Supplementary Materials

Compressed Sensing for Characterization of Tinnitus

Alec Hoyland, Nelson Barnett, Benjamin W. Roop, Adam C. Lammert

This document contains a description of additional experiments to find a performant stimulus generation method and a discussion of sparsity in tinnitus signals. The code for all experiments is freely available at https://github.com/alec-hoyland/tinnitus-project and the data are available upon request.

I. Stimulus Generation

In the context of this paper, a stimulus generation method is a process that generates a random waveform that is:

1) auditorally-distinguishable
2) statistically uncorrelated, and
3) similar to tinnitus percepts.

Additionally, compressed sensing requires that the matrix of stimuli should satisfy the restricted isometry property, which many random matrices do with high probability (*e.g.* Gaussian random matrices) [1], [2].

A. Auditorally-Distinguishable Stimuli

We used mel-frequency binning to ensure that our stimuli were auditorally-distinguishable. The mel scale is a perceptual scale of pitches judged by listeners to be equal in distance from one to another (Fig. 1) [3].

The formula

$$m = 2595 \log_1 0 \left(1 + \frac{f}{700}\right) \tag{1}$$

converts f Hz to m mels.

Furthermore, to reduce the system complexity by more than 80x, we implement tonotopic binning, where the frequency scale is binned along 100 equally mel-spaced bins (Fig. 2).

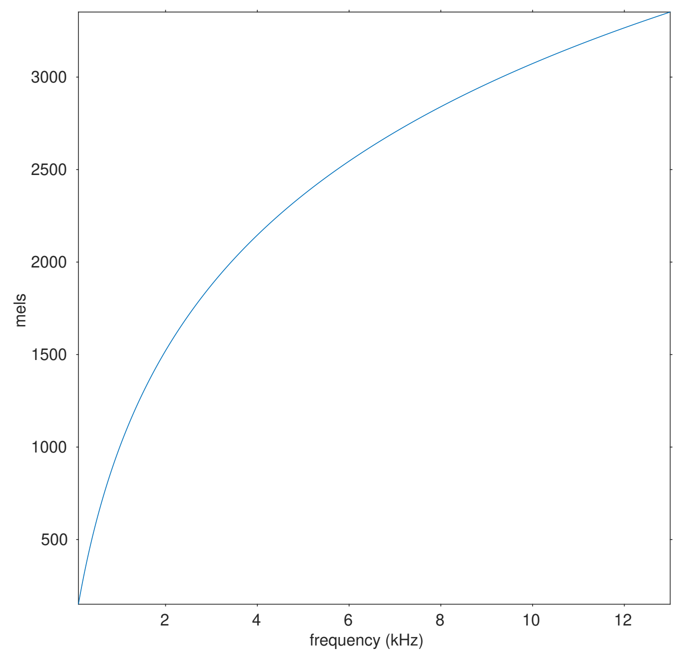

Fig. 1. The relationship between Hz and mels is logarithmic.

II. Hyperparameter Sweep

To determine a suitable stimulus generation method, we performed a hyperparameter sweep over hyperparameters of nine different stimulus generation methods. We used an *in-silico* model of the experiment, matching against an American Tinnitus Association tinnitus example, as in the human experiment.

The methods are as follows:

1) *Bernoulli*: A binned method in which each tonotopic bin has a probability *bin_prob* of being set to 0 dB otherwise it is set to -20 dB.
2) *Brimijoin*: A binned method inspired by [4]. Each bin is assigned an amplitude value chosen from a uniform distribution of discrete values on the interval $[-20, 0]$. We used 6 steps, which is consistent with the original paper.
3) *Gaussian Noise No Bins*: The amplitude of each frequency was determined by Gaussian noise with known mean and variance, which are hyperparameters *amplitude_mean* and *amplitude_var*.
4) *Gaussian Noise*: A binned method where the bin amplitudes were determined by a Gaussian random variable with known mean and variance, *e.g.* $\mathcal{N}(amplitude\_mean$ and $amplitude\_var)$.
5) *Gaussian Prior*: A binned method where the number of filled bins was set by round$(\mathcal{N}(n_bins_filled_mean, n_bins_filled_var))$, and that many bins were filled randomly at 0 dB (unfilled bins were set to -20 dB).
6) *Power Distribution*: A binned method where the amplitude of each bin is drawn from a distribution matching the histogram of amplitudes of the American Tinnitus Association tinnitus examples from their website.
7) *Uniform Noise No Bins*: The amplitude associated with each frequency is determined by a uniform random variable on the interval $[-20, 0]$ dB.
8) *Uniform Noise*: The amplitude associated with each tonotopic bin is determined by a uniform random variable on the interval $[-20, 0]$ dB.
9) *Uniform Prior*: The number of filled bins is determined by drawing an integer from the discrete interval

