Hubble's Constant Assignment

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Introduction

I have divided the entire solution to three parts:

1. Finding the values for v_r and for d
2. Plotting v_r and d in a graph and finding the best fit line.
3. Questions.

Finding the values

The equation connecting redshift, v_r and c is as follows:

$$1 + z = \sqrt{\frac{1 + \frac{v_r}{c}}{1 - \frac{v_r}{c}}}$$

Simplifying this equation we get,

$$(1+z)^2 = \frac{1 + \frac{v_r}{c}}{1 - \frac{v_r}{c}} \tag{1}$$

$$1 + z^2 + 2z = \frac{c + v_r}{c - v_r} \tag{2}$$

$$\frac{z^2 + 2z + 2}{z^2 + 2z} = \frac{c}{v_r} \tag{3}$$

$$v_r = \frac{c.z(z+2)}{z^2 + 2z + 2} \tag{4}$$

The equation for distance in terms of absolute magnitude and apparent magnitude is as follows:

$$m - M = 5logd - 5$$

Simplifying this equation we get,

$$m - M + 5 = 5logd \tag{5}$$

$$\frac{m - M + 5}{5} = logd \tag{6}$$

$$d = 10^{\frac{m-M+5}{5}} \tag{7}$$

The data given about the galaxies, I imported into Python and configured everything into standard lists and got the following result (Relative Velocity is the wrong - It should be Radial Velocity):

Galaxies	Redshift	Absolute Magnitude	Apparent Magnitude
UGC858	0.00795	-21.9	10.9
UGC1554	0.00703	-20.7	11.4
UGC1633	0.01426	-22.2	11.4
UGC3834	0.0068	-20.9	11.4
UGC4888	0.00756	-21.2	11.3
UGC5183	0.00447	-21.3	10.3
UGC6376	0.01426	-23.6	10.4
UGC7122	0.00607	-20.7	11.8
UGC7393	0.01412	-21.6	12.4
UGC7412	0.0038	-20.6	10.2
UGC8033	0.00832	-21.8	10.8
UGC9358	0.00637	-21.8	10.2
UGC11218	0.00496	-21.4	10.0

Then I wrote another Python program to find the values of v_r and v_d using the above formulas and got the following result:

Galaxies	Relative Velocity(in km/s)	Distance(in Mpc)
UGC858	2375.5199222194137	36.3078054770101
UGC1554	2101.587046898659	26.302679918953814
UGC1633	4247.50091735968	52.480746024977336
UGC3834	2033.0641592735774	28.840315031265998
UGC4888	2259.4272031449254	31.622776601683793
UGC5183	1338.0028948091826	20.892961308540407
UGC6376	4247.50091735968	63.09573444801929
UGC7122	1815.4733661998516	31.622776601683793
UGC7393	4206.096779463108	63.09573444801929
UGC7412	1137.8340155792064	14.45439770745925
UGC8033	2485.616996402942	33.11311214825914
UGC9358	1904.9135877022582	25.118864315095824
UGC11218	1484.309805168314	19.054607179632445

