

**Step 2**

$$\begin{array}{ccccc}
 & S^0 \wedge S^0 & \xrightarrow{\text{id}_{\text{Sets}_*|S^0, S^0}^{\otimes, -1}} & S^0 \otimes_{\text{Sets}_*} S^0 & \xrightarrow{\text{id}_{1_{\text{Sets}_*}}^{\otimes, -1} \wedge \text{id}_{S^0}} & 1_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0 \\
 & \uparrow \lambda_{S^0}^{\text{Sets}_*, -1} & & & & \\
 S^0 & \xrightarrow{\lambda_{S^0}'^{-1}} & 1_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0 & & & \\
 & \downarrow \text{id}_{S^0} \wedge [x] & (1) & \downarrow \text{id}_{S^0} \otimes_{\text{Sets}_*} [x] & & \\
 & S^0 \wedge X & \xrightarrow{\text{id}_{\text{Sets}_*|S^0, X}^{\otimes, -1}} & S^0 \otimes_{\text{Sets}_*} X & & \\
 & \uparrow \lambda_X^{\text{Sets}_*, -1} & (2) & & & \\
 X & \xrightarrow{\lambda_X'^{-1}} & 1_{\text{Sets}_*} \otimes_{\text{Sets}_*} X & & & \\
 & \downarrow [x] & & & & \\
 & X & & & & \\
 & & & & & \downarrow \text{id}_{1_{\text{Sets}_*}} \wedge [x] \\
 & & & & & 1_{\text{Sets}_*} \otimes_{\text{Sets}_*} X
 \end{array}$$

Diagram illustrating the relationship between various tensor products and smash products of  $S^0$  and  $X$ , showing the commutativity of the diagram (Step 2).

Key components and maps:

- $S^0$  and  $X$  are the base objects.
- $S^0 \wedge S^0$ ,  $S^0 \wedge X$ ,  $S^0 \otimes_{\text{Sets}_*} S^0$ ,  $S^0 \otimes_{\text{Sets}_*} X$ ,  $1_{\text{Sets}_*} \otimes_{\text{Sets}_*} S^0$ , and  $1_{\text{Sets}_*} \otimes_{\text{Sets}_*} X$  are the target objects.
- Maps include:
  - $\lambda_{S^0}^{\text{Sets}_*, -1}$  (diagonal)
  - $\lambda_{S^0}'^{-1}$  (horizontal)
  - $\lambda_X^{\text{Sets}_*, -1}$  (diagonal)
  - $\lambda_X'^{-1}$  (horizontal)
  - $[x]$  (vertical)
  - $\text{id}_{S^0} \wedge [x]$  (vertical)
  - $\text{id}_{S^0} \otimes_{\text{Sets}_*} [x]$  (vertical)
  - $\text{id}_{1_{\text{Sets}_*}} \wedge [x]$  (vertical)
  - $\text{id}_{\text{Sets}_*|S^0, S^0}^{\otimes, -1}$  (horizontal)
  - $\text{id}_{\text{Sets}_*|S^0, X}^{\otimes, -1}$  (horizontal)
  - $\text{id}_{1_{\text{Sets}_*}}^{\otimes, -1} \wedge \text{id}_{S^0}$  (diagonal)
  - $\text{id}_{1_{\text{Sets}_*}}^{\otimes, -1} \wedge \text{id}_X$  (diagonal)
- Labels (1), (2), (3), (4), and (5) indicate specific points or regions in the diagram.