

Numerical
Ability

Quantitative Aptitude

Part - 1

QUANTITATIVE APTITUDE

1. NUMBER SYSTEMS

Introduction

→ Factors / Divisors

→ Remainder

→ Units place / last digit (Cyclicity)

→ Successive divisors

→ LCM & HCF

→ Base Systems

→ Divisibility

→ Factorial

02/08/2020 : Sunday

Factors / Divisors:

12 → 1, 2, 3, 4, 6, 12. No. of factors = 6

120 → 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120 No. of factors = 16

What if large numbers = ?

$$12 = 3 \times 4$$

$$= 3^1 \times 2^2$$

$$= (3^0, 3^1) (2^0, 2^1, 2^2)$$

No. of factors = $2 \times 3 = 6$

$$1800 = 18 \times 2 \times 5 \times 10$$

$$= 2^2 \times 9^1 \times 5^1 \times 2^1$$

$$= 2^3 \times 3^2 \times 5^1$$

No. of factors = $4 \times 3 \times 3$

$$120 = 12 \times 10$$

$$= 3 \times 8 \times 5$$

$$= 2^3 \times 3^1 \times 5^1$$

No. of factors = $4 \times 2 \times 2 = 16$

1. Write given no. as prime factors

2. Consider each power and add 1

3. Multiply all of them

4. Result is no. of factors

find no. of factors for $N = 13^1 \times 23^2 \times 33^3 \times 43^2$

$$N = 13^1 \times 23^2 \times 3^3 \times 11^3 \times 43^2$$

$$\therefore \rightarrow 2 \times 3 \times 4 \times 4 \times 3$$

$$= 288$$

find no. of odd factors for 1200

Soln $1200 = 12 \times 2^2 \times 25$

$$= 3^1 \times 2^4 \times 5^2$$

$$= 2^4 \times [3^1 \times 5^2]$$

$$\text{Total} \rightarrow 2 \times 3 \times 5 = 30$$

$$\text{consider only odd numbers} \Rightarrow 2 \times 3 = 6$$

If even factors \Rightarrow Total - no. of odd factors

$$= 30 - 6$$

$$= 24$$

$$\{2, 2, 2, 2, 3, 5, 5\}$$

No. of prime factors \Rightarrow Add the powers $= 4 + 1 + 2 = 7$

No. of distinct/different prime factors $\Rightarrow 3 \{2, 3, 5\}$

Odd and Even factors

* 1800 $= 2^3 \times 3^2 \times 5^2$

Total factors = 36 Odd $\Rightarrow 9$

Even $\Rightarrow 27$

Generalised $\Rightarrow N = a^m b^n c^p d^l \dots \dots$

where a, b, c, d are prime numbers

No. of factors $= (m+1)(n+1)(p+1)(l+1) \dots$

Even factors $= (m)(n+1)(p+1) \dots$ (m is 2 say)

Odd factors $= (1)(n+1)(p+1) \dots$

prime factors $= (m+n+p+l) \dots$

distinct prime factors $= \downarrow \downarrow \downarrow \downarrow$
 $1 + 1 + 1 + \dots$

* $N = 8! \rightarrow 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 2^3 \times 7 \times 2 \times 3 \times 5 \times 2^3 \times 3 \times 2$

No. of factors = ?

$$= 2^7 \times 7 \times 3^2 \times 5^1$$

$$\text{Total} \rightarrow 8 \times 2 \times 3 \times 2$$

$$\text{Total} = 96$$

$$\text{Odd} \Rightarrow 12$$

$$\text{Even} = 84$$

$$\text{Prime} = 7 + 1 + 2 + 1 = 11$$

$$\text{Distinct} = 4 \{2, 3, 5, 7\} \quad \checkmark \checkmark \checkmark \checkmark$$

$$\text{No. of perfect square factors} = 2^1 \times (2^2)^1 \times (3^2)^1 \times (5^2)^1$$

$$\rightarrow 4 \times 2 = 8$$

$$\text{perfect square factors for } 1800 = 2^1 \times (2^2)^1 \times (3^2)^1 \times (5^2)^1$$

$$= 2 \times 2 \times 2 = 8$$

No. of factors other than one and itself \Rightarrow total - 2

$$\text{for } 1800 \Rightarrow 36 - 2 = 34$$

$$\text{for } 8! \Rightarrow 96 - 2 = 94$$

Units place / Last digit / Cyclicity / Power cycles

Find units place in the expansion of

$$3, @ (142)^{2123}$$

$2^1 - 2$	$2^5 - 2$	$2^9 - 2$
$2^2 - 4$	$2^6 - 4$	$2^{10} - 4$
$2^3 - 8$	$2^7 - 8$	$2^{11} - 8$
$2^4 - 6$	$2^8 - 6$	$2^{12} - 6$

So whatever the power, You divide power with 4
last two digits

Then $234 \Rightarrow 8$ then units place is 8

$$(b) (432)^{191}$$

$41 \div 4 \Rightarrow 1$ means last digit 2

$$(c) (142)^{1248}$$

$48 \div 4 \Rightarrow 0$ means last digit 6

Note: Whatever we do for 2, the same power cycle gets repeated for 3 & 7 & 8

2, 3, 7, 8

Consider 7 in last place say $(1127)^{1294}$

$7^1 = 7$	$7^5 = 7$
$7^2 = 9$	$7^6 = 9$
$7^3 = 3$	$7^7 = 3$
$7^4 = 1$	$7^8 = 1$

$$(d) (1127)^{1294}$$

$94 \div 4 \Rightarrow 2$ means 9

$$(e) (1135)^{5678}$$

Whatever may power, if last place is 5
then result always 5.

$$(f) (6)^{5619}$$

Whatever may power, if last place is 6
then result always 6.

$3^1 = 3$	$3^5 = 3$
$3^2 = 9$	$3^6 = 9$
$3^3 = 7$	$3^7 = 7$
$3^4 = 1$	$3^8 = 1$

$8^1 = 8$	$8^5 = 8$
$8^2 = 4$	$8^6 = 4$
$8^3 = 2$	$8^7 = 2$
$8^4 = 6$	$8^8 = 6$

Note: This sort of numbers are

0, 1, 5, 6

$$(g) (124)^{127} = 4$$

$4^1 = 4$ | $4^3 = 4 \rightarrow$ power odd then 4
 $4^2 = 6$ | $4^4 = 6 \rightarrow$ power even then 6

$$(h) 9^{142} = 1$$

$9^1 = 9$ | $9^3 = 9 \rightarrow$ odd then 9
 $9^2 = 1$ | $9^4 = 1 \rightarrow$ even then 1

Note: This sort of numbers are 4, 9

Units place for factorial

$$* (122)^{123!} \rightarrow 2^{123!}$$

$123! = 123 \times 122 \dots \underbrace{\dots}_{\text{divided by 4}} \times 4 \times \dots \times 1$

Means Remainder = 0

Means last digit = 6

$$* (33)^{34} \times (35)^{36} + 37^{12}$$

$$3^{34} \times 5^{36} + 7^{12}$$

$$9 \times 5 + 1$$

$$45 + 1 \rightarrow \text{last digit} = 6$$

$$(*) 33^{34} + 35^{36} + 37^{12}$$

$$9 + 5 + 1 \rightarrow \text{last} \Rightarrow 5$$

$$* 1! + 2! + 3! + 4! + 5! \dots + 100!$$

Trick is from 5! all unit places are zero

$$\text{So } 1 + 2 + 6 + 4 \Rightarrow 3$$

$$Q. (131)! \rightarrow \text{last digit} = 0$$

Factorial / Highest Power

* find the highest power of 3 in $29!$

$$29! = 29 \times 28 \times 27 \times \dots \times 4 \times 3 \times 2 \times 1$$

$\{3, 6, 9, 12, 15, 18, 21, 24, 27\} \rightarrow$ multiples of 3

$$= 3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, 3 \times 5, 3 \times 3 \times 2, \dots, 3 \times 3 \times 3, \dots$$

$$\Rightarrow 3^{13}$$

what if number is large ????

So do it like $\left[\frac{29}{3}\right] + \left[\frac{9}{3}\right] + \left[\frac{3}{3}\right]$

$$9 + 3 + 1 = 13$$

Now consider $129! \Rightarrow \left[\frac{129}{3}\right] + \left[\frac{43}{3}\right] + \left[\frac{14}{3}\right] + \left[\frac{4}{3}\right]$

$$43 + 14 + 4 + 1 = 62$$

* highest power of 4 in $129!$

find power of 2^2 (2^2)

so $\left[\frac{129}{2^2}\right] + \left[\frac{64}{2^2}\right] + \left[\frac{32}{2^2}\right] + \left[\frac{16}{2^2}\right] + \left[\frac{8}{2^2}\right] + \left[\frac{4}{2^2}\right] + \left[\frac{2}{2^2}\right]$

$$= 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$2^2 \text{ power} = 127$$

$$\text{so } (2^2)^{63} \times 2$$

$$4^2 \text{ power} = 63$$

for 3 $\rightarrow \left[\frac{129}{3^2}\right] + \left[\frac{25}{3^2}\right] + \left[\frac{5}{3^2}\right] + \left[\frac{1}{3^2}\right]$

$$30 + 1 = 31$$

$$5^2 \text{ power} = 31$$

6's power $\rightarrow (2 \times 3)^6$ power

$$2^{127} \quad 3^{62}$$

6^{62} possible

so 6's power $\Rightarrow 62$

7 prime number

$$8 \rightarrow (2^3)^4 \times 2$$

$$9 \rightarrow (3^2)^3$$

No. of zeroes in factorial

* find the highest power of 10 in $129!$

(or)

find no. of zeroes at the end of $129!$

(or)

find trailing zeroes in $129!$

Solⁿ: 10^c power $\rightarrow (2 \times 5)^c$ power

so $2^{127} \quad 5^{31}$
 10^{31} possible

* find no. of zeroes at end of product

@ $(8 \times 25 \times 21 \times 50 \times 16 \times 35 \times 90)$

$$2 \times 3^2 \times 5^2 \times 3 \times 7 \times 2 \times 5^2 \times 2^4 \times 5 \times 7 \times 3^2 \times 2^3 \times 5^1$$

$$2^7 \times 5^6 \times \dots$$

\checkmark
 10^6 possible

No. of zeroes in Product

* find the highest power of 10 (or) no. of zeroes at the end of product

$$\text{i), } 10 \times 20 \times 30 \times 40 \times \dots \times 200$$

$$1 \times 10 \times 2 \times 10 \times 5 \times 10 \dots \cdot 20 \times 10$$

$$10^{20} \times 1 \times 2 \times 3 \times \dots \cdot 20$$

$$10^{20} \times 20!$$

$$\left[\frac{10}{2} \right] + \left[\frac{10}{5} \right] + \left[\frac{2}{5} \right] + \left[\frac{1}{2} \right] \quad \left[\frac{20}{5} \right] + \left[\frac{4}{5} \right]$$

$$10+5+2+1$$

$$2^{18}$$

$$\text{So } (10)^4 \text{ possible} \rightarrow \text{Totally } 10^{24}$$

$$\text{ii), } 5 \times 10 \times 15 \times 20 \times \dots \cdot 200$$

$$5 \times 1 \times 5 \times 2 \times 5 \times 3 \dots \cdot 5 \times 40$$

$$5^{40} \times 40!$$

$$\left[\frac{20}{2} \right] + \left[\frac{10}{2} \right] + \left[\frac{10}{5} \right] + \left[\frac{2}{5} \right] + \left[\frac{1}{2} \right] \quad \left[\frac{10}{5} \right] + \left[\frac{8}{5} \right] +$$

$$20+10+5+2+1$$

$$38$$

$$\text{So totally } 2^{28} 5^{49}$$

$$10^{38} \text{ possible}$$

$$\text{iii), } 1 \times 2 \times 3^3 \dots \cdot 49^{49}$$

$$1 \times 5^5 \times 10^{10} \times 15^{15} \times 25^{25} \times 20^{20} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$$

$$2^{10} \quad 5^{10} \quad 2^{40} \quad 5^{20}$$

$$\text{No. of } 5's \rightarrow 5+10+15+20+25+30+35+40+45$$

$$5's = 250 \quad 250$$

$$\text{No. of } 2's \rightarrow 2^1 \times (2^2)^4 \times (2^4) \times (2^3)^8 \times (2^1)^9 \times (2^2)^{12} \times (2^4) \dots$$

