FIT3139: Lab questions for week 3

Question 1

Write the following set of equations in matrix form:

$50 = 5x_3 - 6x_2$

$2x_2 + 7x_3 + 30 = 0$

$x_1 - 7x_3 = 50 - 3x_2 + 5x_1$

Use Python/MATLAB to solve for the unknowns. In addition, use it to compute the transpose and the inverse of the coefficient matrix.

Question 2

1. Using Numpy/MATLAB functions, solve $Ax = b$, where:

$$A = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{pmatrix} b = \begin{pmatrix} 0.1 \\ 0.3 \\ 0.5 \end{pmatrix}$$

2. Compute the determinant of A by hand. Compare the determinant of A using Python/MATLAB built-in. Can you reason the discrepancy?

Question 4

Develop a Python/MATLAB script that implements naive Gaussian elimination on square matrices. Test on randomly generated matrices. Implement first the backward substitution algorithm, assuming an upper triangular form. Then implement forward elimination. Then combine.

Question 5

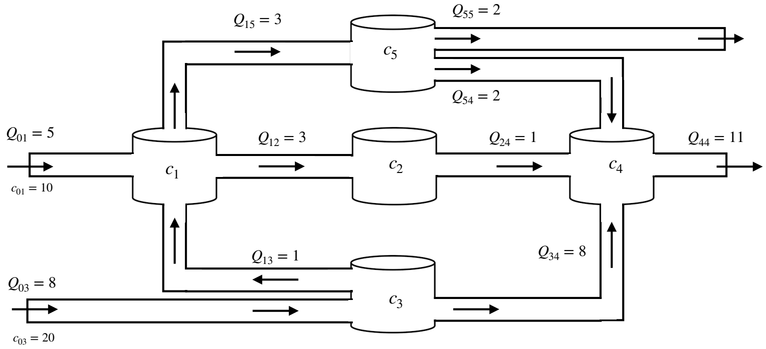
Use the above two functions to factorise A into factors LU. Test on random matrices. Then use all of the above to solve a general linear equation $Ax = LUx = b$. Think about how you want to test your script.

Question 6

This question is about modelling. The following diagram depicts a collection of reactors linked by pipes. The mass flow rate through each pipe is computed as the product of flow (Q) and concentration (c). In the steady state, the mass flow into and out of each reactor must be equal. For example, for the first reactor, a *mass balance* can be written as:

$$Q_{01}c_{01} + Q_{13}c_3 = Q_{15}c_1 + Q_{12}c_1$$

Write balances for each reactor and express the equations in matrix form. Then use Python/MATLAB to solve for the concentrations c_i in each reactor.



Question 6 [1]

Develop, debug and test your own Python/MATLAB script to switch the rows of a matrix using a permutation matrix. Include exception handling to deal with potential erroneous input.

[1] For Q6, Q7 and Q8 please consult the slides from the previous lecture and your demonstrator, if needed.

Question 7

This is not a programming question

Solve $Ax = b$ with:

$$A = \begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 106.8 \\ 177.2 \\ 279.2 \end{pmatrix}$$

using partial pivoting (show each step of the process).

Question 8

Extend the algorithm in Q4 to implement partial pivoting, producing a decomposition PA=LU.

Optional Question

How can you use LU decomposition to compute A^{-1}? What advantages have LU decomposition over Gaussian elimination for this problem? [2].

[2] Note that $L \cdot U \cdot A^{-1} = I$, where I is the identity matrix