Sets

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This chapter (will eventually) contain material on axiomatic set theory, as well as a couple other things.

Contents

2.1 Sets and Functions

2.1.1 Functions

Definition 2.1.1.1.1 ► Functions

A **function** is a functional and total relation.

Notation 2.1.1.1.2 ► Additional Notation for Functions

Throughout this work, we will sometimes denote a function $f\colon X \to Y$ by

$$f \stackrel{\text{def}}{=} [\![x \mapsto f(x)]\!].$$

1. For example, given a function

$$\Phi\colon \operatorname{Hom}_{\mathsf{Sets}}(X, Y) \to K$$

taking values on a set of functions such as $\operatorname{Hom}_{\mathsf{Sets}}(X, Y)$, we will sometimes also write

$$\Phi(f) \stackrel{\text{def}}{=} \Phi([\![x \mapsto f(x)]\!]).$$

2. This notational choice is based on the lambda notation

$$f \stackrel{\text{def}}{=} (\lambda x.\, f(x)),$$

but uses a "$\mapsto$" symbol for better spacing and double brackets instead of either:

   (a) Square brackets $[x \mapsto f(x)]$;
   (b) Parentheses $(x \mapsto f(x))$;

hoping to improve readability when dealing with e.g.:

   (a) Equivalence classes, cf.:
      i. $[\![[x] \mapsto f([x])]\!]$
      ii. $[[x] \mapsto f([x])]$
      iii. $(\lambda[x].\, f([x]))$
   (b) Function evaluations, cf.:
      i. $\Phi([\![x \mapsto f(x)]\!])$
      ii. $\Phi((x \mapsto f(x)))$
      iii. $\Phi((\lambda x.\, f(x)))$

3. We will also sometimes write $-$, $-_1$, $-_2$, etc. for the arguments of a function. Some examples include:

   (a) Writing $f(-_1)$ for a function $f\colon A \to B$.
   (b) Writing $f(-_1, -_2)$ for a function $f\colon A \times B \to C$.
   (c) Given a function $f\colon A \times B \to C$, writing

$$f(a, -)\colon B \to C$$

   for the function $[\![b \mapsto f(a, b)]\!]$.
   (d) Denoting a composition of the form

$$A \times B \xrightarrow{\phi \times \mathrm{id}_B} A' \times B \xrightarrow{f} C$$

   by $f(\phi(-_1), -_2)$.

4. Finally, given a function $f\colon A \to B$, we will sometimes write

$$\mathrm{ev}_a(f) \overset{\text{def}}{=} f(a)$$

for the value of f at some $a \in A$.

For an example of the above notations being used in practice, see the proof of the adjunction

$$(A \times - \dashv \mathrm{Hom}_{\mathsf{Sets}}(A,-))\colon \quad \begin{tikzcd} \mathsf{Sets} \arrow[r, bend left, "A\times-"] \arrow[r, phantom, "\perp"] & \mathsf{Sets}, \arrow[l, bend left, "\mathrm{Hom}_{\mathsf{Sets}}(A,-)"] \end{tikzcd}$$

stated in Constructions With Sets, **??** of **??**.

2.2 The Enrichment of Sets in Classical Truth Values

2.2.1 (-2)-Categories

Definition 2.2.1.1.1 ▶ (-2)-Categories

A (-2)**-category** is the "necessarily true" truth value.[1,2,3]

[1] Thus, there is only one (-2)-category.

[2] A $(-n)$-category for $n = 3, 4, \ldots$ is also the "necessarily true" truth value, coinciding with a (-2)-category.

[3] For motivation, see [**lectures-on-n-categories-and-cohomology**].

2.2.2 (-1)-Categories

Definition 2.2.2.1.1 ▶ (-1)-Categories

A (-1)**-category** is a classical truth value.

Remark 2.2.2.1.2 ▶ Motivation for (−1)-Categories

[1](−1)-categories should be thought of as being "categories enriched in (−2)-categories", having a collection of objects and, for each pair of objects, a Hom-object $\mathrm{Hom}(x, y)$ that is a (−2)-category (i.e. trivial).
As a result, a (−1)-category C is either:[2]

1. *Empty*, having no objects.
2. *Contractible*, having a collection of objects $\{a, b, c, \ldots\}$, but with $\mathrm{Hom}_C(a, b)$ being a (−2)-category (i.e. trivial) for all $a, b \in \mathrm{Obj}(C)$, forcing all objects of C to be uniquely isomorphic to each other.

