FORENSIC CATEGORIES: A FRAMEWORK FOR SQISIGN-LIKE PRIMITIVES

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OVERVIEW

- Motivation
- Background
- Effective Categories
- The Fingerprint
- Instantiation
- Protocols
- Conclusion

Motivation

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- SQIsign without all the algebraic machinery - more accessible conceptually
- A new perspective to formalize such concepts using category theory

Background

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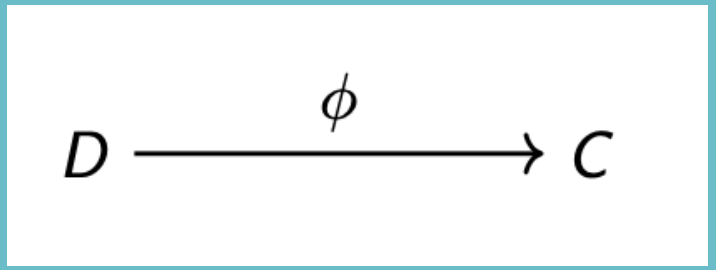

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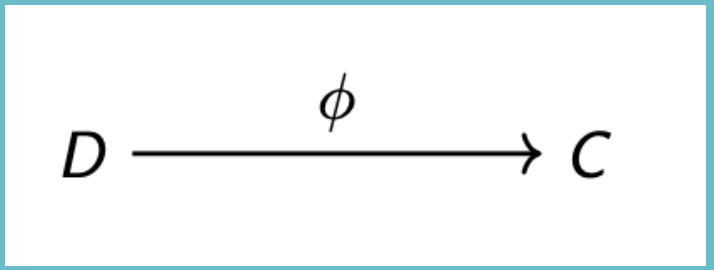
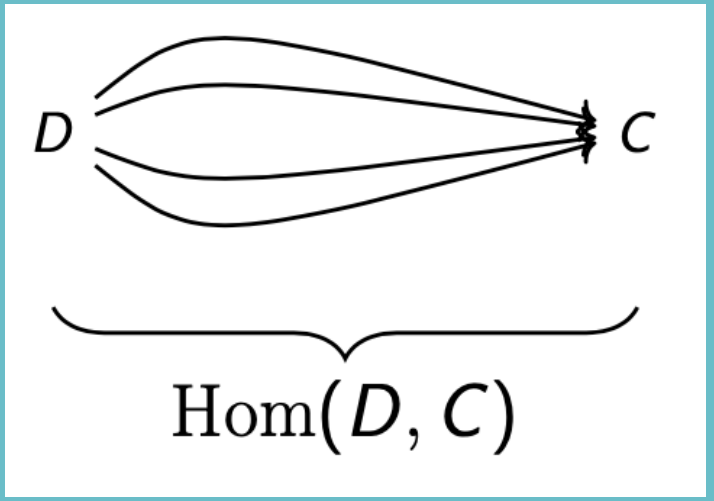
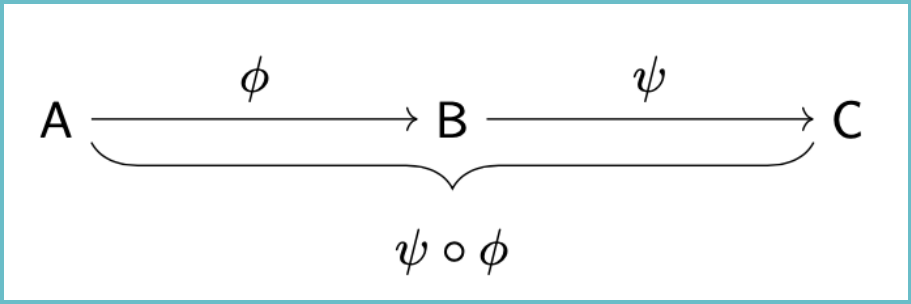
- Every morphism ϕ has a **domain** object D and a **codomain** object C, denoted by $\phi : D \to C$.
- The set of all morphisms with domain D and codomain C is denoted by Hom(D,C); this is called a **homset**





INTRODUCTION TO CATEGORIES

- There exists a **composition law**, written ∘, that maps a morphism $\phi : A \to B$ and a morphism $\psi : B \to C$ to a morphism $\psi \circ \phi : A \to C$, and that is *associative*: $(\psi \circ \phi) \circ \chi = \psi \circ (\phi \circ \chi)$.



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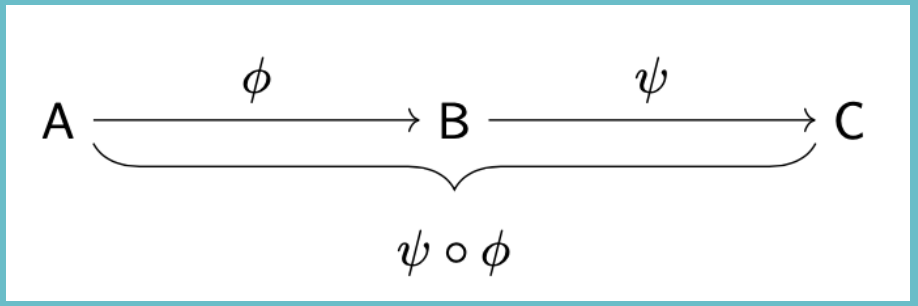
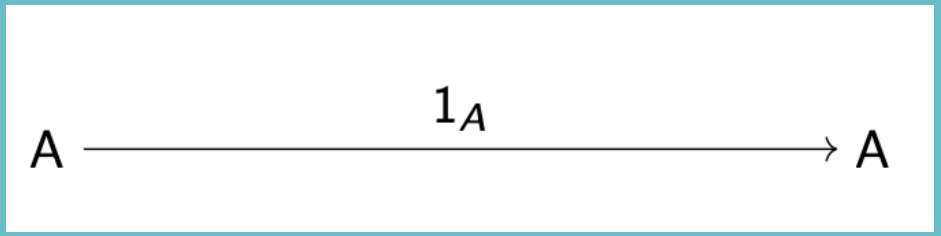


- For every object A, there exists a morphism $1_A : A \to A$ such that $1_A \circ \phi = \phi$ and $\psi \circ 1_A = \psi$ for every $\phi : Z \to A$ and every $\psi : A \to B$



Effective Categories

PRE-EFFECTIVE CATEGORY FAMILY

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- Given n and an object A in $\mathscr{C}_n$, compute the identity morphism $1_A : A \to A$.

