L12. KINEMATICS I

MOTION IN ONE and THREE DIMENSIONS

READING ASSIGNMENT:
Textbook: Chapters 2 and 4

KINEMATICS I
Definitions for 1-D and 3-D motions
Kinematic Equations for 1-D motion
Free Fall
Motion Diagrams

2 DEMONSTRATIONS

Special Presentation: Bio-Bullet 1:
Velocity and Acceleration in Animal Kingdom

15 Problems

Kinematics describes motion while ignoring the agents that caused the motion.

Concept of Motion and Independence of Motion Principle

Motion is a change of position of a body (or body elements) with respect to some selected frame of reference.

Independence of motion principle:
Any complex 3D motion may be described by considering motion along each of the spatial coordinates independently.

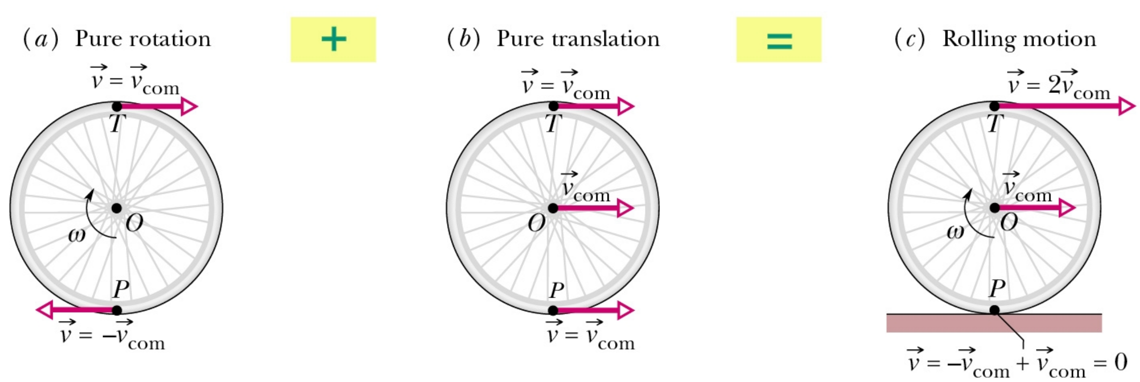
Types of Motion

Translational Motion: all elements of a body undergo the same change of position

Rotational Motion: the body changes orientation in space.

Vibrational Motion: the body oscillates about the equilibrium position.

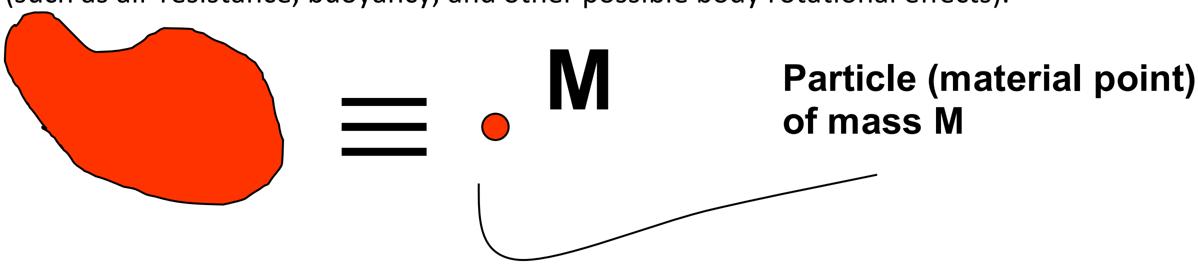
More complex types of motion may be analyzed as a combination of two or more simple motions:



Concept of a Particle (Material Point)

We can effectively describe the TRANSLATIONAL MOTION of a body using the idealization of **a particle** (dimensionless point of the same mass as a body (material point))

By doing this, we are neglecting various effects, which are related to the body's physical dimensions (such as air-resistance, buoyancy, and other possible body rotational effects).



For many of the problems discussed in our introductory course, this is an excellent approximation and we can describe many aspects of motion using this **simplified model.**

Fundamental definitions for 1-D motion:

Position

Displacement $\overrightarrow{\Delta x} = \overrightarrow{x_f} - \overrightarrow{x_i}$

$$\overrightarrow{v(t)} = \lim_{\Delta t \to 0} \overrightarrow{v_{avg}} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{\overrightarrow{dx}}{dt} = \dot{\vec{x}} \qquad \vec{v}_{avg} = \langle \vec{v} \rangle = \frac{displacement}{time\ interval} = \frac{\overrightarrow{\Delta x}}{\Delta t} \qquad \text{average speed} = \frac{\text{total distance}}{\text{time interval}} = \frac{s}{\Delta t}$$

$$\overrightarrow{a_{avg}} = \langle \vec{a} \rangle = \frac{change\ in\ velocity}{time\ interval} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{v_2} - \overrightarrow{v_1}}{t_2 - t_1} \qquad a(t) = \lim_{\Delta t \to 0} a_{avg} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

Fundamental definitions for 3-D motion (vectors!):

