Specific Aims

The **objective** of this proposal is to directly and more completely characterize the perceptual experience of tinnitus using the novel application of a primary method for uncovering latent perceptual representations: reverse correlation. Tinnitus is the perception of ringing, buzzing, or hissing in the ears, which can range from mild and transient to debilitating. In the U.S. alone, it affects 45 million people per year, 30% of which report their tinnitus to be moderate to severe. A primary form of treatment habituation therapy that retrains a patients nervous system by habituating it to external sounds that mimic the perceptual experience of the tinnitus sound. Habituation therapies require accurate estimates of patients tinnitus experience in order to promote patient outcomes. There is a pressing need for new approaches to uncovering individuals perceptual experience of tinnitus, because current estimation methods rely on strongly reductionist assumptions concerning the nature of tinnitus sounds, and produce results that are correspondingly biased and incomplete. Reverse correlation holds promise in this regard, as a primary method used to characterize neural tuning that is known to allow for unconstrained and unbiased estimation of latent neural representations (*e.g.,* neural receptive fields) [30, 34], as well as higher-level cognitive representations, including psychophysical kernels that drive the top-down processes of perception (*e.g.,* face or phoneme recognition) [1, 10, 29, 35], and even abstract psychological categories (*e.g.,* male vs. female faces) [6, 23, 33], all on the basis of fairly simple stimulus-response data [21, 24]. Despite its wide acceptance in other areas of inquiry, reverse correlation has not been brought to bear with respect to uncovering tinnitus experience.

Our **long-term goal** is to directly uncover and more completely characterize the perceptual experience of tinnitus using the demonstrated capabilities of the reverse correlation approach. Our guiding hypothesis is that reverse correlation will produce estimates of tinnitus percepts that are substantially more rich, individualized and unbiased than currently possible using existing approaches. The rationale is that high quality characterization of tinnitus percepts can promote detailed assessment of tinnitus, helping to explain individual variability in tinnitus experience, and ultimately informing individualized treatment through enhanced habituation therapies. The proposed work will simultaneously improve the efficiency of the reverse correlation method itself, further enhancing its potential for adoption in clinical settings. We will test our central hypothesis and attain our objective via the following specific aims:

Aim #1: Uncover and reconstruct cognitive representations of tinnitus

To reconstruct perceptual representations of tinnitus using a novel reverse correlation procedure. Participants with tinnitus will be presented with richly varying stimuli in the context of a yes-no behavioral response paradigm. Perceptual representations can be estimated by regressing observed responses against the stimuli over many trials. Reconstructed representations will be compared to conventional estimates of tinnitus from the same participants. Expected outcome: reconstructed representations of tinnitus will more detailed and less biased than any those obtained through conventional methods.

Aim #2: Develop efficient reconstruction algorithm for cognitive representations using compressive sensing

To enhance the efficiency of the reverse correlation procedure using compressive sensing, which substantially reduces the number of trials required to accurately reconstruct perceptual representations. We will use compressive sensing to more efficiently and more accurately reconstruct cognitive representations using data obtained in Aim #1. Expected outcome: compressive sensing is expected to improve efficiency of the reverse correlation procedure by up to 90%.

Aim #3: Characterize and interpret cognitive representations of tinnitus

To characterize variability in perceptual representations of tinnitus between participants and over time. We will use unsupervised machine learning techniques to reveal latent similarities between tinnitus representations, and subsequently cluster those into distinct categories. Expected outcome: Unsupervised machine learning will identify distinct categories of tinnitus, which will allow us to further improve the effiency and accuracy of uncovering tinnitus representations through improved stimulus generation and representation classification.

A. Significance

Tinnitus, the perception of ringing, buzzing, or hissing in the ears when no external sound is present, is a health condition estimated to affect 10-15% of adults worldwide [16]. The condition is highly heterogeneous and can range from mild and transient to debilitating and constant. In the U.S., tinnitus affects approximately one in ten Americans, with 7.2% of those rating their tinnitus as severe ([3]. Tinnitus can be perceived unilaterally or bilaterally and may differ over time, even within individuals. This heterogeneity in characterization has important implications for research and clinical practice. Identifying patterns in how tinnitus sounds and its relationship to hearing may aid in identifying different forms of tinnitus and revealing their underlying mechanisms. However, the subjective nature of characterizing tinnitus makes it difficult to reliably define and measure [38].

Current methods to determine cognitive representations of tinnitus are inefficient and introduce bias, resulting in a lack of standardization in tinnitus assessment [16]. One common method is the alternate forced-choice (AFC) paradigm task, which asks the patient to determine which of two sounds is closer to their tinnitus percept [12, 13, 15, 19]. Recent computer and methodological advances have made the test more portable and efficient, but the AFC paradigm still relies on assumptions about the subject's tinnitus percept. Tinnitus percepts are too heterogeneous to be represented by pure tones or small-envelope waveforms. Figure 1 displays spectrograms of example tinnitus percepts from the American Tinnitus Association website, showing the heterogeneity of tinnius percepts as well as their richness in frequency spectra and temporal characteristics. Adequately sampling the psychoacoustic space of possible tinnitus percepts via an AFC task is unfeasible. Other approaches have attempted to capture the richness of tinnitus percepts using likeness measures, in which the subject rates whether a presented stimulus is part of their tinnitus percept or not [31]. While these methods capture more features of cognitive representations of tinnitus, they are time-consuming and limited by the aural skills of the subject [38]. Both AFC and likeness measures involve the subject comparing an external stimulus to their cognitive representation of the tinnitus percept, however AFC tasks introduce biases by artificially limiting the stimulus space and likeness measures are time-consuming. *We believe that a novel reverse correlation-based approach will produce efficient and unbiased cognitive representations of tinnitus percepts* [10].