2's obviously greater than 250

So possible zeroes are 250 $\rightarrow 10^{250}$

Remainder - Introduction 03/08/2020 : Monday

$$R\left(\frac{17}{9}\right) \rightarrow +8 \text{ (positive remainder) generally}$$

$$\rightarrow -1 \text{ (negative remainder)}$$

$$R\left(\frac{9+8}{9}\right) \quad R\left(\frac{18-1}{9}\right) \quad \text{(Q) find the remainder when }$$

$$\underline{25 \times 35 \times 45 \times 65 \times 75}$$

$$\text{Solt: } R\left(\frac{2 \quad 3 \quad 1 \quad -1 \quad 11}{25 \times 35 \times 45 \times 65 \times 75} \right)$$

other way

$$2 \times 3 \times 1 \times 10 \times 9$$

$$R\left(\frac{99}{11}\right) \rightarrow R\left(\frac{1}{11}\right)$$

$$2 \times 3 \times 1 \times 1 \times 2$$

$$R\left(\frac{12}{11}\right) \rightarrow R\left(\frac{1}{11}\right) \text{ Remainder} \Rightarrow +1$$

$$\text{Remainder} = +1$$

(Q) find remainder when 17^{99} is divided by 8

$$R\left(\frac{17^{99}}{8}\right) \rightarrow \frac{17 \times 17 \times \dots \times 17}{8} \rightarrow R\left(\frac{1^{99}}{8}\right) \text{ Remainder} = 1$$

* find remainder when 62^{62} is divided by 7

$$R\left(\frac{62^{62}}{7}\right) \Rightarrow R\left(\frac{(-1)^{62}}{7}\right) = R\left(\frac{1}{7}\right) \quad \text{Remainder} = -1 \text{ or } +6$$

* find remainder when 3^{250} is divided by 7

$$R\left(\frac{3^{250}}{7}\right) = R\left(\frac{(-1)^{250}}{7}\right) = R\left(\frac{1}{7}\right) \Rightarrow \text{Rem} \equiv +1$$

* Another way $R\left(\frac{(3^3)^{83} \times 3}{7}\right) = R\left(\frac{(-1)^{83} \times 3}{7}\right) = R\left(\frac{-3}{7}\right) \quad \text{Rem} \Rightarrow +4$

* find $R\left(\frac{21^{53}}{17}\right) \Rightarrow \left(\frac{4}{17}\right)^{53} = \left(\frac{4 \times (-1)^{26}}{17}\right) = \left(\frac{4 \times 1}{17}\right) \Rightarrow \text{Rem} \equiv +4$

Remainder for factorial :

find remainder when $1! + 2! + 3! + \dots + 100!$ is divided by 5

$$R\left(\frac{1! + 2! + 3! + 4! + \overbrace{5! + \dots + 100!}^0}{5}\right)$$

$$R\left(\frac{1+2+3+6+24}{5}\right)$$

$$R\left(\frac{3}{5}\right) \Rightarrow \text{Rem} = +3$$

Q5 $R\left(\frac{1! + 2! + 3! + 4! + 5! + \dots + 100!}{6}\right) \Rightarrow R\left(\frac{1! + 2!}{6}\right)$
 $= R\left(\frac{3}{6}\right)$
 $\text{Rem} \Rightarrow +3$

* find the remainder when $1 \times 1! + 2 \times 2! + 3 \times 3! + 4 \times 4! + \dots + 100 \times 100!$ divided by 5

$$R\left(\frac{1 \times 1! + 2 \times 2! + 3 \times 3! + 4 \times 4! + \overbrace{5 \times 5! + \dots + 100 \times 100!}^0}{5}\right)$$

$$R\left(\frac{1+4+18+96}{5}\right) = R\left(\frac{4}{5}\right) \quad \text{Rem} = +4$$

$$\text{If } R\left(\frac{1 \times 1! + 2 \times 2! + 3 \times 3! + 4 \times 4! + \dots + 100 \times 100!}{6}\right)$$

$$R\left(\frac{1+4}{6}\right) = R\left(\frac{5}{6}\right) \quad \text{So Rem} = +5$$

Note: $R\left[\frac{1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n!}{D}\right] = (D-1) \quad (D \leq n)$

* find the sum of $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + 100 \times 100!$

- (a) 101! (b) 101 \times 101! (c) 101! - 1 (d) none

Check ; $1 \times 1! + 2 \times 2! = 5 \Rightarrow 3! - 1$

$$1 \times 1! + 2 \times 2! + 3 \times 3! = 23 = 4! - 1$$

Note: $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! \Rightarrow (n+1)! - 1$

Remainder-Two powers Addition:

* what is the remainder when $15^{23} + 23^{23}$ divided by 19

$$R\left(\frac{15^{23} + 23^{23}}{19}\right) \Rightarrow \left(\frac{(-4)^{23} + (4)^{23}}{19}\right) = R\left(\frac{0}{19}\right) \quad \text{Rem} \Rightarrow 0$$

* what is remainder when 21^{41} divided by 15

$$R\left(\frac{21^{41}}{15}\right) \Rightarrow R\left(\frac{21 \times 21^{40}}{8 \times 5}\right) \Rightarrow R\left(\frac{2 \times (1)^{40}}{5}\right) = R\left(\frac{2}{5}\right) \quad \text{Rem} \Rightarrow 2$$

↑ coz u divided here with 3 But Wrong

So $2 \times 3 = 6$

Rem = 6

Note: When you have common multiple in both numerators and denominators, you can cancel them to simplify problem. But at the end, you must multiply the result with that multiple.

Successive Divisors

* When 100 is successively divided by 5, 3 & 2 leaves

$$\text{remainder } \frac{0}{}, \frac{2}{}, \frac{1}{}, \frac{0}{}$$

$$5)100(20 \quad \begin{array}{c} 3 \\ \downarrow \end{array} 20(6 \quad \begin{array}{c} 2 \\ \downarrow \end{array} 6(3 \\ \hline 0 \qquad \qquad 2 \qquad \qquad 0)$$

* A number is successively divided by 5, 3 & 2 leaves remainder

0, 2 & 0. final Quotient is 2. find number

$$N = D \times Q + R \\ = 2 \times 2 + 0$$

$$N = 4$$

$$N = 3 \times 4 + 2 \\ = 12 + 2$$

$$N = 14$$

$$N = 5 \times 14 + 0 \\ = 70 + 0$$

$$N = 70$$

* A number is SD by 7, 3 & 5 leaves remainder 4, 1 & 2.

final Quotient is 3. ① find Complete Remainder

② If divisors are reversed then find remainders

$$\text{Soln: } 7[3[5(3)+2]+1]+4 \\ \Rightarrow 368$$

$$\textcircled{1} \text{ Complete Remainder} \rightarrow \frac{368}{7 \times 3 \times 5} = \frac{368}{105} \Rightarrow 53$$

$$\textcircled{2} \text{ Reversed divisors } (5, 3, 7)$$

$$5)368(73 \\ \begin{array}{r} 35 \\ \hline 18 \\ 15 \\ \hline 3 \end{array}$$

$$3)73(24 \\ \begin{array}{r} 6 \\ \hline 13 \\ 12 \\ \hline 1 \end{array}$$

$$7)21(3 \\ \begin{array}{r} 21 \\ \hline 3 \end{array}$$

Alternate Method to find Complete Remainder

$$\text{Divisors} \rightarrow \begin{array}{c} 49 \\ 7 \\ 3 \\ 5 \\ \hline + \\ K \\ + \\ 1 \\ \hline 4 \\ 1 \\ 2 \end{array}$$

$$\text{Remainder} \rightarrow \begin{array}{c} 53 \\ 7 \\ \hline + \\ 5 \\ 3 \\ \hline 7 \end{array}$$

$$\text{Complete Remainder} = 53$$

Number

$$N = (7 \times 3 \times 5)k + 53$$

$$[\because k = \text{final Quotient}]$$

$$= (7 \times 3 \times 5)3 + 53$$

$$N = 368$$

LCM - Introduction

04/08/2020 : Tuesday

$$* \text{LCM}(10, 12, 15) = 2 \times 2 \times 3 \times 5 = 60$$

$$\begin{array}{c} 1 \\ | \\ 10, 12, 15 \\ 2 \\ | \\ 5, 6, 15 \\ 3 \\ | \\ 5, 3, 15 \\ 5 \\ | \\ 5, 1, 1 \\ , , , \end{array} \quad \begin{array}{l} \downarrow \\ \text{Least Common Multiple} \end{array}$$

If this takes time, try multiples of highest number with remaining

$$* \text{LCM}\left(\frac{2}{3}, \frac{4}{5}, \frac{8}{9}\right) = \frac{\text{LCM}(2, 4, 8)}{\text{HCF}(3, 5, 9)} = \frac{8}{1} = 8$$

$$* \text{LCM}\left(\frac{2}{3}, \frac{5}{6}, \frac{8}{9}\right) = \frac{\text{LCM}(2, 5, 8)}{\text{HCF}(3, 6, 9)} = \frac{40}{1} = 40$$

$$* \text{LCM}(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{8}{9}) = 24$$

LCM Applications - Level I

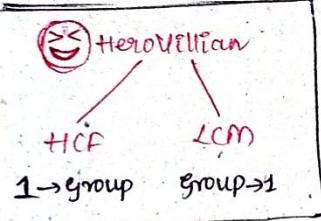
[Together, meeting, simultaneously] Means LCM
Group → One]

- * B1 → 10 sec
- B2 → 6 sec
- B3 → 15 sec

After how much time all bells ring together?

$$\begin{array}{r} 3 | 10, 6, 15 \\ 2 | 10, 2, 5 \\ 5 | 5, 1, 5 \\ \hline 1, 1, 1 \end{array} \Rightarrow 6 \times 5 = 30$$

So after 30 sec



* find the smallest number which is divided by 10, 12, 8 & 15?

$$\text{LCM}(10, 12, 8, 15) = 120$$

30, 60, 90, 120

* find the smallest number which is divided by 10, 12, 8 & 15 leaves remainder 5 in each case?

$$\text{Sol}^n: \text{LCM}(10, 12, 8, 15) + 5 = 125$$

* find the least number which is divided by 10, 12, 8 & 15 leaves remainders 8, 10, 6 & 13 respectively?

Solⁿ: Do apply negative remainder concept then

$$-2, -2, -2, -2 \rightarrow \text{Means } \text{LCM} - 2 = 120 - 2 = 118$$

Applications

* A heap of stones can be made in groups of 7 but when made up into groups of 8, 12 and 16. There are 3 stones left in each case. The no. of stones in heap is

147 | 150 | 137 | 154

$$\text{Sol}^n: \text{LCM}(8, 12, 16) = 48$$

leaves remainder 3 $\Rightarrow 48 + 3 = 51 \rightarrow$ not divided by 7

$$48(2) + 3 = 99$$

$$48(3) + 3 = 147 \checkmark$$

* find the least perfect square which is divisible by 3, 4, 5, 6 and 8

1200 | 2500 | 900 | 3600

$$\text{Sol}^n: \text{LCM}(3, 4, 5, 6, 8) = 120$$

$$\begin{aligned} \text{So } 120k &\rightarrow 2^3 \times 3 \times 5 \\ k &= 2 \times 3 \times 5 \\ &\Rightarrow 120 \times 30 \\ &\approx 3600 \end{aligned}$$

* Six bells begin to toll together and toll at intervals of

2, 4, 6, 8, 10 and 12 seconds

(a) If they all toll simultaneously at 9:20 am then they will again toll together at?

$$\text{LCM}(2, 4, 6, 8, 10) = 120 \text{ seconds} = 2 \text{ minutes}$$

So again at 9:22 am

(b) In 30 min, how many times do they toll together?

$$\frac{30 \text{ min}}{2 \text{ min}} + 1 \Rightarrow 16 \text{ times}$$

↓
Starting

* Three men start together to travel the same way around a circular track of 11 km's in circumference. Their speeds are 4, $5\frac{1}{2}$ and 8 kmph respectively. When will they meet at starting point?

$$\text{Soln: } \text{LCM}\left(4, \frac{11}{2}, 8\right) \Rightarrow \frac{\text{LCM}(4, 11, 8)}{\text{HCF}(1, 2, 1)} = \frac{88}{1} \text{ for 1st one small } \frac{88}{4} = 22 \text{ hrs}$$

Another way

$$\text{time} = \frac{\text{distance}}{\text{speed}} \quad T_A = \frac{11}{4} \text{ hrs} \quad T_B = 2 \text{ hrs} \quad T_C = \frac{11}{8} \text{ hrs}$$

$$\text{LCM}\left(\frac{11}{4}, 2, \frac{11}{8}\right) \Rightarrow \frac{\text{LCM}(11, 2, 11)}{\text{HCF}(4, 1, 8)} = \frac{22}{1} \text{ hrs}$$

* Three rings complete 60, 36 and 24 revolutions in a minute. They start point in their circumference downwards. By what time they come together again in the same position?

