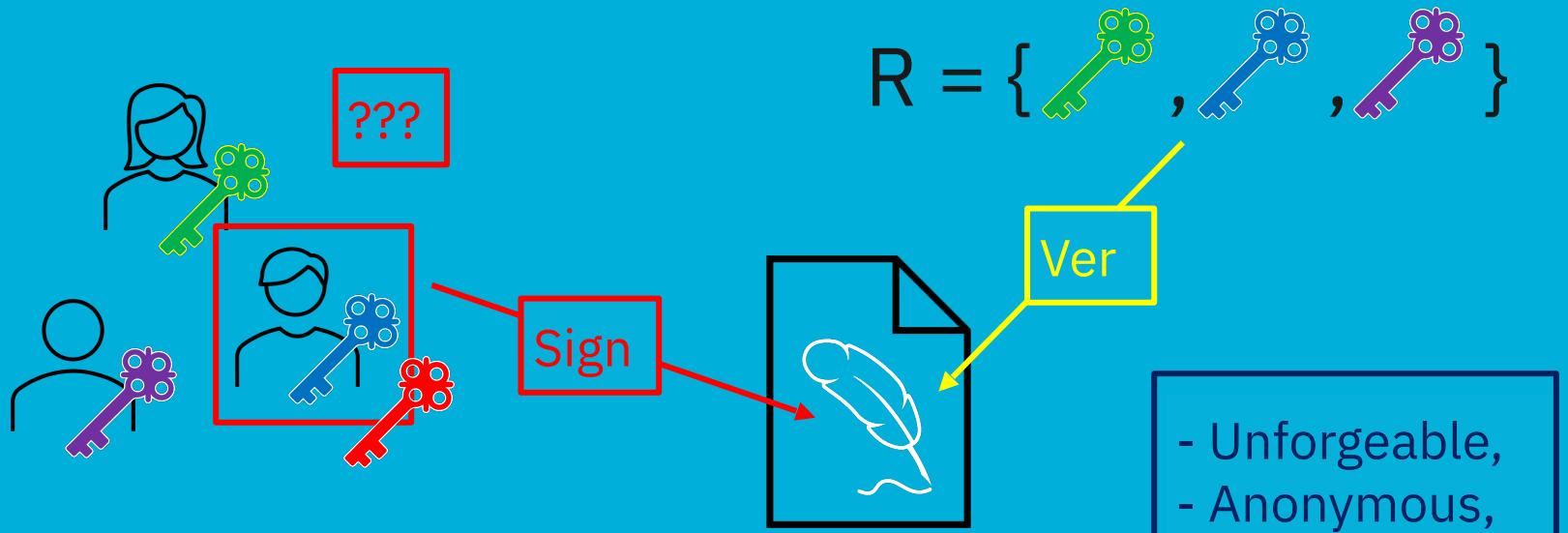
Erebor & Durian

Full Anonymous Ring Signatures from Quaternions and Isogenies

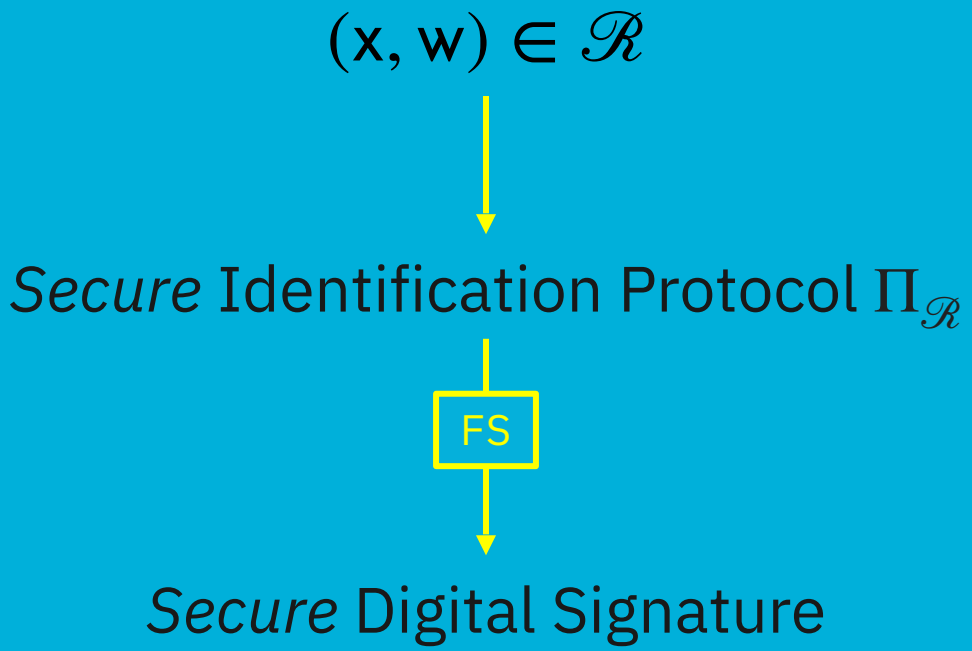
Giacomo Borin, YiFu Lai, Antonin Leroux

Opinions are my own!

Full-Anonymous
Linear
Ring Signatures



Full-Anonymous Linear **Ring Signatures**



Full-Anonymous Linear **Ring Signatures**

$$\mathcal{R}_{\mathrm{OR}} = \left\{((i, \mathrm{w}), (\mathrm{x}_1, \ldots, \mathrm{x}_N)) \mid (\mathrm{w}, \mathrm{x}_i) \in \mathcal{R}\right\}$$

Secure OR-Proof for $\mathcal{R}_{OR}$

Secure Ring Signature

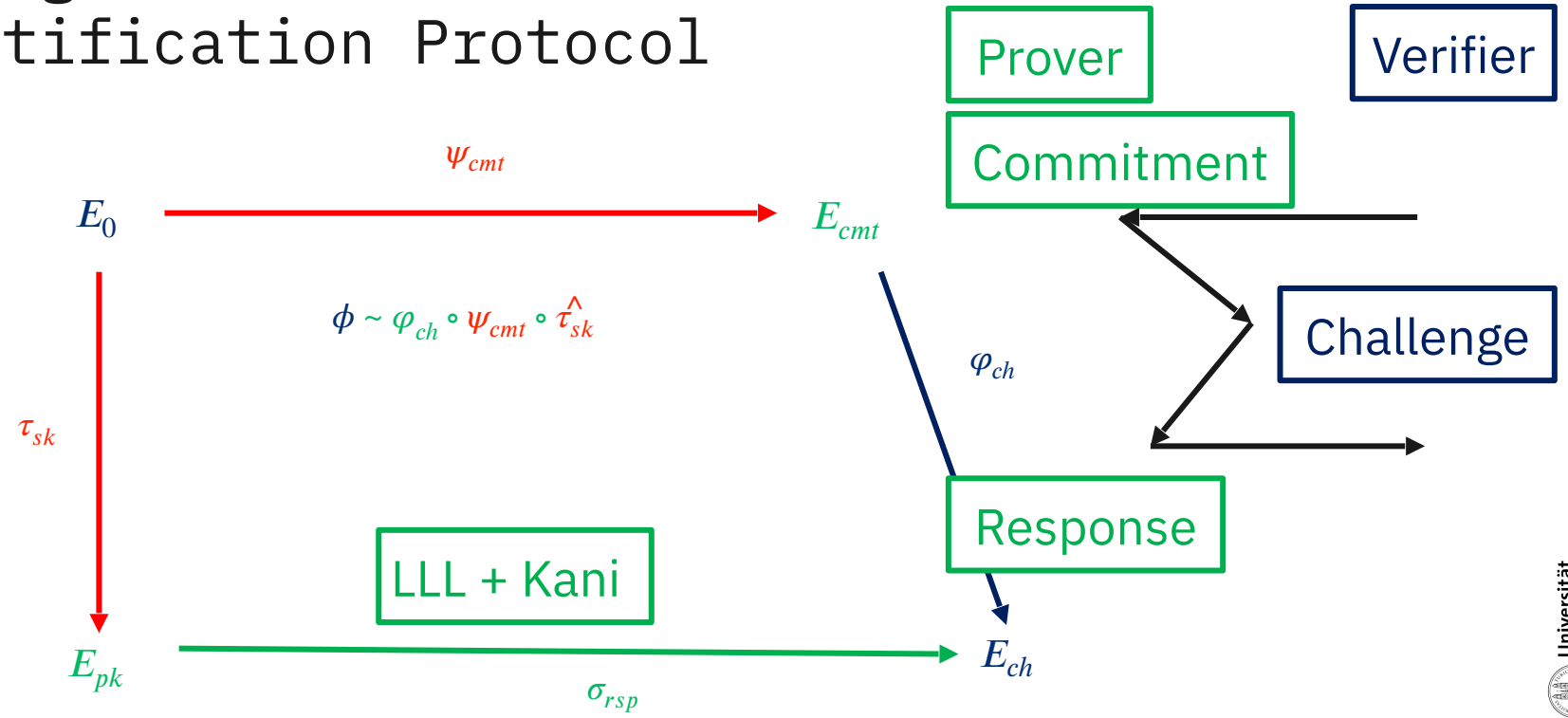
- Sequential,
- Parallel,
- Logarithmic

...