Thus there are only two (−1)-categories up to equivalence:

1. The (−1)-category false (the empty one);
2. The (−1)-category true (the contractible one).

[1]For more motivation, see [**lectures-on-n-categories-and-cohomology**].
[2]See [**lectures-on-n-categories-and-cohomology**].

Definition 2.2.2.1.3 ▶ The Poset of Truth Values

The **poset of truth values**[1] is the poset $(\{\mathsf{true}, \mathsf{false}\}, \preceq)$ consisting of:

- *The Underlying Set.* The set $\{\mathsf{true}, \mathsf{false}\}$ whose elements are the truth values true and false.
- *The Partial Order.* The partial order

$$\preceq\colon \{\mathsf{true}, \mathsf{false}\} \times \{\mathsf{true}, \mathsf{false}\} \longrightarrow \{\mathsf{true}, \mathsf{false}\}$$

  on $\{\mathsf{true}, \mathsf{false}\}$ defined by[2]

$$\begin{aligned} \mathsf{false} \preceq \mathsf{false} &\stackrel{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{false} &\stackrel{\text{def}}{=} \mathsf{false}, \\ \mathsf{false} \preceq \mathsf{true} &\stackrel{\text{def}}{=} \mathsf{true}, \\ \mathsf{true} \preceq \mathsf{true} &\stackrel{\text{def}}{=} \mathsf{true}. \end{aligned}$$

[1]*Further Terminology:* Also called the **poset of** (−1)**-categories**.
[2]This partial order coincides with logical implication.

We also write $\{\mathrm{t},\mathrm{f}\}$ for the poset $\{\mathrm{true},\mathrm{false}\}$.

Proposition 2.2.2.1.5 ▶ Cartesian Closedness of the Poset of Truth Values

The poset of truth values $\{\mathrm{t},\mathrm{f}\}$ is Cartesian closed with product given by[1]

$$
\begin{aligned}
\mathrm{t}\times\mathrm{t} &= \mathrm{t},\\
\mathrm{t}\times\mathrm{f} &= \mathrm{f},\\
\mathrm{f}\times\mathrm{t} &= \mathrm{f},\\
\mathrm{f}\times\mathrm{f} &= \mathrm{f},
\end{aligned}
$$

and internal Hom $\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}$ given by the partial order of $\{\mathrm{t},\mathrm{f}\}$, i.e. by

$$
\begin{aligned}
\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t},\mathrm{t}) &= \mathrm{t},\\
\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t},\mathrm{f}) &= \mathrm{f},\\
\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f},\mathrm{t}) &= \mathrm{t},\\
\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f},\mathrm{f}) &= \mathrm{t}.
\end{aligned}
$$

[1] Note that $\times$ coincides with the "and" operator, while $\mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}$ coincides with the logical implication operator.

Proof 2.2.2.1.6 ▶ Proof of ??

Existence of Products

We claim that the products $\mathrm{t}\times\mathrm{t}$, $\mathrm{t}\times\mathrm{f}$, $\mathrm{f}\times\mathrm{t}$, and $\mathrm{f}\times\mathrm{f}$ satisfy the universal property of the product in $\{\mathrm{t},\mathrm{f}\}$. Indeed, suppose we have diagrams of the form

$$
\begin{tikzcd}
 & P_1 \arrow[dl, bend right, "p_1^1"'] \arrow[dr, bend left, "p_2^1"] & \\
\mathrm{t} & \mathrm{t}\times\mathrm{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathrm{t}
\end{tikzcd}
\quad
\begin{tikzcd}
 & P_2 \arrow[dl, bend right, "p_1^2"'] \arrow[dr, bend left, "p_2^2"] & \\
\mathrm{t} & \mathrm{t}\times\mathrm{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathrm{f}
\end{tikzcd}
\quad
\begin{tikzcd}
 & P_3 \arrow[dl, bend right, "p_1^3"'] \arrow[dr, bend left, "p_2^3"] & \\
\mathrm{f} & \mathrm{f}\times\mathrm{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathrm{t}
\end{tikzcd}
\quad
\begin{tikzcd}
 & P_4 \arrow[dl, bend right, "p_1^4"'] \arrow[dr, bend left, "p_2^4"] & \\
\mathrm{f} & \mathrm{f}\times\mathrm{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathrm{f}
\end{tikzcd}
$$

