Step 1

$$
\begin{tikzcd}
 & S^0 \wedge S^0 \arrow[r, "\mathrm{id}^{\otimes,-1}_{\mathrm{Sets}_*|S^0,S^0}"] \arrow[dd, "\mathrm{id}_{S^0} \wedge [x]"] \arrow[dr, phantom, "(\ddagger)"] & S^0 \otimes_{\mathrm{Sets}_*} S^0 \arrow[dr, "\mathrm{id}^{\otimes,-1}_{\mathbb{1}/\mathrm{Sets}_*} \wedge \mathrm{id}_{S^0}"] \arrow[dd, "\mathrm{id}_{S^0} \otimes_{\mathrm{Sets}_*} [x]"] & \\
S^0 \arrow[ur, "\lambda^{\mathrm{Sets}_*,-1}_{S^0}"] \arrow[rrr, "\lambda'^{,-1}_{S^0}"] \arrow[ddd, "{[x]}"'] \arrow[dr, phantom, "(3)"] & \arrow[dr, phantom, "(1)"] \arrow[ddr, phantom, "(5)"] & \arrow[dr, phantom, "(4)"] & \mathbb{1}_{\mathrm{Sets}_*} \otimes_{\mathrm{Sets}_*} S^0 \arrow[dd, "\mathrm{id}_{\mathbb{1}_{\mathrm{Sets}_*}} \wedge [x]"] \\
 & S^0 \wedge X \arrow[r, "\mathrm{id}^{\otimes,-1}_{\mathrm{Sets}_*|S^0,X}"] \arrow[dr, phantom, "(2)"] & S^0 \otimes_{\mathrm{Sets}_*} X \arrow[dr, "\mathrm{id}^{\otimes,-1}_{\mathbb{1}/\mathrm{Sets}_*} \wedge \mathrm{id}_X"] & \\
X \arrow[ur, "\lambda^{\mathrm{Sets}_*,-1}_{X}"] \arrow[rrr, "\lambda'^{,-1}_{X}"] & & & \mathbb{1}_{\mathrm{Sets}_*} \otimes_{\mathrm{Sets}_*} X
\end{tikzcd}
$$