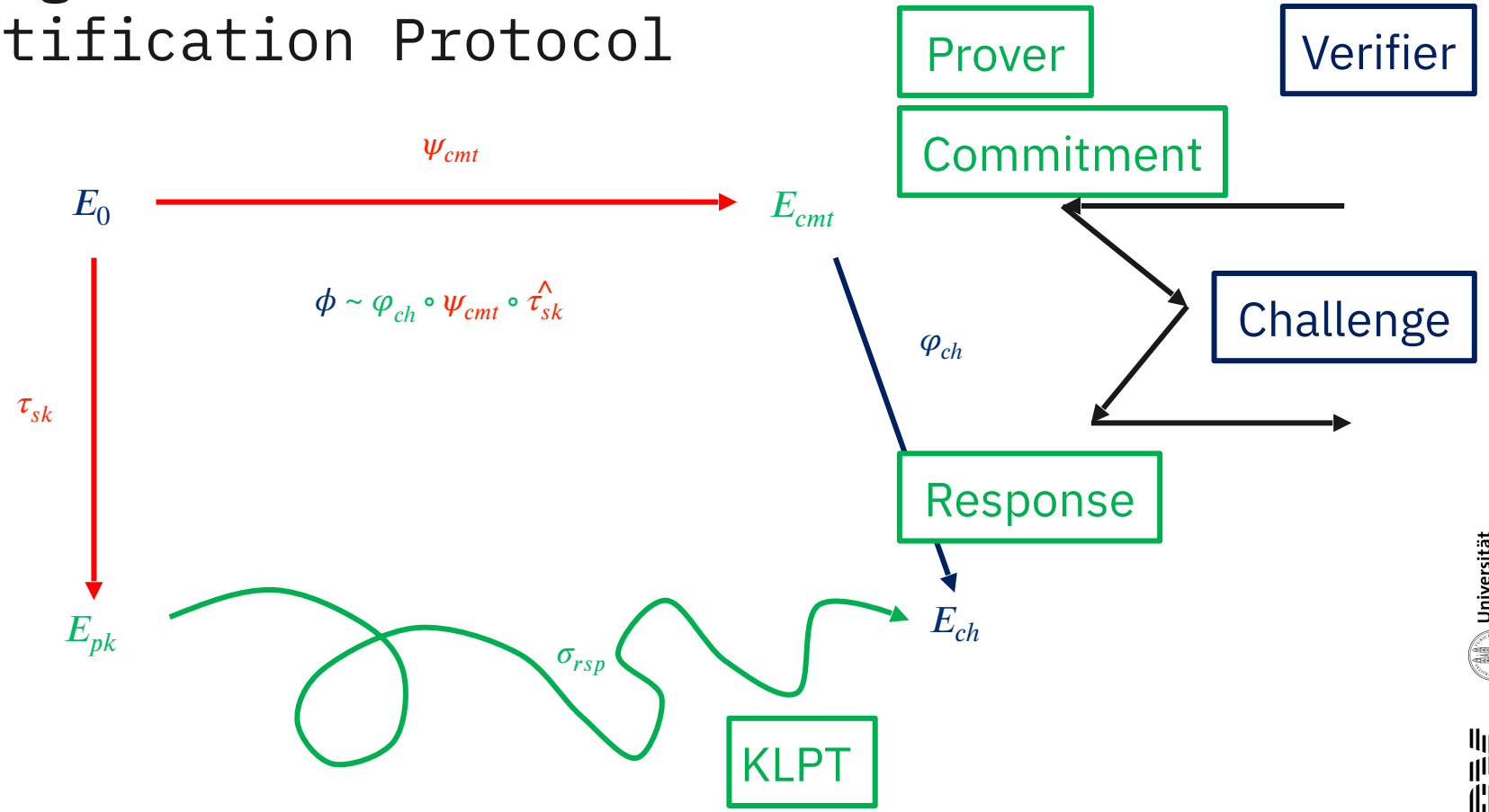
Sequential OR-Proofs

- Framework: **A**be, Masayuki, Miyako **O**hkubo, and Koutarou **S**uzuki. "1-out-of-n signatures from a variety of keys."
- Requires: Sigma Protocol Simulatable in PPT, with Special Honest-Verifier Zero Knowledge property.