where the pr_1 and pr_2 morphisms are the only possible ones (since $\{\mathrm{t},\mathrm{f}\}$ is posetal). We claim that there are unique morphisms making the diagrams

$$\begin{tikzcd} & P_1 \arrow[ld, bend right, "p_1^1"'] \arrow[d, dashed, "\exists!"] \arrow[rd, bend left, "p_2^1"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_2 \arrow[ld, bend right, "p_1^2"'] \arrow[d, dashed, "\exists!"] \arrow[rd, bend left, "p_2^2"] & \\ \mathsf{t} & \mathsf{t}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd} \quad \begin{tikzcd} & P_3 \arrow[ld, bend right, "p_1^3"'] \arrow[d, dashed, "\exists!"] \arrow[rd, bend left, "p_2^3"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{t} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{t} \end{tikzcd} \quad \begin{tikzcd} & P_4 \arrow[ld, bend right, "p_1^4"'] \arrow[d, dashed, "\exists!"] \arrow[rd, bend left, "p_2^4"] & \\ \mathsf{f} & \mathsf{f}\times\mathsf{f} \arrow[l, two heads, "\mathrm{pr}_1"] \arrow[r, two heads, "\mathrm{pr}_2"'] & \mathsf{f} \end{tikzcd}$$

commute. Indeed:

1. If $P_1 = \mathsf{t}$, then $p_1^1 = p_2^1 = \mathrm{id}_\mathsf{t}$, so there's a unique morphism from P_1 to t making the diagram commute, namely id_t.
2. If $P_1 = \mathsf{f}$, then $p_1^1 = p_2^1$ are given by the unique morphism from f to t, so there's a unique morphism from P_1 to t making the diagram commute, namely the unique morphism from f to t.
3. If $P_2 = \mathsf{t}$, then there is no morphism p_2^2.
4. If $P_2 = \mathsf{f}$, then p_1^2 is the unique morphism from f to t while $p_2^2 = \mathrm{id}_\mathsf{f}$, so there's a unique morphism from P_2 to f making the diagram commute, namely id_f.
5. The proof for P_3 is similar to the one for P_2.
6. If $P_4 = \mathsf{t}$, then there is no morphism p_1^4 or p_2^4.
7. If $P_4 = \mathsf{f}$, then $p_1^4 = p_2^4 = \mathrm{id}_\mathsf{f}$, so there's a unique morphism from P_4 to f making the diagram commute, namely id_f.

This finishes the existence of products part of the proof.

Cartesian Closedness

We claim there's a bijection

$$\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(A \times B, C) \cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(A, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(B, C)\big),$$

natural in $A, B, C \in \{\mathsf{t}, \mathsf{f}\}$. Indeed:

- For $(A, B, C) = (\mathrm{t}, \mathrm{t}, \mathrm{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t} \times \mathrm{t}, \mathrm{t}) &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{t}) \\ &= \{\mathrm{id}_{\mathsf{true}}\} \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{t}) \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}\big(\mathrm{t}, \mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathrm{t}, \mathrm{t}, \mathrm{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t} \times \mathrm{t}, \mathrm{f}) &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{f}) \\ &= \emptyset \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{f}) \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}\big(\mathrm{t}, \mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{f})\big).\end{aligned}$$

- For $(A, B, C) = (\mathrm{t}, \mathrm{f}, \mathrm{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t} \times \mathrm{f}, \mathrm{t}) &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{t}) \\ &\cong \mathrm{pt} \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{t}) \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}\big(\mathrm{f}, \mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathrm{t}, \mathrm{f}, \mathrm{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t} \times \mathrm{f}, \mathrm{f}) &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{f}) \\ &\cong \{\mathrm{id}_{\mathsf{false}}\} \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{f}) \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}\big(\mathrm{t}, \mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{f})\big).\end{aligned}$$