Soln: for 1 revolution A \rightarrow 1 sec

$$B \rightarrow \frac{5}{3} \text{ sec}$$

$$C \rightarrow \frac{5}{2} \text{ sec}$$

$$\text{LCM}\left(1, \frac{5}{3}, \frac{5}{2}\right) = \frac{\text{LCM}(15, 5)}{\text{HCF}(1, 3, 2)} \Rightarrow \frac{5}{1} = 5 \text{ sec}$$

LCM - Greatest / Smallest n-digit Numbers:

* find the greatest 5 digit number which is divided by

10, 12, 15 & 8

$$\text{Soln: } \text{LCM}(10, 12, 15, 8) \Rightarrow 120$$

$$\begin{array}{r} 120) 99999(833 \\ \underline{960} \\ 399 \\ \underline{360} \\ 399 \\ \underline{360} \end{array}$$

$$\begin{array}{r} x < 99999 \\ \text{So } 99999 - 39 \\ \underline{99969} \end{array}$$

* find the smallest 5 digit number which is divided by 10, 12, 15 & 8 leaves 3 remainder in each case

$$\text{Soln: } \text{LCM}(10, 12, 15, 8) \Rightarrow 120$$

$$\begin{array}{r} 120) 10000(83 \\ \underline{960} \\ 400 \\ \underline{360} \\ 40 \end{array} \rightarrow \text{So add } 80 \Rightarrow$$

$$\begin{array}{r} x > 10000 \\ 10080 \\ \text{But 3 remainder} \\ 10083 \end{array}$$

HCF - Introduction:

Highest common factor (HCF)
Greatest common divisor (GCD)

* HCF(12, 18, 24)

$$\left(2^2 \times 3, 2 \times 3^2, 2^3 \times 3\right) \quad \left.\begin{array}{l} 12 \\ 18 \\ 24 \end{array}\right\} \text{HCF} \rightarrow 3 \times 2 = 6$$

* HCF(403, 434, 496)

$$\left.\begin{array}{l} 403 \\ 434 \\ 496 \end{array}\right\} \quad \begin{array}{l} 93 \\ 31 \\ 62 \end{array}$$

$$\text{HCF}(31, 62, 93) = (31)$$

* HCF(60, 84, 108)

$$\left.\begin{array}{l} 60 \\ 84 \\ 108 \end{array}\right\} \quad \begin{array}{l} 24 \\ 24 \end{array}$$

$\text{HCF}(24, 24, 48) = 24$ Check here, it is wrong
So now take factors for 24

$$24, 12, 8, 6, 4, 3, 2, 1$$

So try one after one now

HCF Applications - I

[Common, equal, Same] } Clues for GCD

→ group

* A wholesale tea dealer has 408 kgs, 468 kgs and 516 kgs of three different qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.

- (a) 50 (b) 86 (c) 24 (d) 112

$$\text{Soln: HCF } (408, 468, 516)$$

$$\begin{array}{r} 108 \\ 60 \quad 48 \end{array}$$

$$\text{HCF } (48, 60, 108) \rightarrow 24$$

$$So, 12 \text{ kgs}$$

$$\begin{array}{r} 12 \\ 12 \end{array}$$

* The greatest possible length which can be used to measure exactly the lengths 7m, 3m, 85cm, 12m 95cm is:

- (a) 15 cm (b) 25 cm (c) 35 cm (d) 42 cm

$$\text{Soln: GCD } [700 \text{ cm}, 300 \text{ cm}, 85 \text{ cm}, 1295 \text{ cm}]$$

$$\text{GCD } [700, 385, 1295]$$

$$\begin{array}{r} 48 \\ 24 \quad 24 \end{array}$$

Check Options
→ 35 cm

6. find the greatest number which can divide 1354, 1866 and 2762 leaving the same remainder 10 in each case.

- (a) 64 (b) 124 (c) 156 (d) 260

$$\text{Soln: } \text{GCD } (1354, 1866, 2762)$$

$$\text{GCD } [1344, 1856, 2752] \quad \text{Check Options} = 64$$

HCF Applications - II

* Two baskets contain 195 and 250 bananas respectively, which are distributed in equal number among children. Find the largest no. of bananas that can be given, so that 3 bananas are left over from the first basket and 2 from second.

- (a) 4 (b) 18 (c) 8 (d) 6

$$\text{Soln: } \text{GCD } (192, 248) \rightarrow 16 \text{ (Check Options)}$$

* In a seminax the no. of applications in Hindi, English and Maths are 60, 84 and 108 respectively. Find the minimum number of rooms required, where in each room the same number of applications be allotted; and all of them being in same subject.

- (a) 20 (b) 22 (c) 25 (d) 21

$$\text{Soln: L.C.M. } (60, 84, 108) = 12$$

$$\begin{array}{r} 48 \\ 24 \quad 24 \end{array}$$

$$\text{So, 12 per room} \rightarrow \text{Total } \frac{60+84+108}{12} = 21 \text{ rooms}$$

* A merchant has three kinds of milk: 435 litres, 493 litres and 551 litres. Find the number of casts of equal size required to store all the milk with mixing.

- (a) 51 (b) 61 (c) 47 (d) 45

$$\text{Soln: } \text{GCD } (435, 493, 551)$$

$$\begin{array}{r} 116 \\ 58 \quad 58 \end{array}$$

$$\text{Total} = \frac{435+493+551}{29}$$

$$\text{GCD } [88, 58, 116] \rightarrow 29$$

Total = 51

LCM & HCF Properties

$$\text{LCM}(8, 10) = 40$$

$$\text{HCF}(8, 10) = 2$$

$$\text{LCM} \times \text{HCF} = 8 \times 10$$

$$40 \times 2 = 8 \times 10$$

$$80 = 80$$

$$\text{LCM}(8, 10, 12) = 120$$

$$\text{HCF}(8, 10, 12) = 2$$

* The ratio of two numbers is 3:4 and their HCF is 4.

Their LCM is:

- A. 12 B. 16 C. 24 D. 48

Soln: $3x, 4x$ HCF = 4 ————— But $\text{HCF}(3x, 4x) = x$

$$(3x)(4x) = (4)(\text{LCM})$$

$$\text{So } x=4$$

$$12x^2 = 48 \text{ LCM}$$

$$\text{LCM} = 3x^2$$

$$= 3(16)$$

$$\text{LCM} = 48$$

* Three numbers are in the ratio of 3:4:5 and their LCM is 2400. Their HCF is:

- (A) 40 (B) 80

Soln: $a:b:c = 3:4:5$

$$\text{HCF} \cdot (3x, 4x, 5x) = ? (x)$$

$$\text{LCM} = x \times 60$$

$$2400 = x \times 60$$

$$x = 40$$

* HCF of two numbers is 24 and their LCM is 1080. If one of the numbers is 120, find the other.

- (A) 216 (B) 532 (C) 108 (D) 820

Soln: $(24)(1080) = (120)(x)$

$$216 \quad 5$$

$$x = 216$$

* The HCF of two numbers is 32 and their product is 10240. Find their LCM.

- (A) 640 (B) 320 (C) 324 (D) 230

Soln: $(32)(\text{LCM}) = 10240$

$$\text{LCM} = 320$$

* The LCM of two numbers is 48. The numbers are in the ratio 2:3. The sum of the numbers is

- (A) 28 (B) 32 (C) 40 (D) 64

Soln: $2x \times 3x = 48 \times x$

$$6x^2 = 48x$$

$$\text{So } 2x + 3x = 5x = 40$$

$$6x = 48$$

$$x = 8$$

* HCF of two numbers is 24 and their LCM is 1344. If the difference between the numbers is 80, their sum is

- (A) 368 (B) 356 (C) 382 (D) 304

Soln: $(x)(x-80) = (24)(1344)$

$$x^2 - 80x - (24)(1344) = 0$$

$$\text{Using } \sqrt{b^2 - 4ac} = 32256$$

$$(a+b)^2 = (a-b)^2 + 4ab$$

$$= \sqrt{(80)^2 + 4(24)(1344)}$$

$$= \sqrt{129024 + 6000}$$

$$= \sqrt{135424}$$

$$a+b = 368$$

* If the sum of two numbers is 55 and the HCF and LCM of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to

$$(a) \frac{55}{601} \quad (b) \frac{601}{55} \quad (c) \cancel{\frac{11}{120}} \quad (d) \frac{120}{11}$$

$$\underline{\text{Soln}}: \quad a+b=55 \quad LCM=120 \quad HCF=5$$

$$\frac{1}{a} + \frac{1}{b} = ?$$

$$\frac{a+b}{ab} \Rightarrow \frac{\cancel{55}}{(120)(5\cancel{4})} = \frac{11}{120}$$

Number Systems - Units place:

2/3/7/8 → Cyclicity → 4

4/9 → Cyclicity → 2

0/11 5/6 → Cyclicity → 1

Power $\div 4$ Rem. \rightarrow 1, 2, 3 \rightarrow replace same in power
 \rightarrow 0 \rightarrow replace 4 in power

* find unit place $(123)^{125}$

Sol: $125 \div 4 \Rightarrow$ rem 1 then $3^1 \Rightarrow$ last place = 3

Units place - problem

* find last digit in $(2013)^7 + (2015)^9 + (2017)^{11} + (2019)^{13}$

$$\underline{\underline{Sd^n}}: 3^7 + 5^9 + 7^{11} + 9^{13}$$

$$3^3 + 5^1 + 7^3 + 9^1$$

$$7 + 5 + 3 + 9$$

289

So \$1 last digit

Remainders Using Cyclicity:

* find remainder when 9^{99} is divided by 7

$$R\left(\frac{q^2}{\pi}\right) = R\left(\frac{q^2}{\pi}\right) = 4$$

$$71 \div 3 \rightarrow \text{Rem} = 2$$

Remainder related problem

* find remainder when 7^{77} is divided by 15

Soln: $R\left(\frac{777}{15}\right) = 77 \div 4 = 1$ $R\left(\frac{7}{15}\right) = 7$ $R\left(\frac{7 \cdot 7^4}{15}\right) = 7$

$R\left(\frac{7 \cdot (14)^{89}}{15}\right) = 7$ $R\left(\frac{7}{15}\right) = 7$ $R\left(\frac{7^2}{15}\right) = 4$ \vdots

Remainder = 7 Cyclicality = 4 $R\left(\frac{7^3}{15}\right) = 13$ \vdots

General way Rem=7 $R\left(\frac{7^4}{15}\right) = 1$ \vdots

2. AVERAGES

05/08/2020 : Wednesday

Introduction

$$\text{Average} = \frac{\text{Sum of terms}}{\text{No. of terms}}$$

- ① If terms are in A.P ; and no. of terms are odd, then average is the middle number.

Eg: 34 36 38 40 42

$$A = \frac{190}{5} = 38$$

GATE

A number is much as greater than 75 as it is less than 117. find the number.

Soln: $\frac{75}{x} = \frac{x}{117} \Rightarrow x = \frac{75+117}{2}$

Another way $x-75 = 117-x$

Then $x = \frac{75+117}{2}$

$x=96$

- ② Average is a number equidistant from either sides

Averages - Concept I

Natural numbers $\rightarrow n$

1, 2, 3, ..., n

from property $\Rightarrow \frac{n+1}{2}$

Even numbers $\Rightarrow 2, 4, 6, 8, \dots, 2n$

$$A = \frac{2(n+1)}{2} = n+1$$

Odd numbers $\Rightarrow 1, 3, 5, \dots, 2n-1$

$$A = \frac{2n-1+1}{2} = \frac{2n}{2} = n$$

Sum of first n natural numbers $\Rightarrow A$ (no. of terms)

$$= \left(\frac{n+1}{2}\right)(n)$$

$$= \frac{n(n+1)}{2}$$

Similarly for Even = $n(n+1)$

for Odd = n^2

Averages - Concept II

If the terms are in A.P & no. of terms are even;

then take average of the two middle terms (or)

Take average of first and last terms

GATE

In a sequence of 12 consecutive odd numbers, the sum of first 5 terms is 425, what is the sum of last 5 terms?