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- μ is a distribution on objects (supported everywhere) such that when r is a sequence of n uniformly random coins, the statistical distance between μ and the distribution of the output's codomain B is a *negligible function* of n.

EFFECTIVE CATEGORY

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Pre-effective category + Walk + Origin object

EFFECTIVE CATEGORY

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Pre-effective category + Walk + Origin object = Effective Category

Def (Effective category): An **effective category** is a *pre-effective category* together with a *walk* **Walk** and an *origin object* O whose representation can be computed in polynomial time.

The Fingerprint

DEFINITION

Def (Fingerprint): A **fingerprint** fp on a category $\mathscr{C}$ is a collection of maps

$$\mathsf{fp}: \mathsf{Hom}(-,-) \to \mathscr{M} \cup \{ \perp \},$$

such that $\mathscr{M}$ is finite and containing at least two fingerprints, i.e., $|\mathscr{M}| \geq 2$. The special symbol $\perp$ is used to denote those cases when fp is undefined.

Forensic Categories

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Effective category

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Fingerprint
(with special properties)

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Def (Forensic category): A **forensic category** is an *effective category* where the *fingerprint* satisfies the following properties:

- Effective
- Hard (on average)
- Triangularizable
- (Computationally) Walk-Indistinguishable

EFFECTIVE

Given φ, computing $\mathrm{fp}(\varphi)$ is efficient.

HARD (ON AVERAGE)

The language

$$\mathscr{L} = \left\{ (A, (\varphi, \psi)) \mid \exists B \text{ s.t. } \left\{ \begin{array}{l} \varphi, \psi : A \to B, \\ \mathsf{fp}(\varphi) \neq \mathsf{fp}(\psi), \\ \mathsf{fp}(\varphi), \mathsf{fp}(\psi) \neq \bot \end{array} \right\} \right.$$

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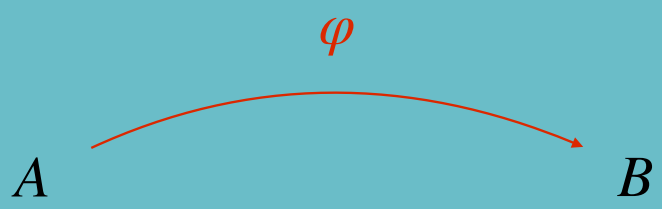

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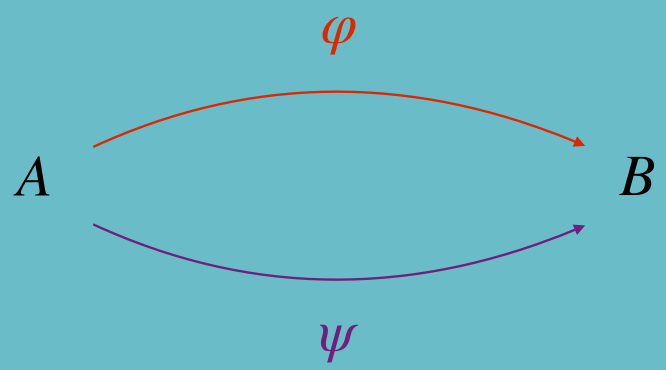

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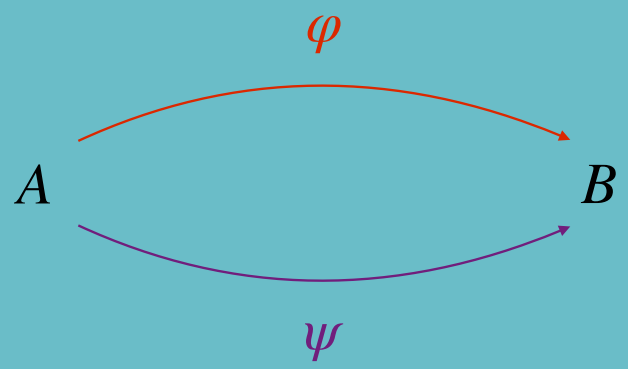

TRIANGULARIZABLE

There exists a PPT algorithm, **Triangle**, that on inputs two morphisms $\varphi\colon O \to A, \psi\colon O \to B$ and a fingerprint $m \in \mathcal{M}$, returns $\chi\colon A \to B$ such that $\mathsf{fp}(\chi) = m$. Furthermore, the distribution of χ only depends on A, B and m.

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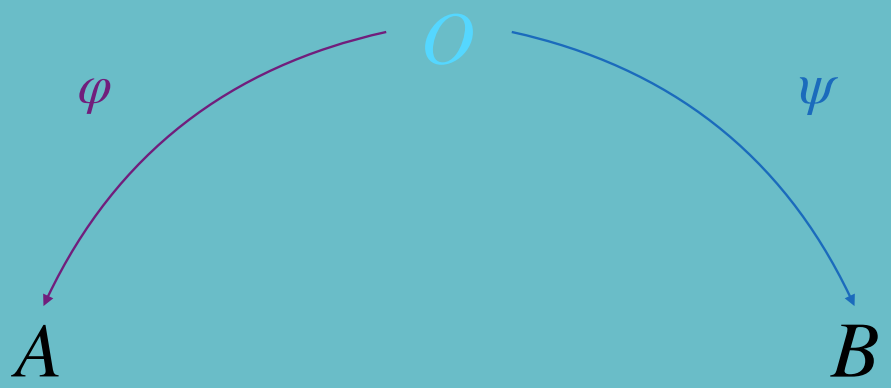