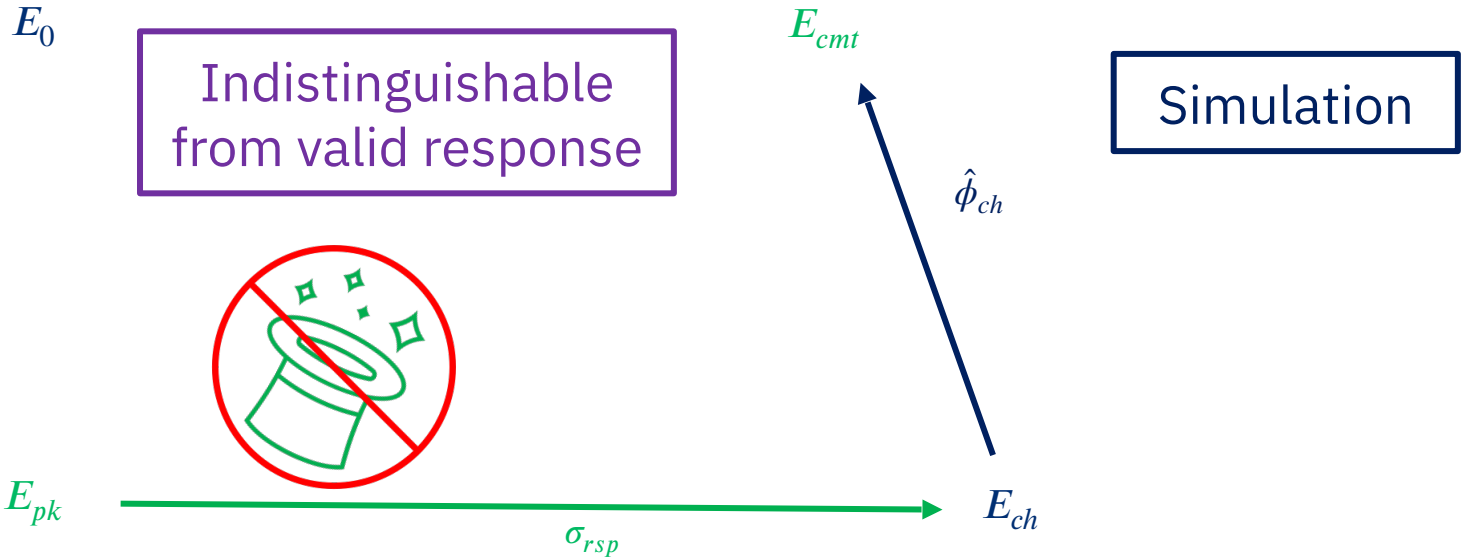
SQISign Identification Protocol



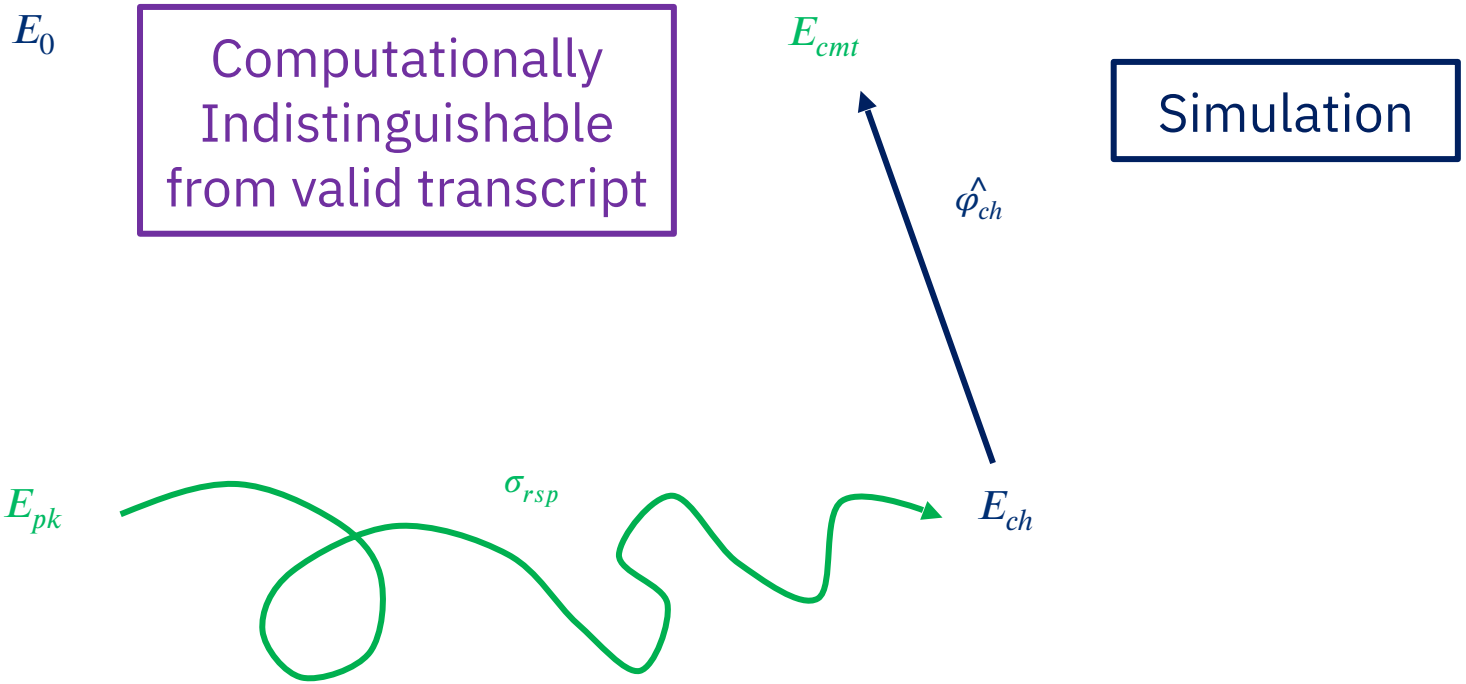
SQISign Identification Protocol



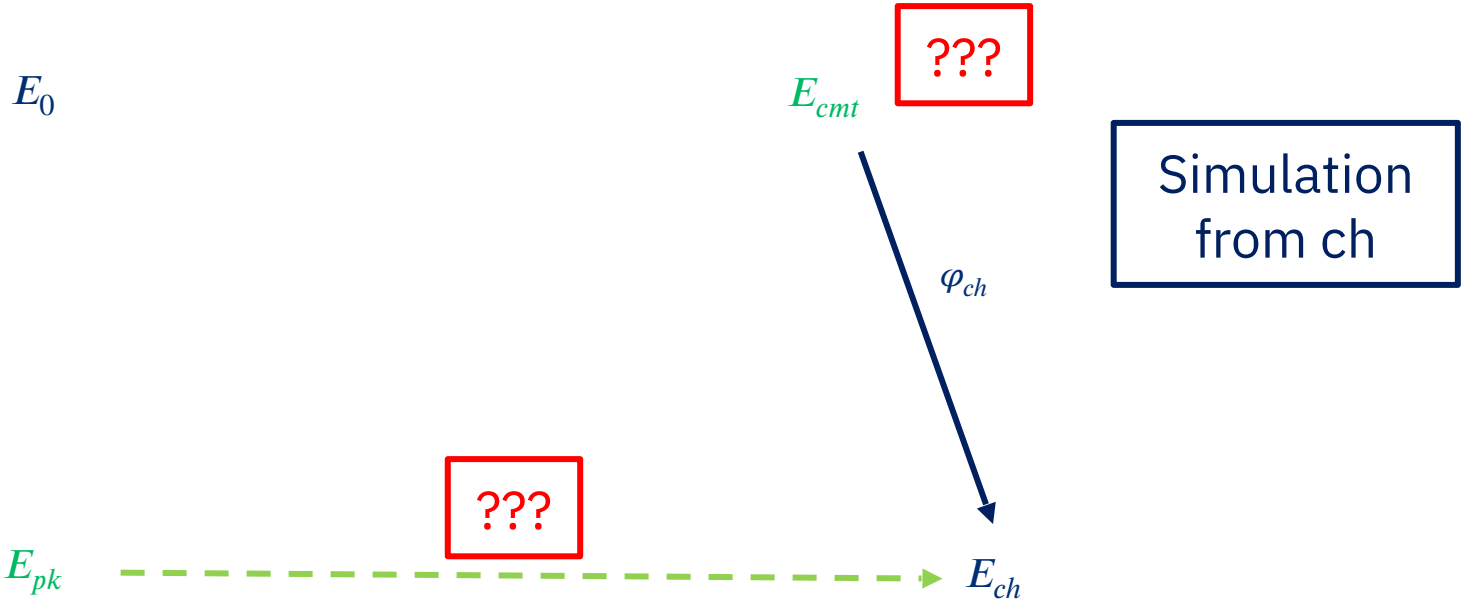
SQISign-HD
HVZK



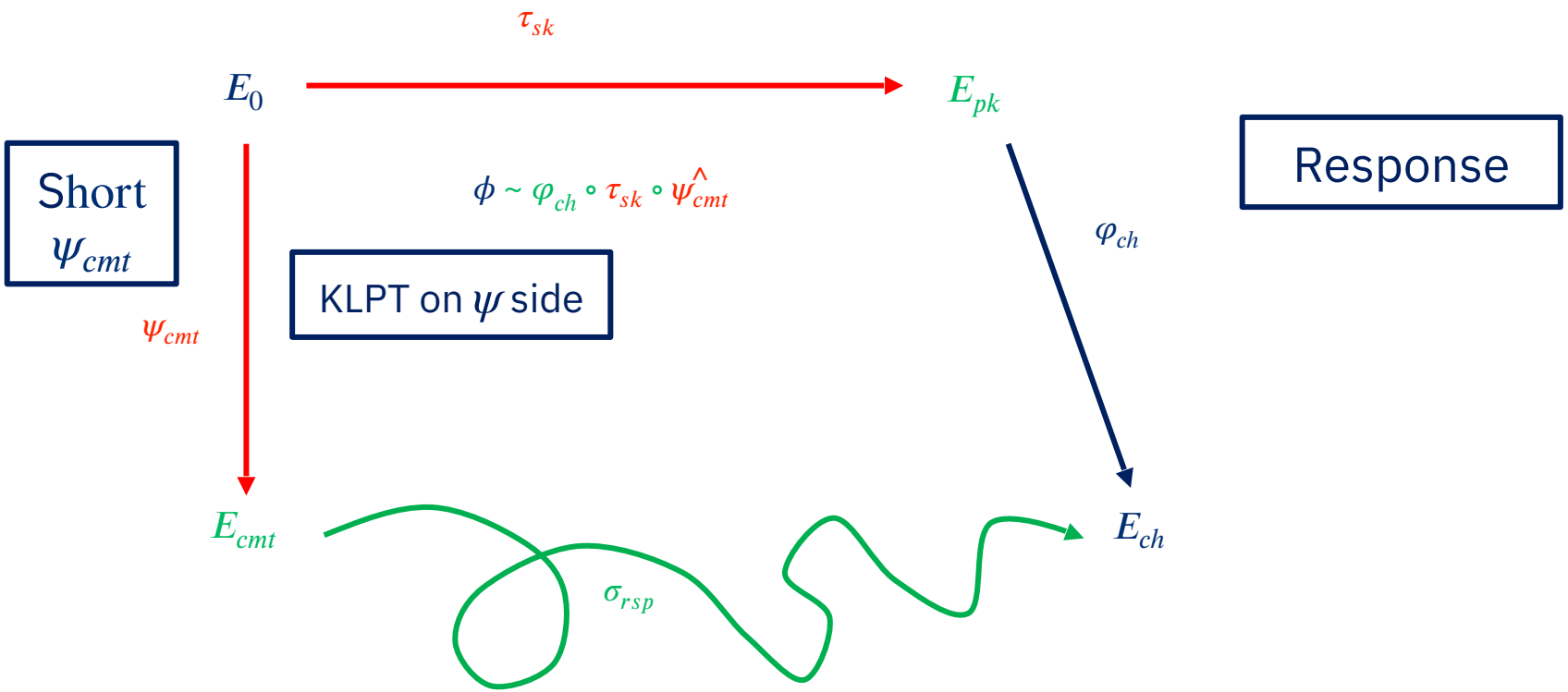
SQISign-KLPT
HVZK



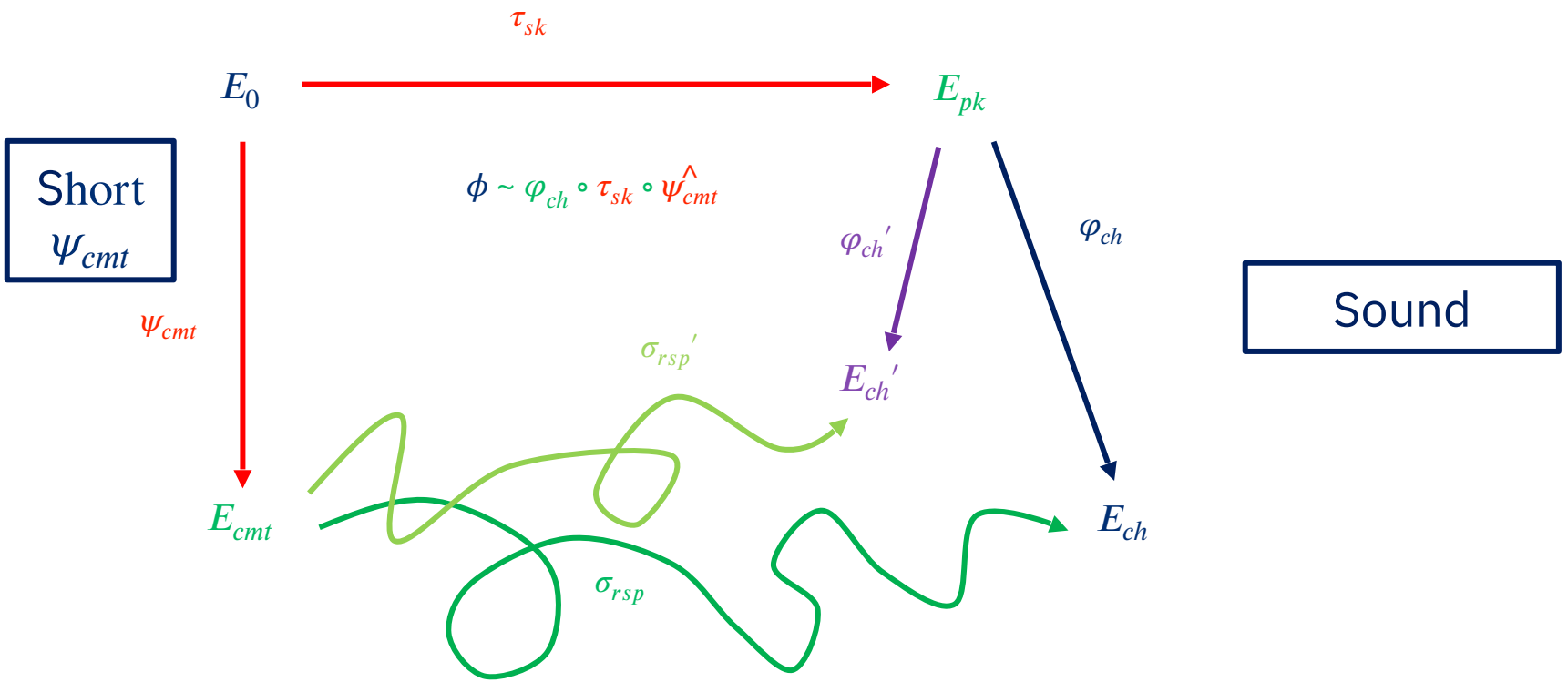