Soln:

$$A = \frac{425}{5} = 85 \quad A = \frac{S}{\text{No.}}$$

$$So \quad 99 \times 5 = S$$

$$S = 495$$

Example:

Q: Sum of 8 consecutive odd numbers is 656. The average of 4 consecutive even numbers is 87. Find the sum of the smallest odd number and second largest even number?

$$\begin{aligned} \text{Soln: } & 75 \quad 81 \quad 83 \quad 89 \quad 84 \quad 88 \quad 90 \\ & A = \frac{656}{8} = 82 \quad B = 87 \times 4 \end{aligned}$$

Smallest odd \rightarrow 75. $\cancel{8} + \cancel{85}$

Largest even \rightarrow 90

$$\text{So } 75+88 = 163$$

Second largest even \rightarrow 88

Averages - Concept I

				Avg
24	30	36	30	
+5	29	35	41	$35 [30+5]$
-7	17	23	29	$23 [30-7]$
$\times 2$	48	60	72	$60 [30 \times 2]$
$\div 6$	4	5	6	$5 [30 \div 6]$

Example: The average age of a couple 7 years ago was 24 yrs. The average age of couple including a child at present is 22 yrs. Find present age of child.

$$\text{Soln: present } \frac{H+W}{2} = 31$$

$$H+W=62$$

$$\text{So Child} = 66 - 62$$

$$= 4$$

$$\text{present } H+W+C = 22 \times 3 = 66$$

Child age is 4 years.

Averages - Concept IV (Including/Excluding)

a. The average height of 12 boys is 143 cm. When a new boy joins the group average increases by 2 cm. Find the height of the new boy?

Soln: Method I

$$\frac{S}{12} = 143$$

$$\text{So } S = (143)(12) = 1716$$

$$= 1885 - 1716$$

$$x = 169 \text{ cm}$$

Method II

$$\begin{array}{c|c} n & a \\ \hline 12 & 143 \\ 13 & 145 \end{array}$$

$$\begin{array}{c|c} 12 & 13 \\ \hline 143 & 145 \\ & 145 \\ & 24 \\ \hline & 169 \text{ cm} \end{array}$$

This is called
T-Approach
(to reduce calculations)

Example:

A batsman in his 16th innings makes a score of 109 runs thereby increases his average score by 4 runs. Find his average after 16th innings?

Soln:

$$\frac{x}{15} = A$$

$$\frac{x+109}{16} = A+4$$

$$15A + 109 = 16A + 64$$

$$A = 109 - 64$$

$$A = 45$$

$$\begin{array}{c|c} 15 & 16 \\ \hline +4 & A \Rightarrow ? \\ +60 & \\ \hline 109 & \end{array}$$

$$109 - 60 = 49$$

$$\text{Average} = 49$$

Averages - Concept I (Replacement)

q. The average height of 7 boys has increased by 3 cm, when one of them whose height is 146 cm is replaced by a new boy. Find the height of the new boy?

$$\text{Soln: } A = \frac{S}{7} \quad A+3 = \frac{S-146+x}{7}$$

$$7x + 21 = 7x + x - 146$$

$$x = 146 + 21$$

$$x = 167 \text{ cm}$$

I method

$$\begin{aligned} x &= 146 + (7 \times 3) \\ &= 167 \text{ cm} \end{aligned}$$

Removed One
Change in heights
Total boys

Averages - Concept V (Combined Average)

Q. The average marks obtained by a group of 18 boys are 72, and that of 12 girls are 91. The average marks of both the groups together are?

$$\text{Soln: } \frac{B}{18} = 72 \quad \frac{G}{12} = 91 \quad \frac{B+G}{30} = ?$$

Generalised:

$$\frac{n_1 a_1 + n_2 a_2}{n_1 + n_2}$$

$$\begin{aligned} \text{I Method} \\ &\frac{(72)(18) + (91)(12)}{30} \\ &= \frac{432 + 364}{10} \end{aligned}$$

I method

$$\begin{array}{c|cc} & 72 & 91 \\ \hline 18:12 & & \\ 3:2 & \frac{72(3) + 91(2)}{3+2} & \end{array} \quad \text{Avg} = 79.6$$

Averages - Exercises

06/08/2020 : Thursday

1. The average marks of a student in 10 papers are 80. If the highest and the lowest scores are not considered, the average is 81. If the highest is 92, find the lowest.

$$\text{Soln: } \frac{S}{10} = 80 \quad \frac{S-92-x}{9} = 81$$

$$800 = 648 + 92 + x$$

$$x = 60$$

2. The batting average for 40 innings of a cricket player is 50 runs. His highest score exceeds lowest score by 172 runs. If these two innings are excluded, the average of remaining 38 innings is 48 runs. The highest score of player is

$$\text{Soln: } \frac{S}{40} = 50 \quad \frac{S+H-L}{38} = 48$$

$$-1824 + 2000 = H+L$$

$$H-L = 172$$

$$H+L = 176$$

$$2H = 348$$

$$H = 174$$

3. The captain of a cricket team of 11 players is 26 years old and wicket keeper is 3 years older. If the ages of these two are excluded, the average age of whole team is one year less than the average age of the whole team? What is the average age of the team?

$$\text{Soln: } \frac{S}{11} = A \quad \frac{S-55}{9} = A-1$$

$$11A - 55 = 9A - 9$$

$$2A = 46$$

$$A = 23 \text{ yrs}$$

4. The avg weight of 45 students in a class is 52 kg. Five of them whose average weight is 48 kg leave the class and 5 students whose avg weight is 54 kg join the class. What is the new avg weight of the class?

Solⁿ: $S = (45)(52)$ So now $\frac{3}{45} \cancel{(45)}(52) - \frac{5}{45} \cancel{(45)}(48) + \frac{2}{45} \cancel{(45)}(54)$

$$\frac{156+2}{3} = \frac{158}{3}$$

$$So \Rightarrow 52 \frac{2}{3}$$

5. Out of 9 persons, 8 persons spent Rs.30 each for their meals. The ninth one spent Rs.20 more than the average expenditure of all the nine. The total money spent by all of them was?

Solⁿ: $8 \times 30 = 240$

$$\frac{240 + (a+20)}{9} = a$$

$$260 + a = 9a$$

$$260 = 8a$$

$$a = \frac{260}{8}$$

So total $9a = \frac{9 \times 260}{8}$

$$9a = 292.50$$

6. The average age of a class of 36 students is 17 years. When the age of the teacher is also included, the avg will be increased by one year. What is the age of teacher?

Solⁿ: $\frac{S}{36} = 17$ $\frac{S+x}{37} = 18$

Method

$$36 \times 17 = 37 \times 18 - x$$

$$x = 666 - 612$$

$$x = 54$$

7. The avg height of 13 boys is 149 cm. If one of them is excluded the average height of the group increases by 2 cm. Find the height of the boy excluded.

Solⁿ: $S = (149)(13)$ $S - x = (151)(12)$

$$x = -(151)(12) + (149)(13)$$

$$x = 125 \text{ cm}$$

Method

Always put smallest on left

8. The avg temp of the town in the first four days of a month was 58° . The average for the second, third, fourth and fifth days was 60° . If the temps of the first and fifth were in ratio, 7:8, then what's the temp on fifth day?

Solⁿ:

Let 1st; 5th $\rightarrow 7x:8x$

$\frac{60}{2} = 30$ $\frac{58}{4} = 14.5$

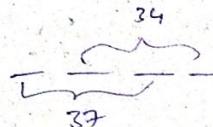
$$\frac{S - 7x + 8x}{4} = 60$$

$$x = 60 \times 4 - 58 \times 4$$

$$x = 8 \rightarrow \text{Means } 8x = 64^{\circ}$$

9. The mean temp of Monday to Wednesday was 37°C and of tuesday to thursday was 34°C . If the temp on thursday was $\frac{4}{5}$ that of monday. The temp on thursday was?

Sol:-



$$5:4 \rightarrow M:T$$

$$\text{similar one} \Rightarrow (37 \times 3) - 5x + 4x = 34 \times 3$$

$$x=9 \rightarrow \text{Thursday} = 36^{\circ}\text{C}$$

10. The avg weight of 3 men A, B and C is 84 kg . Another man D joins the group and the average now becomes 80 kg . If another man E whose weight is 3 kg more than that of D replaces A then the avg weight of B, C, D and E becomes 79 kg . Weight of A is?

$$\text{Soln:- } A+B+C = 84 \times 3$$

$$A+B+C+D = 80 \times 4$$

$$\text{Actually } D = 320 - 252$$

$$D = 68\text{ kg}$$

$$\text{So } E = 71\text{ kg}$$

$$\begin{aligned} B+C+D+E &= 79 \times 4 \\ &\downarrow \\ A-E &= 4\text{ kg} \end{aligned}$$

$$A = E+4$$

$$A = 75\text{ kg}$$

3. RATIO & PROPORTION:

Introduction

A Ratio is just a fraction in its minimum terms without any common factor in it.

Eg: $42:48 \rightarrow \text{fraction}$

$7:8 \rightarrow \text{Ratio}$

$a:b \rightarrow ak:bk$

Properties:

① If the terms of the ratio are multiplied with same number, the value of the ratio remains unaffected

② If the each term of ratio are divided with same numbers, the value of the ratio remains unaffected

$$\begin{array}{c} 7:8 \\ | \\ 1 \end{array} \rightarrow \begin{array}{c} 21:24 \\ | \\ 3 \end{array}$$

$$\begin{array}{c} 7:9 \\ | \\ 2 \end{array} \rightarrow \begin{array}{c} 3.5:4.5 \\ | \\ 1 \end{array}$$

③ When you are dividing the ratio (terms) with some k, then the difference b/w those terms gets multiplied by k.

Ratio - Exercises:

1. Two numbers are in the ratio $7:5$. If the sum of them is 96. find the largest number?

$$\text{Soln: } 12x = 96$$

$$x = 8 \quad 7x = 56 \checkmark$$

2. The ratio of no.of males to no.of females in a club is $7:4$. If there are 84 males in the club, the total number of members in the club is

- (a) 126 (b) 132 (c) 136 (d) 148

$$\text{Soln: } M:F = 7:4$$

$$7x = 84$$

$$\text{so } 4x + 7x = 11x$$

$$x = 12$$

$$\text{Total} = 132$$

3. The sum of money is divided among P, Q & R in the ratio of 2:3:5. If the amount of P and R together is 400/- more than that of Q. What is R's amount?

- (a) 200 (b) 300 (c) 500 (d) 400

$$\text{Soln: } 2x + 5x = 400 + 3x \\ 4x = 400 \\ x = 100$$

P \rightarrow	200
Q \rightarrow	300
R \rightarrow	500

4. An amount of 2430 is divided among A, B and C such that if their shares be reduced by 25, 210, 215 respectively, the remainders shall be in the ratio 3:4:5.

Then, B's share was:

- (a) 605 (b) 790 (c) 800 (d) 810

$$\text{Soln: } A:B:C \\ Ak + Bk + Ck = 2430 \\ A \rightarrow 600 + 5 \\ B \rightarrow 600 + 10 \rightarrow 810 \\ C \rightarrow 1000 + 15$$

B \rightarrow 810

$$Ak - 5 : Bk - 10 : Ck - 15 \\ \downarrow \quad \downarrow \quad \downarrow \\ 2430 - 2400 = 12x \\ -30 \quad \quad \quad x = 200 \\ x = 2.5$$

5. 60 kg of Alloy A mixed with 100 kg of alloy B. If alloy A has lead and tin in 3:2 ratio and Alloy B has tin and copper in 1:4 ratio, then amount of tin in new alloy is:

- (a) 36 kg (b) 44 kg (c) 53 kg (d) 80 kg

$$\text{Soln: } \text{Alloy A} \rightarrow 3:2 \quad (60 \text{ kg}) \quad \text{Alloy B} \rightarrow 1:4 \quad (100 \text{ kg})$$

Lead \rightarrow	36 kg
Tin \rightarrow	24 kg
Copper \rightarrow	80 kg

When Mixed Tin $\rightarrow 24 + 20 = 44 \text{ kg}$

6. 20 litres of mixture contains milk & water in the ratio 5:3. If 4 litres of this mixture is replaced by 4 litres of milk, the ratio of milk to water in the new mixture would be:

- (a) 2:1 (b) 7:3 (c) 8:3 (d) 4:3

$$\text{Soln: } 20 \text{ lts} \rightarrow \frac{100}{8} \text{ milk} \quad 4 \text{ lts} \rightarrow \frac{20}{8} \text{ milk} \\ \rightarrow \frac{60}{8} \text{ water} \quad \rightarrow \frac{12}{8} \text{ water}$$

So Now Milk = $\frac{100}{8} - \frac{20}{8} + 4 = 14$

Water = $\frac{60}{8} - \frac{12}{8} + 0 = 6$

Ratio $\Rightarrow 14:6 = 7:3$

7. Two alloys A and B contain gold and copper in 2:3 and 3:7 by mass respectively. Equal masses of alloys A and B are melted to make alloy C. The ratio of gold to copper in alloy C is (a) 5:10 (b) 7:13 (c) 6:11 (d) 9:13

$$\text{Soln: I say 100 kg; 100 kg} \quad \text{Gold} = 40 \text{ kg} + 30 \text{ kg} \\ \text{Cu} = 60 \text{ kg} + 70 \text{ kg}$$