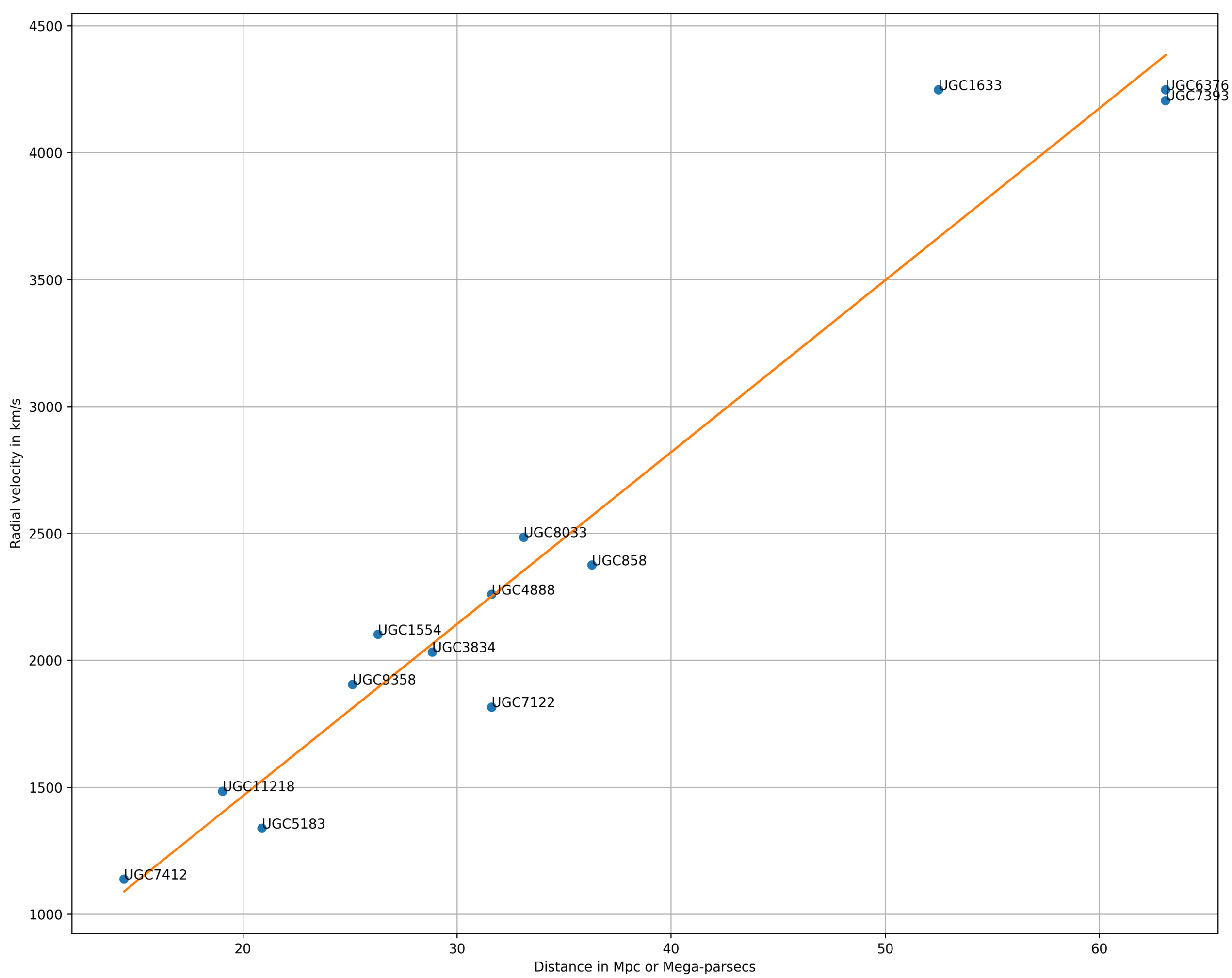
Plotting

Given the above values, I decided to plot the values using the computer. I made a program using python's numpy and matplotlib libraries I made a graph. I have attached the code along with this:

```
// Graph.py
import numpy as np
import array as ar
import matplotlib.pyplot as plt
from matplotlib.offsetbox import AnchoredText

galaxies = [
    "UGC858","UGC1554","UGC1633","UGC3834","UGC4888","UGC5183","UGC6376","UGC7122",
    "UGC7393","UGC7412","UGC8033","UGC9358","UGC11218"
]

z = arr.array('d', [
    0.00795,0.00703,0.01426,0.00680,0.00756,0.00447,0.01426,0.00607,0.01412,0.00380,
    0.00832,0.00637,0.00496
])

m = arr.array('d', [
    10.9,11.4,11.4,11.4,11.3,10.3,10.4,11.8,12.4,10.2,10.8,10.2,10.0
])

M = arr.array('d', [
    -21.9,-20.7,-22.2,-20.9,-21.2,-21.3,-23.6,-20.7,-21.6,-20.6,-21.8,-21.8,-21.4
])

velocity = []
distance = []

l = len(galaxies)
for i in range(l):
    t1 = (z[i]*300000*(z[i]+2))/(z[i]*z[i] + 2*z[i] + 2)
    velocity.append(t1)
    t2 = (10**((m[i]-M[i]+5)/5))/(10**6)
    distance.append(t2)

y = np.array(velocity)
x = np.array(distance)
m, b = np.polyfit(x, y, 1)
```

```
print(m)
print(b)
plt.grid()
plt.plot(x, y, 'o')
plt.plot(x, m*x+b)
for i in range(l):
    plt.annotate(galaxies[i], (x[i], y[i]))
plt.xlabel("Distance in Mpc or Mega-parsecs")
plt.ylabel("Radial velocity in km/s")
plt.show()
```

This returned the following output:



Questions

0.1 What is the general mathematical expression for a straight line?

Solution: The general mathematical expression for a straight line is:

$$y = mx + c$$

where m is the slope of the line and c is the y-intercept. □

0.2 What is the equation for the best-fit straight line that you obtained?

Solution: y = 67.75175389567737x + 109.18424886047119 □

0.3 What does the slope of that line correspond to?

Solution: The slope of a line show $\frac{\Delta y}{\Delta x}$, change in y with respect to x. In this line, the slope shows the change in radial velocity corresponding to the distance. Here, the slope gives the Hubble's constant. □

0.4 The value of the Hubble's constant you obtained in units of km/s/Mpc correct to two decimal places

Solution: The Hubble's constant I obtained from my graph is 67.75. □

0.5 Based on the value of the Hubble's constant obtained, estimate the age of the universe. Write your answers in units of Gigayear (i.e., billion years).

Solution: I found the ratio between km and Mpc to be $3.2408.10^{-20}$. Using this I did $\frac{1}{H_0}$ to find the age of the universe.
I got the age of the universe as:
14431496562.431839 years
$= 14.43149 \times 10^9 years$
$\approx$ 14.43 Gigayears □