SQISign
Special HVZK



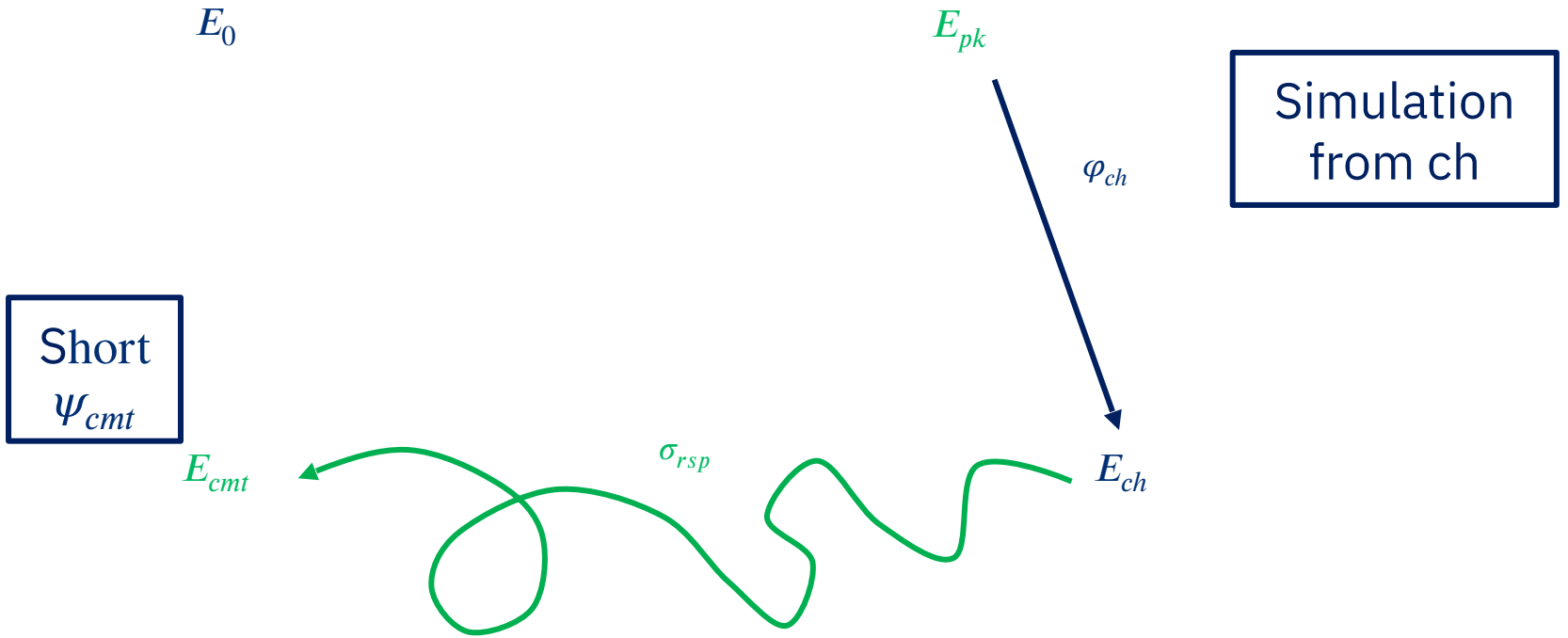
SQISign Modification



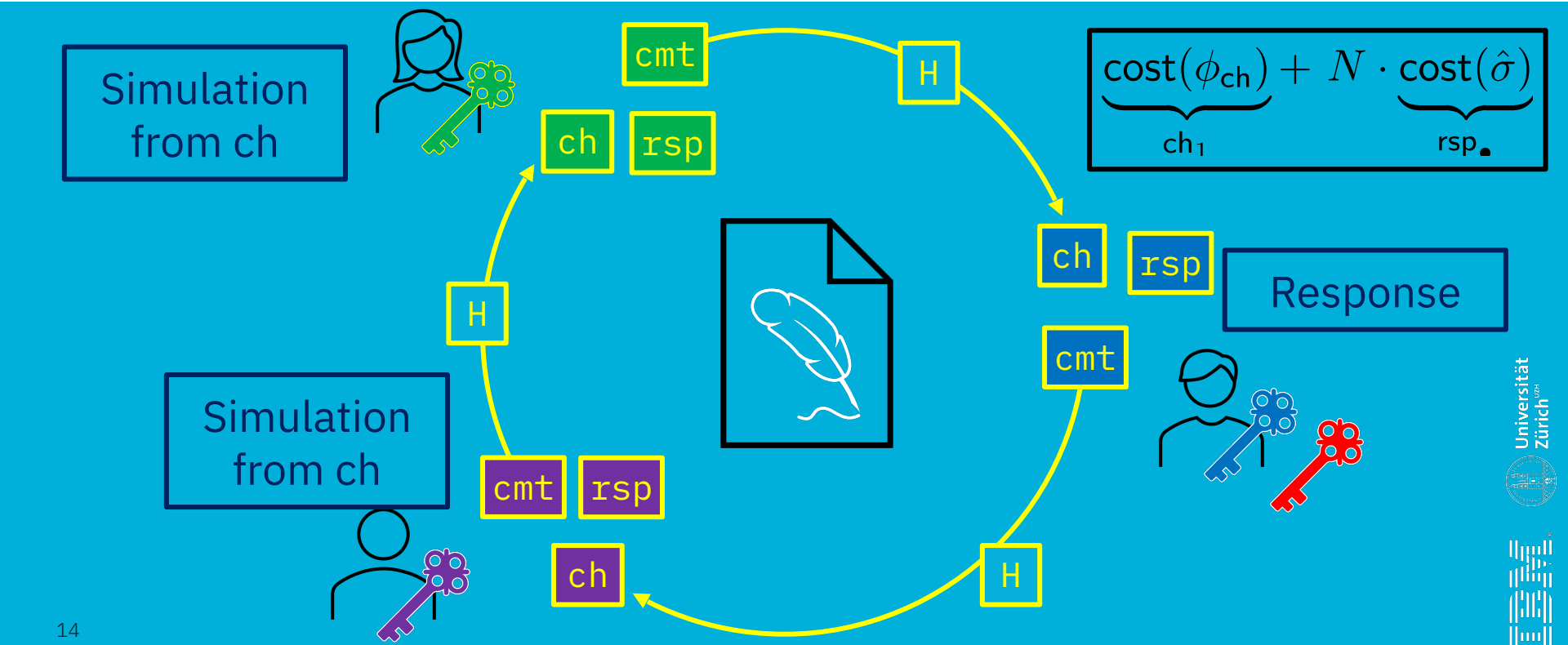
SQISign Modification



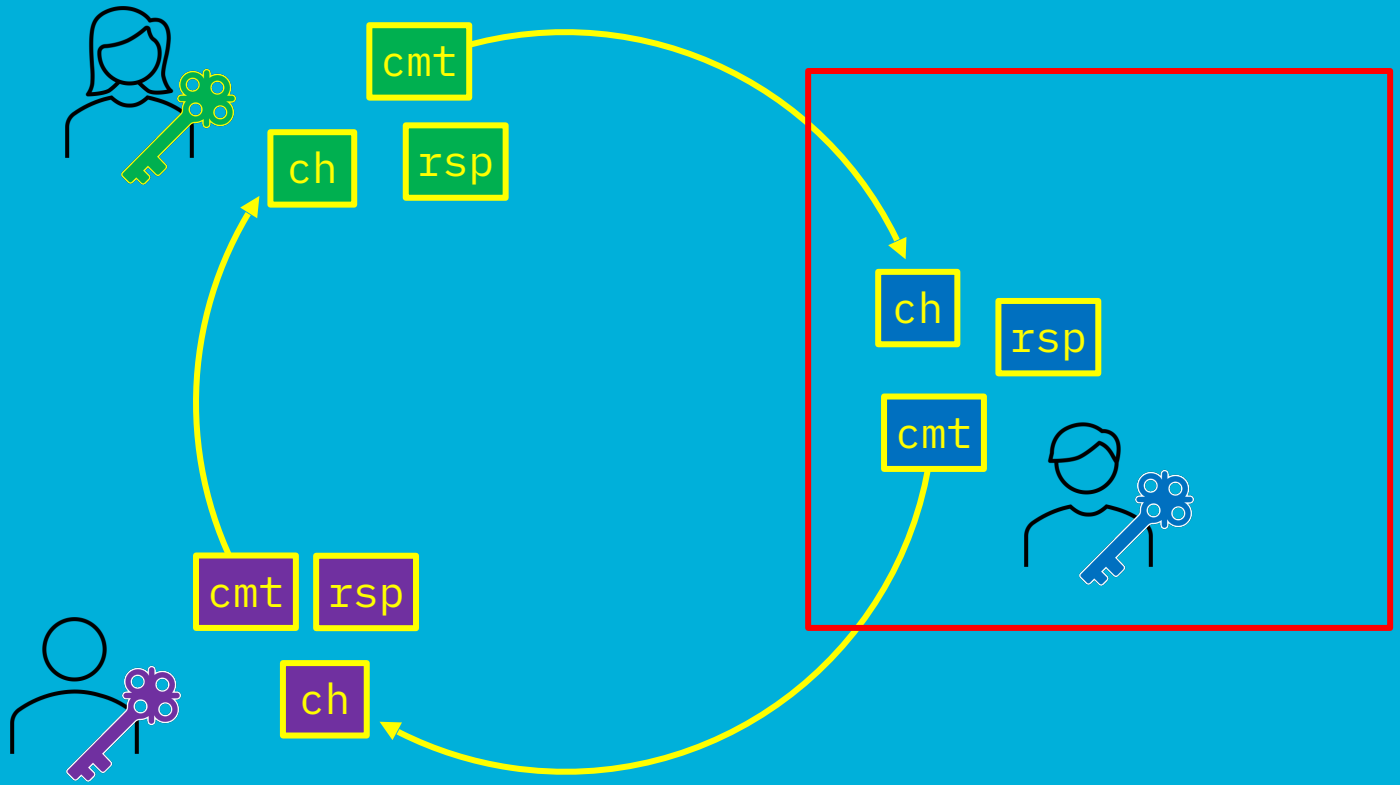
SQISign
Special HVZK



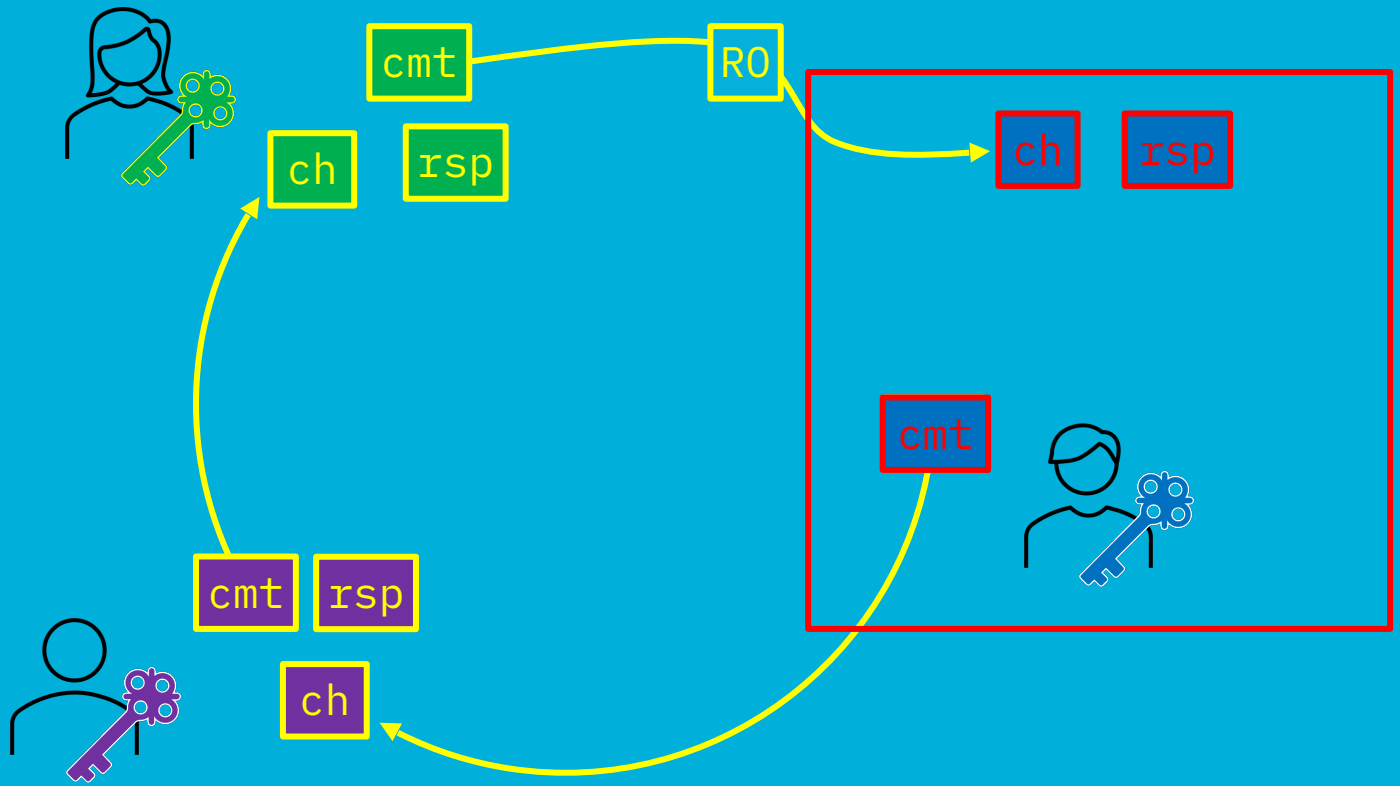
Full-Anonymous **Linear** Ring Signatures