Now $\rightarrow 70:130 = 7:13$

8. If $A:B = 8:15$; $B:C = 5:8$; $C:D = 4:5$ Then $A:D =$

- (a) 2:7 (b) 4:15 (c) 8:15 (d) 15:4

$$\text{Soln: } \frac{A}{B} = \frac{8}{15} \rightarrow B = \frac{15}{8}A$$

$$\frac{B}{C} = \frac{5}{8} \rightarrow C = \frac{8}{5} \times \frac{15}{8} \times A$$

$$\frac{C}{D} = \frac{4}{5} \rightarrow D = \frac{5}{4} \times \frac{8}{5} \times \frac{15}{8} \times A$$

$$\frac{D}{A} = \frac{15}{4} \rightarrow A:D = 4:15$$

Method II

$$\begin{array}{rcl} A:B & 8:15 \\ B:C & & 5:8 \\ C:D & & \overline{A:D} \\ \hline A:B:C:D & \rightarrow & 8:15:10:12 \end{array}$$

$$\rightarrow 8:15:20:30$$

$$\text{So now } A:D = 8:30 = 4:15$$

9. If 510 be divided among A, B & C in such a way that A gets two-thirds of what B gets and B gets one fourth of what C gets, then their shares respectively are

- (a) 120, 240, 150 (b) 60, 90, 360 (c) 150, 300, 60 (d) None

$$\text{Soln: } A = \frac{2}{3}B \quad B = \frac{1}{4}C$$

$$\begin{array}{rcl} A:B & \rightarrow & 2:(3) \\ B:C & & 1:4 \\ \hline A:B:C & \rightarrow & 2:3:12 \end{array}$$

$$\text{So } \frac{510}{17} = 30 \rightarrow 60:90:360$$

10. Salaries of Ravi & Sunil are in the ratio 2:3. If the salary of each is increased by 4000, the new ratio becomes 40:57. What is Sunil's present salary?

- (a) 17000 (b) 20000 (c) 25500 (d) None

$$\text{Soln: } R:S \rightarrow 2:3 \quad R+4000:S+4000 \rightarrow 40:57$$

$$\frac{R}{S} = \frac{2}{3} \quad \frac{\frac{2S}{3} + 4000}{S + 4000} = \frac{40}{57}$$

$$R = \frac{2S}{3}$$

$$40S + 40(4000) = \frac{2S}{3} \times 57 + 57(4000)$$

$$2S = 17(4000)$$

$$S = 34000$$

$$\text{Present Salary} = 38000$$

II. In a mixture of 60 litres, the ratio of milk & water is 2:1. If the ratio is to be 1:2, then the quantity of water to be further added is:

- (a) 20 lts (b) 80 lts (c) 40 lts (d) 60 lts

$$\text{Soln: } \begin{array}{ccc} 60 & \xrightarrow{\text{40 milk}} & \frac{40+0}{20+x} = \frac{1}{2} \\ & \xrightarrow{\text{20 water}} & 80 = 20+x \end{array}$$

$$x = 60 \text{ lts}$$

4. PARTNERSHIP

Introduction:

	A	B
Investment	I ₁	I ₂
Time	t ₁	t ₂
Profit	I ₁ t ₁	I ₂ t ₂

profit always shared in the ratio of contribution of partners to the business

So profit \rightarrow Investment \times time

Eg: A B

Inv \rightarrow 6000 8000

time \rightarrow 12 8

Profit 3(3) : 4(2)
9 : 8

Partnership- Exercises:

1. Yogesh started a business, investing 45000/- After 3 months pranab joined him with capital of 60000/- After another 6 months, Ashot joined with a capital of 90,000/- At the end of the year, they made a profit of 20,000/- What would be Ashot share in it?

(a) 8000 (b) 4000 (c) 3000 (d) 2000

Soln: Y P A

Inv 45000 60000 90000 \rightarrow 12 : 12 : 6

Time 12 9 3

Profit 3(4) : 4(3) : 6(1)

So profit \rightarrow 20,000 \rightarrow 8000 : 8000 : 4000

Ashot share \rightarrow 4000

2. A, B and C enter into partnership A invests some money at the beginning, B invests double the amount after 6 months and C invests thrice the amount after 8 months. If the annual profit be ₹ 27000. C's share is:

(a) 8625 (b) 9000 (c) 10800 (d) 11250

Soln: A B C

I	1	2	3
t	12	6	4

Profit 12 : 12 : 12

1 : 1 : 1

So each get 9000

C's share = 9000.

3. A and B started a business jointly. A's investment was thrice the investment of B and the period of A's investment was 2 times the period of investment of B. If B received 4000 as profit. Then their total profit is:

(a) 16000 (b) 20000 (c) 24000 (d) 28000

Soln: A B

I	3	1
t	2	1

1 \rightarrow 4000

Total \rightarrow 6+1 \rightarrow 28000

Profit	6	:	1
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4. A and B started a business with initial investments in the ratio 14:15 and their profits were in the ratio 7:6. If A invested the money for 10 months. How many months did B invest his money

- (a) 8 (b) 4 (c) 6 (d) 12

Soln:

$$\begin{array}{rcl} I & \text{A} & \text{B} \\ \text{I} & 14 : 15 \\ t & ? : ? \xrightarrow{\frac{1}{2} : \frac{2}{5}} & \text{Means } T_1 : T_2 = 5 : 4 \\ P & 7 : 6 & \end{array}$$

If $t_1 = 10$ months
 $t_2 = 8$ months

5. A and B entered into a partnership investing a 16000 and 12000 respectively. After 3 months, A withdrew 5000 while B invested 5000 more. After 3 more months, C joins the business with a capital of 21000. The share of B exceeds that of C, out of a total profit of 26400 after one year by

- (a) 2400 (b) 3000 (c) 3600 (d) 4800

Soln:

$$\begin{array}{rcl} P & \text{A} & \text{B} & \text{C} \\ \text{I} & (16000 \times 3) & (12000 \times 3) & (21000 \times 6) \\ & (11000 \times 9) & (17000 \times 9) & \\ \hline \text{Profit} & 147000 & 189000 & 126000 \\ & 21 \times 7 & 21 \times 9 & 21 \times 6 \end{array}$$

$$7 : 9 : 6$$

$$\text{Total} \rightarrow 26400 / 22 = 1200$$

Exceeds by 3600

6. A, B and C enter into a partnership. They invest 40000, 80000 and 120000 respectively. At the end of the first year, B withdraws 40000, while at the end of the second year, C withdraws 80000. In what ratio will the profit be shared at the end of 2 years?

- (a) 2:3:5 (b) 3:4:7 (c) 4:5:9 (d) None.

Soln:

$$\begin{array}{rcl} A & \text{B} & \text{C} \\ (40000 \times 3) & (80000 \times 1 + 40000 \times 2) & (120000 \times 2 + 40000) \end{array}$$

Soln:

$$3 : 4 : 7$$

profit $\rightarrow 3:4:7$

7. A is a working partner and B is a sleeping partner in a business. A invests 12000/- and B invests 20000. A receives 10% of the profit for managing and rest being divided in proportion to their capitals out of a total profit of 9600-. The money received by A is

- (a) 4200 (b) 4600 (c) 5100 (d) 4800

Soln: $9600 - 10\% = 8640$

$$A : B \Rightarrow 12000 : 20000$$

$$3 : 5$$

$$\text{So } A \rightarrow 960 + \left(\frac{3}{8}\right) 8640$$

$$4200$$

8. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest \$500 for 6 months, B \$400 for 5 months and C \$1000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The profit earned was \$7400. Calculate the share of B in profit.

- (a) 1900 ~~(b) 2660~~ (c) 2800 (d) 2840

Soln: A B C $7400 - 5\% = 7030$

$$\begin{array}{r} 6500 \times 6 \\ 8400 \times 5 \\ 10000 \times 3 \end{array}$$

$$\begin{array}{r} 65 \times 6 \\ 13 \times 2 \\ 14 \end{array} \quad \begin{array}{r} 84 \times 5 \\ 28 \times 1 \\ 10 \end{array} \quad \begin{array}{r} 100 \times 3 \\ 20 \times 1 \\ 10 \end{array}$$

Profits $\Rightarrow 13 : 14 : 10$

So B's share = $\frac{14}{37} \times 7030$

B's share = 2660

5. PERCENTAGES

Introduction

* percent means

\downarrow
for every 100

Eg: $23 \rightarrow ?$

$25 \rightarrow 100$

$$\frac{23}{25} \times 100 = 92\%$$

Eg: A is what percent of B? (%)

What percent of B is A? (%)

What percent of A is B?

$$\% \Rightarrow \frac{A}{B} \times 100$$

percentages - Concept

① 25 is what percent of 50?

$$\begin{array}{l} 25 - ? \\ 50 - 100 \end{array} \quad \left| \quad \frac{25}{50} \times 100 = 50\%. \checkmark \right.$$

② 60 is what percent of 50?

$$\begin{array}{l} 60 - ? \\ 50 - 100 \end{array} \quad \left| \quad \frac{60}{50} \times 100 = 120\%. \checkmark \right.$$

③ A is by what percent more (less) than B?

$$\frac{\text{Diff b/w A and B}}{B} \times 100$$

Note: - $A > B \neq B < A$

④ 60 is by what percent less than 75?

$$\frac{75-60}{75} \times 100 = 20\%$$

⑤ 75 is by what percent more than 60?

$$\frac{75-60}{60} \times 100 = \frac{15}{60} \times 100 = 25\%$$

⑥ What is $r\%$ of A?

$$\Rightarrow \frac{r}{100} \times A$$

⑦ What is 40% of 1900?

$$\frac{40}{100} \times 1900 = 760$$

⑧ What is 99% of 1648?

$$\frac{99}{100} \times 1648 = 1631.52 \quad (\text{or}) \quad \frac{1}{100} \times 1648 = \dots$$

Use complementary

Example

considered 2400

$$100\% = 2400$$

$$50\% = 1200$$

$$25\% = 600$$

$$12.5\% = 300$$

$$10\% = 240$$

03/09/2020 Friday

$$100\% \text{ of } A = A$$

$$50\% \text{ of } A = \frac{A}{2}$$

$$25\% \text{ of } A = \frac{A}{4}$$

$$12.5\% \text{ of } A = \frac{A}{8}$$

① If 13% of number is 52. What's the number?

$$\begin{array}{r} \times 4 \\ 13\% \longrightarrow 52 \end{array}$$

$$100\% \longrightarrow 400$$

② If 17% is 48 then what is 34%?

$$\begin{array}{r} \times 2 \\ \times 2 \\ 17\% \longrightarrow 48 \\ 34\% \longrightarrow 96 \end{array}$$

③ If 68% is 28 then 85%?

$$68\% = 28$$

$$85\% = ?$$

$$x = \frac{28 \times 85}{68}$$

$$x = 35$$

④ 150% of 1680?

$$\text{means } 100\% + 50\% \Rightarrow 1680 + \frac{1680}{2} = 2520$$

2400

$$10\% \rightarrow 240$$

$$1\% \rightarrow 24$$

$$0.5\% \rightarrow 12$$

$$5\% \rightarrow 120$$

$$0.1\% \rightarrow 48$$

$$0.1\% \rightarrow 2.4$$

$$2.5\% \rightarrow 60$$

$$0.1\% \rightarrow 72$$

$$0.1\% \rightarrow 1.44$$

$$1.25\% \rightarrow 30$$

$$0.1\% \rightarrow 96$$

① 125% of 1240 ② 97.5% of 640? ③ 99% of 1650

$$100\% + 25\% = 100\% - 2.5\%$$

$$1240 + 310 = 640 - 16$$

$$= 1550 \quad = 624$$

$$= 50\% - 1\%$$

$$= 825 - 165$$

$$= 808.5$$

④ 20.5% of x

$$= 20\% + 0.5\%$$

$$= \frac{x}{5} + \frac{x}{200}$$

Percentages - Calculation Tricks

① 78% of 150?

$$So 150\% \text{ of } 78$$

$$\Rightarrow 78 + 39$$

$$= 117$$

② 0.98 × 140?