Position:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} = x \hat{i} + y \hat{j} + z \hat{k} = x \hat{i}_x + y \hat{i}_y + z \hat{i}_z$$

$$\vec{r} = (r_x, r_y, r_z) = (x, y, z)$$

Displacement:

$$\overrightarrow{\Delta r} = \overrightarrow{r_2} - \overrightarrow{r_1}$$

$$\overrightarrow{\Delta r} = (x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}) - (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k})$$

$$\overrightarrow{\Delta r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

Average velocity $\vec{v}_{avg} = \langle \vec{v} \rangle = \frac{displacement}{time\ interval} = \frac{\overrightarrow{\Delta x}}{\Delta t}$

Instantaneous velocity definition

$$\overrightarrow{v(t)} = \lim_{\Delta t \to 0} \overrightarrow{v_{avg}} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\overrightarrow{dr}}{dt} = \dot{\vec{r}}$$

Average acceleration definition

$$\overrightarrow{a_{avg}} = \langle \vec{a} \rangle = \frac{change\ in\ velocity}{time\ interval} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{v_2} - \overrightarrow{v_1}}{t_2 - t_1}$$

Instantaneous acceleration definition

$$\overrightarrow{a(t)} = \lim_{\Delta t \to 0} \overrightarrow{a_{avg}} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{dv}}{dt} = \frac{d}{dt}\left(\frac{\overrightarrow{dr}}{dt}\right) = \frac{d^2 \vec{r}}{dt^2}$$

KINEMATIC EQUATIONS:
<u>Kinematic Equations – valid only for particles moving under constant acceleration!</u>

Given our general definitions of instantaneous velocity and acceleration, it is possible to obtain a set of kinematic equations when acceleration is constant.

$$v_f = at + v_i$$
$$x_f = \frac{1}{2}at^2 + v_i t + x_i$$
$$v_f{}^2 - v_i{}^2 = 2a(x_f - x_i)$$
$$x_f = \frac{1}{2}(v_i + v_f)t + x_i$$

- These kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration!
- One may need to use two or more equations to solve a problem.
- Typically, there is more than one way to solve a problem.

When we deal with 3-D motion with constant acceleration, the same equations describe each coordinate separately!

<u>An Example of 1-D motion with constant acceleration</u>: **<u>Free Fall</u>**

A ***free falling object*** is any object moving under the influence of gravity alone.
$a_y = -g$;

v_{yi}, the initial velocity may be positive, negative or 0

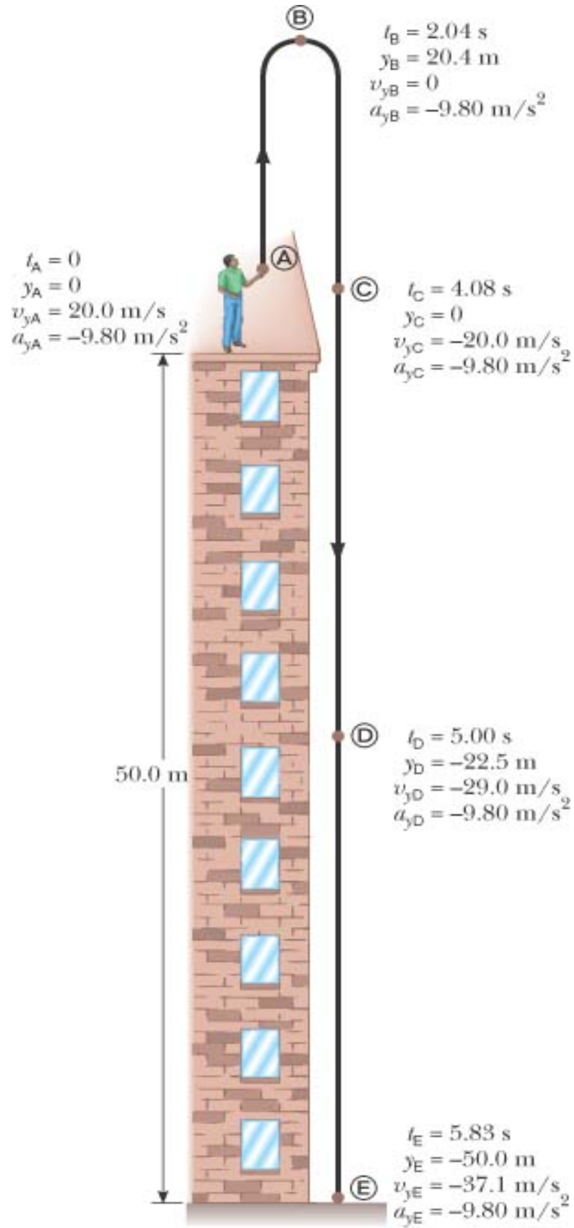
- Dropped – released from rest $v_{yi} = 0$ *
- Thrown downward $v_{yi} < 0$ *
- Thrown upward $v_{yi} > 0$ *

* NOTE: these signs are only valid if we pick the direction of the y axis to be "up"!