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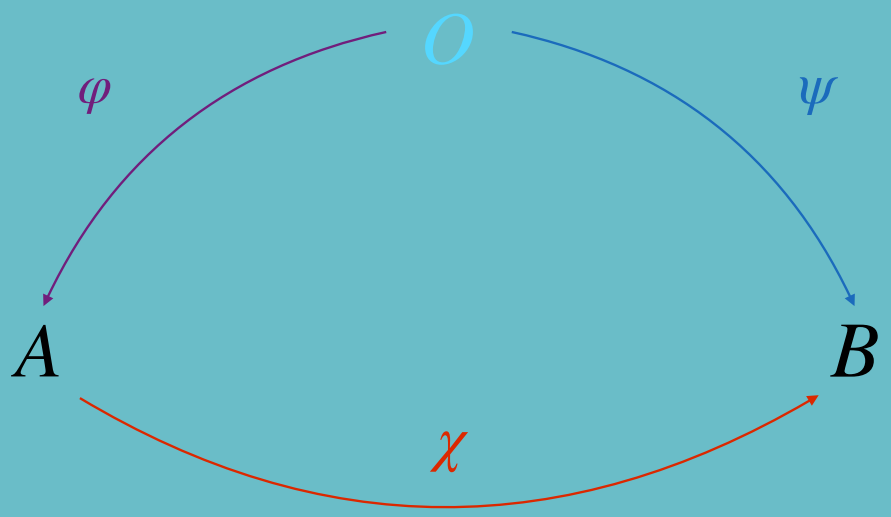

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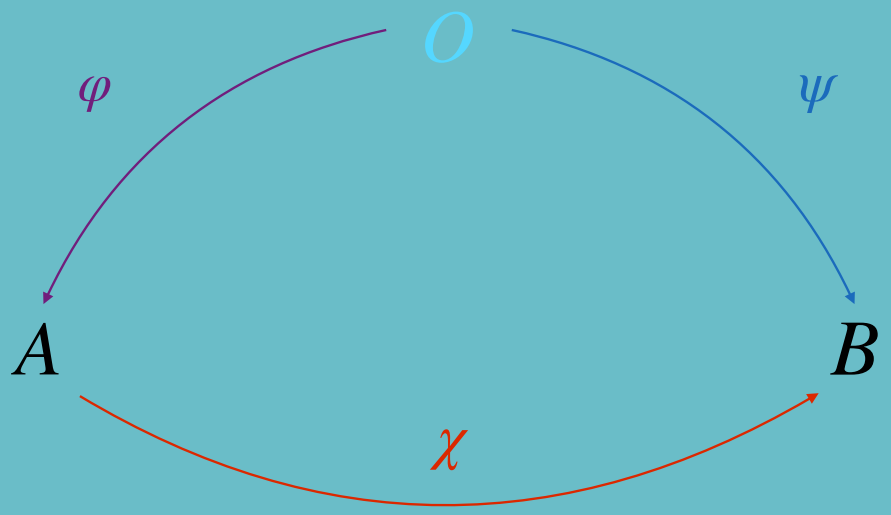

$= m$

(COMPUTATIONALLY) WALK-INDISTINGUISHABLE

There exists a PPT algorithm **SimWalk** which takes an object A as input and returns a morphism $\chi : A \to B$ indistinguishable from the output of **Triangle**.

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Algorithm 1 Oracles of the walk-distinguishing experiment

1: **oracle** $\mathcal{O}^{\varphi:O\to A}_{\mathsf{SimWalk}}$	1: **oracle** $\mathcal{O}^{\varphi:O\to A}_{\mathsf{Triangle}}$
2: $\quad \chi \leftarrow \mathsf{SimWalk}(A)$	2: $\quad (B,\psi) \leftarrow \mathsf{Walk}(O)$
3: $\quad$ **return** χ	3: $\quad m \leftarrow_{\$} \mathcal{M}$
	4: $\quad$ **return** $\chi \leftarrow \mathsf{Triangle}(\varphi,\psi,m)$

An Instantiation of a Forensic Category

CHOOSING AN EFFECTIVE CATEGORY

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Pre-effective category + Walk + Origin object = Effective Category

The category of *supersingular elliptic curves in characteristic* p, written $\mathscr{C}_p$, is the category where

- **objects** are isomorphism classes of supersingular elliptic curves over $\overline{\mathbb{F}}_p$, and
- **morphisms** $\psi\colon E_1 \to E_2$ are isogenies between them.

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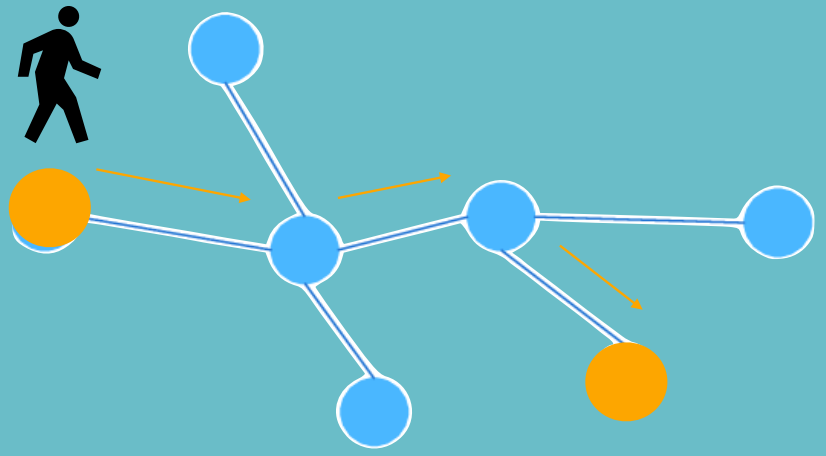
Walk is instantiated by following a (non-backtracking) random walk of length n from the input curve in the ℓ-isogeny graph.