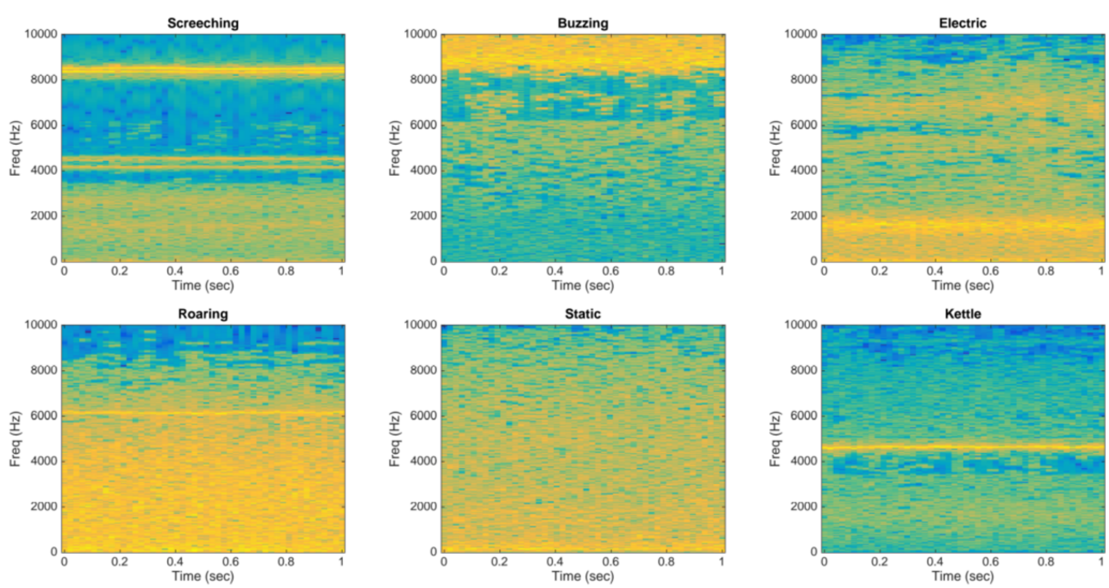

Figure 1: Examples of tinnitus spectrograms from the American Tinnitus Association website [37]. These spectrograms are heterogeneous as well as high-dimensional; the signals contain many frequency bands and vary over time.

Reverse correlation allows for unconstrained and unbiased characterization of latent representations directly from stimulus-response data by eliciting responses to richly varying stimuli, such as white noise [24, 30]. Rich stimuli, by virtue of the fact that they are inherently vague, force the top-down process to exert a clearly measurable influence on responses. Latent representations that drive the top-down process can then be estimated by regressing observed responses against the stimuli over many trials [27]. Reverse correlation, as a method, is powerful enough to characterize any aspect of neurological, cognitive or psychological function that can be modeled as a transductive process [34]. It has become a primary method used to characterize the latent representations encapsulated in neural tuning (*e.g.,* receptive fields; [34]), and is closely related to the widely-used "white noise approach" to characterizing physiological [24] and engineering [21] systems.

Reverse correlation has been increasingly used for inferring higher-level cognitive representations as well, including psychophysical kernels that drive the top-down processes of perception [1, 10, 29, 35], and even abstract psychological categories (*e.g.,* "male" vs. "female" faces; [6, 23, 33]. Recent work in vision has demonstrated that reverse correlation can effectively characterize cognitive representations underlying letter and face recognition [10, 20]. One speech study has also estimated steady-state representations of vowels /a/ and /i/ using a closely-related paradigm to the one proposed here (Fig 2B; [5]). Our proposed use of reverse correlation in the domain of speech expands on classic efforts to understand top-down processing in speech with the presentation of rich, vague stimuli to elicit responses from listeners [39, 40]. These studies provide clear evidence that rich stimuli, such as white noise, are sufficient to engage the top-down process, even if they do not attempt to characterize the latent representations.

Incomplete characterizations of cognitive representations impoverish our scientific understanding of tinnitus, weaken causal explanations for its etiology, and hinder progress towards effective treatments. Reverse correlation shows promise to provide a more complete characterization of cognitive representations of tinnitus and is applicable to other psychophysical domains as well. Directly characterizing complex representations of tinnitus can enable more effective, targeted treatments, reveal insights about subtypes of the condition, and pave the way for new tinnitus-masking assistive devices. Fully characterizing a high-dimensional representation of tinnitus will improve causal explanations for currently unexplained variability in tinnitus experience both between subjects and within single subjects over time.

The premise of the proposed work is that reverse correlation will deliver unbiased estimates of cognitive representations of tinnitus. Furthermore, that compressive sensing will dramatically increase the efficiency of experiments, resulting in convergent cognitive representations in a fraction of the samples.

The evidence for this premise is first based on well-established studies using reverse correlation to derive cognitive representations of sounds and symbols. Reverse correlation has been applied to infer cognitive representations from letters [10], vowel sounds [5], and faces [6, 35]. More broadly, it has been applied to infer the shape of receptive fields in linear transducers and spiking neurons [34]. The reverse correlation paradigm makes minimal assumptions about the derived cognitive representation since the subject is presented with high-dimensional random input. A large number of stimulus-response samples are typically required for accurate reconstruction of cognitive representations using conventional techniques. To address this inefficiency, studies often limit the richness of stimuli, or impose strict constraints on the reconstructions, leading to estimates that are biased or incomplete. However, recent advances in signal processing, most notably a techniques known as compressive sensing, are leading to dramatic improvements the efficiency of traditional sampling.

We propose to develop an advanced signal processing pipeline that will enable us to overcome the inefficiencies of existing reverse correlation methods through the use of compressive sensing, a recent advance in signal processing which has led to dramatic improvements the efficiency of traditional sampling and signal estimation methods [2]. Compressive sensing has recently gained wide recognition in domains such as medical imaging [11, 22], where considerations of efficiency and bias reduction are critical. Compressive sensing holds promise to similarly improve the efficiency of reverse correlation, without the drawback of biasing estimates. By dramatically decreasing the number of trials needed for signal reconstruction, this technique will extend the range of perceptual mechanisms that can be estimated. Moreover, compressive sensing can be directly substituted for conventional, regression-based estimation, with no other required changes to existing experimental protocols. Our ultimate objective is to develop and validate a compressive sensing data processing pipeline - culminating in an open-source software tool that will allow for efficient and accurate reconstruction of latent representations using data obtained via the reverse correlation method.

B. Innovation

The proposed project is innovative for several reasons. First, we will leverage interdisciplinary collaboration between engineering (Adam Lammert) and hearing science (Ben Parrell) to characterize tinnitus to inform clinical care. Second, we will apply reverse correlation to the tinnitus domain in a novel application, which promises to deliver unbiased, high-dimensional estimates of cognitive representations of tinnitus. Third, we will develop a signal processing pipeline to leverage compressive sensing to acquire accurate estimates of cognitive representations of tinnitus using significantly fewer samples. This pipeline will save experimenters and subjects time and will enable researchers to collect data from more subjects and from individual subjects over time. We will release our open-source software tool online with bindings to popular scientific computing languages.

