

**Step 5**

The diagram illustrates the following relationships:

- Top Row:**

$$S^0 \xrightarrow{\rho_{S^0}^{\text{Sets}_*, -1}} S^0 \wedge S^0 \xrightarrow{\text{id}_{\text{Sets}_*|S^0, S^0}^{\otimes, -1}} S^0 \otimes_{\text{Sets}_*} S^0 \xrightarrow{S^0 \text{id}_{S^0} \wedge \text{id}_{\mathbb{1}|\text{Sets}_*}^{\otimes, -1}} S^0 \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*}$$
- Bottom Row:**

$$X \xrightarrow{\rho_X^{\text{Sets}_*, -1}} X \wedge S^0 \xrightarrow{\text{id}_{\text{Sets}_*|X, S^0}^{\otimes, -1}} X \otimes_{\text{Sets}_*} S^0 \xrightarrow{X \text{id}_X \wedge \text{id}_{\mathbb{1}|\text{Sets}_*}^{\otimes, -1}} X \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*}$$
- Vertical and Diagonal Maps:**
  - $S^0 \xrightarrow{[x]} X$  (Left vertical arrow)
  - $S^0 \wedge S^0 \xrightarrow{[x] \wedge \text{id}_{S^0}} X \wedge S^0$  (Middle-left vertical arrow)
  - $S^0 \otimes_{\text{Sets}_*} S^0 \xrightarrow{[x] \otimes_{\text{Sets}_*} \text{id}_{S^0}} X \otimes_{\text{Sets}_*} S^0$  (Middle-right vertical arrow)
  - $S^0 \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*} \xrightarrow{[x] \wedge \text{id}_{\mathbb{1}_{\text{Sets}_*}}} X \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*}$  (Right vertical arrow)
  - $S^0 \xrightarrow{\rho_{S^0}^{\text{Sets}_*, -1}} S^0 \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*}$  (Top horizontal arrow)
  - $X \xrightarrow{\rho_X^{\text{Sets}_*, -1}} X \otimes_{\text{Sets}_*} \mathbb{1}_{\text{Sets}_*}$  (Bottom horizontal arrow)
- Shaded Boxes (Labels):**
  - (1)  $\rho_{S^0}^{\text{Sets}_*, -1}$
  - (2)  $\rho_X^{\text{Sets}_*, -1}$
  - (3)  $[x] \wedge \text{id}_{S^0}$
  - (4)  $[x] \otimes_{\text{Sets}_*} \text{id}_{S^0}$
  - (5)  $\text{id}_{\text{Sets}_*|X, S^0}^{\otimes, -1}$
- Other Labels:**
  - $(\cong)$  above the top horizontal arrow.
  - $(\cong)$  below the bottom horizontal arrow.