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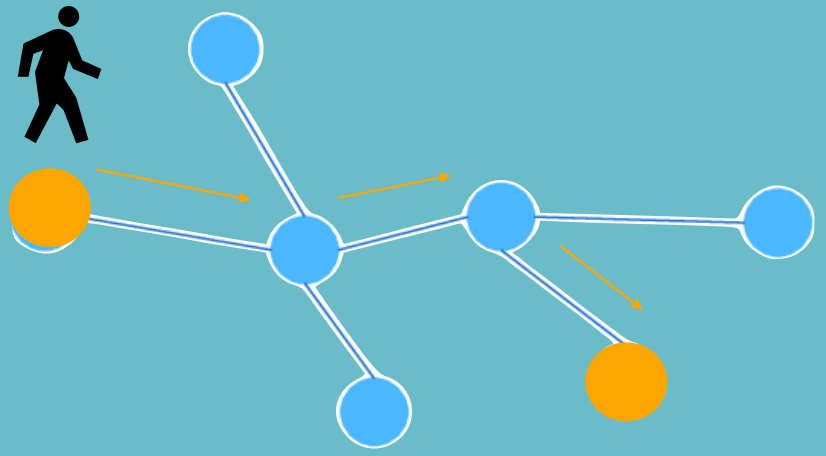
Since this graph is *Ramanujan* the distance between the distribution of the output of the walk and the stationary distribution is a negligible function of the walk's length n.



AN ORIGIN OBJECT $\mathcal{O}$

There exists an algorithm to find a supersingular curve E_0 of known endomorphism ring.

CHOOSING A FINGERPRINT

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Let $\varphi : E \to E'$ be an isogeny and define $\ker(\varphi)[N] = \ker(\varphi) \cap E[N]$. Then,

$$\mathsf{prefp}(\varphi) = \begin{cases} \ker(\varphi)[N], & \text{if } \ker(\varphi)[N] \cong \mathbb{Z}/N\mathbb{Z} \\ \perp, & \text{otherwise.} \end{cases}$$

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$$\ker(\varphi)[\ell^n] = \langle P \rangle = \langle aP_1 + bQ_1 \rangle$$

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$$\mathsf{fp}_{\mathscr{K}}(\varphi) = \begin{cases} (1,b), & \text{if } \ker(\varphi)[N] = \langle P_E + bQ_E \rangle \text{ for } b \in (\mathbb{Z}/N\mathbb{Z}) \\ (a,1), & \text{if } \ker(\varphi)[N] = \langle aP_E + Q_E \rangle \text{ for } a \in (\mathbb{Z}/N\mathbb{Z}) \backslash (\mathbb{Z}/N\mathbb{Z})^{\times} \\ \perp, & \text{otherwise,} \end{cases}$$

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2. Then we compute $\mathsf{fp}_{\mathscr{K}}(\varphi)$ by a generalised discrete logarithm computation, taking time polynomial in $\log(N)$

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Strategy: Prove that if $\mathsf{fp}_{\mathscr{K}}$ does not satisfy the Hard on average property, then ONEEND is not hard on average either (hence ISOGENY is not hard in the worst case)

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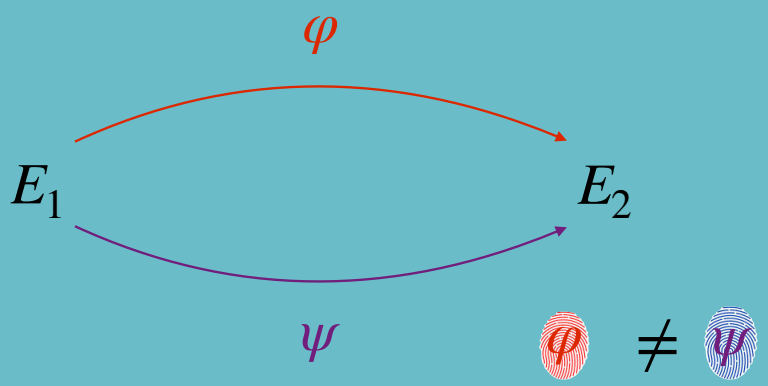
- **Solve** ONEEND: Let E_1 be uniformly random.
  - run $(\varphi, \psi) \leftarrow \mathscr{A}(E_1)$



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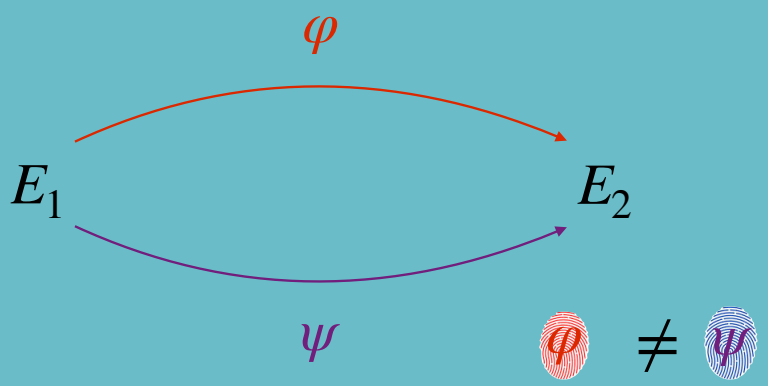

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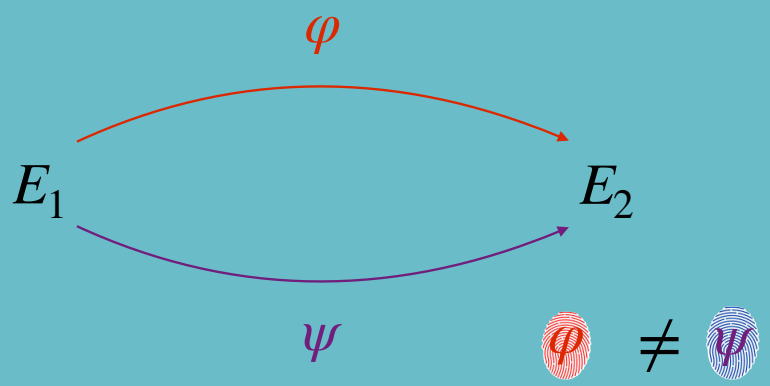