C. Approach

In the proposed project, we will use reverse correlation to reveal unbiased high-dimensional representations of tinnitus and further characterize the condition using dimensionality-reduction and density-based clustering. We will use data regarding the space of tinnitus Representations to iteratively improve our stimulus generation and representation classification algorithms. In *Aim #1*, we will develop and run an experimental paradigm in which tinnitus patients listen to noisy stimuli and perform an alternate forced-choice task answering the question, "Does this sound like your tinnitus?" Instead of using limited, preconstructed stimuli, we will randomly sample from a distribution. Reverse correlation frees us from the methodological constraint of imposing strong *a priori* assumptions about the tinnitus percept. In *Aim #2*, we will develop an open-source signal processing pipeline to reconstruct cognitive representations of tinnitus using compressive sensing. Compressive sensing holds promise to dramatically reduce the number of samples needed for accurate reconstruction, enabling us to run more experimental subjects for Aim #1, and retest subjects to examine the stability of cognitive representations of tinnitus over time. *Aim #3* contextualizes reconstructed cognitive representations of tinnitus by visualizing underlying subtypes or clusters on a lower dimensional manifold. We will employ dimensionality reduction and clustering algorithms to represent and visualize the data, providing direct insight into commonalities and variability of tinnitus experience in and between subjects. Furthermore, we will use information about the distribution of representations to iteratively improve our stimulus generation and data-driven subtype classification processes.

C. Aim #1: Uncover and reconstruct cognitive representations of tinnitus

In this aim, we will identify high-dimensional cognitive representations of tinnitus using reverse correlation. Current methods for estimating cognitive representations of tinnitus fall broadly into three primary categories, each of which has limitations (Table 1).

Two common approaches are alternate forced choice tasks with constructed stimuli and likeness measures. In the former experimental paradigm, the experimenter presents participants with multiple samples of auditory stimuli that vary in well-characterized and often highly-constrained ways, along specific dimensions (*e.g.*, pure tone frequency, width of spectral envelope) that are known to be, or are hypothesized to be, relevant to tinnitus representation or masking [14, 38]. The subject then makes a choice between stimuli. While new methodological advances in experiment design and technology have streamlined this approach [13, 15, 19], it is still constrained by *a priori* assumptions about the nature of tinnitus percepts. In contrast, likeness approaches use subjective judgment tasks, in which the subject rates how much (if at all) a presented stimulus is part of or masks their tinnitus percept [31]. While likeness measures provide more complete reconstructions of tinnitus cognitive representations, likeness tests are time-consuming and generally not used in clinical practice [38].

A third, more recent approach to uncovering cognitive representations is to employ neuroimaging (*e.g.*, fMRI) to identify neural activation patterns indicative of tinnitus. In investigations of functional connectivity in tinnitus patients, resting-state fMRI measures were found to be replicable and reliable, though no explicit representations have been proposed [17]. This approach is also limited, in that it can only uncover representations that have a definite, localized seat in the brain. Cognitive representations, by contrast, are commonly understood to be constructs that need not have such a localized neural seat by definition, and which can best be revealed through analyzing behavior.

The highly-constrained nature of stimuli used in the forced selection approach necessarily results in recovered representations that are correspondingly constrained and typically low-dimensional. It has long been known, however, that representations limited to only a few dimensions are insufficient to account for the full richness of tinnitus experiences [14, 38]. Subjective judgment experiments yield more informative reconstructions but are time-consuming to employ. The nature of subjective judgment approach is such that it is best suited to uncover relations between perceptual categories, rather than characterizing cognitive representations directly, and thus provides limited information about how perceptual similarity comes about. While some of the recovered dimensions along which stimuli differ in these tasks correlate with frequently assumed dimensions of representation (*e.g.*, frequency and pitch to mask tinnitus percept), some recovered dimensions of contrast have no known characterization. Moreover, this approach is fundamentally limited by the precise stimuli presented to participants, and therefore may not systematically explore the entire space of perceptually-meaningful variables.

Approach	**Stimulus**	**Measurements**	**Output**
Forced selection	Constructed stimuli, constrained variation	Subjective choice	Representation with defined acoustic properties
Likeness measures	Constructed stimuli, constrained variation	Subjective ratings	Frequency spectra
Neural monitoring	N/A	Neural signals (*e.g.*, fMRI)	Correlation maps in localized brain regions, functional connectivity maps
Reverse Correlation	Constructed stimuli, unconstrained variation	Response classification (*e.g.*, present/absent)	Representations best explaining classification behavior

Table 1: Comparison of approaches for estimating cognitive representations. Each approach has benefits and limitations regarding stimuli, measurements, and output. The proposed approach, featured in the final row, is the only approach that can directly uncover a time-frequency map of the cognitive representation (a spectrogram) that best explains tinnitus percepts.

We propose to implement a reverse correlation approach for recovering cognitive representations of speech which allows for unconstrained, direct characterization of latent representations from yes-no responses to randomly-generated stimuli. Participants will be asked to classify auditory stimuli, which vary in their time-frequency content, as containing or not containing their tinnitus percept. This is a transductive task which we model as comparing a stimulus to a latent "cognitive template" in a top-down process (Figure 2). The latent representation that drives that top-down process can then be estimated by regressing observed responses against the stimuli over many trials [27]. We use random stimuli, *i.e.,* stimuli where the frequency magnitudes are randomly chosen, in order to sample the space of percepts without biasing towards any particular representation, a problem with other approaches. We will consider both time-varying and stationary stimuli, *i.e.,* where the spectral content of the stimuli changes over time and where it remains constant.