Soundness & weak HVZK → Security against impersonation → Unforgeability

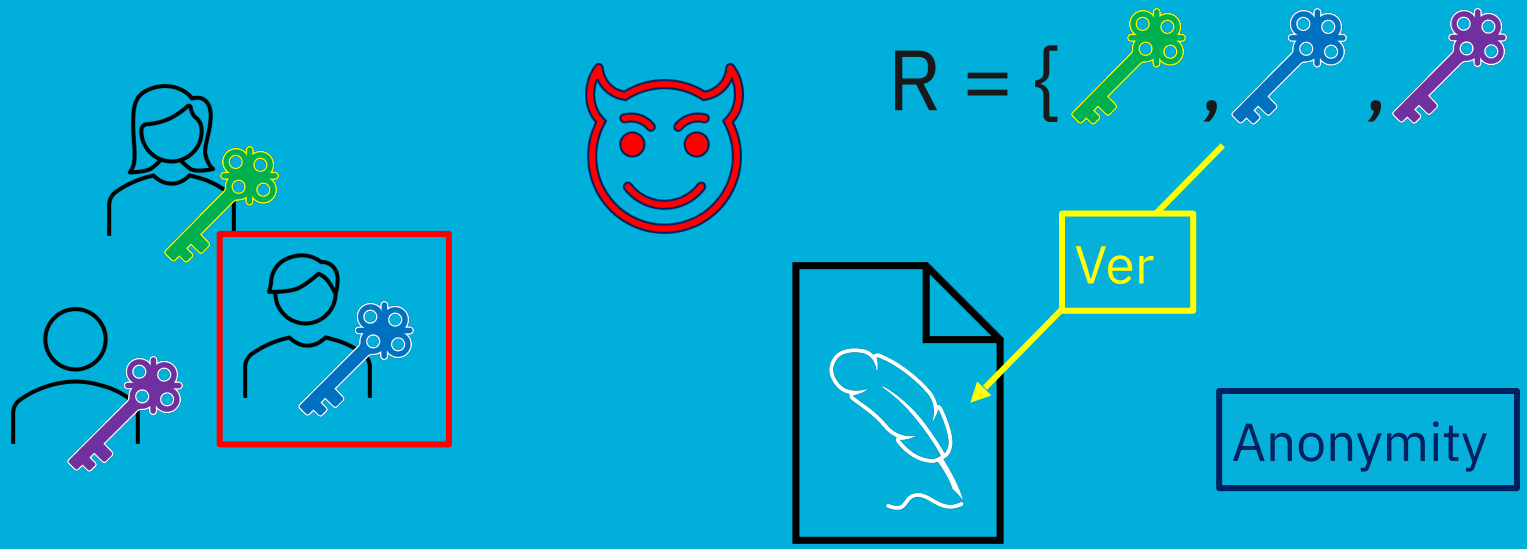


Soundness & weak HVZK → Security against impersonation → Unforgeability



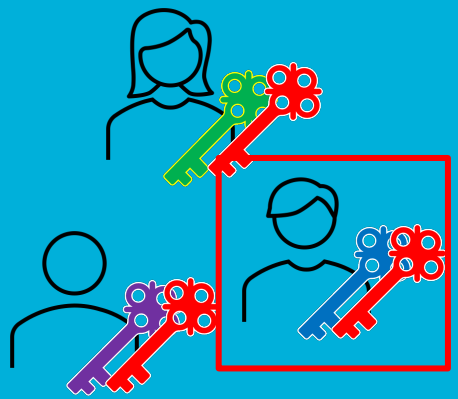
Full-Anonymous

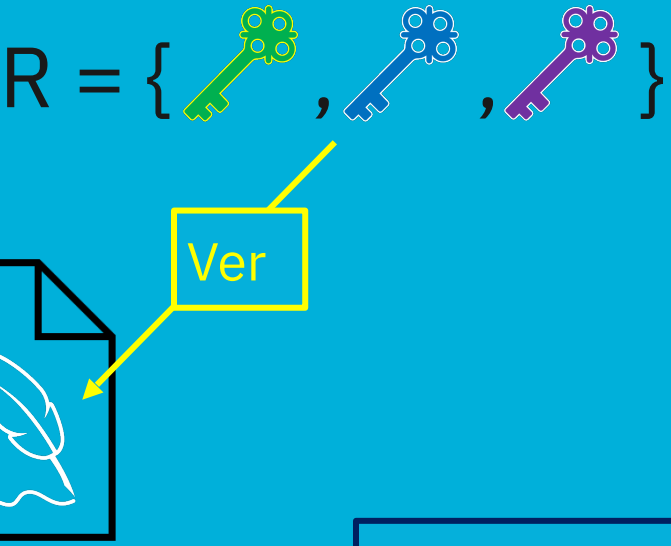
Linear
Ring Signatures



Full-Anonymous

Linear
Ring Signatures





Full Anonymity

Full-Anonymous Linear Ring Signatures

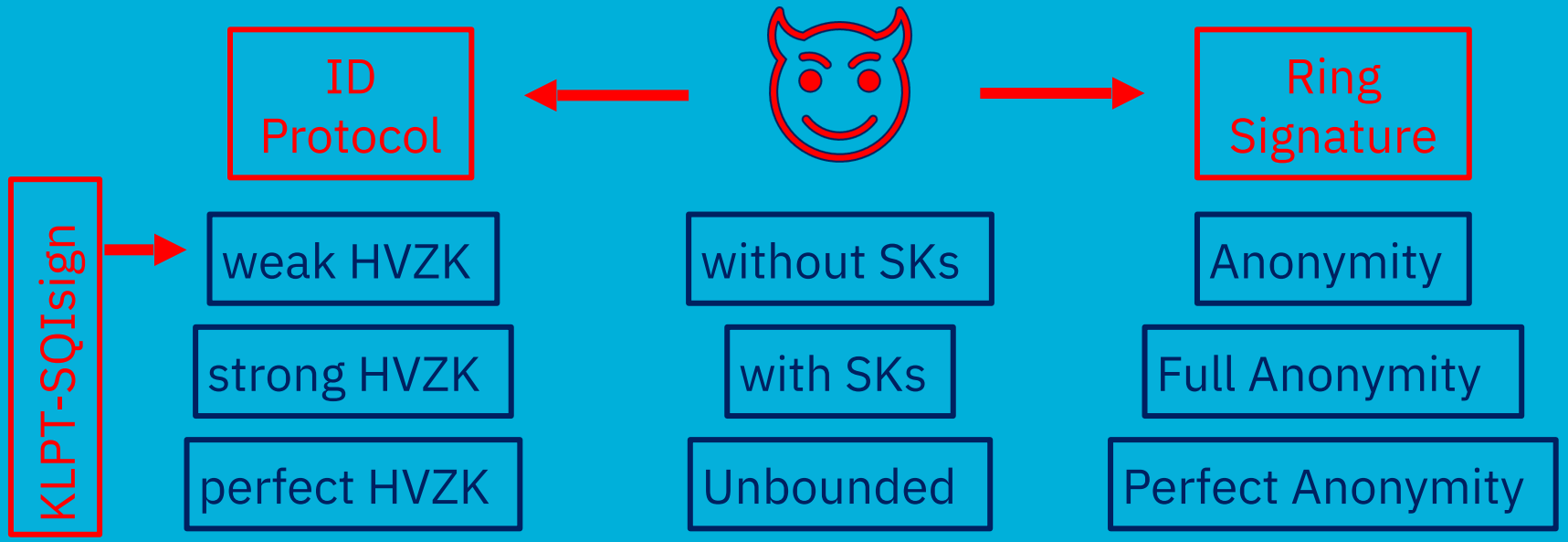
Relevant for:

- Designated Verifier Signatures;
- Deniable Key Exchanges;
- Deniable AKEM;