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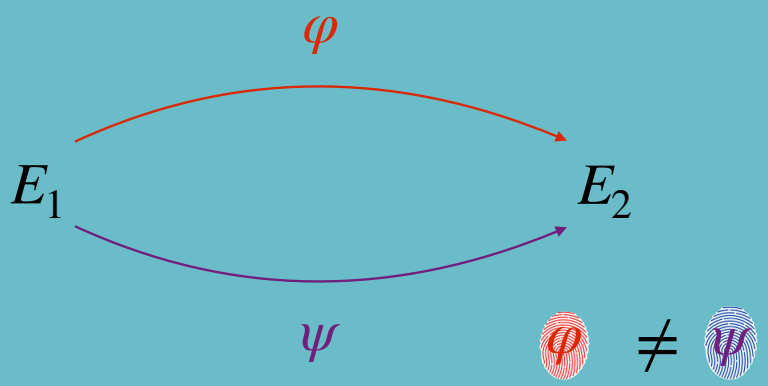

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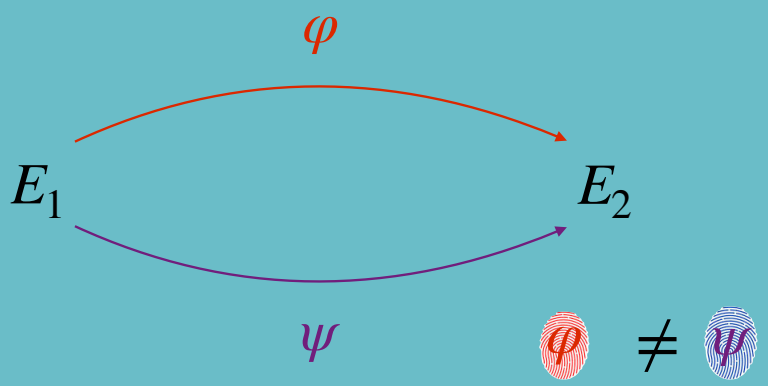

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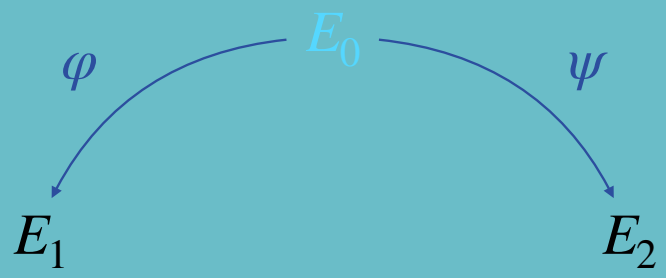

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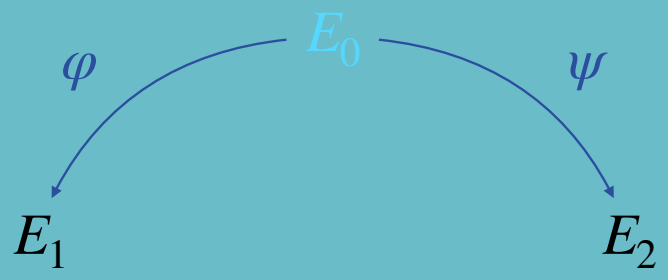

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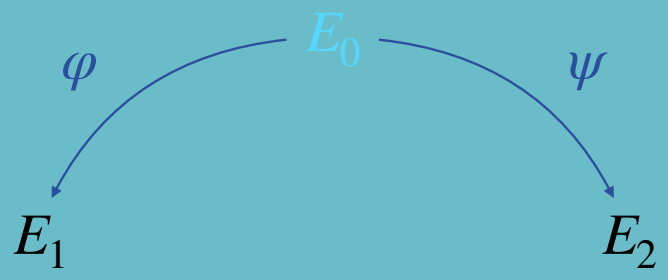

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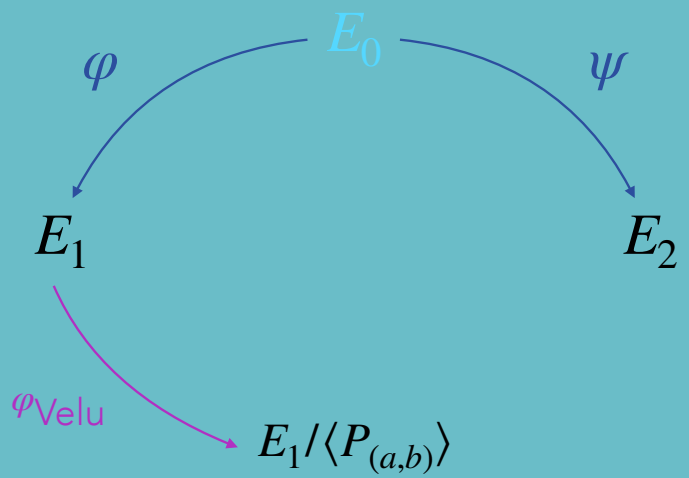