Reverse correlation has been applied across multiple fields to reveal underlying representations by application of unconstrained, noisy input. It has been used to characterize low-level latent representations encapsulated in neural tuning (*e.g.,* receptive fields) [34] and has been used to successfully recover cognitive representations underlying visual perception [10] Both existing experimental work (*cf.,* Fig 2B in [5]) and our preliminary simulation experiments (Figure 5) suggest that this method can be successfully translated into the auditory domain. Reverse correlation allows us (*a*) to uncover more complex, potentially higher-dimensional representations than would be possible using more focused, constrained stimuli, (*b* to recover directly the nature of the representations themselves rather than the relation between the tinnitus percept and a masking sound, and (*c*) to recover cognitive representations of the tinnitus percept directly rather than a neural correlate. In short, reverse correlation will be able to provide deeper understanding of cognitive representations of tinnitus percepts than is possible

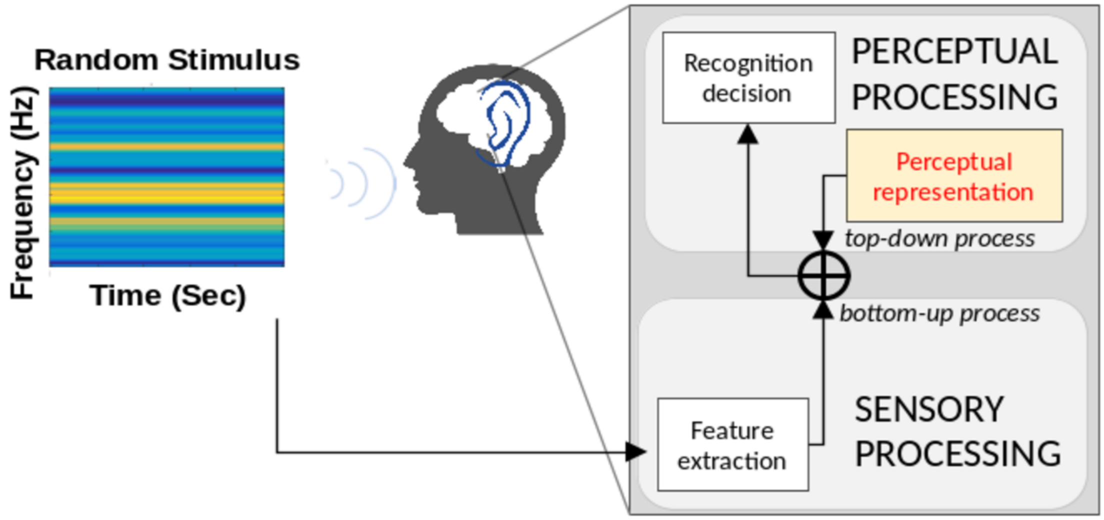

Figure 2: Model of the discrimination task as a transductive process. Random stimuli is filtered by the subject using low-level sensory mechanisms. The perceptual discrimination task is modeled as a top-down process that compares the sensory information to a perceptual representation "template" to determine if the stimulus belongs to that perceptual representation.

with conventional methods.

C.1. Method

The proposed method for uncovering perceptual representations can be divided into a sequence of three main parts: (1) construction of vague, "random" stimuli for presentation to subjects, (2) experimental procedure and data collection, and (3) reconstruction of perceptual representations from subject responses.

Stimuli construction Stimuli will be generated randomly, in keeping with prior work in the visual domain [10]. The meaning of "random" with respect to the stimuli presented to subjects refers specifically to the shape of spectral envelope in the frequency domain. This shape will be determined randomly for all stimuli and will not be informed by prior knowledge or statistics over canonical examples of tinnitus. These random spectral envelopes will be used as the basis for building spectrograms that sound like tinnitus percepts.

Our preliminary testing suggests that truly random envelopes often result in the perception of colored noise. Minimal additional spectral-temporal constraints may need to be imposed on stimuli such that participants will interpret the stimuli as intending to be tinnitus percepts. This is similar to the approach taken in some visual domain experiments, where bandlimited random stimuli, or stimuli with partial superimposed templates, are presented [35]. The basic approach will be to use inverse Fourier transforms to generate 0.5-s noise stimuli including frequencies from 0.1 to 22 kHz [5]. We will impose minimal constraints on the stimuli generation process, in order to selectively generate stimuli approaching the heterogeneous samples of tinnitus percepts provided by the American Tinnitus Association on their website [37].

Subject Recruitment Subjects will be recruited from the clinical population by our clinical collaborator and will be assessed by the Tinnitus Severity Scale, which measures the negative impact tinnitus has on a subject's quality of life. We will not screen for unilateral or bilateral tinnitus but will only include subjects with primary (*i.e.*, subjective, not somatosensory) tinnitus.

Experimental procedure During the experiment, subjects will be asked to perform an computerized alternate forced-choice task, selecting whether the presented stimulus sounds similar to their tinnitus percept or not. They will be told that the stimuli are tinnitus-like sounds corrupted by noise and that not all will sound like their tinnitus percept. Each signal will be presented to subjects over headphones at an audible level ($\approx$ 75 dB SPL) after which the subject will select either "yes" or "no" via keyboard presses. The next trial will begin after a 500-ms pause. Subjects will be asked to perform the task in 80-trial blocks with a rest period in-between. We will ask subjects to perform 20 blocks at minimum, which should take about a half an hour. Subjects can perform the task for more blocks at their choice, and will be compensated for their participation.

Reconstructing representations We will reconstruct cognitive representations of tinnitus from the data by taking the least-squares fit to the behavioral data (Figure 3). This is equivalent to interpreting the cognitive representation as a "classification spectrogram" generated by subtracting the average of the negative trials from the average of the positive trials [10]. We theorize that in the underlying transductive cognitive process, that the classification spectrogram is used as a template vector against which stimuli are compared in a linear matching model of response generation. While we acknowledge that this reconstruction approach is inefficient, requiring many samples to acquire a coherent reconstruction, it is well-studied in the literature and will serve as a baseline against which our efficient reconstruction algorithm in Aim #2 will be compared.

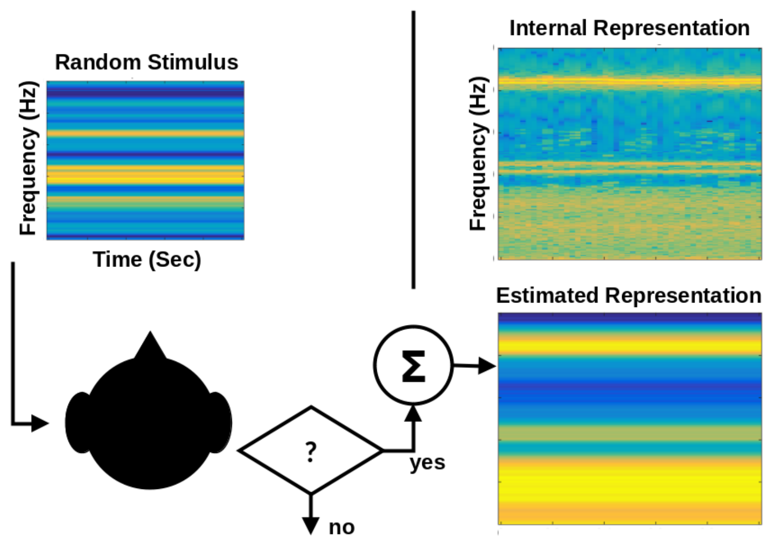

Figure 3: Schematic overview of experiments proposed in Aim #1. Subjects are presented with random stimuli, which is compared to a latent internal representation. Reverse correlation is used to reconstruct an estimate of the latent internal representation over many trials. Simulation results are shown, demonstrating proof-of-concept for the proposed reverse correlation paradigm.

