$A = 19500 = P\left(1 + \frac{30}{100}\right)$

$P = \frac{19500 \times 100}{130}$

$P = 15000$

II Method

P — ₹15000

A_3 — 19500 } 3000 → 1500

A_5 — 22500 } ↓ 2 yrs, ↓ 1 yr

$R \rightarrow \frac{1500}{15000} \times 100 = 10\%$

5. Nitin borrowed some money @6% p.a for first 3 years, 9% p.a for the next 5 yrs, 13% p.a for the period beyond 8 years. If the total interest paid by him at the end of eleven years is Rs. 8160. Find sum.

Soln:

100 —(6% for 3 yrs)→ 118 —(9% for 5 yrs)→ 163 —(13% for 3 yrs)→ 202

×80 ↓ 8000

100+102 ×80 ↓ 8160

Exercises

1.

	P	I	A
@R%	P	$\frac{PTR}{100}$	$P + \frac{PTR}{100}$
@R%+1%	P	$\frac{PT[R+1]}{100} = \frac{PTR}{100} + \frac{PT1}{100}$	$P + \frac{PTR}{100} + \frac{PT1}{100}$

Eg1: ₹800 becomes ₹956 in 3 yrs at a certain rate of simple interest. If the rate of interest is increased by 4%. What amount will ₹800 become in 3 yrs?

(a) ₹1020-80 (b) ₹1025 (c) ₹1052 (d) Data Inadequate

Soln: $A = 956 + \frac{(800)(3)(4)}{100}$

$= 956 + 96$

$= 1052$

R% = 20% p.a

	P	I
* P	100	20
3P	300	60

2. The simple interest on a sum of money will be ₹600 after 10 years. If the principal is trebled after 5 yrs what will be the total interest at the end of the tenth years?

(a) 600 (b) 900 (c) 1200 (d) 1500

Soln: $I = TR/yr$

$600 = (10)R/1yr$

$R/yr = 60$

If $P \rightarrow 3P$ after 5 yrs

$I \rightarrow 3I = 1800$

So $\Rightarrow (60)(5) + (180)(5)$

$I_{total} = 300 + 900$

$I = 1200$

3. Nitin borrowed some money . . . . . .

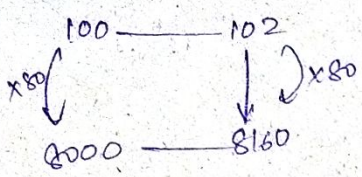
. . . . . Same que in Eg:5 [Except I] . . .

. . . . . how much he borrow?

(a) ₹8000 (b) ₹10000 (c) ₹12000 (d) Data Inadequate

Soln: $\frac{P}{100}$ $\frac{I}{(6\% \times 3 + 9\% \times 5 + 13\% \times 3) = 102\%}$

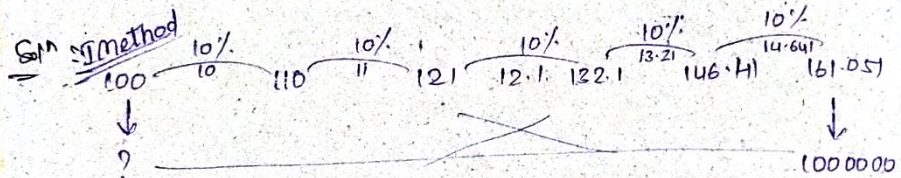
$\frac{I}{102}$



4. Leila aspires to buy a car worth Rs. 10,00,000 after 5 years. What is the minimum amount in Rupees that she should deposit now in a bank which offers 10% annual rate of interest, if the interest was compounded anually?

(a) 500,000 (b) 6,21,000 (c) 6,66,667 (d) 7,50,000

Soln: I method



$$\Rightarrow \frac{100 \times 1000000}{161.051} \approx \frac{1000}{161} \times 100000$$

approxly 6,21,000

II method

$$P\left(1+\frac{10}{100}\right)^5 = 1000000$$

$$P\left(\frac{11}{10}\right)^5 = 1000000$$

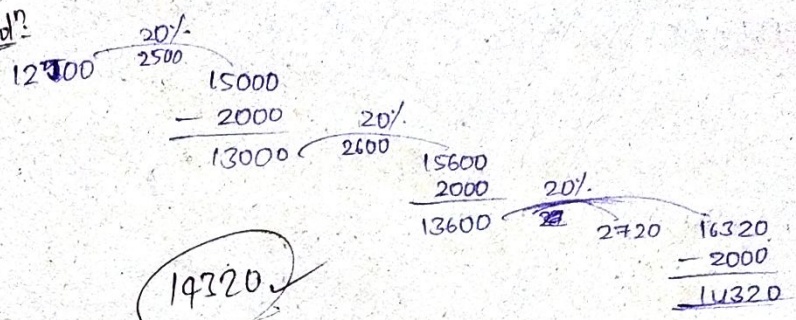
$$P \times \frac{161051}{100000} = 1000000$$

$$P \approx 6,21,000$$

5. A man borrows ₹ 12,500 at 20% compound interest At the end of every year he pays 2000 as part repayment. How much does he still owe after three such installments

(a) 12000 (b) 12864 (c) 15600 (d) None

Soln:



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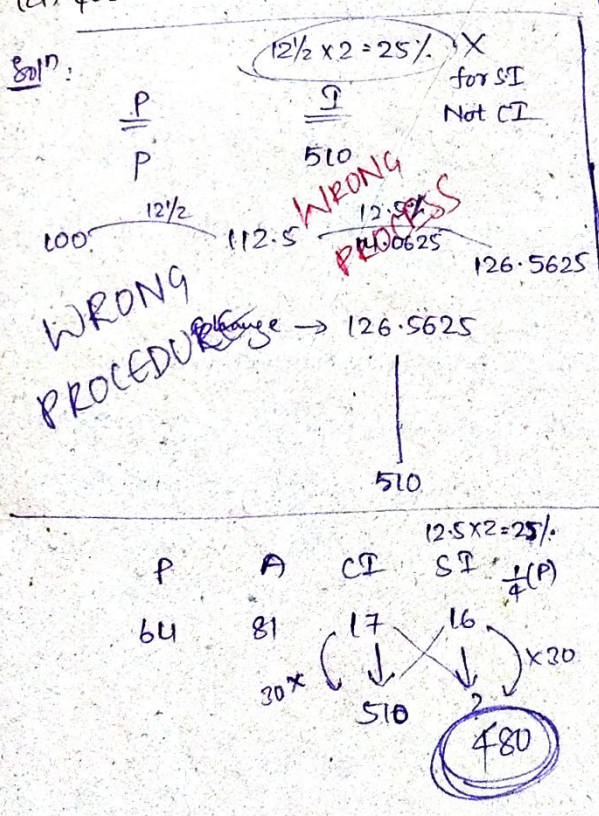
6. If the compound interest on a sum for 2 yrs at 12½ % per annum is ₹ 510, the simple interest on the same sum at the same rate for the same period of time is

(a) 400 (b) 450 (c) 460 (d) 480

Soln:



COMPOUND INTEREST:-

$A = P\left(1+R\%\right)^n$

$A = P\left(1+R\%\right)^n$

$A = P\left(1+\frac{1}{8}\right)^2$

$A = P\left(\frac{9}{8}\right)^2$

$A = P\left(\frac{81}{64}\right)$

P : A = 64 : 81

8. ALLIGATION & MIXTURES

Introduction

Always $30 \leq x \leq 90$

Mixing	rice ₹90/kilo	rice ₹30/kilo	
1 : 1	$\frac{90(1)+30(1)}{2}$		= 60/kilo
1 : 3	$\frac{90(1)+30(3)}{4}$		= 45/kilo

Exercises

1. The cost of Type1 rice is ₹15 per kg and Type2 rice is ₹20 per kg. If both Type1 and Type2 are mixed in the rate of 2 : 3, then the price per kg of the mixed variety of rice is :

(a) ₹18 (b) ₹18.50 (c) ₹19 (d) ₹19.50

Sol^n:- $\frac{(2)(15)+(3)(20)}{5} = 18/kg$

2. A family consists of grandparents, parents and three grandchildren. The average age of grandparents is 67 yrs, that of the parents is 35 yrs and that of the grandchildren is 6 yrs. What is average age of the family?

(a) 28 (b) 31 (c) 32 (d) None

Sol^n: $= \frac{(67)(2)+(35)(2)+(6)(3)}{7} = 31\frac{5}{7}$ yrs

3. A library has an average of 510 visitors on Sundays and 240 on other days. The avg no. of visitors per day in a month of 30 days beginning with a Sunday is

(a) 250 (b) 276 (c) 280 (d) 285

Sol^n: $\Rightarrow \frac{(5)(510)+(25)(240)+(120)(240)}{30}$

$\Rightarrow \frac{2550+6000}{30}$

$= 285$

4. Three papers were set in an examination and the maximum marks per paper were in the ratio 1 : 2 : 2 respectively. If a student obtained 50% in 1st paper, 60% in the 2nd and 65% in the third, what percent did he obtain overall?

(a) 58.3%
(b) 66.66%
(c) 33.33%
(d) 60%

Sol^n: $\Rightarrow \frac{(100)(\frac{1}{2})+(200)(\frac{2}{5})+(200)(\frac{13}{20})}{500} \times 100$

$= \frac{50+120+130}{500} \times 100$

$= \frac{300}{500} \times 100$

$= 60\%$

5. One percent of the people country X are taller than 6ft. Two percent of the people of country Y are taller than 6ft. There are thrice as many people in country X as in

country Y. Taking both countries together, what is the percentage of people taller than 6 ft

(a) 3 (b) 2.5 (c) 1.5 (d) 1.25

Soln:

$$\% \Rightarrow \frac{\frac{1}{100}(300) + \frac{2}{100}(100)}{400} \times 100$$

$$\Rightarrow \frac{5}{400} \times 100$$

$$\Rightarrow 1.25$$

6. The ratio of the number of boys and girls who participated in an examination is 4:3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed is 90. The percentage of boys who passed is ——

(a) 90.00 (b) 80.50 (c) 55.50 (d) 72.50

Soln:

$$\frac{80}{100} = \frac{400\left(\frac{x}{100}\right) + 300\left(\frac{90}{100}\right)}{700}$$

$$560 = 4x + 270$$

$$4x = 290$$

$$x = 72.50$$

7. One quality of wheat at ₹9.30 per kg mixed with another quality at a certain rate in the ratio 8:7. If the mixture so formed be worth ₹10 per kg, what is the rate per kg of second quality of wheat?

(a) 10.30 (b) 10.60 (c) 10.80 (d) 11

Soln:

$$\frac{8(9.30) + 7(x)}{15} = 10$$

$$x = \frac{150 - 74.4}{7}$$

$$= \frac{75.6}{7}$$

$$x = 10.80/\text{kg}$$

8. Tea worth ₹126 per kg and ₹135 per kg are mixed with a third variety in ratio 1:1:2. If the mixture is worth ₹153 per kg, price of third variety/kg will be:

Soln: (a) 169.5 (b) 170 (c) 175.5 (d) 180

Soln:

$$153 = \frac{126 + 135 + 2x}{4}$$

$$\frac{612 - 261}{2} = x$$

$$x = 351/2 = 175.5$$

9. There are two varieties of rice costing Rs.98 per kg and Rs.71 per kg. In what proportion must these two varieties of rice be mixed such that the resultant mixture is worth Rs.86 per kg?

(a) 5:4 (b) 4:5 (c) 1:1 (d) 2:1

Soln:

$$86 = \frac{98 + x(71)}{1+x}$$

$$86x + 86 = 71x + 98$$

$1st = 12$

$t = \frac{12}{15}$

$t = \frac{4}{5}$

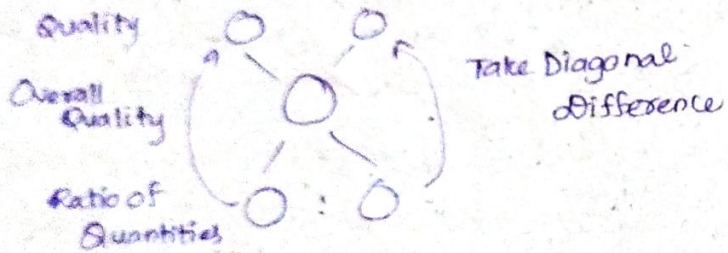
So $1 : t \Rightarrow 5 : 4$

10. There are two varieties of rice costing Rs.98 per kg

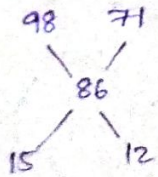
. . . . . .