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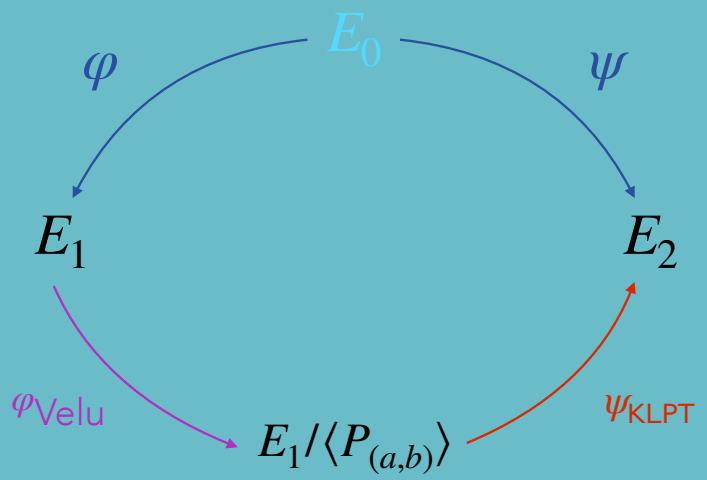

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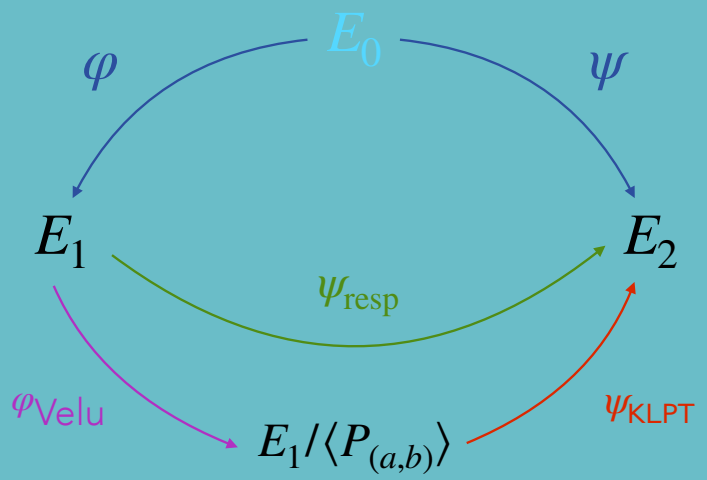

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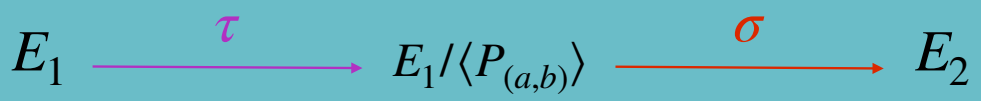

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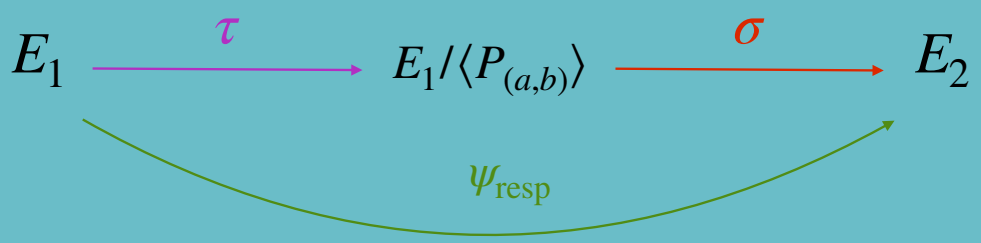

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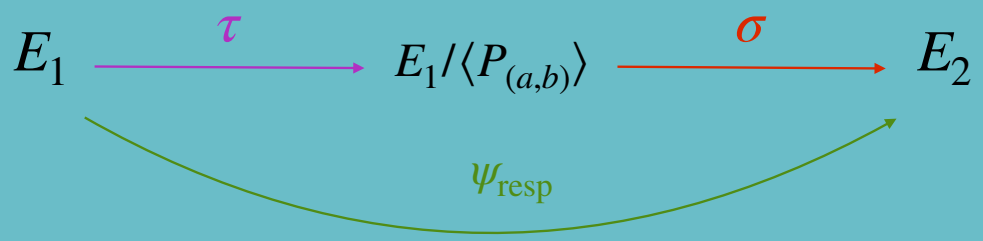

Assumption: *The output of* RSigningKLPT*, is computationally indistinguishable from uniformly random cyclic isogenies of same degree and same domain.*

Digital Signature

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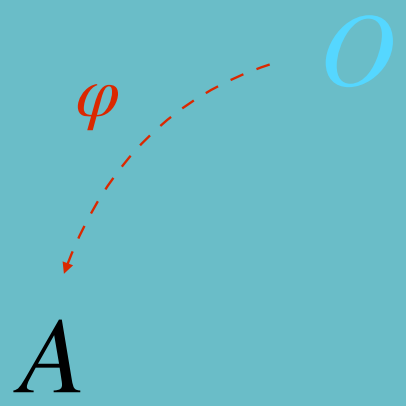

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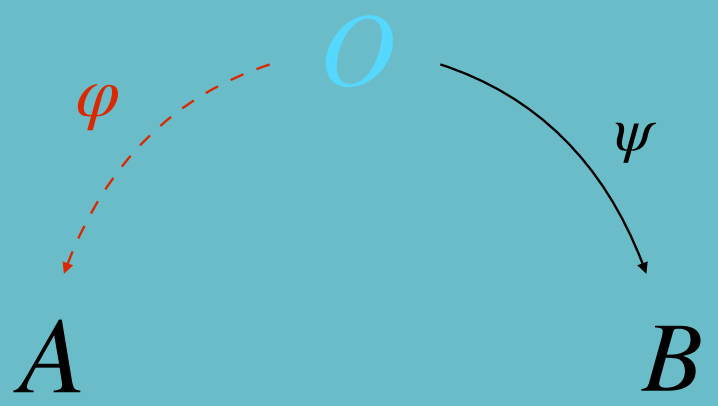

