FIT3139: Lab questions for week 5

Question 1

A model of insect populations (in which all adults are assumed to die before next breeeding) leads to the difference equation

$N_{k+1} = \frac{\lambda N_k}{1+aN_k}$

where λ and a are positive constants.

- Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.
- What is the general shape of the graph of $R(N_k)$?
- Express the unrestricted growth rate r and the carrying capacity K, for this model, in terms of the parameters a and λ.

Question 2

Another alternative model for restricted population growth is given by:

$$N_{k+1} = N_k e^{a(1-N_k/K)}$$

where K is the carrying capacity and a is a positive constant.

- Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the variable growth rate $R(N_k)$.
- Sketch $R(N_k)$ and compare it with $R(N_k)$ of a discrete logistic equation.
- Show that $a = \log(r+1)$ where r, the unrestricted growth rate, is defined as the limit of $R(N_k)$ as $N_k \to 0$.
- Use a computer to iterate numerically the above model when $K = 1000$, $N_0 = 100$ and you should use your own choice of values of a. Describe the types of behaviour which appear for different values of a. In particular:

Question 3

A model which has been used to analyse insect populations, a modification of the one introduced in Question 1, is:

$$N_{k+1} = \frac{\lambda N_k}{(1+N_k)^b}$$

for the insect population N_k.

Using a computer, sketch solutions for the following values of the parameters:

	λ	b
Moth	1.3	0.1
Mosquito	10.6	1.9
Potato Beetle	75.0	3.4

Test the above with your own choices of N_0. Observe the behaviour when you vary N_0.

Question 4

For the discrete logistic equation with $K = 1000$ examine N_{100} as you vary N_0 slightly in each of the following cases:

- $r = 0.5$
- $r = 2.3$
- $r = 3$

Start with $N_0 = 100$ and then try $N_0 = 101$ and $N_0 = 99$. What happens for each value of r?

Question 5

In a host-parasite system, a parasite searches for a host on which to deposit its eggs. Define:

- N_k = Number of host species in kth breeding season
- P_k = Number of parasite species in kth breeding season
- f = fraction of hosts not parasitized
- c = average number of eggs laid by parasite which survive
- λ = host rate, given that all adults die before their offspring can breed.

1. Show that N_k and P_k satisfy:
   $N_{k+1} = \lambda f N_k$
   and
   $P_{k+1} = cN_k[1 - f]$
2. If $f = e^{-\gamma P_k}$ numerically simulate this model for $\gamma = 0.068$, $c = 1$ and $\lambda = 2$. What observations can you gather from this simulation?