<u>Exercise:</u> Attributing signs to the kinematic equations: general approach

FREE FALL TEXTBOOK ANALYSIS:

- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s2
  - g decreases with increasing altitude
  - g varies with latitude
  - 9.80 m/s2 is the average at the Earth's surface
- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with $a_y = -g$ = -9.80 m/s2



Example from the textbook

- Initial velocity at A is upward (+) and acceleration is g (-9.8 m/s2)
- At B, the velocity is 0 and the acceleration is g (-9.8 m/s2)
- At C, the velocity has the same magnitude as at A, but is in the opposite direction
- The displacement is –50.0 m (it ends up 50.0 m below its starting point)

Exercise:
READING OF THE x(t), v(t), AND a(t) MOTION DIAGRAMS

Suggested Problems

1. Particle moves from initial position r_i to final position r_f in 2.0s

$$\vec{r_i} = 2\hat{i} - 6\hat{j} + \hat{k} \;\; ; \;\; \vec{r_f} = 4\hat{i} + 7\hat{j} - \hat{k}$$

a) Find average velocity vector during this move, b) Find the magnitude of the average velocity vector

2. Time dependent vector r(t) describes the position of the Particle A at any instant of time:

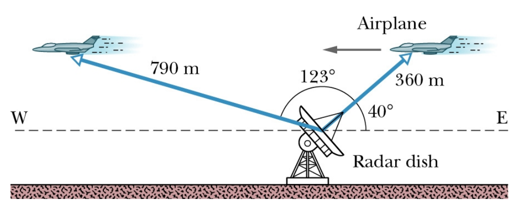
$$\overrightarrow{r_i(t)} = 2t^2\,\hat{i} - 6t\,\hat{j} + 2\sqrt{t}\,\hat{k}$$

a) Find x ,y, z components of instantaneous velocity v(t) vector.
b) Find magnitude of the velocity vector at time t=2s

3. An ion's position vector is initially **r** = 5.0**i** - 6.0**j** +2.0**k**, and 10s later it is **r** = -2.0**i** + 8.0**j** -2.0**k**, all in meters. What is its average velocity during this time interval?

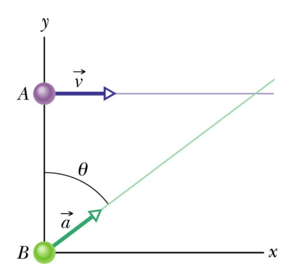
4. The position of an electron is given by **r** = (3.00t) **i** – (4.00t) **j** +(2.00)**k**,/ t in second and r in meters /
a) What is the electron's velocity **v**?
b) What is electron's velocity at t=2s (in unit-vector notation)?
c) What is a magnitude and angle relative to the positive direction of the x axis?

5 A radar station detects an airplane approaching directly from the east. At first observation the range to the plane is 360m at 40° above the horizon.
The airplane is tracked for another 123o in the vertical east-west plane, the range at final contact being 790 m.
Find the displacement of the airplane during the period of observation.



6 An object A moves along the x-axis according to the equation: x(t) = 3.0t^2+3.3t+10.4, where x is in meters, and t is in seconds. The other object (object B) moves along the x-axis with constant speed +9 m/s.
When does A have the same instantaneous velocity as B ? State your answer to nearest 0.01s.

7 Particle A moves along the line y=30 m with a constant velocity v of magnitude 3.0m/s and directed parallel to the positive x axis. Particle B starts at origin with zero speed and constant acceleration a (of magnitude0.40m/s2) at the same instant that particle A passes the y axis. What angle θ between a and the positive axis would result in a collision between these two particles?



8 The ball is shot vertically with the initial velocity of 7 m/s from the 13m level. What is the maximum height it will reach? Neglect the air resistance, and take g=9.8 m/s^2. Give the answer to the nearest tenth of a meter.

9 The ball is shot vertically with the initial velocity of 3 m/s from the 10m level. How long it will take for the ball to reach the ground? Neglect the air resistance, and take g=9.8 m/s^2. Give the answer to the nearest hundredth of a second.

10 A window cleaner uses an external elevator while moving between the floors of a skyscraper in downtown Manhattan. At the moment when the elevator is at 5 m above the ground, moving up with the speed of 15 m/s and acceleration of 1 m/s^2, a small brush is dropped from the elevator. How long before it will hit the ground? Round your answer to the nearest tenth of a second.

11 Motion of the rocket is given by the following equation: $x=2+1t+7t^2+3t^3$ where t is in seconds, x is in meters. At 17s a small object detaches itself from the body of the rocket. What is the initial height of this object? Round your answer to the nearest tenth of a meter.

12 The maximum height from which a person can safely jump is 2.45 m.
What is the maximum allowable landing speed for a parachutist

13 A body covers 64% of the total distance fallen in the last second. From what height did it fall?

14 Cheetah can reach 105km/h in 2 s and maintain this speed for 15 s. After this time it must rest. An antelope can reach 90km/h in 2s and sustain it for a long time. Suppose they are initially separated by 100m and the antelope reacts in 0.5s.
a) can cheetah get the antelope?
b) if not, how close does it get?
Assume both start from the rest.

15 A climber can estimate the height of a cliff by dropping a stone and noting the time at which he hears the impact on the ground. Suppose this time is 2.5s. Find the height of the cliff under the following conditions:
a) by assuming the speed of sound is large enough to be ignored
b) by taking the speed of the sound to be 330m/s.