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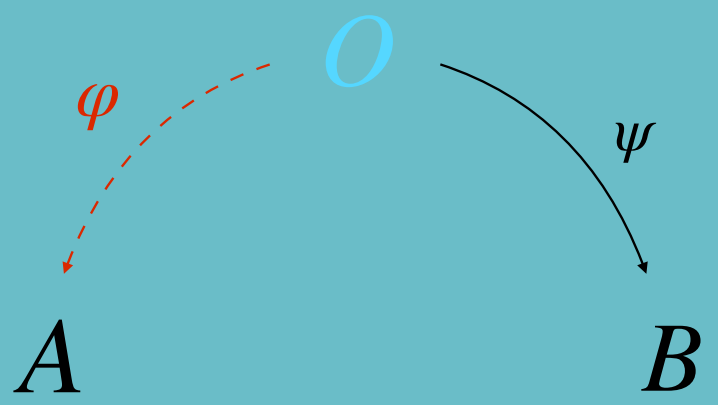

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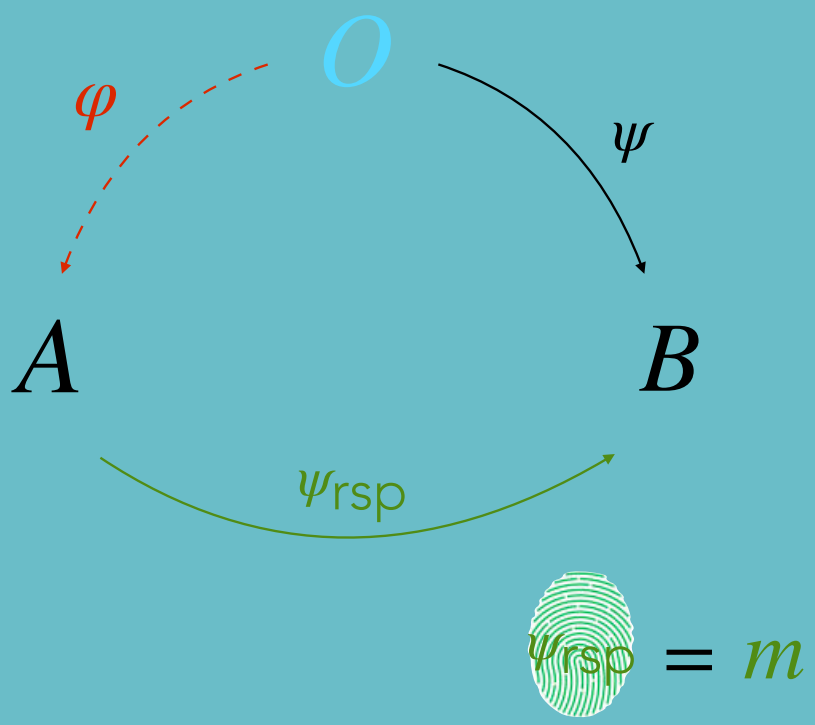

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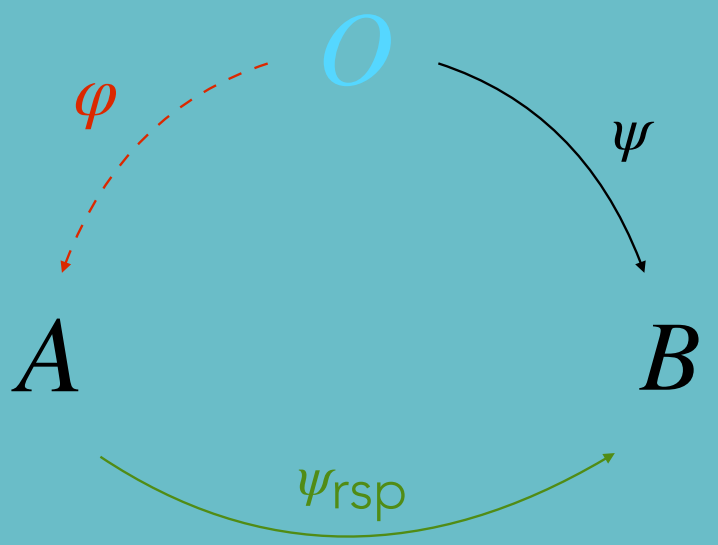

Verification. The verifier accepts if ψ_{rsp} has A for domain, B for codomain, and if $\mathsf{fp}(\psi_{\mathsf{rsp}}) = m$.

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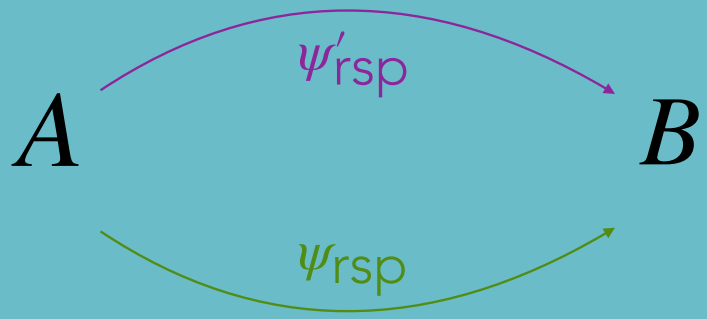

