FIT3139: Lab questions for week 8

Question 1

Write a function to sample a uniform random variable defined in the interval $[a, b)$. Both, a an b should be parameters of your function.

Question 2

Use a Montecarlo approach to estimate the area under the curves:

1. $f(x) = \sin \pi x$ in the unit square.
2. $f(x) = \frac{1}{x+1}$ in the unit square.

Use the above programs to estimate the values of π and $\log 2$ respectively.

Question 3

Revise the Buffon's needle problem handled in Week 7 where we computed the axiomatic probability of a needle of length $r > 1$ will land crossing some line, given a floor with equally spaced parallel lines, where each pair of lines are a unit distance apart.

1. Derive the expression of the axiomatic probability if the needle had a length $r > 1$?
2. Use the Monte carlo method to find the probability of the needle crossing some line when $r > 1$.

Question 4

Write a function that takes as an argument a positive and continuous function $f(x)$, in an interval $[a, b]$. Your function should estimate, using Montecarlo, the value of $\int_a^b f(x)dx$. The function should also raise an error if the given function is not positive in the interval of interest.

Use your function to evaluate 3 difficult integrals of your own choice.

Question 5

This questions is about rejection sampling. In this exercise, we want to sample from a distribution characterized as $\hat{f}(x) = \sqrt{\frac{2}{\pi}} \exp(-x^2/2)$ when $x \geq 0$.

1. Before anything, plot and visualize this distribution for varying values of $x \geq 0$.
2. To apply rejection sampling on this distribution, choose the negative exponential distribution as the proposal distribution. You will have to make a decision on the values for the constant c and the rate parameter λ of the exponential distribution you have considered for the proposal distribution, such that $c \times p(x) \geq \hat{f}(x)$. After making this choice, plot your $c \times p(x)$ against the plot of $\hat{f}(x)$.
3. After this, implement the rejection sampling approach to sample randomly from $\hat{f}(x)$. One way to check your sampling approach is consistent is to plot a histogram of sampled values (say, in intervals of 0.5 of the sampled points) and compare them against the plot of $\hat{f}(x)$.

Question 6

Write a Monte Carlo simulation script for the stochastic version of the epidemic Susceptibles-Infectives-Recovered (SIR) model discussed early in Week 8.[1]

[1] Refer to the Gillespie's algorithm – use initial values: $\beta = \delta = 0.0002$, $f = 0.0005$, $\alpha = 0.1$, $S_0 = 500$, $I_0 = 25$, $R_0 = 4475$