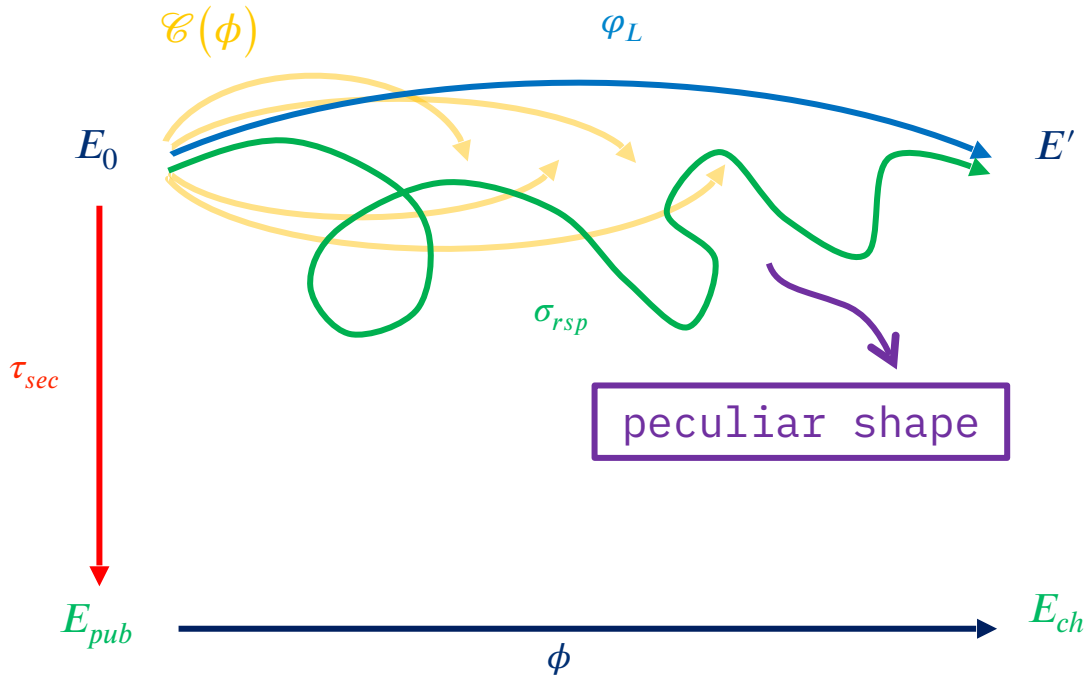
Some References:

- *Phillip Gajland, Jonas Janneck, and Eike Kiltz*. Ring signatures for deniable AKEM: Gandalf's fellowship.
- *Keitaro Hashimoto, Shuichi Katsumata, Kris Kwiatkowski, and Thomas Prest*. An efficient and generic construction for Signal's handshake(X3DH): Post-quantum, state leakage secure, and deniable.
- *Jacqueline Brendel, Rune Fiedler, Felix Günther, Christian Janson, and Douglas Stebila*. Post-quantum asynchronous deniable key exchange and the Signal handshake

Full-Anonymous
Linear
Ring Signatures



KLPT machinery



Geometric World

Supersingular Curves

Isogenies

Endormorphisms

Deuring Correspondence

$$i^2 = -1$$
$$j^2 = -p$$
$$ij = -ji$$

Quaternion World

Geometric World

Maximal Orders in the Algebra

Ideals

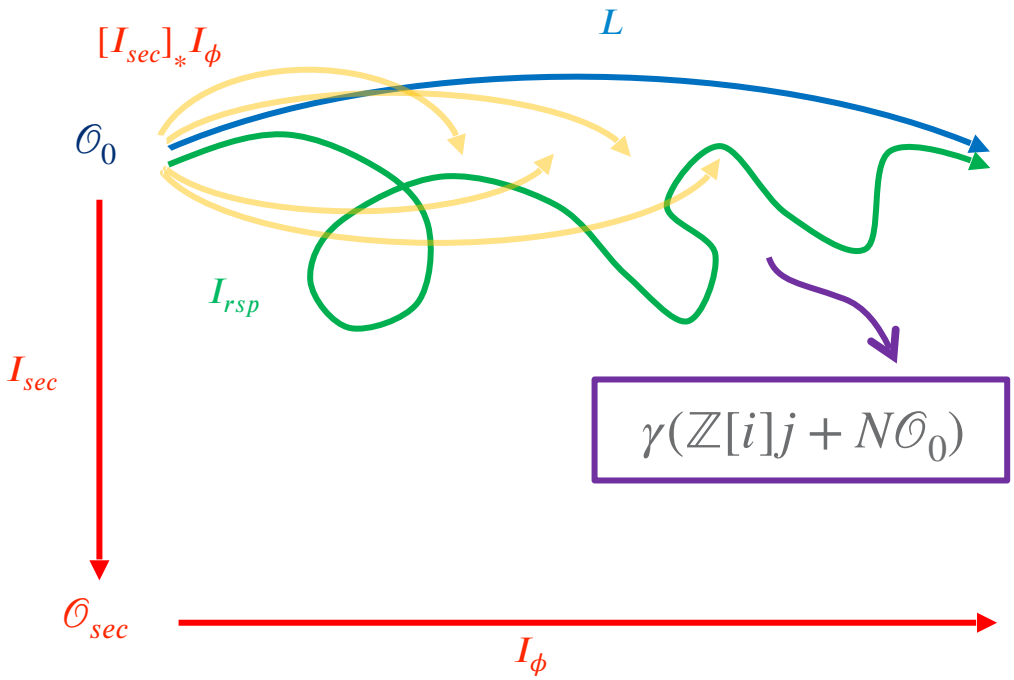
Quaternions

Supersingular Curves

Isogenies

Endormorphisms

KLPT machinery



Quaternion World

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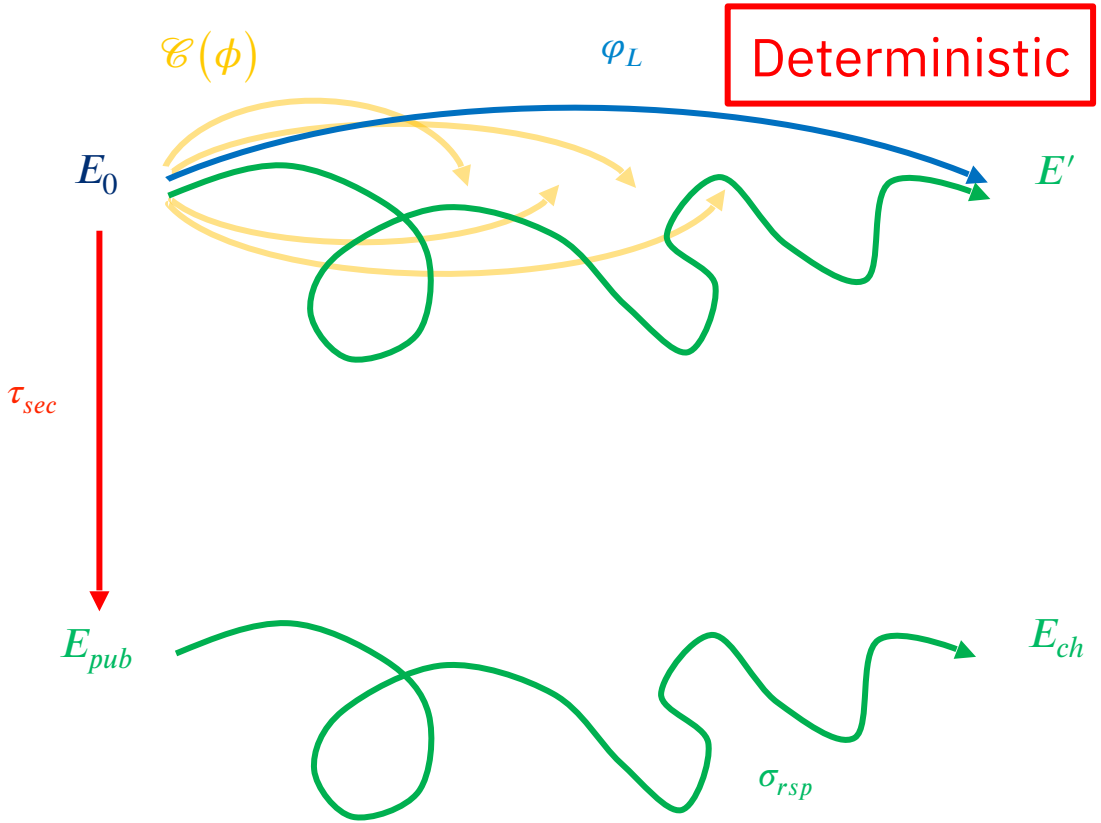
KLPT Distinguisher



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Supersingular Curves

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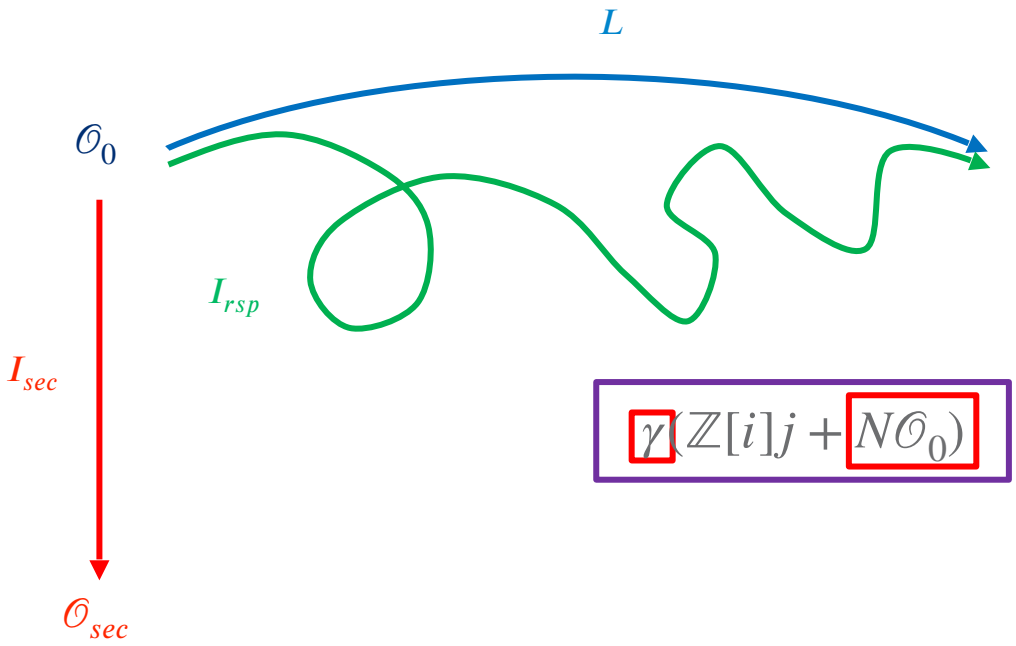
KLPT Distinguisher



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Maximal Orders in the Algebra