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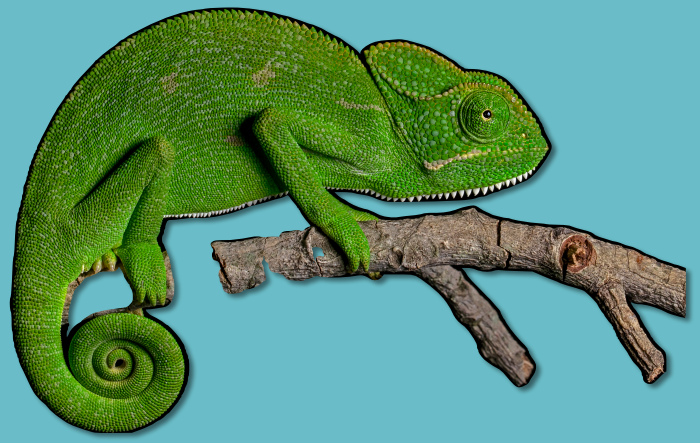
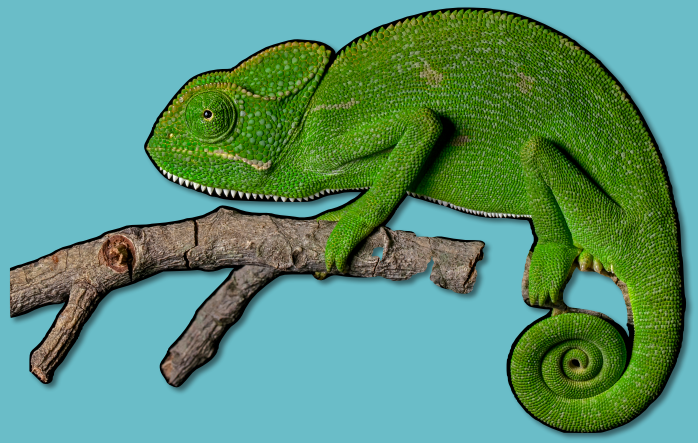
Comment: This construction does not quite capture the HD variants of SQIsign

Chameleon Hash Function





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CHash: On input a public key **pk** and a message **msg**, outputs a hash h and a *check value* (aka *randomness*) c.

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CHKG: On input the public parameters **pp** outputs a private-public key pair (**sk, pk**).

CHash: On input a public key **pk** and a message **msg**, outputs a hash h and a *check value* (aka *randomness*) c.

CHCheck: On input a public key **pk**, a message **msg**, a check value c, and a hash h, outputs a bit b, indicating whether the hash h is valid.

WHAT IS A CHAMELEON HASH FUNCTION?

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CHange: On input a key pair (**sk, pk**), a pair of messages **msg, msg′**, a check value c and a hash h, outputs a new check value c'.

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(**Weak) Collision-Resistance**: without knowledge of the secret key, it is hard to find pairs (msg, c) and (msg′, c') with msg ≠ msg′ both passing CHCheck for the same hash h.

(**Strong) Indistinguishability**: it is infeasible to distinguish between check values produced by CHash and those produced by CHange

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Algorithm 4 Chameleon hash function from a camouflaged fingerprint

function CHKG(pp)
  $(A_{\mathsf{pk}}, \varphi_{\mathsf{sk}}) \leftarrow \mathsf{Walk}(O)$
  return $\varphi_{\mathsf{sk}},\ A_{\mathsf{pk}}$

function CHash($A_{\mathsf{pk}}, \mathsf{msg}$)
  $(B_h, \psi_c) \leftarrow \mathsf{SimWalk}(A_{\mathsf{pk}})$
  $m_h \leftarrow H(\mathsf{msg}) \star \mathrm{fp}(\psi_c)$
  return $\psi_c,\ (B_h, m_h)$

function CHeck($A_{\mathsf{pk}}, \mathsf{msg}, \psi_c, (B_h, m_h)$)
  Parse domain and codomain of $\psi_c : A \to B$
  return $H(\mathsf{msg}) \star \mathrm{fp}(\psi_c) = m_h$ **and** $A = A_{\mathsf{pk}}$ **and** $B = B_h$

function CHange($\varphi_{\mathsf{sk}}, A_{\mathsf{pk}}, \mathsf{msg}, \mathsf{msg}', \psi_c, (B_h, m_h)$)
  if not CHeck($A_{\mathsf{pk}}, \mathsf{msg}, \psi_c, (B_h, m_h)$) **then return** $\perp$
  return $\psi_c' \leftarrow \mathsf{Triangle}(\varphi_{\mathsf{sk}}, \psi_c \circ \varphi_{\mathsf{sk}}, H(\mathsf{msg}')^{-1} \star m_h)$

CORRECTNESS

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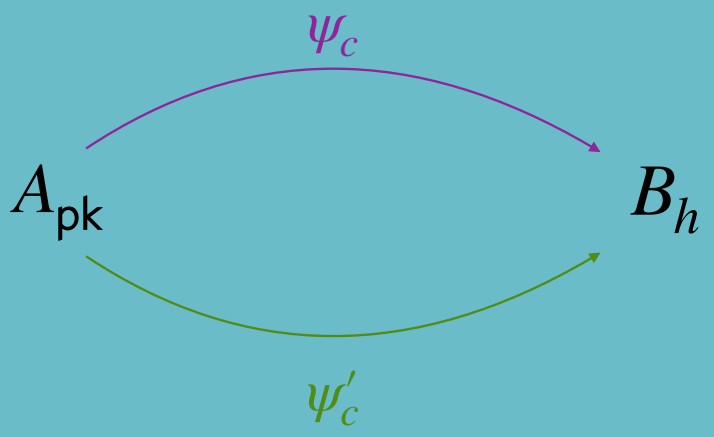
An adversary **adv** that wins the weak collision-resistance game computes $\mathsf{msg}, \psi_c, \mathsf{msg}', \psi_c', (B_h, m_h)$ such that

$$\mathsf{CHCheck}(A_{\mathsf{pk}}, \mathsf{msg}, \psi_c, (B_h, m_h)) = \mathsf{CHCheck}(A_{\mathsf{pk}}, \mathsf{msg}', \psi_c', (B_h, m_h)) = 1.$$



$$H(\mathsf{msg}) \star \mathsf{fp}(\psi_c) = m_h = H(\mathsf{msg}') \star \mathsf{fp}(\psi_c')$$

1. $H(\mathsf{msg}) = H(\mathsf{msg}')$, but $\mathsf{msg} \neq \mathsf{msg}'$, thus **adv** has produced a collision for H

2. $H(\mathsf{msg}) \neq H(\mathsf{msg}') \implies \mathsf{fp}(\psi_c) \neq \mathsf{fp}(\psi_c')$, witness for the hard language

STRONG INDISTINGUISHABILITY

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The proof relies on the **Walk Indistinguishable** property

Conclusion

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Thank You!