. . . . . Same previous Question

Sol": Solving Using Alligation Diagram method



So in this problem ⟶



$\Rightarrow 5 : 4$

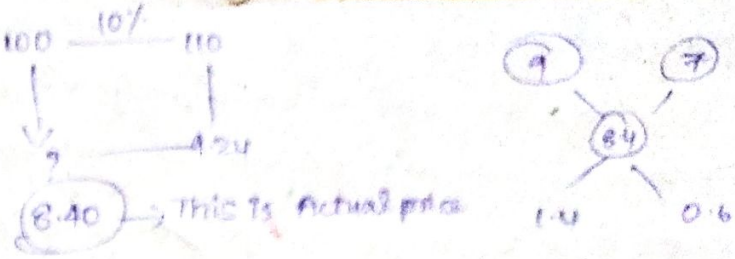
11. How many kilograms of sugar costing ₹9 per kg must be mixed with 27 kg of sugar costing ₹7 per kg So that there may be a gain of 10% by selling the mixture at ₹9.24 per kg?

(a) 36 kg (b) 42 kg

(c) 54 kg (d) 63 kg

Sol": $(9.24) = \frac{b(9) + 27(7)}{27 + b}$

Not Actual price

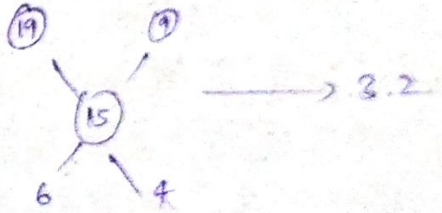


Mixed proportions ⟶ 7 : 3

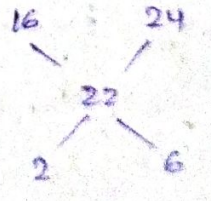
$\Rightarrow 63 : 27$

12. Gold is 19 times heavier than water and copper is 9 times heavier than water. In what proportions must these two metals be mixed to prepare an alloy, which is 15 times heavier than water

(a) 3:4 (b) 3:2 (c) 4:3 (d) 2:1



13. A goldsmith has two qualities of gold one is of 16 carats purity and the other is of 24 carats purity. In what proportion must these two varieties be mixed to prepare a gold ornament of 22 carats purity?

Sol": 

1 : 3

(a) 3:4

(b) 3:2

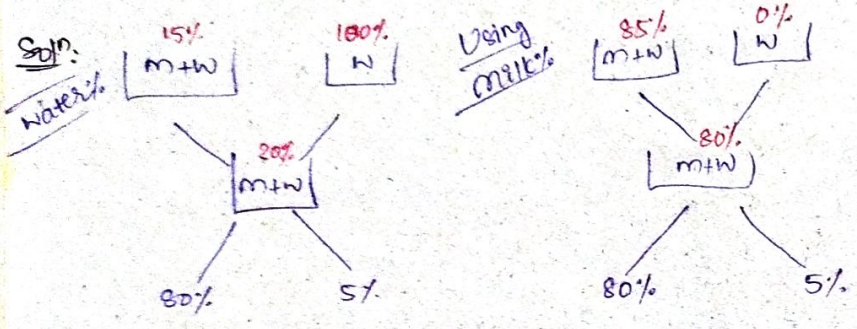
(c) 1:3

(d) 2:1

14. A mixture of 48 lts of milk and water contains 15% of water. How much more water is to be added to it to make the water 20% of resulting mixture

(a) 3.75 lts (b) 3.25 lts (c) 3.50 lts ~~(d)~~ 3.00 lts

Soln:



16 : 1

↓

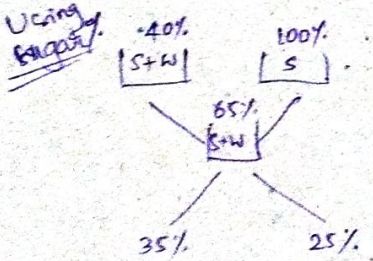
48 : 3

16 : 1

48 : 3

15. 420 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 65% in solution?

Soln: (a) 275 gm (b) 150 gm ~~(c)~~ 300 gm (d) 450 gm



300 gms must be added

7 : 5

420 : 300

16. A solution of sugar syrup has 15% sugar. Another solution has 5% sugar. How many litres of the second solution must be added to 20 lts of first solution to make a soln of 10% sugar?

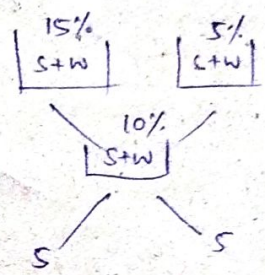
(a) 10 lts

(b) 5 lts

(c) 15 lts

~~(d)~~ 20 lts

Soln: Sugar%



20 lts must be added

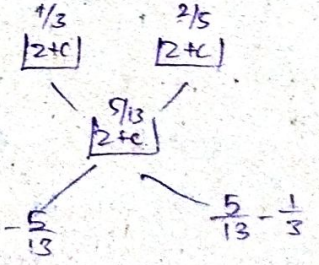
1 : 1

20 : 20

17. In an alloy, zinc and copper are in the ratio 1:2. In second alloy they are in ratio 2 : 3. In what ratio should these two alloys be mixed to form a new alloy in which two elements are in ratio 5:8

~~(a)~~ 3:10 (b) 10 : 3 (c) 6 : 7 (d) 7 : 6

Soln: Zinc%

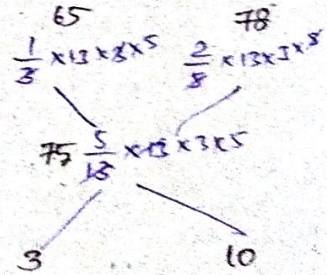


$\frac{2}{5} - \frac{5}{13}$ $\qquad$ $\frac{5}{13} - \frac{1}{3}$

$\frac{1}{65} : \frac{2}{39}$

39 : 130

3 : 10



In order to eliminate fractions and make it easy, So multiplied like this

18. A container originally contains 10 lts of pure spirit from this container 1 ltr of spirit is replaced with 1 ltr of water. Subsequently, 1 ltr of mixture again replaced with 1 ltr of water and this process repeated 1 more time. How much spirit left in container?

(a) 7.58
(b) 7.84
(c) 7
(d) 7.29

Soln: 10S + 0W
↓
9S + 1W
↓
8.1S + 1.9W
↓
(8.1 − 0.81) + (1.9 − 0.19 + 1)W
↓
7.29S + 2.71W ✓

* If there is x ltrs of spirit, and y ltrs of spirit replaced by water for n times then

Spirit left $= x\left(\frac{x-y}{x}\right)^n$

⇓

x = 10 y = 1 n = 3

$S \Rightarrow 10\left(\frac{9}{10}\right)^3$

$= 10 \times \frac{729}{1000}$

S = 7.29 lts

19. 8 ltrs are drawn from a cask full of wine and is then filled with water. This operation performed 3 more times. The ratio of the quantity of wine now left in cask to that of water is 16:65. How much wine did the cask hold originally?

(a) 18 (b) 24 (c) 32 (d) 42

Soln: x = ? y = 8 n = 4

Given $\frac{S}{x-S} = \frac{16}{65}$

$\frac{x}{S} = \frac{65+16}{16}$

$S = \frac{16}{81}x$

$S = x\left(\frac{x-y}{x}\right)^4$

$\frac{16}{81} = \left(\frac{x-8}{x}\right)^4$

$\left(\frac{2}{3}\right)^4 = \left(\frac{x-8}{x}\right)^4$

2x = 3x − 24

x = 24

9. TIME AND WORK

Intro Work → 40 chairs

Efficiency / Capacity = 4 chairs/day

Time taken $= \frac{40}{4} = 10$ days

So **Time = $\frac{\text{Work}}{\text{Efficiency}}$** $T = \frac{W}{E}$

Basic Questions:

Q. A, B & C can do a work in 10 days, 15 days & 12 days respectively. How long will it take together to do a work?

Soln: Let us say → Work = 60 units [∵ LCM(10, 12, 15) = 60]

So now A → 10 days → 6 units/day

B → 15 days → 4/day

C → 12 days → 5/day

Totally A + B + C ⇒ 15/day

So $\frac{\text{Work}}{\text{Time}}$ = Efficiency

$\frac{60}{\text{Time}} = 15/\text{day}$ $\quad T \propto \frac{1}{E}$

Time = 4 days

Q) P can complete work in 12 days working 8 hrs/day. Q can complete same work in 8 days working 10 hrs/day. If both P and Q work together 8 hrs/day, in how many days can they complete the work?

(a) 5 5/11 (b) 5 6/11 (c) 7 (d) 4

Soln: my method

P ⟶ 96 hrs ⟶ $\frac{W}{96}$/hr ⟶ $\frac{W}{12}$/day

Q ⟶ 80 hrs ⟶ $\frac{W}{80}$/hr ⟶ $\frac{W}{10}$/day

In hours ⟶ $\frac{W}{96} + \frac{W}{80} = W\left(\frac{80+96}{80 \cdot 96}\right) = W\left(\frac{176}{80 \cdot 96}\right) = W\left(\frac{11}{480}\right)$

88

40 48

10

// ⟶ $\frac{W}{12} + \frac{W}{10} = W\left(\frac{22}{120}\right) = W\left(\frac{11}{60}\right)$

Time ⟶ $\frac{60}{11} = 5\frac{5}{11}$

Sir's method

P ⟶ 12 days - 8 hrs/day

Q ⟶ 8 days - 10 hrs/day ⟶ Manipulate ⟶ 80 hrs = 10 days 8 hrs/day

P+Q ⟶ ? 8 hrs/day

So now ⟶ P 12 days

Q 10 days

P+Q = ? Let W = 60

So P ⟶ 5/day Q ⟶ 6/day

Total = 11/day

So Time = $5 \cdot \frac{5}{11}$

Q. 5 skilled workers can build a wall in 20 days & semiskilled workers // // // 25 days & 10 unskilled // // // 30 days. If a team has 2 skilled, 6 semiskilled and 5 unskilled workers, how long takes to build wall?

(a) 20 days (b) 10 (c) 16 (d) 15

Soln: my method

5 S ⟶ 20 days 1 S ⟶ $\frac{W}{100}$

8 SS ⟶ 25 days 1 SS ⟶ $\frac{W}{200}$

10 U ⟶ 30 days 1 U ⟶ $\frac{W}{300}$

One day ⟶ $\frac{2W}{100} + \frac{6W}{200} + \frac{5W}{300}$

$\frac{6W + 9W + 5W}{300} = \frac{20W}{300} = \frac{W}{15}$

They takes 15 days

Q. A alone can do 80% of work in 20 days. A & B together completed remaining work in 4 days. How long will B alone can do it?

Soln: A ⟶ $\left(\frac{80W}{100}\right)$ ⟶ 20 days

1 day ⟶ $\frac{80W}{100 \times 20} = \frac{W}{25}$

(A+B) ⟶ $\frac{20W}{100}$ ⟶ 4 days

A+B ⇒ $\frac{20W}{100 \times 4} = \frac{W}{20}$

B = $W\left(\frac{1}{20} - \frac{1}{25}\right) = \frac{W}{100}$ per day

So 100 days

Q. A and B can do a work in 12 days, B and C can do the same work in 15 days, A and C together in 20 days. How long (days) it takes A, B and C together to finish it?