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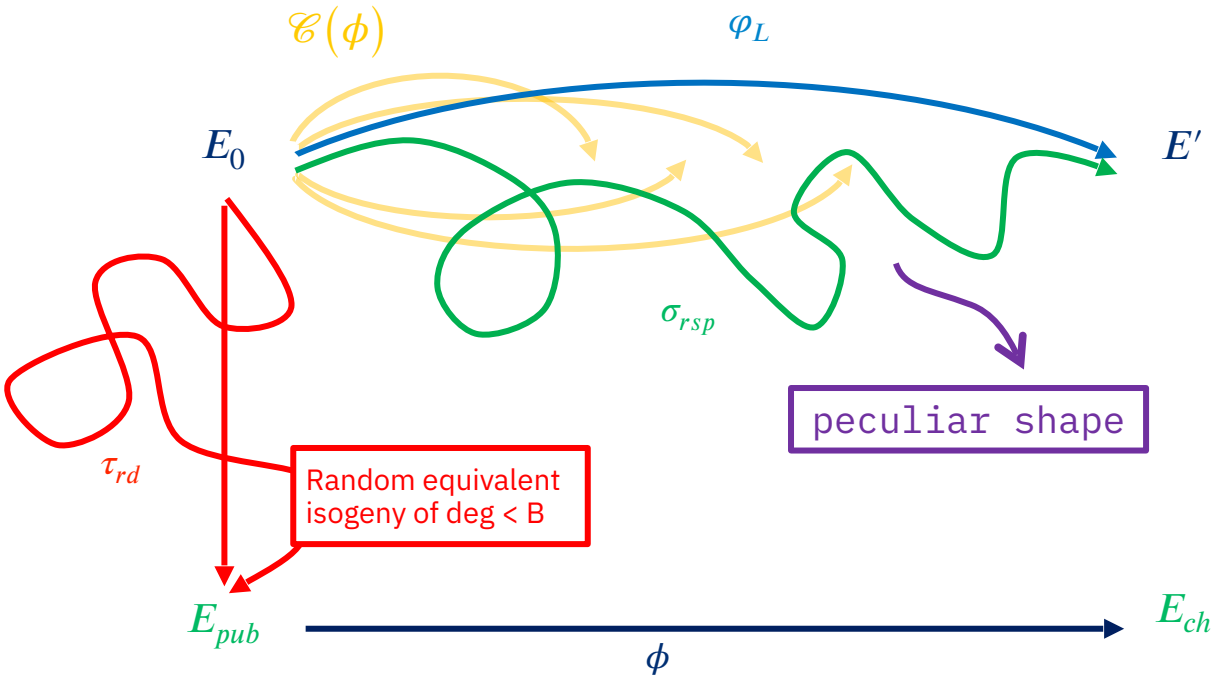
Full-Anonymous Linear Ring Signatures

Problems:

- knowledge of the Eichler Order
- knowledge of the *Class* in which we are working on
- `EquivalentPrimeIdeal(`*Class*`)` is a deterministic KLPT step

Randomized KLPT



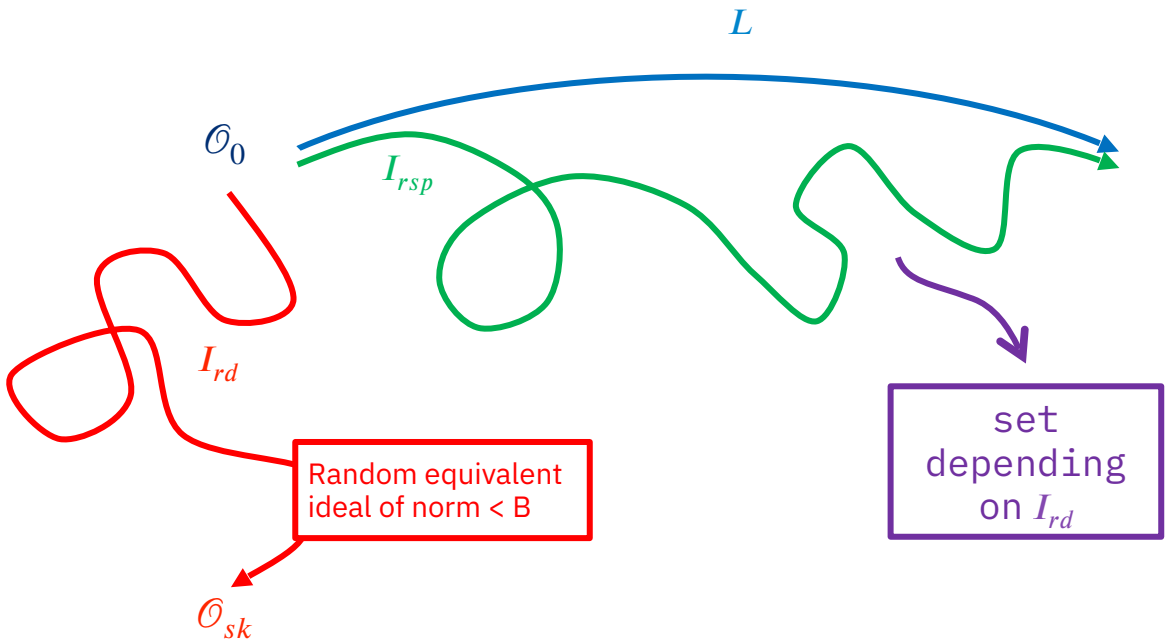
Geometric World

Supersingular Curves

Isogenies

Endormorphisms

Randomized KLPT



Quaternion World

Maximal Orders in the Algebra

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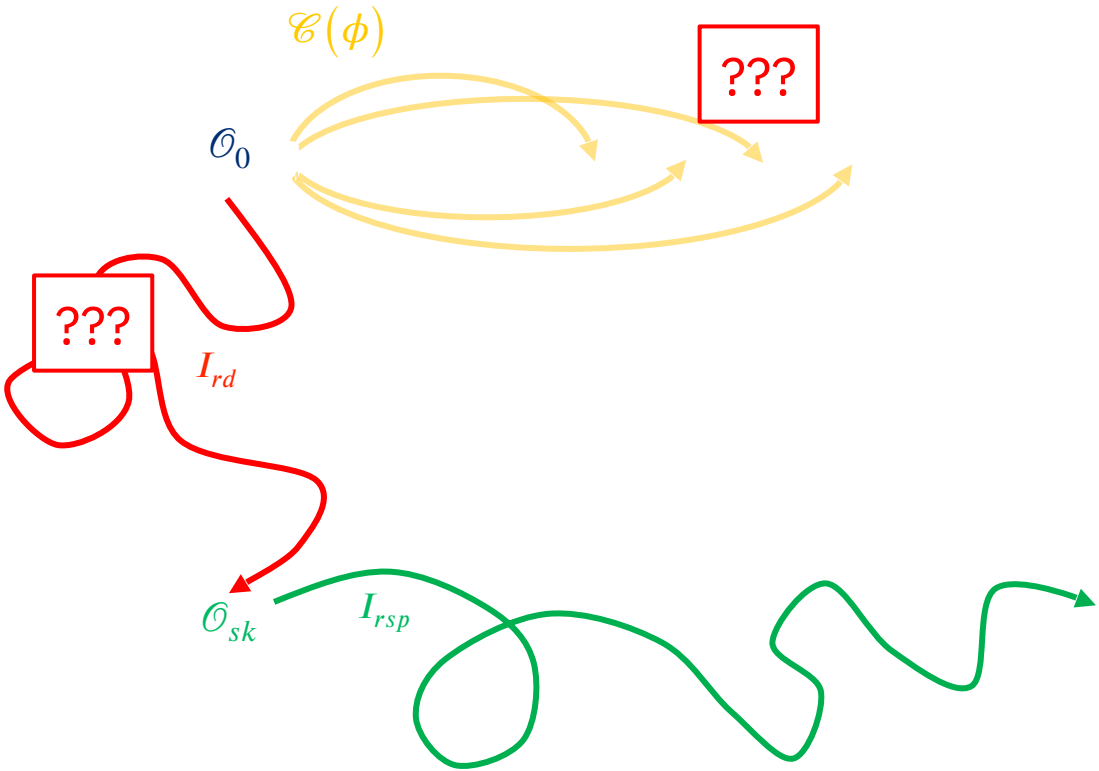
More formally…

Given E, a random curve of known endomorphism ring $\mathcal{O}$, test if a cyclic left $\mathcal{O}$-ideal I_η of norm D lie in the union:

$$\left\{ [I_\psi]_* I_\iota \mid \iota \in \mathcal{P}_{\deg(\psi)}, \boxed{\psi \in \texttt{Iso}_B(E_0, E)} \right\} ;$$

where $\mathcal{P}_{\deg(\psi)}$ is defined as in SQIsign.

Randomized KLPT



Quaternion World

Maximal Orders in the Algebra

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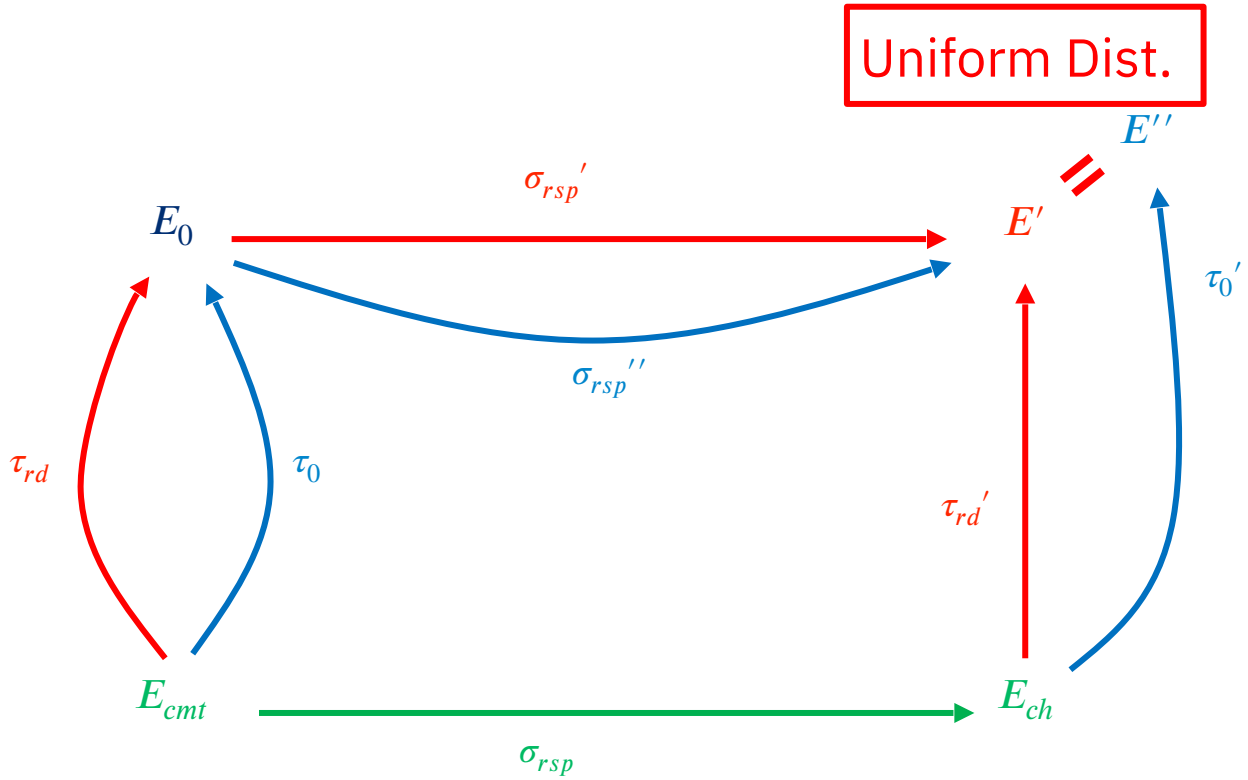
Randomized KLPT