C.2. Aim #2: Develop an efficient reconstruction algorithm for cognitive representations using compressive sensing

In this aim, we employ advanced signal processing techniques to overcome critical inefficiencies in reverse correlation that limit its scope and impact.

Current attempts to characterize perceptual mechanisms using reverse correlation are limited in scope due to inefficiencies inherent to conventional methods. While reverse correlation allows for relatively unconstrained and unbiased estimation of latent representations using straightforward stimulus-response data [24, 30], the number of stimulus-response samples required for accurate estimation is typically very large [27]. This inefficiency limits the feasibility of applying reverse correlation more broadly, as subject participation must be maintained over long timelines.

The impact of this inefficiency extends well beyond the present proposal. This fundamental sampling problem affects similar efforts to estimate higher-level cognitive representations and psychophysical kernels that drive top-down processes of perception [1, 10, 29, 35]. Since reverse correlation has broad applicability for characterizing any aspect of neurological, cognitive, or psychological function that can be modeled as a transductive process [34], the inefficiencies of reverse correlation impact efforts to characterize phenomena as fine-grained as latent neural representations (*e.g.*, neural receptive fields) [34] and as high-level as abstract psychological categories (*e.g.*, "male" vs. "female" faces) [6, 23, 33]. To combat this efficiency limitation, studies often (*a*) limit the richness of the stimuli (*e.g.*, by only allowing specific aspects of the stimulus to vary) [9], or (*b*) impose some constraints on the inferred representations, for instance by smoothing the raw estimates [10]. The constraints imposed by these approaches ultimately defeat the full power of reverse correlation [28], leading to estimates that are biased or incomplete.

We propose to develop an advanced signal processing pipeline that will enable us to overcome the inefficiencies of existing reverse correlation methods through the use of *compressive sensing*, a recent advance in signal processing which has led to dramatic improvements in efficiency over conventional sampling and signal estimation methods [2]. Compressive sensing has recently gain wide recognition in domains such as medical imaging [11, 22], where considerations of efficiency and bias reduction are critical. Compressive sensing holds promise to similarly improve the efficiency of reverse correlation without the drawback of biasing estimates. Compressive sensing will extend the range of possible cognitive and perceptual mechanisms that can be estimated by dramatically decreases the number of trials needed for signal reconstruction. Since compressive sensing can be directly substituted for conventional, regression-based estimation, it can be directly substituted into existing experimental protocols without requiring other changes. Our ultimate objective is to develop and validate a compressive sensing data processing pipeline culminating in an open-source software tool that will allow for efficient and accurate reconstruction of latent representations using data obtained via the reverse correlation method.

Compressive sensing framework Many natural signals, x, including latent representations, are compressible, meaning that they can be represented by a sum of a small number of functions from an appropriately-chosen basis set ($s = \Psi^T x$, for basis Ψ and weights s). A key insight of compressive sensing is that response variables, y, stem from a process of comparing stimuli to latent representations ($y = \Phi x$, for measurement Φ) (Figure 4). It is then possible to estimate the latent representation using only a small number of measurements by acquiring the basis function representation directly (*i.e.*, $y = \Phi\Psi s$) via sparse optimization (to find s). The responses can be continuous (*e.g.*, firing rates), ordinal similarity scores [42], or binary [4, 32], as in the proposed experiments here. In practice, sparse representations can be found even when the chosen basis domain is quite general and incorporates no prior knowledge of the signal's characteristics (*e.g.*, the discrete cosine transform or wavelet transform). Critically, it has been shown that using random stimuli to elicit and subsequently observe responses is a highly effective way to ensure accurate reconstruction of latent representations within the compressive sensing framework [7, 8, 41]. In other words, the somewhat unusual model of sampling assumed by compressive sensing maps directly onto reverse correlation.

Simulations and preliminary evidence We have conducted simulation studies to begin validating compressive sensing for improving the efficiency of reverse correlation. For example, using a template time-frequency representation, x, as a proxy for the latent representation of interest, we generate plausible yes/no subject responses, $y \in \{-1, 1\}$), based on the similarity between the template and a random stimuli (*i.e.*, $y = \mathrm{sign}(\phi x)$ for stimuli ϕ). Estimation of the template is performed using (*a*) convention regression-based reverse correlation (*i.e.*, $\hat{x}_c = n^{-1} X^{\mathrm{T}} y$) [10], or (*b*) compressive sensing.

Figure 5 shows example results from our preliminary simulation studies. In this example, we attempt to reconstruct a spectrally-rich signal. Four reconstructions are shown, corresponding to two sample sizes (12,500 and 100,000 samples) and two reconstruction methods (conventional linear regression and compressive sensing). The signal quality obtained using fewer samples via compressive sensing is effectively equivalent to that obtained using eight times more samples via conventional reconstruction. When allowed to operate on the full complement of 100,000 samples, compressive sensing shows +21% improved accuracy over conventional reconstruction. If