→ 98% of 140

140% of 98

or 150% - 10%

→ 100% - 2%

$$= 140 - 2.8$$

$$= 137.2$$

③ 2.61 × 1450?

$$\frac{261}{100} \times 1450$$

261% of 1450

$$= 200\% + 50\% + 10\% + 1\%$$

$$= 2900 + 725 + 145 + 14.5$$

$$= 3784.5$$

A property we should know

$$a\% \text{ of } b = b\% \text{ of } a$$

Examples

① If 35% of a number is 105. find number

$$\begin{array}{rcl} 35\% & \xrightarrow{x3} & 105 \\ & \text{So } \rightarrow 300 \\ 100\% & \xrightarrow{\quad} & ? \end{array}$$

② 52 when added to 87% of number, the result is number itself. then find number.

$$\begin{array}{l} \text{means } 13\% \text{ is } 52 \\ \text{So } 100\% \text{ is } 400 \end{array}$$

③ A student needs to get 91% of aggregate marks to pass. A student who got 392 marks was declared fail by 5%. find max aggregate marks in the exam.

$$\begin{array}{l} \text{Soln: So } 5\% \rightarrow 49 \\ 100\% \rightarrow 980 \end{array}$$

$$\text{So max marks} = 980$$

④ In an exam, a student must get 35% marks to pass. A student who got 96 marks declared fail by 16 marks. find max aggregate marks in the exam.

- (a) 285 (b) 320 (c) 300 (d) 425

$$\begin{array}{l} \text{Soln: } 35\% \rightarrow 96+16=112 \\ 100\% \rightarrow ? \end{array}$$

$$\begin{aligned} \frac{35}{100} &= \frac{112}{x} \\ x &= \frac{112 \times 100}{35} \\ x &= 320 \end{aligned}$$

⑤ In an examination a student who got 22% of marks was declared fail by 52 marks. Another who got 45% of marks got 40 marks more than pass mark. find max aggregate mark.

$$\begin{array}{l} \text{Soln: } x-52 \rightarrow 22\% \\ x+40 \rightarrow 45\% \end{array}$$

$$\text{We want M} \rightarrow \frac{22M}{100} + 52 = \frac{45M}{100} - 40$$

$$\begin{array}{c} 22\% \quad 45\% \quad 92 = \frac{23M}{100} \\ \downarrow \qquad \downarrow \qquad \downarrow \\ 52 \qquad \qquad \qquad 40 \\ 23\% = 92 \end{array}$$

$$\begin{array}{l} M = \frac{9200}{23} \\ M = 400 \end{array}$$

$$\begin{array}{l} \frac{100}{22}[x-52] = \frac{100}{45}[x+40] \\ 45x - 52(45) = 22x + 22(40) \\ 18x = 74(40) \\ x = \frac{74 \times 40}{18} \\ x = \frac{1480}{9} \end{array}$$

$$\begin{array}{ll} (b) 23\% \rightarrow 92 & (c) 23\% \rightarrow 92 \\ ?\% \rightarrow 52 & 35\% \rightarrow 140 \\ 13\% & \\ \text{So } 22+13 & \Rightarrow 35\% \end{array}$$

Percentages- Concept IV

* A is increased by $\alpha\%$ $\rightarrow A + \alpha\% \text{ of } A$

$$A \left(1 + \frac{\alpha}{100}\right) = A \left(\frac{100+\alpha}{100}\right)$$

If decreased by $\alpha\%$ $\rightarrow A - \alpha\% \text{ of } A$

$$A \left(\frac{100-\alpha}{100}\right)$$

* If A is increased by $\alpha\%$ for n times $\Rightarrow A \left[\frac{100+\alpha}{100}\right]^n$

If A is decreased by $\alpha\%$ for n times $\Rightarrow A \left[\frac{100-\alpha}{100}\right]^n$

Examples

① A number when increased by 40% becomes 980.

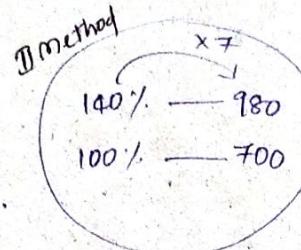
What's the number?

Soln:

$$980 = A \left(\frac{100+40}{100} \right)$$

$$\frac{980 \times 100}{140} = A$$

$$So \quad A = 700$$



② The population of a town increases by 20% during first year and then decreases by 10% during second year. If the population at the end of 2 years is 75600. How much was it at the beginning?

Soln:

$$100 \rightarrow 120 \rightarrow 108$$

$$\downarrow \qquad \qquad \downarrow$$

$$? \qquad \qquad 75600$$

$$x 700$$

$$108 \rightarrow 75600$$

$$100 \rightarrow ?$$

$$70000 \checkmark$$

II method

$$P \left(\frac{100+20}{100} \right) \left(\frac{100-10}{100} \right) = 75600$$

$$P \left(\frac{120 \times 90}{100 \times 100} \right) = 75600$$

$$P = 70000$$

③ The population of a new city is 5 million and is growing at 20% every year. In how many years will it become doubled this growth rate?

$$100 \rightarrow 120 \rightarrow 144 \rightarrow 172.8 \rightarrow 207.36$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$5 \text{ million} \qquad \qquad \qquad 3 \text{ to 4 yrs} \qquad 6 \text{ million +}$$

- (a) 2 to 3 yrs (b) 3 to 4 yrs (c) 4 to 5 yrs (d) 5 to 6 yrs

III method

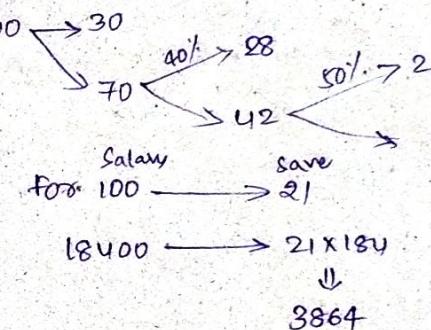
$$5000000 \left(\frac{100+20}{100} \right)^n = 100000000$$

$$\left(\frac{6}{5} \right)^n = 2$$

Not Recommended process

④ Gaurav spends 30% of his monthly income on food articles, 40% of the remaining on conveyance and clothes and saves 50% of remaining. His salary is 18400. How much money does he save every month?

Soln:



5. From the salary of an officer, 10% is deducted as house rent, 15% of the rest he spends on children education and 10% of the balance he spends on clothes. After this expenditure, he left with 1377. What's the salary?

Soln:

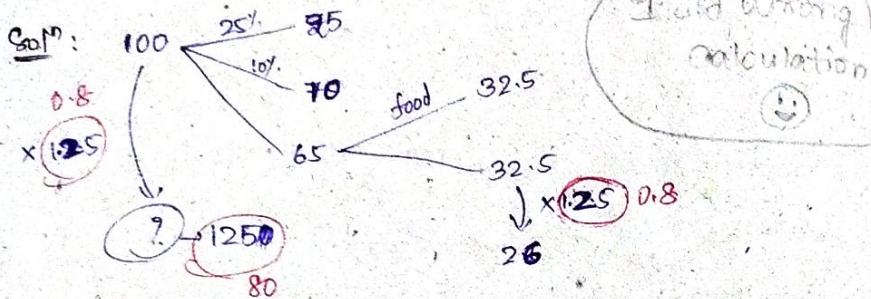
$$100 \xrightarrow{\text{rent } 10} 90 \xrightarrow{15\%} 76.5 \xrightarrow{10\%} 68.85$$

$$x 20 \quad (100 \rightarrow 68.85) \quad 200 \times 20$$

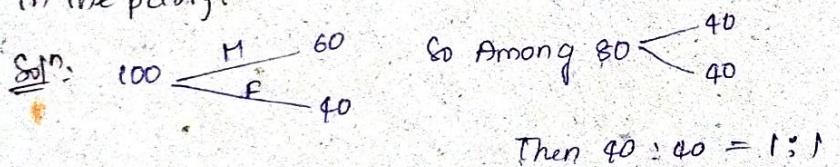
$$? \rightarrow 1377$$

$$\text{Salary} \Rightarrow 2000$$

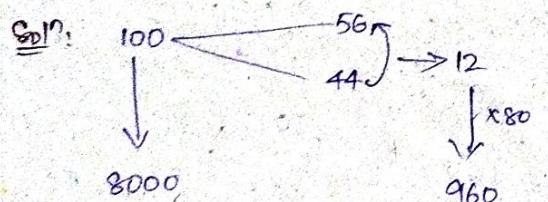
⑥ A person has some money with him. 25% stolen in bus and 10% is lost through a hole in pocket. 50% of remainder is spent on food. He then purchases a book worth Rs. 26 from the remainder. He walks back home because all his money over. What is initial amount?



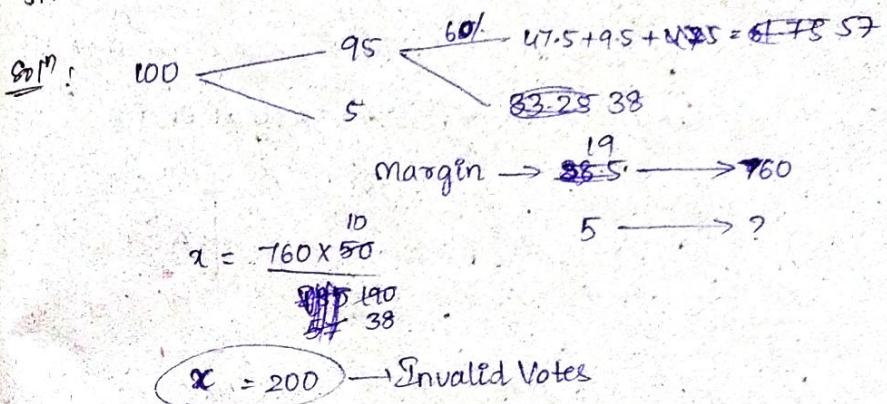
⑦ In a party, 60% of the invited guests are male and 40% are female. If 80% of invited guests attended the party and if all the invited female guests attended, what would be ratio of male to females among attendees in the party?



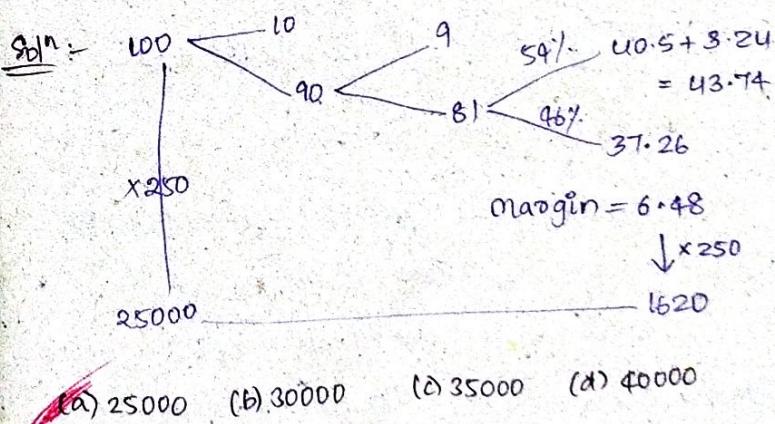
⑧ In an election between two candidates, a person who got 56% of the total votes won election by a majority of 960 votes. Find total votes.



⑨ In an election b/w 2 candidates, a person who got 60% of the valid votes won the election by a margin of 760 votes. If 95% of the polled votes are valid, find no. of invalid votes in election?



⑩ 10% of the voters did not cast their vote in election b/w 2 candidates. 10% of the votes polled were found invalid. The successful candidate got 54% of the valid votes and won by a majority of 1620 votes. The no. of voters enrolled on the voter list was?



Percentages - Concept V

If a number A is $\gamma\%$ more or less than B then

$$A = (B \pm \gamma\% \text{ of } B)$$

Examples

- (1) If A's income is 25% more than that of B then
B's income is by what percent less than that of A's?

Soln: $A = 125\% \text{ of } B \quad A = \frac{125}{100} B$

$$\text{So } B = \frac{100}{125} A$$

$$B = 0.8A$$

$$B = 80\% \text{ of } A$$

B is 20% less than A

$$\boxed{\begin{array}{l} A > B \\ B < A \end{array}}$$

- (2) Two numbers are respectively 10% and 20% less than third number. The first number is by what percent more than the second?

Soln:- Third 100 ————— 2nd 1st
 100 80 90

$$\rightarrow \frac{90}{80} \times 100 = \frac{900}{8} 112.5 = 100 + 12.5$$

So 1st number is 12.5% more than second

- (3) On a shelf, the 1st row contains 25% more books than the second row, and the third row contains 25% less books than the second row. If the total number of books in all the three rows are 600. Find books in 1st row?

Soln: Let 2nd row be 100 3rd row $\rightarrow 75$
1st row $\rightarrow 125$

$$\begin{array}{rcl} 100 + 125 + 75 & = & 300 \\ \downarrow \times 2 & & \downarrow \times 2 \\ 200 & & 600 \end{array}$$

14) percentage change

* It is always taken on initial value

$$* \% \text{ change} = \frac{\text{change}}{\text{Initial value}} \times 100$$

Ex: $\frac{1997}{1996} \quad 1997$
① 100 ————— 130
 $+30$

$$\frac{30}{100} \times 100 = 30\% \uparrow$$

② 250 ————— 300
 -50

$$\frac{50}{250} \times 100 = 20\% \uparrow$$

Examples

- (1) A number is first increased by 30% and then decreased by 20% . Find overall % change in the number.