(a) 10 (b) 9 (c) 5 (d) None

Soln: $A+B \rightarrow \frac{W}{12}$/day

$B+C \rightarrow \frac{W}{15}$/day

$A+C \rightarrow \frac{W}{20}$/day

$A+B+C = \frac{1}{2}\left[\frac{W}{12}+\frac{W}{15}+\frac{W}{20}\right]$

$= \frac{W}{2}\left[\frac{5+4+3}{60}\right]$

$A+B+C = \frac{W}{10}$ per day

So 10 days

Alternate Days

Q. A and B alone can do a piece of work in 8 and 18 days respectively. In how many days the work will be completed if they both work on alternate days starting with B?

(a) 6 5/9 days (b) 5 days (c) 10 7/9 days (d) 10 9/2 days

Soln: My method $A \rightarrow \frac{W}{8}$/day $B \rightarrow \frac{W}{18}$/day.

BABAB.....

first 2 days $\rightarrow \frac{W}{8}+\frac{W}{18} = W\left(\frac{26}{18\times 8}\right)^{13}_{9}$

$= W\left(\frac{13}{72}\right)$

If they do not mention take default ABABAB....

Same work every 2 days $\Rightarrow$ means $\rightarrow \left(\frac{65}{72}\right)W$ after 10 days

Next day $B \rightarrow \frac{4W}{72} \rightarrow \left(\frac{69}{72}\right)W$

12th day $A \rightarrow \frac{9W}{72} \rightarrow \left(\frac{3}{72}\right)+\left(\frac{6}{72}\right)$

After 12th day $\rightarrow$ Only $\frac{1}{3}$rd of day is enough.

So total $\rightarrow 11\frac{1}{3}$ days

If starts with A then,

11th day $\rightarrow \frac{68W}{72}+\frac{9W}{72}$

$\left(\frac{65+7}{72}W\right) + \frac{2W}{72}$

Work done in $\frac{7}{9}$ part of the day

So Answer would be $10\frac{7}{9}$ days

Join/Left

Q. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last?

(a) 6 days (b) 10 days (c) 15 days (d) 20 days

Soln: $X \rightarrow \frac{W}{20}$ $Y \rightarrow \frac{W}{12}$

LCM (12, 20) = 60

W = 60 units

X = 3 units Y = 5 units

X for 4 days = 12 units

X+Y = 8/day So Time = $\frac{48}{8}$ = 6 days

Total $\rightarrow$ 4 days + 6 days = 10 days

Q. A, B and C can alone complete a work in 10, 12 and 15 days respectively. A and C started the work and after working for 4 days, A left and B joined. In how many days the total work was completed?

(a) $6\frac{5}{9}$ days (b) $6\frac{2}{9}$ days (c) 6 days (d) $5\frac{4}{9}$ days (e) $7\frac{2}{9}$ days

Soln: A → 6 units/day W = 60 units
B → 5
C → 4

A + C → 10 units completed in 4 days

B + C → 9 units → 2 days + $\frac{2}{9}$ days $\frac{20}{9} = 2 + \frac{2}{9}$

Total = $6\frac{2}{9}$ days

Work & Wages

Wages ∝ Work

Q. A alone can do a piece of work in 6 days and B alone in 8 days. A & B undertook to do it for Rs. 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?

(a) 375 (b) 400 (c) 600 (d) 800

Soln: A → $\frac{24}{6} = 4$ u/D B → 3 u/D Total → 24 u

A + B → 7 u/D

A + B + C → 8 u/D means C → 1 u/D

So A : B : C = 4 : 3 : 1

C = $\left(\frac{1}{8}\right)(3200) = 400$

Efficiency Method

Q. A and B can do a job together in 7 days. A is $1\frac{3}{4}$ as efficient as B. The same job can be done by A alone in

(a) $9\frac{1}{3}$ days (b) 11 days (c) $12\frac{1}{4}$ days (d) $16\frac{1}{3}$ days

Soln: B → $\frac{4}{4}$ (E_B) A → $\frac{7}{4}$ (E_A)

$E_A = \frac{7}{4} E_B$

So $E_A : E_B = 7 : 4$

Sir Method

A + B → 7 days

We know Time $\propto \frac{1}{\text{Efficiency}}$

$\frac{T_{A+B}}{T_A} = \frac{E_A}{E_A + E_B}$

$\frac{7}{T_A} = \frac{4}{11}$

$T_A = 11$ days

let for one day → 11 u/D

Work = 7 days → 77 u

We took $E_A = 7$ u

So $T = \frac{77}{7}$

$= 11$ days

Q. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is:

(a) 15 days (b) 16 days (c) 18 days (d) 25 days

Soln: $T_S = 20$ days $E_S : E_T = 4 : 5$

$T \propto \frac{1}{E}$

$\frac{T_S}{T_T} = \left(\frac{E_S}{E_T}\right) \Rightarrow \frac{20}{T_T} = \frac{4}{5}$

$T_T = 16$ days

Chain Rule:

Work ∝ Members/Machines Used (∵ Remaining All are constant)

Work ∝ Hours/day } Work ∝ Time

Work ∝ Days

Work ∝ Efficiency

So Combindly → W ∝ MDHE

$$\text{So } \frac{M_1D_1H_1E_1}{W_1} = \frac{M_2D_2H_2E_2}{W_2}$$

(or)

$$\frac{M_1T_1E_1}{W_1} = \frac{M_2T_2E_2}{W_2}$$

Problems

Q: 6 boys working 5 hours/day to bind 1500 Note's in 8 days. At the same amount of binding notes. In what period of time can 4 boys bind 1600 Note's for working 8 hours a day?

(A) 7 days (B) 8 days (C) 11 days (D) 14 days

Soln:

$$\frac{M_1E_1T_1}{W_1} = \frac{M_2E_2T_2}{W_2}$$

$$\frac{6(5)(8)}{1500} = \frac{4(8)(D)}{1600}$$

D = 8 days

Q. A contract is to be completed in 56 days if 104 identical robots work, each working at 8 hours/day. After 30 days, 2/5 of the work is completed. How many additional robots should be deployed so that the work will be completed in the scheduled time, each robot now working 9 hrs/day?

(A) 160 (B) 150 (C) 24 (D) 56

Soln:

$$\frac{(104)(30)(8)E}{\frac{2W}{5}} \quad \frac{2}{5}W = (104)(30)(8)$$

$$\frac{x(26)(9)}{\frac{3W}{5}}$$

$$\frac{104 \times 30 \times 8}{2} = \frac{x \times 26 \times 9}{3}$$

x = 160

Total ⇒ 160

Additional → 160 − 104 = 56

Men Days

10 Men → 10 days (×2)

5 Men → ? 20 days (×5)

2 Men → 50 days (÷2)

20 Men → 5 days (÷2, ×2, ÷5)

More men → less time

less men → More time

Q. 20 men can complete a piece of work in 14 days. 7 men started the work and after 20 days, 7 more men joined the work. In how many days the remaining work will be completed?

(a) 18 (b) 20 (c) 8 (d) 12 (e) 10

Soln: 20 men → 14 days 1 men = $\frac{W}{14\times20}$

7 men → $\frac{7W}{14\times20}\times20 = \frac{W}{2}$

14 men → $\frac{14W}{14\times20}\times T = \frac{W}{2}$

$T = 10$ days

Total → 20+10 = 30 days

Q. A particular no. of guys finished a piece of work in 80 days. If there were 10 guys more, that could be completed in 10 days less. How many guys were exactly there?

(a) 90 (b) 55 ~~(c)~~ 70 (d) 50

MD means mendays

Soln: $(xM)(80D) \Rightarrow 80x$ MD

$(x+10M)(70D) \rightarrow (70)(x+10)$ MD

$80x = 70x + 700$

$10x = 700$

$x = 70$ guys

Q. 4 men and 6 women can complete a work in 8 days, while 2 men and 7 women can complete it in 10 days. In how many days will 10 women complete it?

(A) 35 ~~(B)~~ 40 (C) 45 (D) 50

Soln: 4M + 6W → 8 days

3M + 7W → 10 days ⇒ Work = Time × Efficiency

$8(4E_m+6E_w) = 10(3E_m+7E_w)$

$16E_m + 24E_w = 15E_m + 35E_w$

$E_m = 11E_w$

$\frac{E_m}{E_w} = \frac{11}{1}$ ⇒ $E_m \rightarrow 11$ U/D $E_w = 1$ U/D

So Work = (8)(44+6) = 400 units

So ⇒ $\frac{400}{10} = 40$ days

Time & Work - Men Days

Q: 10 men can complete a work in 10 days, while 12 women can complete a work in 10 days. In how many days will 15 men and 6 women complete it?

~~(A)~~ 5 (B) 10 (C) 15 (D) 20

Soln: 10 men — 10 days 1 Men ⇒ $\frac{W}{100}$/day

12 women — 10 days 1 Women ⇒ $\frac{W}{120}$/day

So $15M+6W \Rightarrow \frac{15W}{100}\times\frac{3}{20} + \frac{6W}{120}\times\frac{}{20}$/day

SIR METHOD

10 men = 12 women

5 men = 6 women

here 15 men + 6 women

= 15 men + 5 men

= 20 men

↓

only 5 days ✓

$\frac{4W}{20}$/day

$\frac{W}{5}$/day

So 5 days

10. PIPES & CISTERNS

Introduction:

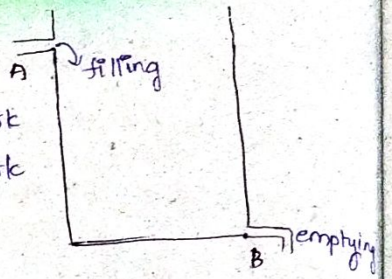
11/08/2020 : Tuesday

Similar to Time & Work

But here Negative work Introduced

Here pipe A can fill tank ⟹ +ve work

pipe B can empty ⟹ -ve work



Fill / Empty

Q. pipe A and B can fill the tank 20 min and 30 min and pipe C can empty the full tank in 40 min. If all pipes are opened, find the time taken to fill the tank?

(a) 15 1/7 min (b) 17 1/7 min (c) 12 1/7 min (d) 11 1/7 min

Soln: $E_A = \frac{W}{20}$ $E_B = \frac{W}{30}$ $E_C = \frac{-W}{40}$

Let W = 120

A → 6 B → 4 C → 3

So Work done in 1 min ⟹ 6 + 4 - 3 = 7

Here W → Tank capacity

Total $\frac{120}{7}$ = 17 mins $\frac{1}{7}$ min

Q. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

(A) 6 min to empty (B) 6 min to full

(C) 9 min to empty (D) 9 min to fill

Soln: $A \to \frac{W}{10}$ $B \to \frac{W}{6}$ Let W = 60

A → +6 B → -10

So for a min ⟹ -4

Given Tank filled → $\left(\frac{2}{5}\right)(60) = 24$

So $-\left(\frac{24}{4}\right) = 6$ min to empty

In the previous question, if A can fill in 6 minutes and B can empty it in 10 minutes. Then?

A → +10 B → -6

for a min → +4

But To get filled → $\left(\frac{3}{5}\right)(60) = 36$

So $+\left(\frac{36}{4}\right) = 9$ min to fill

Q. Through an inlet, a tank takes 8 hrs to get filled up. due to leak in the bottom, it takes 2 hrs more to get it filled completely. If the tank is full, how much time will the leak take to empty it?

(A) 16 hrs (B) 20 hrs (C) 32 hrs (D) 40 hrs.

Soln: Only A → $\frac{W}{8}$ $A + B = \frac{W}{10}$ Here Let W = 40

$-B = \frac{W}{8} - \frac{W}{10}$ A → 5 A + B = 4

B = -1

$B = \frac{-W}{40}$

Takes 40 hrs for B

Q. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of the solution R in the liquid in the tank after 3 minutes?

(A) 11/16 (B) 3 6/11 (C) 1 6/11 ~~(D) 6/11~~

Soln: $A \rightarrow \frac{60}{30} \rightarrow +2$ — P

$B \rightarrow \frac{60}{20} \rightarrow +3$ — Q

$C \rightarrow \frac{60}{10} = +6$ — R

Let W = 60 lts

for 1 min → 6+3+2 = 11

for 3 min → (33 lts) So → $\frac{6+6+6}{33} \rightarrow \frac{18}{33} \rightarrow \frac{6}{11}$

Time to empty

Q. Two pipes A and B can each fill a tank in 40 and 60 minutes respectively. There is an outlet C. If all the 3 pipes are opened, the tank will be filled in 30 minutes. How much time will it take for C alone to empty the full tank?