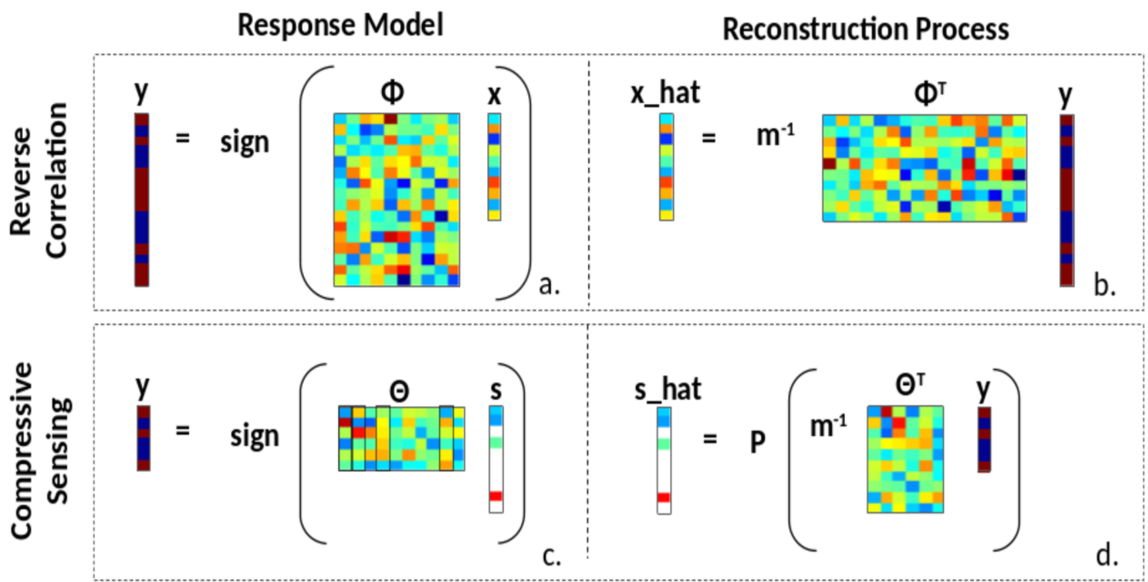

Figure 4: Comparison of reverse correlation and compressive sensing. (*a*) In reverse correlation, the vector of subject responses is modeled as resulting from the multiplication of a latent representation vector, x, and a stimulus matrix, Φ, which can be thought of as a similarity calculation between the latent representation and a vector representation of each presented stimulus. (*b*) An estimate of the latent representation, $\hat{x}$, is then reconstructed by regressing responses against the stimuli. (*c*) In compressive sensing, the vector of subject responses is modeled as resulting from the multiplication of a sparse latent representation vector, s, and a compressive sensing matrix, Θ. The compressive sensing matrix is formed by multiplying a matrix of basis functions by the stimulus matrix, $\Theta = \Phi\Psi$, which amounts to a similarity calculation between the stimuli and the known basis functions. (*d*) An estimate of the sparse latent representation, $\hat{s}$, is then reconstructed by regressing the responses against the compressive sensing sparse representations and known basis functions, $\hat{x} = \Psi\hat{s}$. Note that the response vector, y, in compressive sensing is generally assumed to contain many fewer entries than in reverse correlation without sacrificing reconstruction accuracy.

further simulation studies and validation on real data continue to align with these preliminary results, the number of trials required for accurate estimation using reverse correlation paradigms could be reduced to 12.5% of the current standard. Such a drastic increase in time- and cost-effectiveness would substantially increase the method's potential for widespread use.

Validation of compressive sensing We will validate the proposed use of compressive sensing for relevant cognitive and behavioral data. Validation will be accomplished through simulation studies (*e.g.,* expanding on the simulation results discussed herein) and through analysis of data collected in Aim #1. We will assess outcomes by examining estimation performance as a function of sample size, incuding (*a*) reconstruction accuracy with few samples, (*b*) gains in high-sample reconstruction accuracy, and (*c*) minimal sample size to achieve high-end convergent accuracy. The basis for these assessments will be a comparison with the conventional, full-sample reconstruction as described in Aim #1, since the true gold standard representation is unavailable. Potential improvements of compressive sensing reconstruction will be assessed using signal quality metrics that do not require direct comparison (*e.g.,* peak signal-to-noise ratio). This work will determine optimal parameters for compressive sensing in tinnitus percept reconstruction. We expect these parameters to depend on stimulus type, response noise, and the type of tinnitus experienced by the subject. Considered parameters will include appropriate input/output representations, reconstruction algorithm, and basis type/sparsity.

Development of an open-source software tool We will develop and distribute an open-source software tool for conducting auditory perceptual experiments using reverse correlation, including stimulus presentation, re-

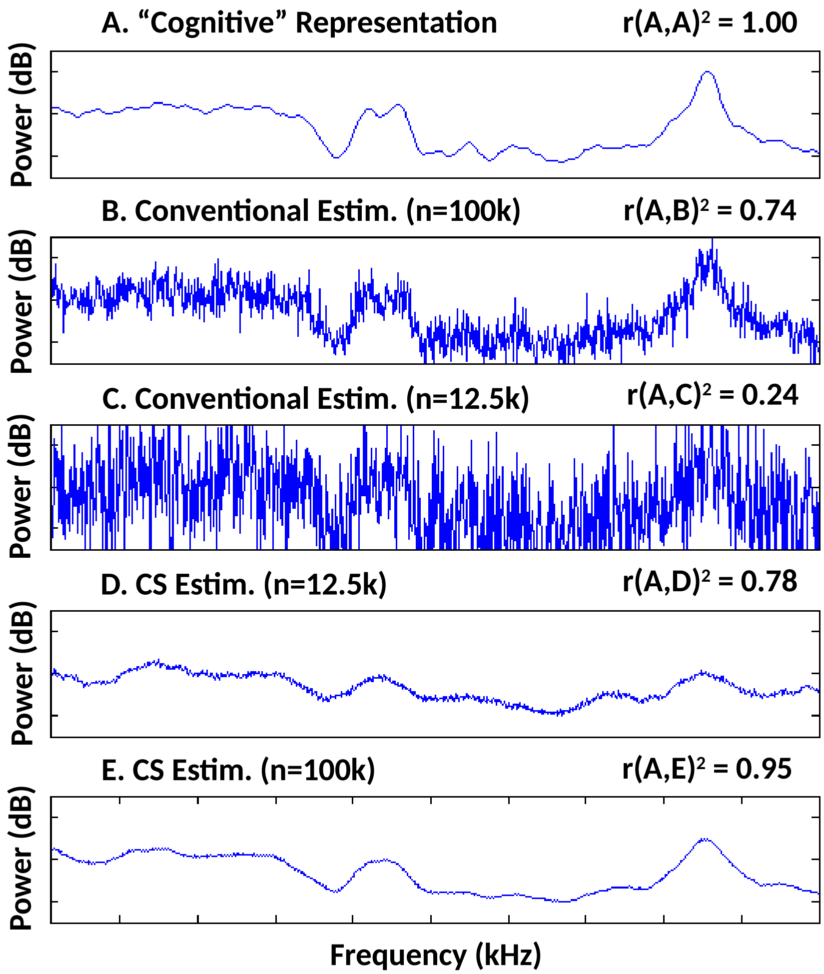

Figure 5: Compressive sensing (CS) delivers accurate signal reconstruction from sampling with a comparatively small number of samples. The latent representation in (*A*) is estimated in (*B-E*). In (*B-C*) conventional regression-based estimation is used and in (*D-E*) compressive sensing is used. The number of samples, n, is displayed along with the correlation coefficient, r^2, between the estimated reconstruction and the original signal, as a measure of reconstruction quality.

