightarrow{\text{id}_{S^0}^{\otimes, -1}} & S^0 \otimes_{S^0} S^0 & \xrightarrow{\text{id}_{\mathbb{1}/S^0}^{\otimes, -1} \wedge \text{id}_{S^0}} & \mathbb{1}_{S^0} \otimes_{S^0} S^0 \\
 & \nearrow \lambda_{S^0}^{S^0, -1} & & \nearrow & & \\
 S^0 & \xrightarrow{\quad} & \lambda_{S^0}'^{-1} & \xrightarrow{\quad} & \mathbb{1}_{S^0} \otimes_{S^0} S^0 \\
 & \downarrow \text{id}_{S^0} \wedge [x] & (1) & \downarrow \text{id}_{S^0} \otimes_{S^0} [x] & & \\
 & S^0 \wedge X & \xrightarrow{\text{id}_{S^0}^{\otimes, -1}} & S^0 \otimes_{S^0} X & & \\
 & \nearrow \lambda_X^{S^0, -1} & (2) & \nearrow \text{id}_{\mathbb{1}/S^0}^{\otimes, -1} \wedge \text{id}_X & & \\
 X & \xrightarrow{\quad} & \lambda_X'^{-1} & \xrightarrow{\quad} & \mathbb{1}_{S^0} \otimes_{S^0} X \\
 & \downarrow [x] & & & & \downarrow \text{id}_{\mathbb{1}_{S^0}} \wedge [x]
 \end{array}$$

Diagram illustrating the relationships between various tensor products and smash products of  $S^0$  and  $X$ , labeled as Step 1.

The diagram shows the following maps and identities:

- $S^0 \xrightarrow{\lambda_{S^0}^{S^0, -1}} S^0 \wedge S^0$
- $S^0 \wedge S^0 \xrightarrow{\text{id}_{S^0}^{\otimes, -1}} S^0 \otimes_{S^0} S^0$
- $S^0 \otimes_{S^0} S^0 \xrightarrow{\text{id}_{\mathbb{1}/S^0}^{\otimes, -1} \wedge \text{id}_{S^0}} \mathbb{1}_{S^0} \otimes_{S^0} S^0$
- $S^0 \xrightarrow{\lambda_{S^0}'^{-1}} \mathbb{1}_{S^0} \otimes_{S^0} S^0$  (labeled (1))
- $S^0 \wedge S^0 \xrightarrow{\text{id}_{S^0} \wedge [x]} S^0 \wedge X$  (labeled (3))
- $S^0 \wedge X \xrightarrow{\text{id}_{S^0}^{\otimes, -1}} S^0 \otimes_{S^0} X$  (labeled (2))
- $S^0 \otimes_{S^0} S^0 \xrightarrow{\text{id}_{S^0} \otimes_{S^0} [x]} S^0 \otimes_{S^0} X$  (labeled (4))
- $X \xrightarrow{\lambda_X^{S^0, -1}} S^0 \wedge X$
- $X \xrightarrow{\lambda_X'^{-1}} \mathbb{1}_{S^0} \otimes_{S^0} X$  (labeled (5))
- $X \xrightarrow{\text{id}_{\mathbb{1}/S^0}^{\otimes, -1} \wedge \text{id}_X} \mathbb{1}_{S^0} \otimes_{S^0} X$
- $\mathbb{1}_{S^0} \otimes_{S^0} S^0 \xrightarrow{\text{id}_{\mathbb{1}_{S^0}} \wedge [x]} \mathbb{1}_{S^0} \otimes_{S^0} X$
- $S^0 \xrightarrow{[x]} X$