(A) 124 (B) 122 (C) 121 ~~(D)~~ 120 min

Soln: $A \rightarrow \frac{W}{40}$ $B \rightarrow \frac{W}{60}$ $C \rightarrow ?$

Let W = 120

$A + B + C = \frac{W}{30}$

$A \rightarrow +3$ $B \rightarrow +2$

$A + B + C = +4$

So $C = -1$ So $\frac{W}{-1} \rightarrow -120$

C takes 120 min to empty alone

Tank Capacity

Q. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 6 gallons per minute All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is?

(A) 210 gallons (B) 220 ~~(C)~~ 240 (D) 230

Soln? $A \rightarrow \frac{W}{20}$ per min $B \rightarrow \frac{W}{24}$ per min $C \rightarrow 6$ per min

$A + B + C = \frac{W}{15}$ per min

$\frac{W}{20} + \frac{W}{24} - 6 = \frac{W}{15}$

$\frac{6W + 5W - 8W}{120} = 6$

$\frac{W}{40} = 6$

$W = 240$ gallons

More fill pipe - Less Time

Q. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?

~~(A)~~ 4 hrs 45 min ~~(B)~~ 3 hrs 45 min

(C) 5 hrs 45 min (D) 5 hrs 45 min

Soln: 3 hrs over → Half tank filled $\frac{W}{6}$/hour

So now Total 4 taps → $\frac{2}{\cancel{4}W}$/hour

$\left(\frac{2W}{3}\right)x = \frac{W}{2}$

$x = \frac{3}{4}$ hrs = 45 min

So 3 hrs 45 min fills full tank

Efficient/Fast

Q. One pipe can fill a tank 3 times as fast as another pipe. If together the two pipes can fill the tank in 36 min. then, the slower pipe alone will be able to fill the tank in ?

(A) 120 min (B) 130 min (C) 140 min (D) 144 min

Soln:

$3E + E = \frac{W}{36}$

$4E = \frac{W}{36}$

Slower → $E = \frac{W}{144}$ → 144 min

faster → $3E = \frac{W}{48}$ → 48 min

$T \propto \frac{1}{E}$

$\frac{T_s}{T_s + T_f} = \frac{E_s + E_f}{E_s}$

$\frac{T_s}{36} = \frac{4E}{E}$

$T_s = 144$ min

Alternate Hours

Q. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, that tank will be full in ?

(A) 4 hr (B) 7 hr (C) 5 hr (D) 6 hr

Soln: A → 5 B → 4 C → 3 v/hr W → 60

This repeats $\begin{pmatrix} 5+4 \\ 5+3 \end{pmatrix} \begin{matrix} \rightarrow 9 \\ \rightarrow 8 \end{matrix} = 17$

$\frac{60}{17}$ = 3 times $\frac{9}{17}$

So 6 hrs + 1 hour

Total → 7 hours

Open/Close

Q. Two pipes P and Q can fill a tank in 6 hours and 4 hours respectively, while a third pipe R can empty the tank in 12 hours. Initially, P and R are open for 4 hours, then P is closed and Q is opened. After 6 more hours R is closed. The total time taken to fill the tank (in hrs) is ——

(A) 13.5 (B) 14.5 (C) 15.5 (D) 16.5

Soln: P → +6 Q → +4 R → −3 W → 36

① P & R open 4 hrs ⇒ $4(6-3) = 12$

② Q & R open 6 hrs → $6(4-3) = 6$

③ R closed → $(+4)(x) = 18$

$x = 4.5$

Totally → 4 + 6 + 4.5 = 14.5 hours

11. TIME, SPEED AND DISTANCE

Introduction:

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$T \propto \frac{1}{S}$ (D is constant)

generally speed → kmph, m/s, mph

$1 \text{ kmph} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{60 \times 60 \text{ sec}} = \frac{5}{18}$ m/sec

* 1 kmph = $\frac{5}{18}$ m/s

* 1 m/s = $\frac{18}{5}$ kmph

Q. A train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75kms. How long did it take to reach its destination 600 km from the starting point?

(A) 6 hrs 21 min (B) 6 hrs 24 min

(C) 7 hrs 22 min (D) 6 hrs

Sol^n: $T = \frac{75 \text{ km}}{100 \text{ kmph}} = \frac{3}{4}$ hrs for 75 kms } This continues

3 min Rest

for once; 48 min But $\frac{600}{75} = 8$.

$48 \times 8 = \frac{384}{60 \text{ min}} = 6 \text{ hrs } 24 \text{ min}$ ✗✗

↓

Last 3 min of Rest should not considered

So 6 hrs 21 min only

Q. A truck covers a distance of 550 meters in 1 minute whereas a train covers a distance of 33 kms in 45 minutes. What is the ratio of their speed?

(A) 2:1 (B) 1:2 (C) 4:3 (D) 3:4

Sol^n: Truck → 550 m/min ; Train 33000 m/45 min

$\frac{550}{1} : \frac{33000}{45}$ (5, 3-20)

15 : 20

3 : 4

Q. A man in a train notices that he can count 21 telephone posts in one minute. If they are known to be 50m apart, at what speed is the train travelling?

Sol^n: In one minute:

50(20) = 1000m distance = 1 km

Time = 60 seconds = $\frac{1}{60}$ hr

↓

60 hrs

Speed = $\frac{1000}{60}$ m/s

$= \frac{200}{1000} \times \frac{3}{18}$ kmph (60, 5)

= 60 kmph

(A) 61 kmph (B) 56 kmph

(C) 63 kmph (D) 60 kmph

Q. A man takes 5 hrs 45 min in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. The time he would take to walk both ways is

(A) 11 hrs (B) 6 hrs 45 min (C) 7 hrs 45 min (D) 9 hrs 20 min

Sol^n: W↑ + R↓ = 5 hrs 45 min

R↑ + R↓ = 3 hrs 45 min

W↑ + W↓ = ? 7 hrs 45 min

If walk replaced with Ride

↓

Gain = 2 hrs

Both walk replaced with Ride

Gain = 4 hrs

Q. The speed of a bus increases by 2 km after every one hour. If the distance travelling in the first one hour was 35 km. What was the total distance travelled in 12 hours?

(A) 422 km (B) 552 km (C) 502 km (D) 492 km

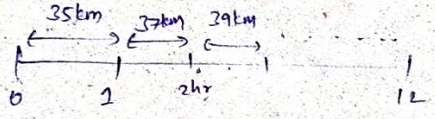
Soln:- $a = 2$ kmph first hour ⟶ 35 km

$S = ut + \frac{1}{2}at^2$ $S \longrightarrow \frac{35\text{km}}{1\text{hr}}$

WHY WRONG? $= (35)(12) + \frac{1}{2}(2)(12)^2$ So $u = 35$ kmph

$= 420 + 144$

$= 564$ km



Sum of terms in AP

$= \frac{\text{no of terms}}{2}[1^{st} + \text{last term}]$

$= \frac{12}{2}[35 + 57]$

$= 6[92]$

$= 552$ km

This is in A.P

35, 37, 39 .....

n^{th} last term $= a + (n-1)d$

$= 35 + (12-1)(2)$

$= 57$

Distance is Constant

Q. If I travel at 5/4 of my speed. I reach my office 6 minutes early. What is the original duration of time I take to reach office?

original (A) 30 min (B) 24 mins (C) 35 min (D) None

Soln: S = constant Speed $\propto \frac{1}{\text{time}}$

$\frac{S_1}{S_2} = \frac{t_2}{t_1}$

$\frac{4}{5} = \frac{t_1 - 6}{t_1}$

$4t_1 = 5t_1 - 30$

$t_1 = 30$ min $\Longrightarrow$ $t_2 = 24$ min

Q. Walking at $\frac{3}{4}$ of his normal speed, Mike is 16 minutes late in reaching his office. The usual time taken by him to cover the distance b/w his home and his office is

(A) 48 min (B) 60 min (C) 42 min (D) 62 min

Soln:- $S_1t_1 = S_2t_2$

$S_1t_1 = \left(\frac{3}{4}S_1\right)(t_1 + 16)$

$t_1 = \frac{3}{4}t_1 + 12$

$\frac{t_1}{4} = 12 \Rightarrow t_1 = 48$ min

Q. A man started 15 minutes late and by travelling at a speed which is 5/4 th of his usual speed reached his office 20 mins early. What is the usual time of the journey?

(A) 85 mins (B) 100 mins (C) 135 mins (D) 175 mins

Soln:- $S_1t_1 = S_2t_2$

$S_1t_1 = \left(\frac{5}{4}S_1\right)(t_1 - 35)$

$\frac{t_1}{4} = \frac{5 \times 35}{4}$

$t_1 = 175$ min

$\frac{5}{4}S \Rightarrow \frac{4}{5}T$

$T - \frac{T}{5}$ ↓ 20 mins +15 mins

So $\frac{T}{5} = 35$

$T = 175$ min

Q. 3 persons are walking from A to B with a speed of 4:3:5. Find the time ratios of 3 persons to reach B

(A) 4:3:5
(B) 5:3:4
(C) 15:20:12
(D) 15:12:20

Soln: Let P → 40 m/s
Q → 30 m/s
R → 50 m/s

Let S = 60 m

Then $T_P = 15$
$T_Q = 20$
$T_R = 12$

15 : 20 : 12

Time Saving

Q. A worker reaches his work place 15 minutes late by walking at 4 kmph from his house. The next day he increases his speed by 2 kmph and reaches in time. Find the distance.

(a) 2 km (b) 6 km (c) 8 km (d) 3 km

Soln: $S_1 t_1 = S_2 t_2$

$4(t_2 + 15) = (6)(t_2)$

$4t_2 + 60 = 6t_2$

$t_2 = 30$ min

$S_2 = 6$ $t_2 = 30$ min

Sp Distance = 6 kmph × $\frac{1}{2}$ hr

$S = 3$ km

Time method

Time Saving = $T_{4\text{kmph}} - T_{6\text{kmph}}$

$15\text{ min} = \frac{D}{4} - \frac{D}{6}$

So $D = 3$ km

Q: A man started from home, travelling at a speed of 12 kmph reach railway station by 3 pm, at a speed of 8 kmph reach by 5 pm.

(i) Find distance from home to Railways? (in kms)

(ii) At what speed he has to travel to reach Railways by 4 pm

(a) 9.6 km/hr (b) 10 km/hr (c) 11 km/hr (d) None

Soln: (i) Time saving = 2 hours = $T_8 - T_{12}$

$2 = \frac{S}{8} - \frac{S}{12}$

$2 \times 48 = 6S - 4S$

$S = 48$ km

(ii) $1\text{ hr} = \frac{48}{8} - \frac{48}{k}$

$8k = 48k - 48(8)$

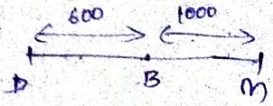
$5k = 48$

$k = \frac{48}{5} = 9.6$ km/hr

Time is Constant

Q. A train leaves Delhi for Mumbai, a distance of 1600 km and at the same time another train leaves mumbai for Delhi. These trains meet at Bhopal at a distance of 600 km from Delhi. What is the ratio of their speed?

(a) 3:5 (b) 5:3 (c) 3:4 (d) None



Soln: $\frac{600}{S_1} = \frac{1000}{S_2}$

$\frac{S_1}{S_2} = \frac{3}{5}$ So 3:5

Without Stoppages

Q. The average speed for an entire journey is 60 kmph without considering the stoppages. When the stoppages are considered the average speed becomes 48 kmph. How many minutes per hours on an average were the stoppages?