sponse collection, and representation reconstruction, characterization, and comparison as described in Aim #3. This software package will include both standard reconstruction methods as well as an implementation of the compressive sensing framework, informed by the findings in this Aim, for use by the research community. We will develop interfaces for widely-used programming languages (including MATLAB and Python) in addition to command-line executables. Making the platform open-source provides opportunities for the community to make improvements and enables researchers to tailor these methods to their specific use-cases.

Reverse correlation has the potential to uncover latent representations underlying perception, and transform our understanding of perceptual mechanisms at various levels of investigation: neural, cognitive and psychological. However, in order for this potential to be fully realized, the fundamental inefficiency of reverse correlation paradigms must be overcome. Compressive sensing holds promise to overcome this limitation by dramatically improving the efficiency of reverse correlation, enabling its extension to perceptual mechanisms that are out of reach using current methods. The work proposed here will enable researchers to access the promise of compressive sensing, broadening the impact of reverse correlation.

C.3. Aim #3: Characterize and interpret cognitive representations of tinnitus

Our approach in this goal is to characterize the properties of uncovered representations through computational modeling. In Aim #1, we will have collected tinnitus percept data using the reverse correlation method. In Aim #2, we will have reconstructed high-dimensional representations of tinnitus percepts using both conventional and compressive sensing methods. In this Aim, we will characterize the space of representations to investigate correspondence between data-driven subtypes and qualitative subtypes, evaluate the stability of tinnitus percepts over time, and accelerate the diagnostic process.

Data-driven tinnitus subtypes Surveys of tinnitus patients have identified sixteen qualitative subtypes of tinnitus [26, 36, 38], but these categories are limited by the fuzziness of linguistic categories and the musical ability of the tinnitus patients. Tinnitus subtypes are used to better explain and quantify the condition, however qualitative subtypes are ill-defined. While one tinnitus patient we interviewed, who has a degree in music performance, was able to identify her tinnitus as sounding like a particular note played on the crotales, a niche percussion instrument, most tinnitus patients lack the depth of vocabulary to explain their tinnitus percept that precisely. Accordingly, linguistic description of tinnitus can be vague and overlapping. For instance, two of the subtypes in [38] are "high tension wire" and "transformer noise." Is "high tension wire" meant in the colloquial sense of high-voltage power lines, or does it mean the reverberation of a tightly-pulled string? If the latter, then how is the sound of a high-voltage power line disambiguated from the sound of a distribution transformer, which is mounted on a utility pole and connected directly to the high-voltage wires? Instead of linguistic categories, we propose to use latent variable analysis to reveal data-driven subtypes of tinnitus.

We will perform a quantitative analysis of tinnitus representations based on latent variable analysis methods to reveal underlying similarities between representations. Latent variable methods relate observed variation (*e.g.,* in tinnitus representations) to latent variables that capture the observed variation (*e.g.,* major variation types across tinnitus percepts). We will use principle component analysis (PCA) and uniform manifold approximation and projection (UMAP; [25]). Applying these dimensionality-reduction algorithms to a collection of representations will identify major modes of variation across those representations, uncovering latent similarities between representations. Clusters of data points become readily visible in the lower-dimensional manifold generated by these unsupervised machine learning methods, each of which represents a data-driven tinnitus subtype (Figure 6). We will characterize these subtypes, identifying their properties and assigning them descriptive names. New data can be mapped into the lower-dimensional manifold, allowing representations from new subjects to be classified into these subtypes.

Improving stimuli generation through data-driven distributions In Aim #1, we generate stimuli by randomly sampling Fourier coefficients from a flat distribution to generate a spectrum. We enforce only minimal constraints on the stimuli (*e.g.,* that the frequency content is audible). This has the benefit of reducing bias in the reconstructed representations, but results in experiments that take longer to converge to a good reconstruction. Previous studies using reverse correlation have attempted to reduce noise in the reconstructed representation (and thus converge much faster) by carefully selecting stimuli biased towards a certain representation. For example, Smith and colleagues [35] devised a task to uncover cognitive representations of faces using reverse correlation, using pictures of human faces distorted by noise as stimuli for the task. While this allowed the experimenters to perform fewer trials to achieve good reconstructions, it introduced bias of what experimenters think human faces look like (by selecting the pictures), defeating one of the main advantages of reverse correlation.

We propose to improve stimuli generation by exploiting our data from subsequent experiments. Each putative reconstruction that we uncover leads to a new known data point hypothesized to lie on the manifold of tinnitus representations. We can treat this manifold much like a distribution and sample our stimuli from it, resulting in stimuli that sound "less random" and more like actual tinnitus percepts. This closed-loop experimental protocol means that our stimuli generation becomes more tinnitus-like with every reconstruction, without introducing experimenter bias *a priori*.

We will implement a variational autoencoder (VAE), a type of unsupervised machine learning model that learns a latent representation of data as well as how to generate new data that appears similar to data the model