Geometric World

Supersingular Curves

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Instantiations: Fixing the parameters

Random Ideal Degree Bound B

$$\#\mathtt{Iso}_B(E_0, E) = \frac{0.61}{\log(B)} \frac{B^2}{p} \approx 2^\lambda$$

$$B \geq 2^{\frac{\lambda}{2}} p^{\frac{1}{2}}$$

Instantiations: Fixing the parameters

Resp Isogeny Degree Bound D

$$D = 3\log(p) + 3\log(B) + O\left(\log\log(p)\right)$$

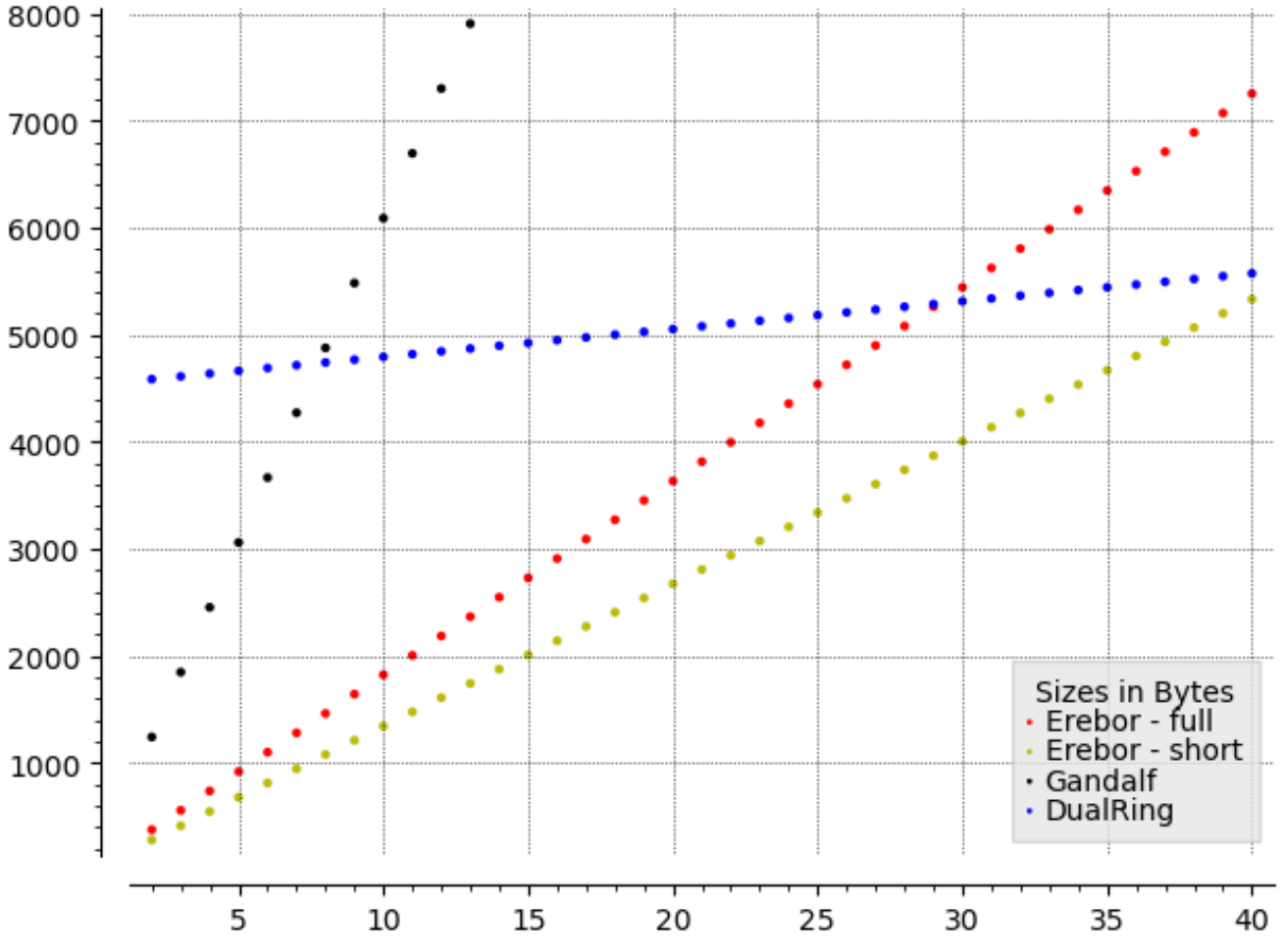
$$\log(D) \approx \frac{9}{2}\log(p) + 3\frac{\lambda}{2} + O\left(\log\log(p)\right) \approx \frac{21}{4}\log(p)$$

+ 40% wrt SQIsign (3.75log(p))

Instantiations: Final Results

NIST I	**Signing** (MC)	**Ver.** (MC)	**Signature size** (B)
Erebor-*full*	$(N-1)\cdot 42 + 3203$	$N \cdot 42$	$16 + N \cdot 181$
Erebor-*short*	$(N-1)\cdot 30 + 2408$	$N \cdot 30$	$16 + N \cdot 133$

Instantiations: Comparisons



Logarithmic-sized Perfect-Anonymous Ring Signature: Durian

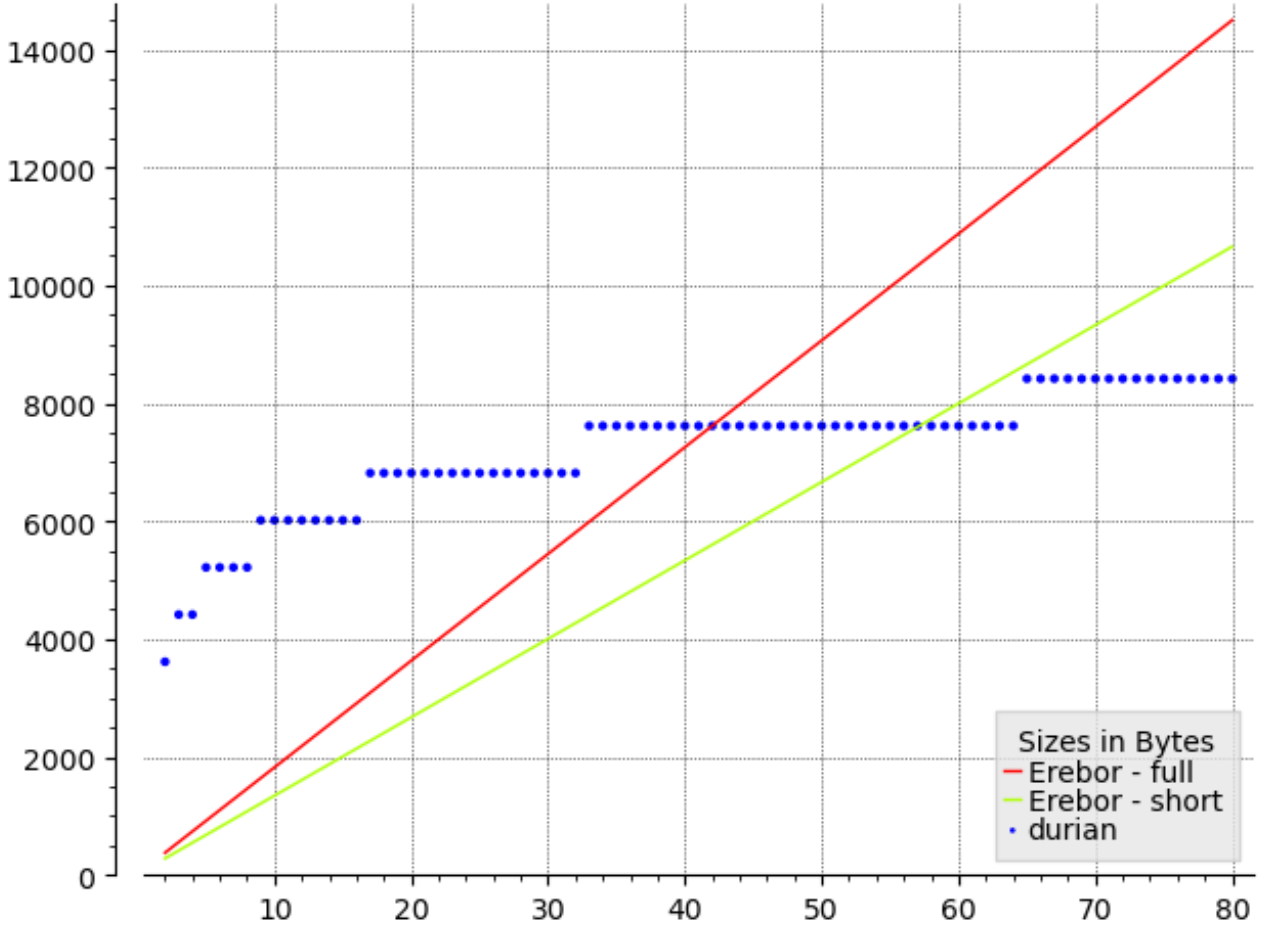
Combine the GPS signature with the framework from Calamari and Falafl

Improve the GPS signature base construction

Best full-anonymous log-size isogeny Ring Signature for NIST I:

4.08 KB for 2 users,
6.29 KB for 8 users,
9.87 KB for 1024 users

Erebor vs Durian: Sizes Comparisons



193 rounds

2024/1185

gbor.in/
erebor