(a) 10 min (b) 12 min (c) 16 min (d) None

Soln: $S_{WOS} = 60$ kmph $S_{WS} = 48$ kmph

Due to stoppages; — 12 kmph lost

$$\frac{12 \text{ kmph}}{60 \text{ kmph}} = \frac{1 \text{ hr}}{5 \text{ hr}} = \frac{60 \text{ min}}{5 \text{ hr}} = 12 \text{ min/hr}$$

12 kmph lost of 60 kmph means 1 hr lost of 5 hours

12 min lost of one hour

$$\Rightarrow \left(\frac{S_{WOS} - S_{WS}}{S_{WOS}}\right) \times 60 \text{ min/hr}$$

12. PROBLEMS ON TRAINS

Introduction:-

(or) Relative Speed

Train Crossing Stationary Object ($S_o = 0$)

S_t = Speed of Train

L_t = length of train

Speed = S_t

Eg: Tree, pole, man standing, platform, station, Stationary train

Train crossing moving object ($S_o > 0$)

S_o - Speed of object

L_o - Length of object

Relative Speed here

Eg: Another train, man moving/running

If they are in opposite direction : $S_1 + S_2$

Same direction ← $|S_1 - S_2|$

Relative Speed → Applicable when only 2 bodies are in motion.

Distance - Pole/platform.

case 1: Train crossing Tree/pole/man standing/man moving

Here Distance = L_t

case 2: Train crossing platform/bridge/station/another train

Distance = $L_t + L_o$

*When you find two moving; then assume one freezed and then check it out so that would be easy for you.

case 3: Train crossing a man standing on platform of length 1000m. Distance = L_t

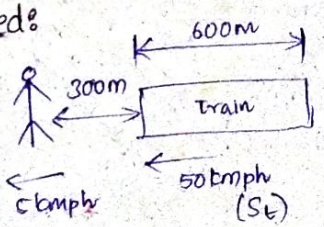
case 4: Train1 crossing a man sitting in train 2

Distance = L_{t_1}

Same ⇒ A man in train 2 observes train 1 crosses him

Relative Speed:

Case 5:

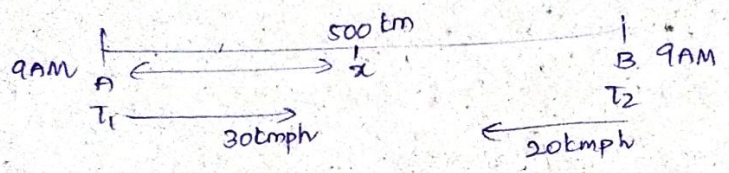


Total Distance = 300 + 600 = 900m

R. Speed = 50 - 5 = 45 kmph = 12.5 m/s

So Time = $\frac{D}{R.S} = \frac{900}{12.5} = 72$ sec

Case6 : Train1 meeting Train2



$$\frac{x}{30} = \frac{500-x}{20}$$

$$20x = 500\times30 - 30x$$

$$50x = 500\times30$$

$x = 300$ km

$$\text{Time} = \frac{500}{\text{Relative Speed}}$$

$$= \frac{500}{30+20}$$

$T = 10$ hrs

Stationary Objects - Examples

Q. A train takes 10 seconds to cross a man standing on a platform and 44 seconds to cross the platform. What is the length of the platform if the speed of the train is 72 kmph?

(A) 440 m (B) 570 m (C) 680 m (d) None

Soln:

$$T_t = \frac{D}{S} \rightarrow 10\,\text{sec} = \frac{L_t}{72\,\text{kmph}}$$

$$L_t = 72 \times \frac{5}{18} \times 10\ \text{m}$$

$$L_t = 200\,\text{m}$$

To cross platform $44 = \frac{200+L_o}{20}$

$$44(20) - 10(20) = L_o$$

$L_o = 680$ m

Q. A train running at the speed of 20 m/s, crosses a railway platform in 20 seconds. If the length of the train is 100 m, what is the length of platform?

(a) 300m (b) 100m (c) 200m (d) None

Soln: $S_t = 20$ m/s

$$20 = \frac{100+x}{20}$$

$$400 - 100 = x$$

$x = 300$ m

Q. A train requires 7 seconds to pass a pole while it requires 25 seconds to cross a stationary train which is 378 m long. Find speed of train...

(a) 75.6 kmph (b) 75.4 kmph (c) 76.2 kmph (d) 21 kmph

Soln:

$$7 = \frac{L_t}{S_t} \qquad 25 = \frac{L_t + 378}{S_t}$$

$$7S_t = 25S_t - 378$$

$$378 = 18S_t$$

$$S_t = \frac{378}{18}\ \text{m/s}$$

$$S_t = 21 \times \frac{18}{5}\ \text{kmph} \quad (3.6)$$

$S_t = 75.6$ kmph

Moving Objects - Examples

Q: Find the time taken by a train 150 m long running at a speed of 63 kmph to cross another train of length 100m long running at a speed of 45 kmph in the same direction.

(a) 25 sec (b) 50 (c) 75 (d) 100 sec

Soln:

$$T = \frac{150+100}{63-45}$$

$$T = \frac{250\ m}{18\ kmph}$$

$$= \frac{250}{18 \times \frac{5}{18}} \frac{m}{m/s}$$

$$T = 50\ sec$$

Q: Two trains of length 150m and 250m run on parallel lines when they run in the same direction it will take 20 seconds to cross each other and when they run in opposite direction it will take 5 seconds. Find the speeds of the two trains [kmph].

(a) 180 and 108 (b) 272 and 211 (c) 123 and 828 (d) None

Soln:

$$\frac{150+250}{20} = |S_1 - S_2| \qquad \frac{150+250}{5} = |S_1 + S_2|$$

$$20 = |S_1 - S_2| \qquad 80 = |S_1 + S_2|$$

(50 & 30) in m/s $\quad 50 \times \frac{18}{5}$ & $30 \times \frac{18}{5}$

180 & 108

Q: Train and a bike are coming from opposite directions, if train speed is 20 m/s, it takes 20 sec for them to completely cross each other. If the length of train is 900m. what is speed of bike?

(a) 24.5 m/s (b) 25 m/s (c) 26 m/s (d) None

Soln:-

$$20 = \frac{900}{R.S} \qquad R.S = 45\ m/s$$

So $S_b = 45 - 20$

$S_b = 25\ m/s$

Q: Akash left the White house and travelled toward the desert. Two hours later Julio left travelling at 35 km/h in an effort to catchup Akash. After travelling for 3 hours Julio finally caught up. Find Akash's average speed.

(a) 21 km/h (b) 31 kmph (c) 41 kmph (d) None

Soln:- I method

105 km $\longrightarrow$ 35×3 Julio travelled in total

In 5 hours, Akash $\rightarrow$ 105 km

So Speed $= \frac{105}{5}$

$S_a = 21$ kmph

II method

$D_a = D_J$

$S_a \times t_a = S_j \times t_j$

$S_a \times 5 = 35 \times 3$

$S_a = 21$ kmph

Trains Meeting

Q. Two trains for Mumbai leave Delhi at 6 am and 6.45 am and travel at 100 kmph and 136 kmph respectively. How many kilometers from Delhi will the two trains be together.

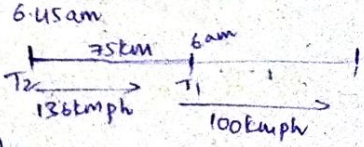
(a) 262.4 km (b) 260 km (c) 283.33 km (d) None

Soln:-

Distance $\rightarrow$ Same

$S_1 t_1 = S_2 t_2$

$(100)(T) = (136)(T - 45)$



$136 \times 45 = 36T$
$34 \quad 5 \quad 4$

$T = 170 \text{ min}$

$S = \frac{D}{T} \Rightarrow 100 \text{ kmph} = \frac{D}{170 \text{ min}}$

Using Relative Speed;

$T = \frac{75}{136 - 100}$

$T = \frac{75}{36} = 2\frac{1}{12} \text{ hrs}$

$D_{T_2} = S_{T_2} \times T_{T_2}$

$= 136 \times \frac{25}{12}$

$D_{T_2} = 283.33 \text{ km}$

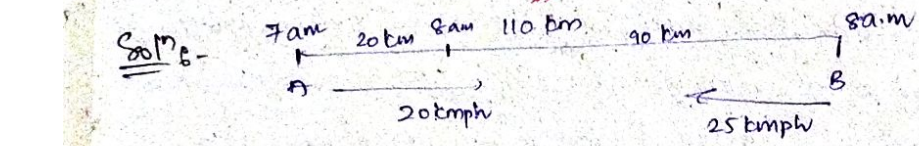
$D = 100 \times 2\frac{5}{6}$

$D = 283.33 \text{ km}$

Q. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m and travels towards B at 20 kmph. Another train starts from B at 8 a.m and travels towards A at 25 kmph. At what time will they meet?

(a) 9 a.m (b) 10:30 a.m (c) 10 a.m (d) 11 a.m

Soln:-



At 8 a.m D → 90 km

R.S → 20 + 25 = 45 kmph

$T = \frac{90}{45} = 2 \text{ hrs}$

So 10 A.M

Q. Two trains are running at 40 km/hr and 22 km/hr respectively in the same direction. Fast train completely passes a man sitting in the slower train in 50 seconds. What is the length of fast train?

(a) 230 m (b) 290 m (c) $230\frac{2}{9}$ m (d) 250 m

Soln:-

$50 = \frac{L}{40 \times \frac{5}{18}} \Rightarrow L = 50 \times 40 \times \frac{5}{18}$

Take Relative Speed

$= 1000$

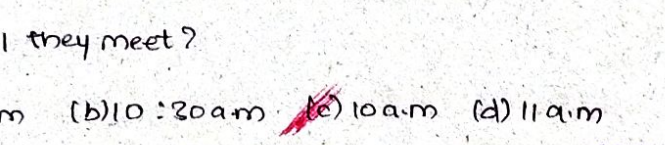
$50 = \frac{L}{18 \times \frac{5}{18}} \Rightarrow L = 50 \times 5$

$L = 250 \text{ m}$

Q. A train X starts from Meerut at 4 p.m and reaches Ghaziabad at 5 p.m while another train Y starts from Ghaziabad at 4 p.m and reaches Meerut at 5:30 pm. The two train cross each other at ?

(a) 4:36 pm (b) 4:42 pm (c) 4:48 pm (d) 4:50 pm

Soln:-



$T_x = 1 \text{ hr} \quad T_y = 1.5 \text{ hr}$

$T_x : T_y = 2 : 3$

$S_1 = \frac{D}{1} = D \text{ kmph} \quad S_2 = \frac{D}{\frac{3}{2}} = \frac{2D}{3} \text{ kmph}$

$T = \frac{D}{D + \frac{2D}{3}} = \frac{D}{\frac{5D}{3}} = 36 \text{ min}$

4:36 PM

Assume → 300 km Distance

R.S → 300 kmph ~~200 kmph~~

R.S → 100 kmph

$T \rightarrow \frac{300}{500}$

~~0.6~~ hrs

$\rightarrow \frac{6}{10} \times 60 \text{ min}$

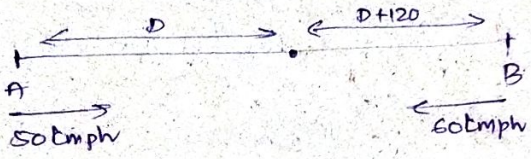
36 min

→ 4:36 PM

Q. Two cars starts from A and B and travel towards each other at the speed of 50 kmph and 60 kmph respectively. At the time of their meeting the second car has travelled 120 km more than the first, the distance between A and B is?

(a) 600 kms (b) 3120 kms (c) 1320 kms (d) 720 kms

Soln:-



Sir Method

1 hour

A B

50 km 60 km

10 km

So for 120 km

↓

12 hrs

$= 50 \times 12 + 60 \times 12$

$= 110 \times 12$

$= 1320$ km

$t_a = t_b$

$\frac{D}{50} = \frac{D+120}{60}$

$60D = 50D + (50)(120)$

$10D = 6000$

$D = 600$ km

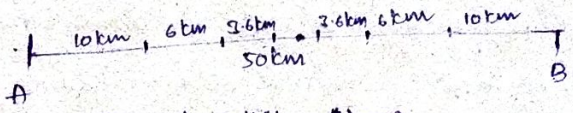
Total Distance $= 600 + 600 + 120 = 1320$ kms

Train 1, 2 & Bird

Q. Two trains start from stations A and B spaced 50 kms apart at the same time and speed. As the trains start, a bird flies from one train towards the other and on reaching the second train, it flies back to the first train. This is repeated till the trains collide. If the speed of the trains is 25 km/h and that of the bird is 100 km/h. How much did the bird travel till the collision.