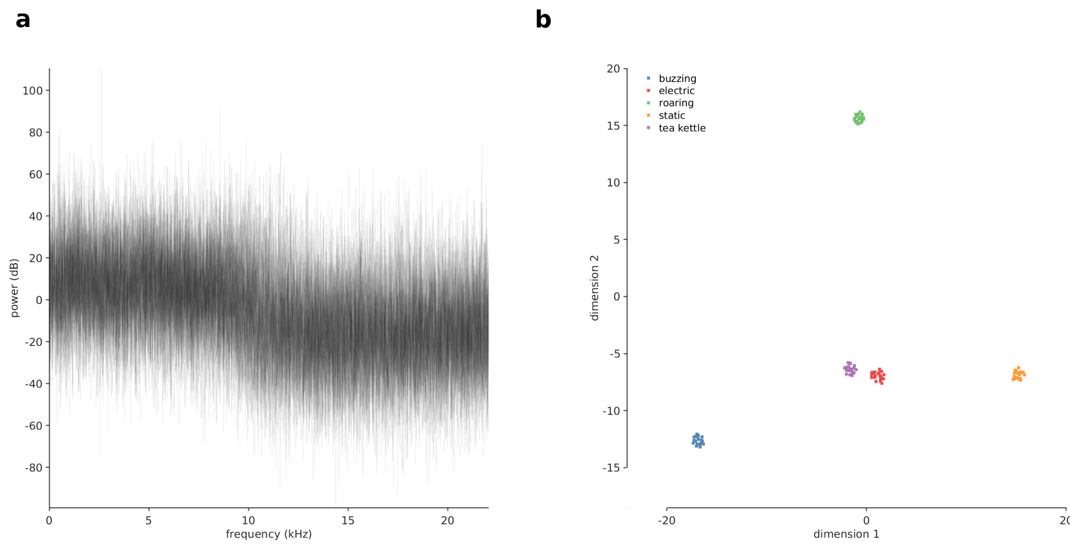

Figure 6: Tinnitus representations with different spectral characteristics cluster in a lower dimensional manifold. Tinnitus representation waveforms were generated using examples from the American Tinnitus Association website with added Gaussian noise [37]. (*a*) shows the frequency spectra for representative waveforms. (*b*) The waveforms are dimensonally-reduced using UMAP from 8,193 dimensions to 2. The waveforms cluster in this lower dimensional space, revealing latent similarities.

has been presented with before. A VAE consists of an encoder artificial neural network which constructs a latent representation as a zero-mean, unit-variance normal-distributed vector and a decoder artificial neural network that reconstructs the input (Figure 7). By training the VAE on reconstructed representations, forcing the encoder to create a latent representation and the decoder to reproduce the input from the latent representation, the model will learn the manifold of tinnitus percepts. By sampling the latent vector, we can generate new stimuli informed by the representations the model has learned. One limitation of VAEs is that sampling from the latent distribution tends to produce new data points that appear noisy. In our experiment, this is a benefit, as noisy, random stimuli is crucial to reverse correlation. Through this generative model, we will create stimuli that appear closer to actual tinnitus percepts by learning directly from the data without introducing external experimenter bias.

Acceleration of diagnosis through subtype identification While we anticipate remarkable performance gains in the number of samples required for accurate representation through compressive sensing, it may be clinically useful to identify only the subtype of tinnitus (*i.e.,* which cluster on the lower-dimensional manifold the representation belongs to) to some convergent accuracy. If only the subtype is needed, we believe the number of samples required for convergent accuracy can be further reduced. We will devise an algorithm to perform the data-driven tinnitus subtyping “online” while the subject performs the task. When convergent accuracy is reached, the task will automatically conclude. By this process, both patient and clinician time is optimized, while still providing high-fidelity quantitative medical data for use in diagnosis and treatment.

Stability of tinnitus percepts over time It is an open question in tinnitus research, whether tinnitus percepts remain stable over time [17]. Using data from subjects who returned to perform the experiment multiple times over the course of several weeks, we will examine representations derived from each experiment instance to quantitatively evaluate drift in the tinnitus representation over time. We will assess measures of variance and spectral distance between representations computed during different temporally-separated experiment sessions.

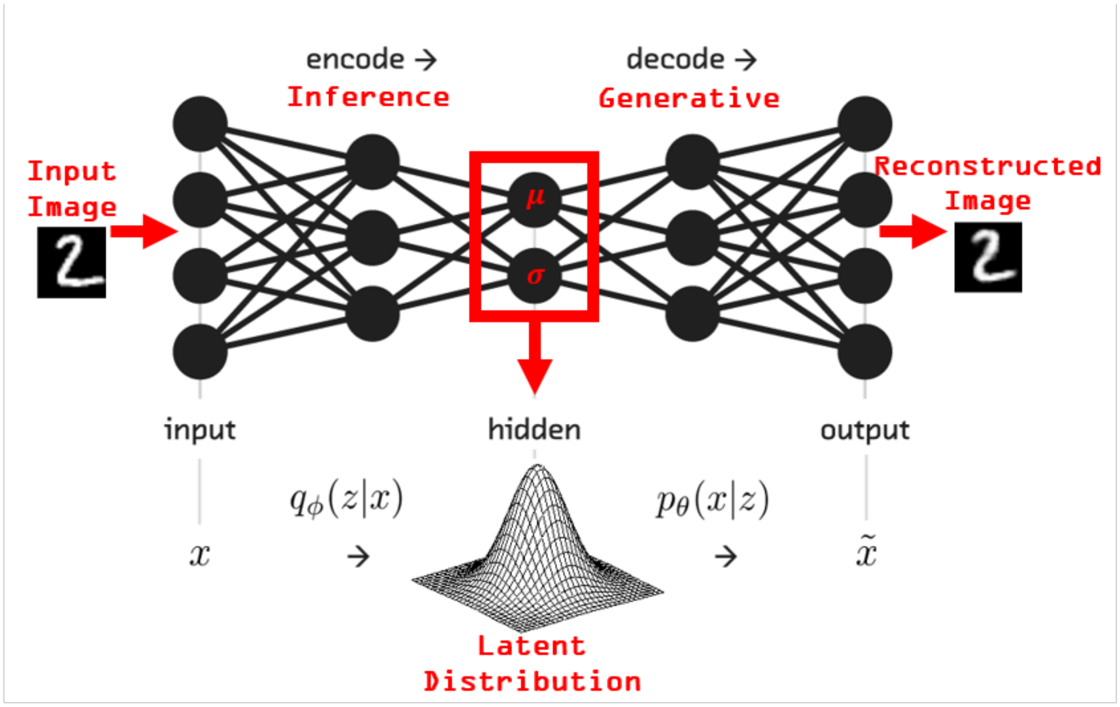

Figure 7: Diagram of a variational autoencoder (VAE) learning a representation of the digit "2". VAEs assume that the data, x, are generated by a directed graphical model, $p_\theta(x|z)$, and that the encoder is learning an approximation $q_\phi(z|x)$, where ϕ and θ denote the parameters of encoder and decoder respectively. The latent representation, z, is interpreted as originating from a multivariate Gaussian distribution. By sampling from the latent distribution, new data can be generated. Figure from [18].

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