- For $(A, B, C) = (\mathrm{f}, \mathrm{t}, \mathrm{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f} \times \mathrm{t}, \mathrm{t}) &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{t}) \\ &\cong \mathrm{pt} \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{f}, \mathrm{t}) \\ &\cong \mathrm{Hom}_{\{\mathrm{t},\mathrm{f}\}}\big(\mathrm{f}, \mathbf{Hom}_{\{\mathrm{t},\mathrm{f}\}}(\mathrm{t}, \mathrm{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{t}, \mathsf{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{t}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \{\mathrm{id}_{\mathsf{false}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{t}, \mathsf{f})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{t})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{f}, \mathsf{t}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{pt}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{t})\big).\end{aligned}$$

- For $(A, B, C) = (\mathsf{f}, \mathsf{f}, \mathsf{f})$, we have

$$\begin{aligned}\mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}\times\mathsf{f}, \mathsf{f}) &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &= \{\mathrm{id}_{\mathsf{false}}\}\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\\ &\cong \mathrm{Hom}_{\{\mathsf{t},\mathsf{f}\}}\big(\mathsf{f}, \mathbf{Hom}_{\{\mathsf{t},\mathsf{f}\}}(\mathsf{f}, \mathsf{f})\big).\end{aligned}$$

Since $\{\mathsf{t}, \mathsf{f}\}$ is posetal, naturality is automatic (Categories, **??** of **??**). □ ./../pictures/trans-fl

2.2.3 0-Categories

Definition 2.2.3.1.1 ▶ 0-Categories

A 0-**category** is a poset.[1]

[1]*Motivation:* A 0-category is precisely a category enriched in the poset of (-1)-categories.

Definition 2.2.3.1.2 ▶ 0-Groupoids

A 0-**groupoid** is a 0-category in which every morphism is invertible.[1]

[1]That is, a *set*.

2.2.4 Tables of Analogies Between Set Theory and Category Theory

Here we record some analogies between notions in set theory and category theory. The analogies relating to presheaves relate equally well to copresheaves, as the opposite X^{op} of a set X is just X again.

Remark 2.2.4.1.1 ► Basic Analogies Between Set Theory and Category Theory

The basic analogies between set theory and category theory are summarised in the following table:

Set Theory	Category Theory
Enrichment in $\{\mathrm{true}, \mathrm{false}\}$	Enrichment in Sets
Set X	Category $\mathcal{C}$
Element $x \in X$	Object $X \in \mathrm{Obj}(\mathcal{C})$
Function $f \colon X \to Y$	Functor $F \colon \mathcal{C} \to \mathcal{D}$
Function $X \to \{\mathrm{true}, \mathrm{false}\}$	Copresheaf $\mathcal{C} \to \mathsf{Sets}$
Function $X \to \{\mathrm{true}, \mathrm{false}\}$	Presheaf $\mathcal{C}^{\mathrm{op}} \to \mathsf{Sets}$

Remark 2.2.4.1.2 ► Analogies Between Set Theory and Category Theory: Powersets and Categories of Presheaves

The category of presheaves $\mathsf{PSh}(\mathcal{C})$ and the category of copresheaves $\mathsf{CoPSh}(\mathcal{C})$ on a category $\mathcal{C}$ are the 1-categorical counterparts to the powerset $\mathcal{P}(X)$ of subsets of a set X. The further analogies built upon this are summarised in the following table:

Set Theory	**Category Theory**
Powerset $\mathcal{P}(X)$	Presheaf category $\mathsf{PSh}(\mathcal{C})$
Characteristic function $\chi_{\{x\}} \colon X \to \{\mathsf{t}, \mathsf{f}\}$	Representable presheaf $h_X \colon \mathcal{C}^{\mathsf{op}} \hookrightarrow \mathsf{Sets}$
Characteristic embedding $\chi_{(-)} \colon X \hookrightarrow \mathcal{P}(X)$	Yoneda embedding よ $\colon \mathcal{C}^{\mathsf{op}} \hookrightarrow \mathsf{PSh}(\mathcal{C})$
Characteristic relation $\chi_X(-_1, -_2) \colon X \times X \to \{\mathsf{t}, \mathsf{f}\}$	Hom profunctor $\mathrm{Hom}_{\mathcal{C}}(-_1, -_2) \colon \mathcal{C}^{\mathsf{op}} \times \mathcal{C} \to \mathsf{Sets}$
The Yoneda lemma for sets $\mathrm{Hom}_{\mathcal{P}(X)}(\chi_x, \chi_U) = \chi_U(x)$	The Yoneda lemma for categories $\mathrm{Nat}(h_X, \mathcal{F}) \cong \mathcal{F}(X)$
The characteristic embedding is fully faithful, $\mathrm{Hom}_{\mathcal{P}(X)}\big(\chi_x, \chi_y\big) = \chi_X(x, y)$	The Yoneda embedding is fully faithful, $\mathrm{Nat}(h_X, h_Y) \cong \mathrm{Hom}_{\mathcal{C}}(X, Y)$
Subsets are unions of their elements $U = \bigcup_{x \in U} \{x\}$ or $\chi_U = \operatorname*{colim}_{\chi_x \in \mathcal{P}(U)} (\chi_x)$	Presheaves are colimits of representables, $\mathcal{F} \cong \operatorname*{colim}_{h_X \in \int_{\mathcal{C}} \mathcal{F}} (h_X)$

Remark 2.2.4.1.3 ▶ Analogies Between Set Theory and Category Theory: Categories of Elements

We summarise the analogies between un/straightening in set theory and category theory in the following table:

Set Theory	**Category Theory**
Assignment $U \mapsto \chi_U$	Assignment $\mathcal{F} \mapsto \int_{\mathcal{C}} \mathcal{F}$
Un/straightening isomorphism $\mathcal{P}(X) \cong \mathsf{Sets}(X, \{\mathsf{t}, \mathsf{f}\})$	Un/straightening equivalence $\mathsf{PSh}(\mathcal{C}) \overset{\text{eq.}}{\cong} \mathsf{DFib}(\mathcal{C})$

Remark 2.2.4.1.4 ▶ Analogies Between Set Theory and Category Theory: Functions Between Powersets and Functors Between Presheaf Categories

We summarise the analogies between functions $\mathcal{P}(A) \to \mathcal{P}(B)$ and functors $\mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$ in the following table:

Set Theory	**Category Theory**
Direct image function $f_* \colon \mathcal{P}(X) \to \mathcal{P}(Y)$	Direct image functor $F_* \colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$
Inverse image function $f^{-1} \colon \mathcal{P}(Y) \to \mathcal{P}(X)$	Inverse image functor $F^* \colon \mathsf{PSh}(\mathcal{D}) \to \mathsf{PSh}(\mathcal{C})$
Direct image with compact support function $f_! \colon \mathcal{P}(X) \to \mathcal{P}(Y)$	Direct image with compact support functor $F_! \colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$

Remark 2.2.4.1.5 ▶ Analogies Between Set Theory and Category Theory: Relations and Profunctors

We summarise the analogies between functions relations and profunctors in the following table:

Set Theory	**Category Theory**
Relation $R \colon X \times Y \to \{\mathrm{t}, \mathrm{f}\}$	Profunctor $\mathfrak{p} \colon \mathcal{D}^{\mathsf{op}} \times \mathcal{C} \to \mathsf{Sets}$
Relation $R \colon X \to \mathcal{P}(Y)$	Profunctor $\mathfrak{p} \colon \mathcal{C} \to \mathsf{PSh}(\mathcal{D})$
Relation as a cocontinuous morphism of posets $R \colon (\mathcal{P}(X), \subset) \to (\mathcal{P}(Y), \subset)$	Profunctor as a colimit-preserving functor $\mathfrak{p} \colon \mathsf{PSh}(\mathcal{C}) \to \mathsf{PSh}(\mathcal{D})$

Appendices

A Other Chapters

Preliminaries

1. Introduction

Sets

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7. Relations
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Category Theory

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Extra Part

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