(a) 50 kms (b) 60 kms (c) 100 kms (d) 80 kms

Soln:-



Time to collide = 1 hour

Meeting point (B, T_2) $\rightarrow \frac{50 \text{ km}}{125 \text{ kmph}} = \frac{2}{5}$ hour

Next $\longrightarrow$ (T_1, Bird) $\rightarrow \frac{30 \text{ km}}{125 \text{ kmph}} = \frac{6}{25}$ hour

(T_2, Bird) $\rightarrow \frac{18 \text{ km}}{125 \text{ kmph}} = \frac{18}{125}$ hour

Sir Method

$D_{Bird} = S_{Bird} \times T_{Bird}$

$= 100 \text{ kmph} \times t_{Bird}$

$\frac{10.8 \text{ km}}{125 \text{ kmph}} = \frac{108}{1250}$ hour

G.P.

Time taken by bird is that the time taken for trains to collide. Then only it stops moving.

So $T_{collision} = \frac{50}{50} = 1$ hour

$D_{Bird} = 100 \text{ kmph} \times 1 \text{ hr}$

$D = 100$ km

13. AVERAGE SPEED:-

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Average (or) mean} \Rightarrow \frac{\text{Total Sum}}{\text{No. of}}$$

$$\text{Average speed} = \frac{\frac{\text{Total Distance}}{\text{No. of}}}{\frac{\text{Total Time}}{\text{No. of}}}$$

So Average (or) mean Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$

Q: A man travelled from A to B for 2 hrs which is 40 km distance, B to C which is 60 km distance for 3 hrs find the average speed for the whole journey.

(a) 20 kmph (b) 40 kmph (c) 50 kmph (d) None

Soln:- $\text{Avg} = \frac{40+60}{2+3} = \frac{100}{5} = 20$ kmph

Q: A bus travels 3 hrs at 60 kmph, 4 hrs at 50 kmph and next 5 hrs at 50 kmph. Find its average speed.

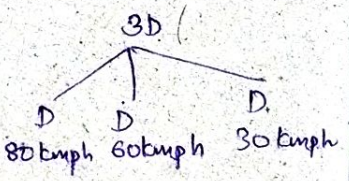
Soln:- $\text{Avg} = \frac{3(60)+(50)(4)+(50)(5)}{3+4+5}$

$= \frac{180+450}{12}$

$= \frac{630}{12} = 52.5$ kmph

Q. A person divides his total route of journey into three equal parts and decides to travel the three parts at the speeds of 80 kmph, 60 kmph and 30 kmph respectively. What is the average speed during the journey?

Soln: Wrong $\text{Avg} = \frac{80+60+30}{3}$



$\text{Avg} = \frac{3D}{\frac{D}{80}+\frac{D}{60}+\frac{D}{30}}$

$= \frac{3D}{D\left(\frac{3+4+8}{240}\right)}$

$= \frac{8\times 200}{15}$ (48)

Avg = 48 kmph

Q. A person travels a distance D. He travelled first half of the distance with speed 15 kmph. Next one third of distance with speed 20 kmph and remaining distance with speed 30 kmph. Calculate Average speed.

Soln:- $\text{Avg} = \frac{D}{\frac{D}{2}(15)+\frac{D}{3}(20)+\frac{D}{6}(30)}$

$= \frac{1}{\frac{45+40+30}{6}} \quad \frac{1}{\frac{1}{30}+\frac{1}{60}+\frac{1}{180}}$

$= \frac{180}{6+3+1}$

$= \frac{180}{10} = 18$ kmph

Avg = 18 kmph

Q. A person travels on cycle from A to B, half of the journey with 30 km/hr and remaining with 20 km/hr. He takes 10 hrs to travel from A to B. What's the distance?

Soln:- $D = (30)(5)+(20)(5)$

WHY NOT THIS? $= 150+100$

$= 250$ km

→ Coz they didn't say equal time

$$\frac{\left(\frac{D}{2}\right)}{30} + \frac{\left(\frac{D}{2}\right)}{20} = 10$$

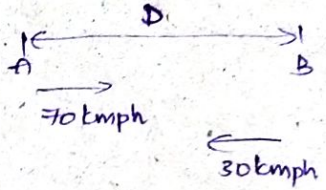
$$D\left(\frac{1}{60} + \frac{1}{40}\right) = 10$$

$$D\left(\frac{100}{2400}\right) = 10$$

$$D = 240\text{km}$$

Q. A person travels on scooter from A to B with 70 kmph and returns with 30 kmph. If he takes total 10 hrs to travel, what is the distance from A to B?

Soln:



$$\frac{D}{70} + \frac{D}{30} = 10$$

$$D\left(\frac{70+30}{2100}\right) = 10$$

$$D\left(\frac{100}{2100}\right) = 10$$

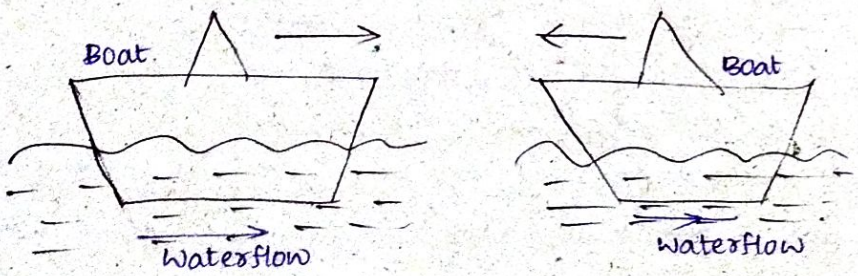
$$D = 210\text{ km}$$

If:

$$\text{Avg} = \frac{210+210}{10}$$

$$\text{Avg} = 42\text{ kmph}$$

14. BOATS AND STREAMS:



Speed of Current/Stream (S_s) → Velocity/rate

Speed of Waterflow

Speed of Boat in Still Water (S_b)

Speed of boat and Speed of waterflow Same direction

↳ downstream (or) along with stream

Speed of Downstream $= S_b + S_s$

Boat and waterflow opposite direction

↳ upstream (or) against stream

Speed of Upstream $= S_b - S_s$

Problems

Q: If boats rate in still water is 20 km/hr and stream rate is 10 km/hr, and it takes him 10 hrs to row to a place and back, then how far is the place?

(a) 75 kms (b) 60 kms (c) 70 kms (d) 40 kms

Soln: $S_b = 20$ kmph $S_s = 10$ kmph

$$\frac{D}{S_b + S_s} + \frac{D}{S_b - S_s} = 10\text{ hrs}$$

$$\frac{D}{30} + \frac{D}{10} = 10$$

$$D\left(\frac{40}{300}\right) = 10$$

$$D = 75\text{ kms}$$

Q: If boat's rate in still water is 10 kmph and river is flowing with a certain speed, it takes him 20 hrs to row to a place and back and the length of river is 75 kms. find the velocity of water

(a) 10 kmph (b) 15 kmph (c) 7 kmph (d) 5 kmph

Soln: $S_b = 10$ kmph

$$\frac{75}{S_b+S_s} + \frac{75}{S_b-S_s} = 20$$

$$75\left(\frac{10+S+10-S}{100-S^2}\right) = 20$$

Use options in the case of high values

$$S^2 = 25$$

$$S = 5 \text{ kmph}$$

Q: If Boat's rate in still water is 20kmph and it takes him thrice as long as to row up as to row down the river. Find the stream rate

(a) 10 km/hr (b) 12km/hr (c) 5 km/hr (d) 8 km/hr

Soln:

$$T_U = 3T_D$$

$$\frac{D}{S_b-S_s} = 3\cdot\frac{D}{S_b+S_s}$$

$$(20+S) = 3(20-S)$$

$$20+S = 60-3S$$

$$4S = 40$$

$$S = 10 \text{ kmph}$$

Q. A man rows downstream 32 km and 14 km upstream If he takes 6 hrs to cover each distance, then the velocity (in kmph) of the current is

(a) 1/2 (b) 1 (c) 1 1/2 (d) 2

Soln:

$$\frac{32 \text{ km}}{S_b+S_s} = 6 \qquad \frac{14 \text{ km}}{S_b-S_s} = 6$$

$$2S_s = \frac{32}{6} - \frac{14}{6}$$

$$2S_s = \frac{18}{6}$$

$$S_s = 1.5 \text{ kmph}$$

Q: Speed of a boat in still water is 16 kmph. If it can travel 20 km downstream in the same time as it can travel 12 km upstream, the rate of stream is

(a) 1 kmph (b) 2 kmph (c) 4 kmph (d) 5 kmph

Soln:- $S_b = 16$ kmph

$$\frac{20}{16+S} = \frac{12}{16-S}$$

$$5 \quad 3$$

$$80-5S = 48+3S$$

$$32 = 8S$$

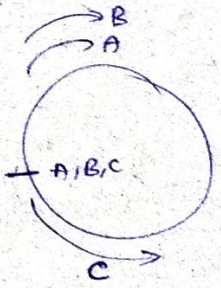
$$S = 4 \text{ kmph}$$

15. CIRCULAR MOTION AND RACE

Introduction

Race & Games

Linear Race — Circular Race



* meeting at starting points
* first ever meeting
* No. of distint meeting points

16. CIRCULAR RACE - PROBLEMS

1. Consider A, B & C are moving in same direction.

$S_A = 9$ kmph, $S_B = 6$ kmph, $S_C = 15$ kmph.

If length of circular track is $\frac{1000\text{m}}{1\text{km}}$, find time taken for

i, first ever meeting point

ii, meeting at starting point?

Sol^n: i, T_{AB} first ever meeting $= \frac{\text{Rt}}{R \cdot S} = \frac{1\text{ km}}{3\text{ kmph}} = 20$ min

T_{BC} first ever meeting $= \frac{\text{Rt}}{R \cdot S} = \frac{1\text{ km}}{9\text{ kmph}} = \frac{20}{3}$ min

A, B, C first ever meeting $\longrightarrow$ LCM $(T_{AB}, T_{BC}) \Rightarrow 20$ min

ii, 1 round for A $\Rightarrow \frac{1}{9}$ hrs

1 round for B $\Rightarrow \frac{1}{6}$ hrs

1 round for C $\Rightarrow \frac{1}{15}$ hrs

$$\text{LCM}(T_A, T_B, T_C) = \text{LCM}\left(\frac{1}{9}, \frac{1}{6}, \frac{1}{15}\right)$$

$$= \frac{\text{LCM}(1,1,1)}{\text{HCF}(9,6,15)}$$

Meeting at starting $= \frac{1}{3}$ hr $= 20$ min

2 Consider A & B are moving in opp-direction. Both are started from same point. Find time taken for

(a) first ever meeting point?

(b) meeting at starting point?

(c) find no of distinct meeting points?

$\text{Rt} = 1$ km

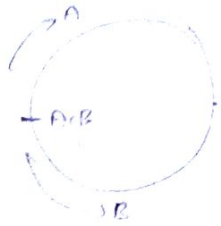
$S_A = 6$ kmph

$S_B = 9$ kmph

Sol^n: given Rt $= 1$ km

i, $T_{AB} = \frac{\text{Rt}}{R \cdot S}$

$= \frac{1}{15}$ hr $= 4$ min



ii, 1 round for A $\rightarrow \frac{1}{6}$ hr

1 round for B $\rightarrow \frac{1}{9}$ hr

Meet at Start $= \frac{\text{LCM}(1,1)}{\text{HCF}(6,9)}$

$= \frac{1}{3}$ hr $= 20$ min

iii, A & B $\rightarrow$ opp direction

$S_A : S_B = 6 : 9$

$= 2 : 3$

No of distinct points meet $= 2 + 3$

$= 5$

If suppose – same direction

$S_A : S_B = 6 : 9$

$= 2 : 3$

No of distinct points $= 3 - 2$

$= 1$

* According to direction, first ever meeting and no of distinct meetings may change but meeting at starting point does not change

3. A, B and C walk around a circle 3 km in circumference at the rates of 200, 150 and 125 metres per min respectively. If they all started together, B & C in same direction & A in opp direction, then find time taken for

i, first ever meeting point?