Soln: $100 \xrightarrow{+30\%} 130 \xrightarrow{-20\%} 104$

$$\% \text{ change} = \frac{4}{100} \times 100 \Rightarrow 4\%$$

I method

$$x \xrightarrow{+30\%} 1.3x \xrightarrow{-20\%} 1.04x$$

$$\text{So change \%} = \frac{1.04x - x}{x} \times 100 = 4\%$$

A number is decreased by 25%, and increased by 20%.

Overall percentage change in number?

$$\text{Soln: } 100 \rightarrow 75 \rightarrow 90$$

$$\% \text{ change} = \frac{10}{100} \times 100 = 10\%$$

A number 10% 10% ↓ 10% ↓ then

$$\text{Soln: } 100 \xrightarrow{10\%} 90 \xrightarrow{10\%} 81 \xrightarrow{10\%} 72.9$$

$$\frac{27.1}{100} \times 100 \Rightarrow 27.1\%$$

② If the side of a square is increased by 30%. find percentage change in the area of the square.

$$\text{Soln: } S \rightarrow S^2$$

$$\text{I method } \frac{13S}{10} \rightarrow \left(\frac{13}{10}\right)^2 S^2$$

$$\text{change} = \frac{\left(\frac{13}{10}\right)^2 - 1}{1} \times 100$$

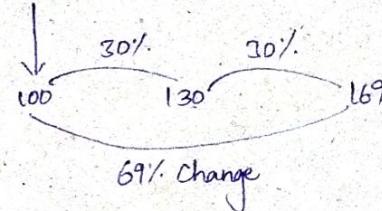
$$\Rightarrow \frac{13^2 - 10^2}{10^2} \times 100$$

$$\Rightarrow (23)(3)$$

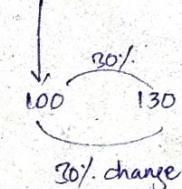
$$\approx 69\%$$

II method

$$A = S^2$$



$$P = 4S^2$$



③ If the price of sugar increased by 25%, then by what percent must the consumption of sugar be decreased such that the expenditure on sugar remains constant?

$$\text{Soln: } 100 \text{ pps} \rightarrow 125$$

$$80 \text{ gms} \rightarrow 100 \quad \% \rightarrow \frac{20}{100} \times 100 \Rightarrow 20\%$$

III method

$$E = P \times Q$$

$$\downarrow 25\% \downarrow$$

$$100 \xrightarrow{25\%} 125 \xrightarrow{25\%} 156 \xrightarrow{25\%} 195 \xrightarrow{25\%} 243 \xrightarrow{25\%} 304$$

$$\text{So} \rightarrow \frac{25}{125} \times 100 = 20\%$$

III method

$$P \propto \frac{1}{Q}$$

$$P \rightarrow x : y \quad P \rightarrow 100 : 125$$

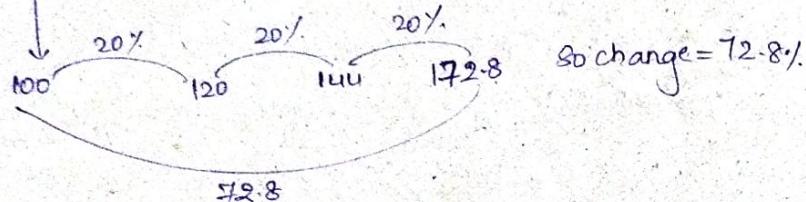
$$Q \rightarrow y : x \quad Q \rightarrow 125 : 100$$

$$\text{So} \quad \frac{25}{125} \times 100 = 20\%$$

Percentage Change - Geometrical figures:

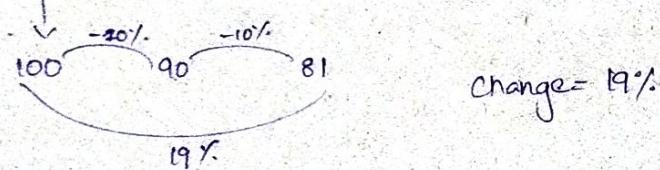
CUBE

$$V = s^3 \quad s \uparrow = 20\%$$



CIRCLE

$$A = \pi r^2 \quad r \downarrow = 10\%$$

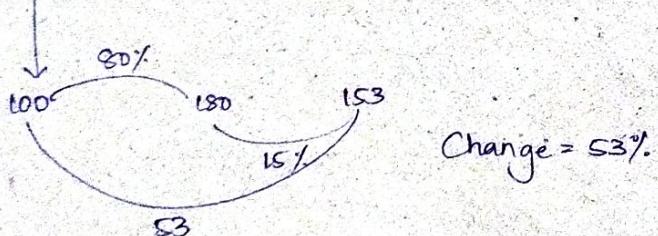


RECTANGLE:

$$A = l \times b$$

$l \uparrow 80\%$

$b \downarrow 15\%$



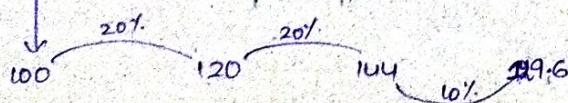
CYLINDER

$$V = \pi r^2 h$$

$\pi \uparrow 20\%$

$h \downarrow 10\%$

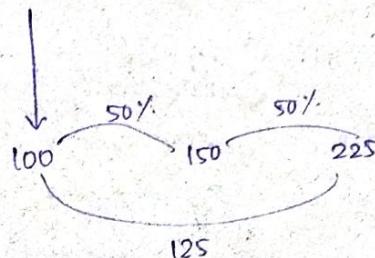
Change = 29.6%



CONE

$$V = \frac{1}{3} \pi r^2 h$$

base $\rightarrow 50\%$
radius



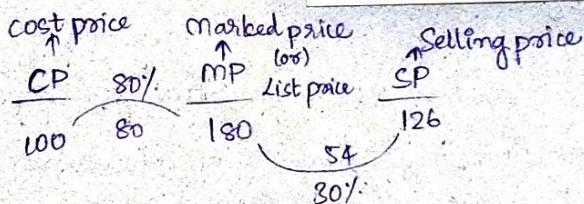
Change $\rightarrow 125\%$

Note: When h is not given any change, take it as a constant. All the constants are neglected at start power of the terms judges how many times the change must be performed.

6. PROFIT, LOSS AND DISCOUNT

Introduction

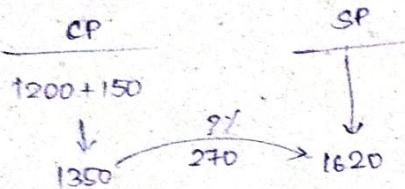
10/08/2020 : Saturday



1. Mark up % is always calculated on Cost price.
2. Discount % is always calculated on Marked price.
3. profit/loss % is always calculated on cost price.
4. Money spent on transportation, packing, overhead, expenses etc. should be added to cp

Examples

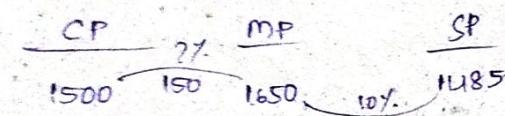
①



$$\text{gain \%} = \frac{270}{1350} \times 100 \\ = 20\%$$

* When there is no discount mentioned in the question, then
Selling price = Marked price

②



$$\text{markup percent} = \frac{150}{1500} \times 100 \\ = 10\%$$

$$\text{Loss \%} = \frac{15}{1500} \times 100 \\ = 1\%$$

* If markup % and Discount % are equal, then there would always be a loss in the transaction

③ If the profit obtained by selling an article for Rs. x is equal to the loss incurred when it is sold for Rs. y

then

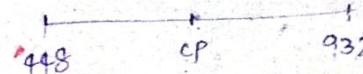
$$\boxed{\text{CP} = \frac{x+y}{2}}$$



$$x - \text{CP} = \text{CP} - y$$

$$\boxed{\text{CP} = \frac{x+y}{2}}$$

④



$$\text{Then } \text{CP} = \frac{932+448}{2}$$

$$\boxed{\text{CP} = 690}$$

④ If profit obtained by selling an article for Rs. 1920 is equal to the loss incurred when it is sold for Rs. 1280. Then at what price must the article be sold to gain 25%?

$$\underline{\text{Soln}}: \quad \text{S.P} = 1920 \quad \text{S.P} = 1280$$

↓ ↓
profit ← → loss

$$\text{C.P} = \frac{1920+1280}{2} = 1600$$

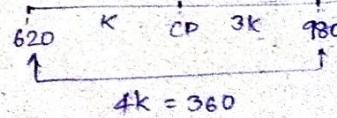
$$25\% \text{ gain} \rightarrow 125\%(1600) = 2000$$

⑤ The profit obtained by selling an article for Rs. 980 is equal to thrice the loss incurred when it is sold for Rs. 620. Find CP?

$$\underline{\text{Soln}}: \quad \text{I method} \quad 980 - \text{C.P} = 3(\text{C.P} - 620)$$

II method

✓ Do this.



$$980 - x = 3x - 1860$$

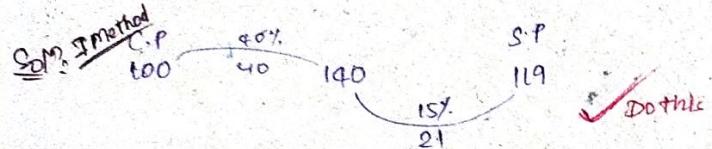
$$4x = 2860$$

$$\boxed{x = 715}$$

$$k = 90$$

$$\text{So C.P} = 620 + 90 = 710$$

⑥ A merchant marks up the price of an article by 40% and sells it at a discount of 15%. Find gain or loss %?



$$\text{Gain} \rightarrow \frac{19}{100} \times 100 = 19\%$$

II Method

C.P.	M.P.	S.P.
x	$\frac{4}{5}x$	$1.19x$
$= 1.4x$		

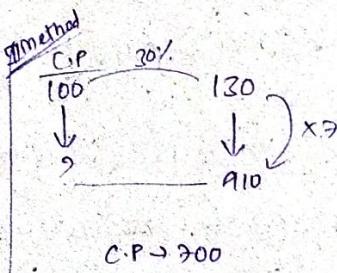
$$\text{Gain} \rightarrow \frac{1.19x - x}{x} \times 100 = 19\%$$

⑦ By selling an article for Rs. 910, a merchant gains 30%. Find C.P.

$$\text{Soln: } \frac{910 - \text{C.P.}}{\text{C.P.}} = \frac{30}{100}$$

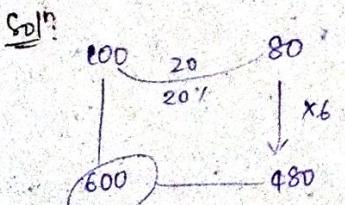
$$\frac{910}{\text{C.P.}} = \frac{130}{100}$$

$$\text{C.P.} = \frac{910 \times 100}{130} = 700$$



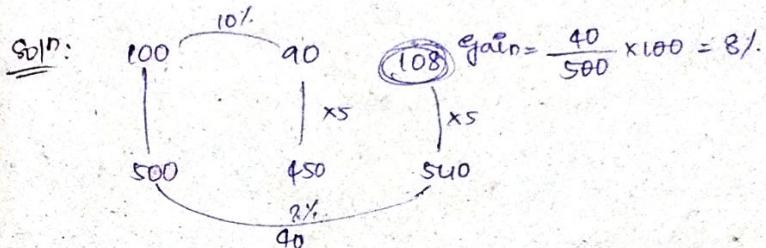
$$\text{C.P.} \rightarrow 700$$

⑧ By selling an article for Rs. 480 a merchant loses 20% at what price must this be sold to gain 30%?

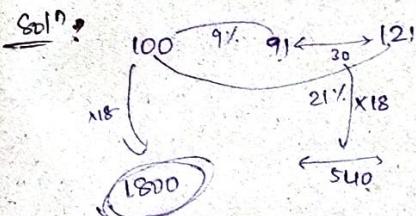


So → Sold at 780

⑨ A merchant loses 10% by selling an article for Rs. 450. What would be his gain or loss %, if he sells it for Rs. 540?



⑩ A merchant sold an article at a loss of 9%. Had it been sold for Rs. 540 more he would have gained 21%. CP?