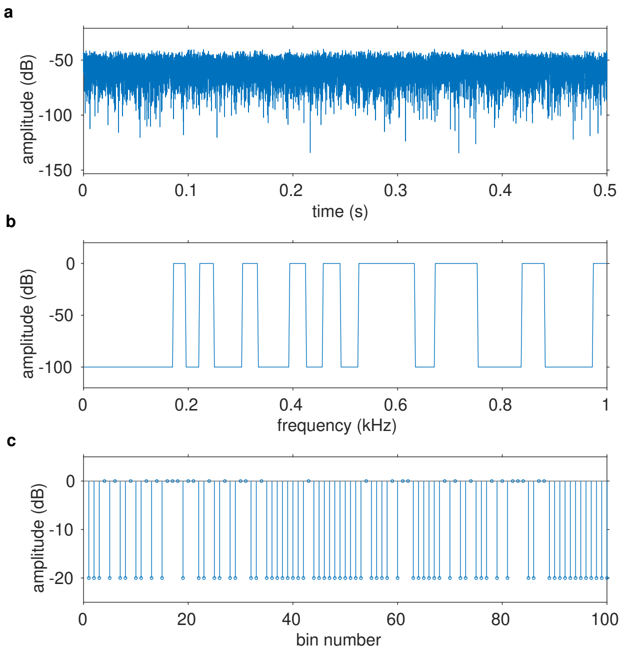

Fig. 2. Example stimulus. **(a)** shows the waveform of the stimulus, **(b)** the frequency spectrum from which the waveform was generated, and **(c)** the 100-dimensional bin representation.

$[min\_bins, max\_bins] \in \mathbb{Z}^+$. Then, that many bins are set to 0 dB, and all other bins are set to -20 dB. *min_bins* and *max_bins* are hyperparameters.

9 methods and 8 hyperparameters were varied in the hyperparameter sweep grid search (Table I). Methods were evaluated *in-silico* using the model:

$$y = \text{sgn}(\Psi * \bar{x}) \tag{2}$$

where $y \in \mathbb{R}^n$ is the binary response vector, $\Psi \in \mathbb{R}^{n \times d}$ is the stimulus matrix, and $\bar{x} \in \mathbb{R}^d$ is the true target signal (the spectrum of the ATA tinnitus example).

Each stimulus generation method, with different hyperparameters, was evaluated for target signals: buzzing, electric, roaring, screeching, static, and tea kettle.

HYPERPARAMETERS

Name	Values	Unit
n_bins	10, 30, 100, 200, 300	n.d.
bin_prob	0.1, 0.3, 0.5, 0.8	n.d.
amplitude_mean	-35	dB
amplitude_var	5, 10, 20	$\sqrt{\text{dB}}$
n_bins_filled _mean	1, 3, 10, 20, 30	n.d.
n_bins_filled_var	0.01, 1, 3, 10	n.d.
min_bins	1, 3, 10, 20, 30	n.d.
max_bins	10, 20, 30, 50	n.d.

TABLE I. Hyperparameter values tested in the parameter sweep. Not all methods use all hyperparameters, and some combinations of hyperparameters are invalid (*e.g.* when *min_bins* > *max_bins*). n.d. means "non-dimensional" and refers to a unitless number.

Reconstructions were computed using linear regression and compressed sensing and reconstruction accuracy was measured using Pearson's r^2 vs. the target signal.

REFERENCES

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