

Step 5

Diagram illustrating a commutative diagram for Step 5, showing relationships between various tensor products and smash products of S^0 and X .

The diagram consists of the following nodes and arrows:

- Top-left node:** S^0
- Top-middle node:** $S^0 \wedge S^0$
- Top-right node:** $S^0 \otimes_{\text{Sets}_*} S^0$
- Bottom-left node:** X
- Bottom-middle node:** $S^0 \wedge X$
- Bottom-right node:** $S^0 \otimes_{\text{Sets}_*} X$
- Far-right node:** $\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0$ (top) and $\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} X$ (bottom)

Key arrows and labels:

- $S^0 \xrightarrow{\lambda_{\text{Sets}_*, -1}^{S^0}} S^0 \wedge S^0$ (blue arrow)
- $S^0 \xrightarrow{\lambda_X^{\text{Sets}_*, -1}} S^0 \wedge X$ (blue arrow, highlighted with gray box)
- $S^0 \wedge S^0 \xrightarrow{\text{id}_{\text{Sets}_* | S^0, S^0}^{\otimes, -1}} S^0 \otimes_{\text{Sets}_*} S^0$
- $S^0 \wedge X \xrightarrow{\text{id}_{\text{Sets}_* | S^0, X}^{\otimes, -1}} S^0 \otimes_{\text{Sets}_*} X$ (highlighted with gray box)
- $S^0 \otimes_{\text{Sets}_*} S^0 \xrightarrow{\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} \text{id}_{S^0}^{\otimes, -1}} \mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0$ (blue arrow)
- $S^0 \otimes_{\text{Sets}_*} X \xrightarrow{\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} \text{id}_X^{\otimes, -1}} \mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} X$ (blue arrow, highlighted with gray box)
- $S^0 \xrightarrow{[x]} X$ (vertical arrow, labeled (3))
- $S^0 \wedge S^0 \xrightarrow{\text{id}_{S^0} \wedge [x]} S^0 \wedge X$ (vertical arrow, labeled (1))
- $S^0 \wedge X \xrightarrow{[x]} X$ (vertical arrow)
- $S^0 \otimes_{\text{Sets}_*} S^0 \xrightarrow{\text{id}_{S^0} \otimes_{\text{Sets}_*} [x]} S^0 \otimes_{\text{Sets}_*} X$ (vertical arrow, labeled (4))
- $S^0 \otimes_{\text{Sets}_*} X \xrightarrow{[x]} X$ (vertical arrow)
- $\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0 \xrightarrow{\text{id}_{\mathbb{1}_{\text{Sets}_*}} \wedge [x]} \mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} X$ (vertical arrow, labeled (5))
- $S^0 \xrightarrow{\lambda'_{S^0}^{-1}} \mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0$ (horizontal arrow, labeled (1))
- $S^0 \xrightarrow{\lambda'_X{}^{-1}} \mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} X$ (horizontal arrow, labeled (2))

Additional labels and structures:

- (\neq) is placed between $S^0 \wedge S^0$ and $S^0 \otimes_{\text{Sets}_*} S^0$.
- (\neq) is placed between $S^0 \wedge X$ and $S^0 \otimes_{\text{Sets}_*} X$.
- Gray boxes highlight the maps $\lambda_X^{\text{Sets}_*, -1}$, $\text{id}_{\text{Sets}_* | S^0, X}^{\otimes, -1}$, and $\mathbb{1}_{\text{Sets}_*} \otimes_{\text{Sets}_*} \text{id}_X^{\otimes, -1}$.