⑪ By selling an article at 5% discount a merchant gains 33%. Find markup %?

$$\text{Soln: } \frac{95x}{100} = 133$$

$$x = \frac{20}{100 \times 133} = \frac{2}{133}$$

$$\text{C.P.} \rightarrow 100 \rightarrow 140 \rightarrow S.P. ← 133$$

5% ↓ 33% ↑

$$x = 100$$

⑫ If C.P. of 8 articles is equal to S.P. of 10 articles. Then find gain or loss %.

Soln: $8 \cdot \text{C.P.} = 10 \cdot \text{S.P.}$

$$\frac{\text{C.P.}}{\text{S.P.}} = \frac{5}{4}$$

$$\text{Loss \%} = \frac{S-C}{S} \times 100$$

$$\text{Loss} = 20\%$$

- (13) By selling 10 articles a merchant gains the S.P. of 2 articles - find gain%?

$$\underline{\text{Soln}}: \frac{2 \cdot SP}{10 \cdot CP} \times 100 = ? \text{ gain}$$

$$\begin{aligned} & \frac{2 \times \frac{5}{4} \times 100}{10} = ? \\ & \frac{5}{2} \times 100 = ? \\ & \Rightarrow \frac{400}{25} \\ & \Rightarrow 16\%. \end{aligned}$$

- (14) A merchant professes to sell the goods at cost price but uses a false weight of 800 gm instead of 1 kilo
find gain%?

$$\underline{\text{Soln}}: \text{SP of } 800 \text{ gm} = \text{CP of } 1000 \text{ gm}$$

$$\begin{aligned} \text{Gain} \rightarrow & \frac{200 \text{ gm}}{800 \text{ gm}} \times 100 \\ & = 25\%. \end{aligned}$$

$$10 \cdot SP - 10 \cdot CP$$

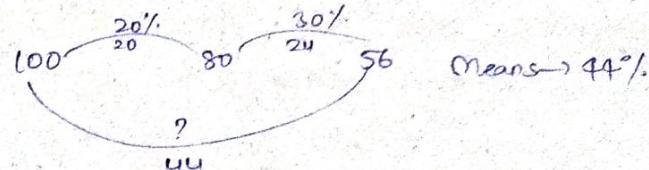
$$\begin{array}{c} 10 \cdot C.P - 10 \cdot S.P \\ \cancel{10 \cdot S.P} \\ \hline 2 \cdot S.P \end{array}$$

$$10 \cdot CP = 8 \cdot SP$$

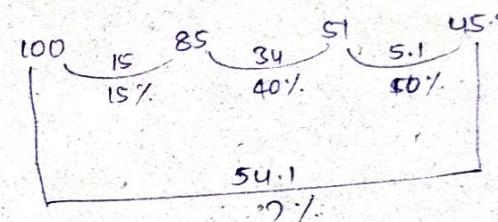
$$\frac{CP}{SP} = \frac{5}{4}$$

Successive Discounts

Eg: two successive discounts of 20%, 30% equals to how much single discount



Eg: Again 15%, 40%, 10%.



Exercises

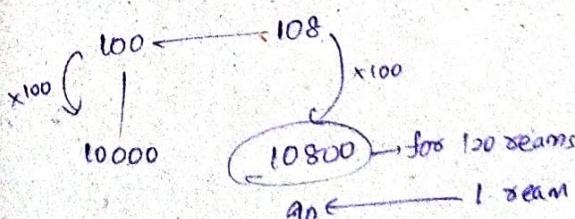
1. Sarans purchased 120 reams of paper at ₹.80 per ream.

He spent ₹.280 on transportation, paid octroi at the rate of 40 paise per ream and paid ₹.72 to the coolie.

If he wants to have a gain of 8%, what must be the selling price per ream?

Soln: (a) ₹. 86 (b) ₹. 87.48 (c) ₹. 89 (d) ₹. 90

$$(120)(80) + 280 + (0.4)(120) + 72 \rightarrow \text{C.P.} = ₹10000 \text{ for } 120 \text{ ream}$$



2. A shopkeeper purchased 150 identical pieces of calculators at the rate of ₹ 250 each. He spent an amount of ₹ 2500 on transport & packing. He fixed the labelled price of each calculator at ₹ 320. However he decided to give a discount of 5% on the labelled price. What is the percentage profit earned by him?

- (a) 14%. (b) 15%. (c) 16%. (d) 20%.

$$\text{Soln: } (150)(250) + 2500 = ₹ 40000$$

$$\text{markup price} = (320)(150) = ₹ 48000$$

$$\begin{array}{r} \\ \downarrow \\ 5\% \\ \hline \text{Selling price} \end{array}$$

$$\frac{5600}{40000} \times 100 = 14\%$$

3. The manufacturer of a certain item can sell all the can produce at S.P. of ₹ 60 each. It costs him ₹ 40 in materials and labour to produce each item and he has overhead expenses of ₹ 2000 per week in order to operate the plant. The numbers of units he should produce and sell in order to make a profit of at least ₹ 1000 per week

- (a) 200 (b) 250 (c) 300 (d) 400.

$$\text{Soln: } \cancel{\text{S.P.}} - \cancel{\text{C.P.}} = \cancel{\text{Profit}}$$

$$-1000 = (x(60) + 2000) - (x(40))$$

$$x(20) = 4000$$

$$x = 200$$

4. A person divided an amount of ₹ 1,00,000 into two parts and invested in two different schemes. In one he got 10% profit and in the other he got 12%. If the profit percentages are interchanged with these investments, he would have got ₹ 120 less. Find the ratio between his investments in the two schemes.

- (a) 47:53 (b) 9:16 (c) 11:14 (d) 37:63

Soln: I Method

$$\frac{x}{x} : \frac{100000-x}{x} \quad x : 100000-x$$

$$\begin{array}{ccc} 10\% & & 12\% \\ \downarrow & & \downarrow \\ 12\% & & 10\% \end{array}$$

$$\frac{x}{10} + \frac{(100000-x)12}{100} - \left(\frac{12x}{100} + \frac{(100000-x)}{10} \right) = 120$$

$$\frac{10x}{100} + 12000 - \frac{12x}{100} - \frac{12x}{100} - 10000 + \frac{10x}{100} = 120$$

$$2000 - \frac{4x}{100} = 120$$

$$1880 = \frac{x}{25} \quad x = (25)(1880)$$

$$x = 47000$$

So 47:53

II Method

$$x+y = 100000$$

$$[(\text{no. of } x + 112\% \text{ of } y)] - [112\% \text{ of } x + 110\% \text{ of } y] = 120$$

$$\frac{110x}{100} - \frac{112x}{100} + \frac{112y}{100} - \frac{110y}{100} = 120$$

$$\frac{24}{100} - \frac{2x}{100} = 12000$$

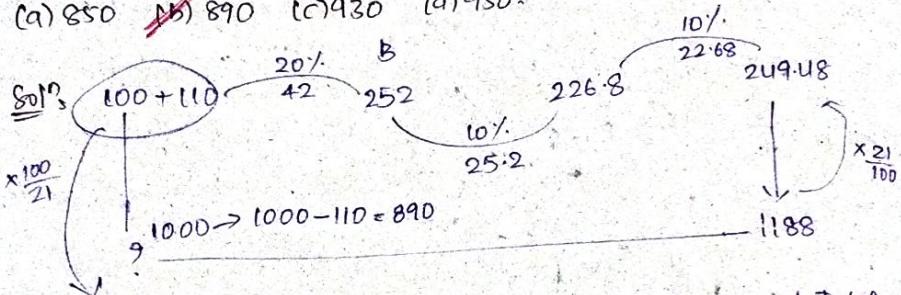
$$y-x = 6000$$

Simplifying $y = 53000$

So 47:53

5. A bought a radio set and spent ₹ 110 on its repairs. He then sold it to B at 20% profit. B sold it to C at a loss of 10%. and C sold it for ₹ 1188 at a profit of 10%. What is the amount of which A bought the radio set.

- (a) 850 (b) 890 (c) 930 (d) 950.

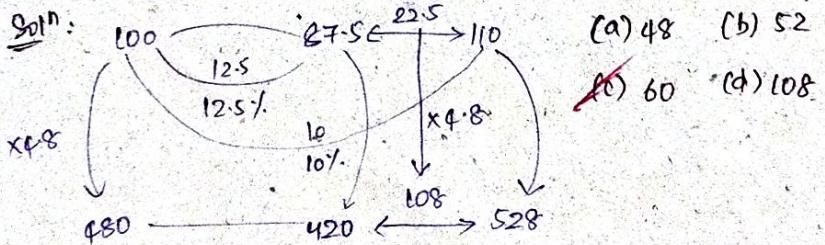


I assumed the only price A bought for is 100 and Total cost price became 210, but due to that process lengthy.

~~out of~~ So, if you assume Total Repairs and Buying included 100, then you will find the answer easier and then you subtract the extras from the final value.

6. A fruit seller sold a basket of fruits at 12.5% loss.

He had sold it for ₹ 108 more, he would have made a 10% gain. What is the loss in Rupees incurred by the fruit seller?



$$\text{Loss} \Rightarrow 1480 - 120 = 60$$

7. SIMPLE INTEREST & COMPOUND INTRST

Concept-I

P - principal / sum

R% - Rate of Interest per year

T - Time period in years

I - Interest

A - Amount = (P+I)

S.I	P	I	A
	100	20	120
		20	140
			160
			180

Eq:	P	I	A
	16800	4200	21000

So S.I for
1 year \rightarrow 20%
2 yrs \rightarrow 40%
3 yrs \rightarrow 60%

$$\text{So finally } I = \frac{PTR}{100}$$

$$\begin{aligned} \text{Amount} \Rightarrow P+I &= P + \frac{PTR}{100} \\ &= P\left(1 + \frac{TR}{100}\right) \end{aligned}$$

Examples

1. A man invested ₹ 48000 @ $7\frac{1}{2}\%$ p.a. S.I for 6 years. Find the amount at the end of 6 years?

$$\begin{array}{ccc} \text{S.I.} & \underline{P} & \underline{I} & \underline{A} \\ & 48000 & 21600 & 69600 \end{array}$$

$$7\frac{1}{2} \times 6 = 45\%$$

I method

$$A = (10000) \left[1 + \frac{16(7.5)}{100} \right]$$

$$= 10000 \times 1.165$$

$$A = 11650$$

Q. What sum of money invested at 6% pa SI amounts to Rs. 4720 in 2 yrs?

Sol.

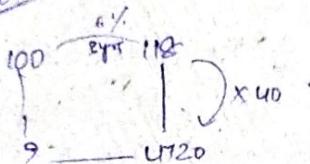
I method

$$4720 = P \left(1 + \frac{16(3)}{100} \right)$$

$$P = 4720 \times \frac{100}{116}$$

$$P = 4000/-$$

II method



$$S_0 = 4000$$

Concept II

P

I

$$P @ R\%$$

$$\frac{PTR}{100}$$

A

$$P \left(1 + \frac{TR}{100} \right)$$

$$P @ R\% + x\%$$

$$\frac{PT(R+x)}{100} = \frac{PTR}{100} + \frac{PTx}{100} \quad P \left(1 + \frac{TR+Tx}{100} \right)$$

Examples

1. A sum of Rs 200 amounts to Rs 956 in 2 yrs at a certain rate of simple interest. If the rate of interest is 4% higher, find the amount.

Sol. $A = 956 + \frac{(200)(13)}{100}$

$$= 956 + 96$$

$$= 1052$$

2. A sum of money invested at simple interest triples itself in 25 years. Find rate of interest.

Sol:

P

R%

$$P$$

$$R$$

$$3P = P \left(1 + \frac{25R}{100} \right)$$

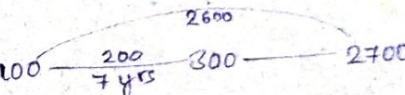
$$2 = \frac{25R}{100}$$

$$R = 8\%$$

Let $P = 100$ \downarrow
SI for 25 yrs $\rightarrow 200$
 $1y \rightarrow 8$
So $R\% = 8\% \text{ pa}$

3. A sum of money invested at simple interest triples itself in 7 years. Then in how many years will it become 27 times itself at the same rate?

Sol:



$$7 \text{ yrs} \rightarrow 200$$

$$1 \text{ yr} \rightarrow \frac{200}{7}$$

$$\frac{200}{7} \times x = 2600$$

$$x = \frac{2600 \times 7}{200}$$

$$x = 91$$

4. A sum of money invested at simple interest amounts to Rs. 19500 in 3 years and to Rs. 22500 in 5 years. Find sum, rate of interest.

Sol:

$$\frac{3x13}{100+3R} = \frac{19500}{100+5R}$$

$$\frac{15}{100+5R} = \frac{22500}{100+5R}$$

$$1300 + 65R = 1500 + 45R$$

$$R = 10\%$$