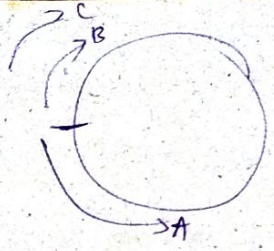
ii, Meet at starting point?

iii, no of distinct meeting points?

Soln: (i) $T_{AB} = \frac{Lt}{R.S}$

$= \frac{3000m}{350\,m/min}$

$= \frac{60}{7}\ min$



$T_{BC} = \frac{Lt}{R.S}$

$= \frac{3000\,m}{25/min}$

$= 120\,min$

first meeting → $LCM\left(\frac{60}{7}, \frac{120}{1}\right) = \frac{LCM(60,120)}{HCF(7,1)}$

$= 120\ min$

(ii) Meet at Starting point

1 round of A $= \frac{3000}{200}\ min = 15\,min$

1 round of B $= \frac{3000}{150}\ min = 20\,min$

1 round of C $= \frac{6000}{\frac{600}{125}}\ min = 24\,min$
25

So Starting point ⇒ LCM(15,20,24)

$= 120\ min$

(iii) A & B meeting points — B & C meeting points

$S_A : S_B = 4:3$ — $S_B : S_C = 6:5$

Opp. direction — same direction

So 4+3=7 — So 6-5=1

HCF of these Values (7,1) = 1

* If time for first ever point and starting meet point is same, then distinct meets = 1.

17. RACES AND GAMES

Introduction

12/08/2020 : Wednesday

* Linear Race
* Circular Race/Track
* In 100m race, A wins/beats B by 20m

Completed Race

A = 100m B = 80m

Same time - 1t sec

* In 100m race, A beats B by 5 sec

A = 100m B = 100m

t sec t+5 sec

* In 100m race, A beats B by 20m (or) 5 sec.

A → 100m	B = 80m	A → 100m	B = 100m
t sec	t sec	t	t+5 sec

$Speed_B = \frac{20m}{5sec} = 4m/sec$

* In 100m race, A gives 10m start to B

A = 100m B = 90m

Problems

1Q. In a 1km race A can beat B by 100m and B can beat C by 60m. In the same race A can beat C by

(A) 124 m (B) 164 m (C) 144 m (D) 154 m

Soln: Total → 1000m A = 1000m B = 900 m

t = constant $S_A : S_B = 10 : 9$

Again t = k $S_B : S_C = 100 : 94$

So $S_A : S_C = 1000 : 846$

A : B : C 1000 : 900 : 846

A : C ⟶ 1000 : 846

⟹ 154 m

2. In a game A can give B 20 points in 60 and C 18 points in 90. How many points can C give B in a game of 120?

(A) 20 (B) 22 (C) 18 (D) 40

Soln In 60 points game; A → 60 B = 40

90 point game; A → 90 C = 72

120 point game; C → 120 B = ?

$$\frac{C}{B} = \frac{C}{A} \times \frac{A}{B} = \frac{\cancel{72}^{6}}{\cancel{90}_{5}} \times \frac{\cancel{120}^{4}}{\cancel{60}_{5}}$$

$$\frac{C}{B} = \frac{6}{5}$$

$$\frac{C}{B} = \frac{120}{100}$$

So C gives 20 points

3. In a game A can give B 40 points, ~~in 60 and C 18 points~~ A can give C 64 points, and B can give C 30 points. How many points to make the game?

(A) 200 (B) 220 (C) 180 (D) 240

Soln:

A —40→ B A —64→ C B —30→ C

Let x be game points

$$\frac{A}{B} = \frac{x}{x-40} \qquad \frac{A}{C} = \frac{x}{x-64} \qquad \frac{B}{C} = \frac{x}{x-30}$$

$$\text{So } \frac{x}{x-30} = \frac{x-40}{\cancel{x}} \times \frac{\cancel{x}}{x-64}$$

$$\frac{x}{x-30} = \frac{x-40}{x-64}$$

$$x^2 - 64x = x^2 - 70x + 1200$$

$$6x = 1200$$

$$x = 200$$

4. P and Q run a kilometer and P win by 1 minute. P and R run a kilometer and P wins by 375 m. Q and R run a kilometer and Q wins by 30 seconds. Find the time taken by each runner to run a kilometer.

(A) 150 sec, 210 sec, 240 sec (C) 100 sec, 160 sec, 90 sec

(B) 120 sec, 160 sec, 200 sec (D) 140 sec, 200 sec, 230 sec

Soln: P → 1000m Q → 1000m P R Q → 1000 R → 1000

t_1 $t_1 + 60$ 1000m 625m t_2 $t_2 + 30$

$$\frac{S_P}{S_Q} = \frac{t_1 + 60}{t_1} \qquad \frac{S_P}{S_R} = \frac{1000}{625} \qquad \frac{S_Q}{S_R} = \frac{t_1 + 90}{t_1 + 60}$$

$$\frac{1000}{625} = \left(\frac{t_1 + 60}{t_1}\right)\left(\frac{t_1 + 90}{t_1 + 60}\right)$$

$1000t_1 = 625t_1 + 625(90)$

$375t_1 = 625(90)$

15 25 30

3 5

$t_P = 150$ sec

$t_Q = 210$ sec

$t_R = 240$ sec

5. P can run 100m in 20 second and Q in 25 seconds. P beats Q by

(A) 10m (B) 20m (C) 25m (D) 12m

Soln:- $S_P = \frac{100}{20} = 5$ m/s $\quad D_q = (4)(20) = 80$ m

$S_q = \frac{100}{25} = 4$ m/s $\quad$ So beats by 20m

6. Two men A and B run a 500m race. A having 140m start their speeds are 3:4. Then, A wins by:

(A) 10m (B) 20m (C) 40m (D) 60m

Soln:- A → 360m B → 500m

let $S_A : S_B \Rightarrow 30$ m/s : 40 m/s

Then A wins in 12 seconds

By 12 seconds, B travels 480m

So wins by 20m

7. A runs 1½ times as fast as B can. If A gives B a start of 50m, how far must the winning post be in order that A and B reach at the same time

(A) 150m (B) 120m (C) 125m (D) 180m

Soln:- $\frac{S_A}{S_B} = \frac{3}{2}$ A ↓ Dm B ↓ D-50m

Time same → So $\frac{S_A}{D_A} = \frac{S_B}{D_B}$

$\frac{3}{2} = \frac{D}{D-50}$

$3D - 150 = 2D$

$D = 150$m

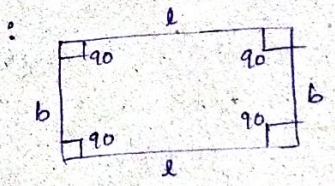
18. AREA & VOLUMES - MENSURATION:

Introduction:

- 2D
  - Interior & Exterior Angles
  - Area
  - perimeter
  - Quadrilateral
    - Rectangle
    - Square
    - parallelogram
    - Rhombus
    - Triangle
    - Pentagon
    - Trapezium
    - Hexagon
    - Circle
- 3D
  - Sphere
  - Cylinder
  - Cone
  - Cube
  - Cuboid
  - Volume
  - TSA
  - LSA
  - CSA

Quadrilateral:

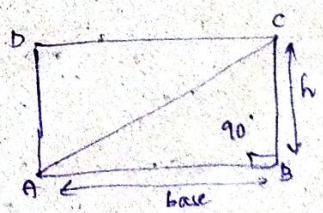
* Rectangle:



perimeter = 2(l+b)

Area = l x b

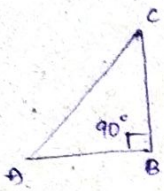
* Triangle (Right angled Triangle):



ΔABC

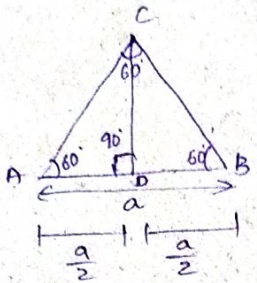
Area = $\frac{1}{2}(bh)$

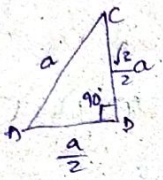
Right Angled Triangle



ΔABC Area $= \frac{1}{2} \times AB \times BC$

Equilateral Triangle





$AC^2 = AD^2 + CD^2$

$CD = \sqrt{\frac{3}{4}a^2}$

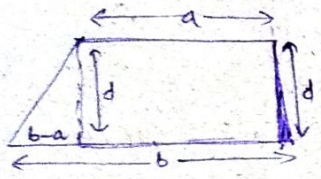
$CD = \frac{\sqrt{3}}{2}a$

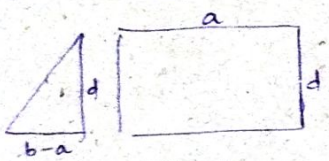
$Area_{ET} = \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$

$Area = \frac{\sqrt{3}}{4}a^2$

Trapezium:





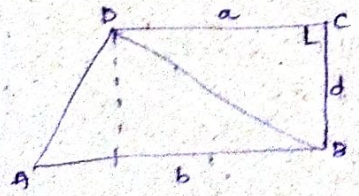
$Area = \frac{1}{2}d(b-a) + a \times d$

$= \frac{d}{2}[b-a+2a]$

$Area = \frac{(a+b)d}{2}$

* So Area $= \frac{1}{2}$ [Sum of parallel sides][Distance b/w pardld sides]

Method-II

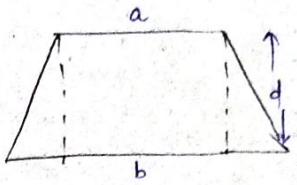


$Area = \Delta BCD + \Delta ABD$

$= \frac{1}{2}a \times d + \frac{1}{2}b \times d$

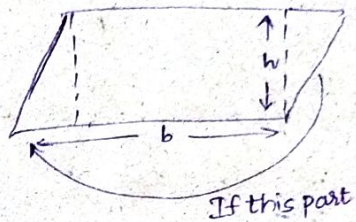
$Area = \frac{d}{2}(a+b)$

Another shape of Trapezium



Also Area $= \frac{1}{2}(a+b)d$

Parallelogram:



Area $= b \times h$

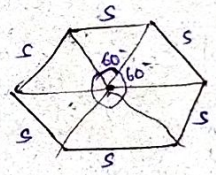
perimeter $= 2(b+h)$ ⇐ Something Fishy

If this part is added there it forms Rectangle

So Area = base × height

Hexagon:

(Regular) (All equal sides)



Area $= 6$ [Area of equilateral Δles]

$= 6\left[\frac{\sqrt{3}}{4}s^2\right]$

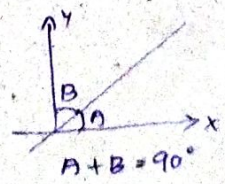
$= \frac{6}{4}s^2 \cdot \cot 30°$

$= \frac{6}{4}s^2 \cot \frac{180°}{6}$

Area of Regular polygon 'n' sides ⇒ $\frac{n}{4}s^2 \cot \frac{180°}{n}$

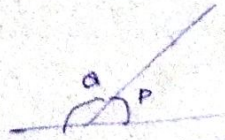
Angles:

Complementary $\angle$les Supplementary $\angle$les Corresponding $\angle$les



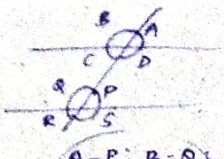
$A + B = 90°$

So A & B - complementary



$P + Q = 180°$

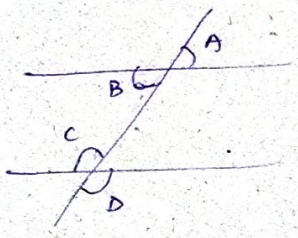
P & Q → Supplementary



A = P; B = Q;
C = R; D = S

Corresponding

Alternate Angles:



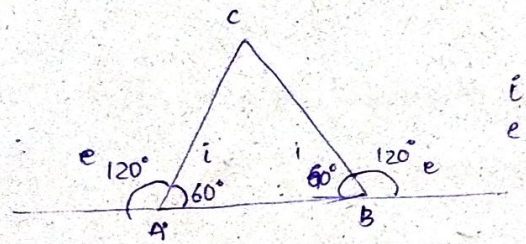
$\angle A = \angle B$

$\angle C = \angle D$

They are all Congruent

Alternate Angles

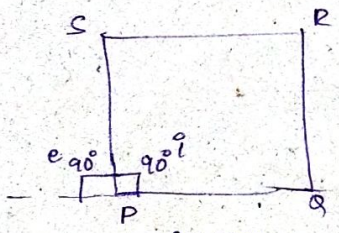
Interior & Exterior Angles:



$i = 60°$

$e = 120°$

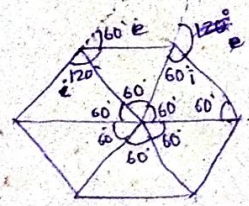
Equilateral Δle



$i = 90°$

$e = 90°$

Square

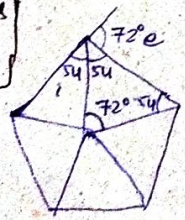


$i \to 120°$

$e \to 60°$

Hexagon

* Sum of Interior angles $= (n-2)180°$



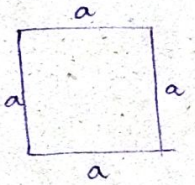
$i \to 108°$

$e \to 72°$

Pentagon

* Sum of Exterior Angles in Polygon $= 360°$

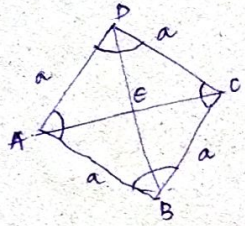
SQUARE:



Area $= a^2$

Perimeter $= 4a$

Rhombus:



$AC \neq BD$

$\angle le \neq 90°$

Area $= \Delta ACD + \Delta ABC$

$= \frac{1}{2} \times AC \times DE + \frac{1}{2} AC \times BE$

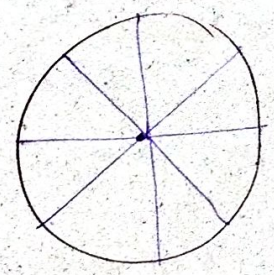
$= \frac{AC}{2}(DE + BE)$

$= \frac{1}{2} \times AC \times BD$

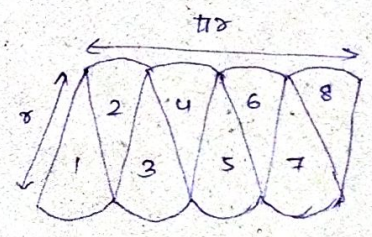
Area $= \frac{1}{2}$ (product of Diagonals)

perimeter $= 4a$

Circle:



Circumference $= 2\pi r$

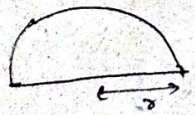


Looks like parallelogram

So, Area $= (\pi r)(r) = \pi r^2$

$A = \pi\left(\frac{d^2}{4}\right)$

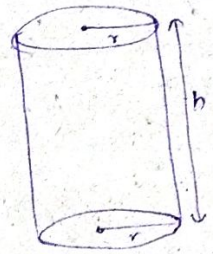
Semi Circle



Area $= \frac{\pi r^2}{2}$

perimeter $= \pi r + 2r$

Cylinder:

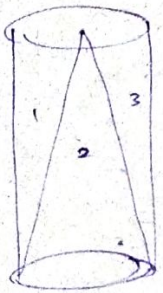


Area = $(\pi r^2) \times h$

$= \pi r^2 h$

Volume = $\pi r^2 h$

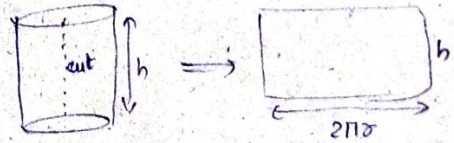
Cone:



$\frac{1}{3}$rd part of Cylinder is cone

So Area = $\frac{1}{3}\pi r^2 h$

Cylinder Continuition



Lateral Curved Surface Area

$= 2\pi r h$

Top and Bottom Circles Area = $\pi r^2 + \pi r^2$

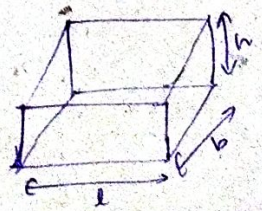
$= 2\pi r^2$

So Total Surface Area = $2\pi r^2 + 2\pi r h$

Cuboid

2D 3D

Rectangle → Cuboid



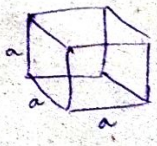
Volume = $l \times b \times h = lbh$

Cube:

2D 3D

Square → Cube

Volume = a^3



$V = a^3$

Total Surface Area

$= 2(lb + bh + lh)$

Lateral Surface Area

$= 2(bh + lh)$

Total Surface Area

$= 6a^2$

Lateral Surface Area

$= 4a^2$

Sphere:

2D 3D

Circle → Sphere

Volume = $\frac{4}{3}\pi r^3$

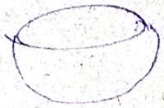
Total Surface Area = $4\pi r^2$

Sphere

Hemisphere

2D 3D

Semicircle → Hemisphere



Total Surface area = $2\pi r^2 + \pi r^2$

of

Hemisphere $= 3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$

Volume of sphere

$= \frac{2}{3}$ (Volume of Cylinder)

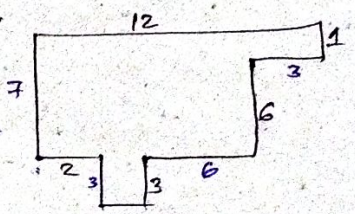
$= \frac{2}{3}(\pi r^2 h)$ $[h = 2r]$

$= \frac{4}{3}\pi r^3$

Perimeter problems

Find the perimeter of the decagon with give dimensions

(a) 32 (b) 34 ~~(c)~~ 44 (d) 22

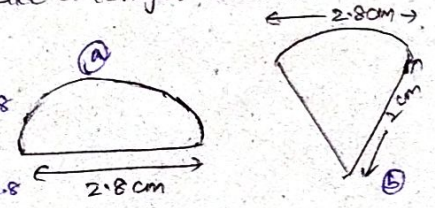


Soln: Total = 12+7+2+3+1+3+6+6+3+1

$= 44$

2. An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a longer round?

Soln:



peri of a = $\pi(2.8\text{cm}) + 2.8$

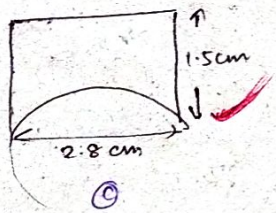
$= \frac{22}{7} \times \frac{2.8\text{cm}}{2} + 2.8$

$= \frac{8.8}{2}\text{cm} = 4.4\text{cm} + 2.8\text{cm}$

$= 7.2\text{ cm}$

peri of b = 4.4 + 4

= 8.4 cm



peri of C = 1.5 + 1.5 + 2.8 + π(1.4)

= 5.8 + 4.4

= 10.2 cm

Perimeter and Area Problem:

Q. Find area and perimeter of semicircle of radius 7 cm

Soln: Area = $\frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7 \times 7}{2} = 77\text{ cm}^2$

Perimeter = $(\pi r + 2r) = 22 + 14 = 36\text{cm}$

2. The perimeters of a circle, a square and an equilateral triangle are equal; which one of following is true?

(a) Circle has largest area

(b) Square has largest area

(c) Eq. Δle has largest area

(d) All three have same area

Soln:- $2\pi r = 4a = 3s$ $\quad \frac{2\pi r}{12\pi} = \frac{4a}{12\pi} = \frac{3s}{12\pi}$

$\frac{r}{6} = \frac{a}{3\pi} = \frac{s}{4\pi}$

Areas → Circle : Square : Δle

$\pi(36) : 9\pi^2 : \frac{\sqrt{3}}{4}(16\pi^2)$

$36\pi : 9\pi^2 : 4\sqrt{3}\pi^2$

113.09 : 88.82 : 68.37

Circle has largest area

3. Area of an equilateral Δle is $\sqrt{3}$. What the perimeter?

(a) 2 (b) 4 (c) 6 (d) 8

Soln: $\frac{\sqrt{3}}{4}a^2 = \sqrt{3}$ $\quad$ perimeter = 3a = 6 cm

$a = 2\text{ cm}$

4. A window is made up of a square portion and an equi Δle portion above it. The base of the triangle is the side of square. If perimeter of window is 6m. Area of window in m^2 is:

(a) 1.43 (b) 2.06 (c) 2.68 (d) 2.88

Soln: peri = 3a + 2a = 6m; 5a = 6m; a = 1.2m

Area = $a^2 + \frac{\sqrt{3}}{4}a^2$

$= \left(1 + \frac{\sqrt{3}}{4}\right)(1.2)^2$

$= 2.06\text{ m}^2$

5. In a 400m race 'A' beats 'B' by 40m or 5 seconds then

(a) Find B's speed in kmph (b) B's time to reach the race?

(c) A's time to reach the race (d) A's speed?

Soln:

A	B	A	B
↓	↓	↓	↓
400	360	t	t+5

(a) $S_b = \frac{40}{5} = 8 \times \frac{18}{5} = 28.8$ kmph

(b) 50 seconds

(c) 45 seconds

(d) 8.88 m/s

5. A wire is in the form of a circle of radius 3.5 cm is bent in the form of a rectangle, whose l:b = 6:5. what's area?

(A) 60 cm² (B) 30 cm² (C) 45 cm² (D) 15 cm² (E) None

Sol^n: $2\pi(3.5) = 2(11x)$ $\quad 6x + 5x$

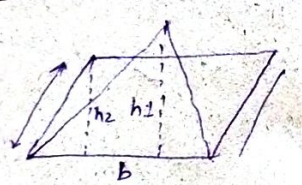
$2 \times \frac{22}{7} \times 0.5 = 2 \times 11 \times x$

$x = 1$ cm $\quad l = 6$ cm $\quad b = 5$ cm $\quad$ Area $= 30$ cm²

Area problems

1. A triangle with altitude h_1 and a parallelogram with altitude h_2 are constructed on the same base such that their areas are equal. Then relation b/w h_1 and h_2 is:

(A) $h_1 = 2h_2$ (B) $2h_1 = h_2$ (C) $h_1 = 3h_2$ (D) None



Sol^n: Area of ▱ = Area of △le

$b \times h_2 = \frac{1}{2} \times b \times h_1$

$h_1 = 2h_2$

2. The area of the largest triangle than can be inscribed in a semicircle of radius 'r' units is

(A) $3r^2$ (B) $2r^2$ (C) r^2 (D) $4r^2$

Sol^n: Area $= \frac{1}{2}(2r)(r)$

$= r^2$

3. The parallel sides of a trapezium are 32 m and 20 m and distance b/w them is 15 m. Area of trapezium is

(a) 290 m² (b) 390 m² (c) 400 m² (d) 160 m²

Sol^n: $A = \frac{d}{2}(a+b)$

$A = \frac{15}{2}(20+32)$

$= 15 \times 26$

$A = 390 m^2$

4. There is a rectangular Garden whose length and width are 60m x 20m. There is a walkway of uniform width around garden. Area of walkway is 516m². Find width of walkway?

(a) 1 (b) 2 (c) 3 (d) 4

Sol^n:- Let width be $\frac{x}{2}$ Then

$(l+x)(b+x) - (l)(b) = 516$

$(60+x)(20+x) - 1200 = 516$

$1200 + 80x + x^2 = 516 + 1200$

$x^2 + 80x - 516 = 0$

$x^2 + 86x - 6x - 516 = 0$

$(x-6)(x+86) = 0$

$x = 6, -86$

So $\frac{x}{2} = 3$