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The transitionProbablity matrix is

$$
\begin{array}{c|cccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\
1 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\
2 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\
3 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\
4 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\
5 & 1/2 & 0 & 0 & 0 & 1/2 & 0
\end{array}
$$

$$
{}^{2} = \begin{pmatrix}
0 & 0.5 & 0 & 0 & 0 & 0.5 \\
0.5 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0.5 \\
0 & 0 & 0 & 0 & 0.5 & 0 \\
0.5 & 0 & 0 & & &
\end{pmatrix}
\times
\begin{pmatrix}
0 & 0.5 & 0 & 0 & 0 & 0.5 \\
0.5 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0.5 \\
0.5 & 0 & 0 & 0 & 0.5 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0.5 & 0 & 0.25 & 0.25 & 0.25 & 0 \\
0 & 0.5 & 0.25 & 0.25 & 0.25 & 0.25 \\
0.25 & 0 & 0.5 & 0.25 & 0.25 & 0 \\
0.25 & 0 & 0.25 & 0 & 0.25 & 0.5 & 0 \\
0 & 0.25 & 0 & 0.25 & 0.25 & 0 & 0.5
\end{pmatrix}
$$

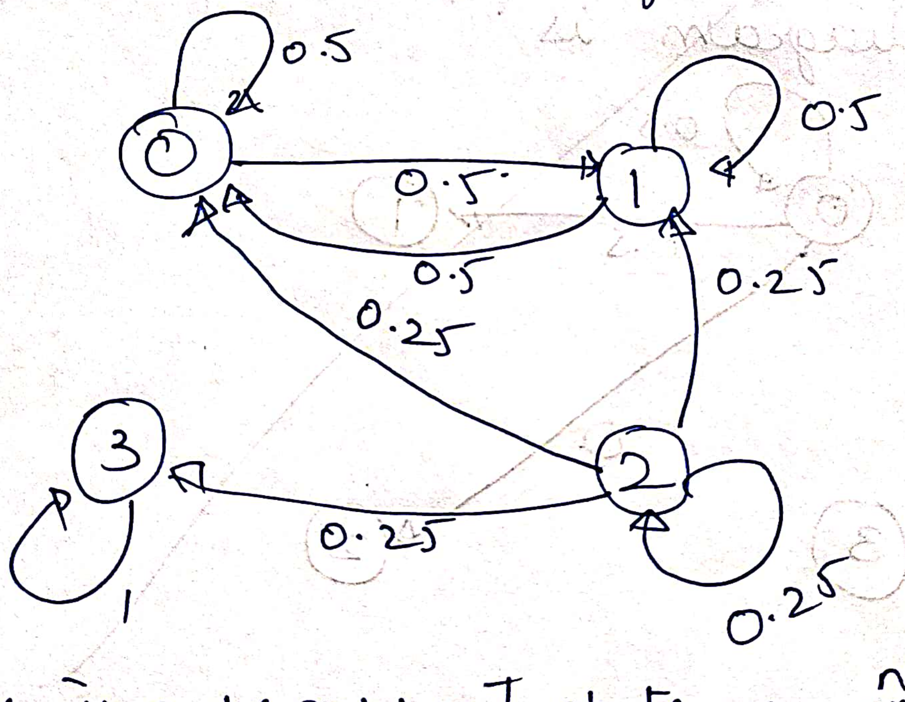
$$\therefore P^2 = \begin{bmatrix} 0.5 & 0. & 0.25 & 0.25 & 0.25 & 0. \\ 0 & 0.5 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0 & 0.5 & 0.25 & 0.25 & 0 \\ 0.25 & 0.25 & [illegible] & 0.5 & 0 & 0.25 \\ 0.[illegible] & 0.[illegible] & 0.25 & 0 & 0.5 & 0 \\ 0 & 0.25 & 0 & 0.25 & 0 & 0.5 \end{bmatrix}.$$

(2)

Q 2

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & \multicolumn{4}{c}{\left[\begin{array}{cccc} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{array}\right]} \\ 1 & & & & \\ 2 & & & & \\ 3 & & & & \end{array}$$

The transition state diagram is



If there is a recurrent state, then $\cancel{\sum_{i=0}^{n} f_{ii}^{(n)} = 1}$

. for i=0. $\sum_{i=0} P_{ii}^{(i)} = 1$

For i = 0.

$P_{00}^{(1)} = 0.5$

$P_{00}^{(2)} = 0.5 \times 0.5 = 0.25$.

$P_{00}^{(3)} = 0 \to 1, 1 \to 1, 1 \to 0.$

$= 0.5 \times 0.5 \times 0.5 = 0.125$.

$P_{00}^{(4)} = 0$.

$P_{00}^{(5)} = 0.5 \times 0.5 \times 0.5 \cdots$

$P_{00}^{(6)} =$

$\therefore P_{00}^{(1)} + P_{00}^{(2)} + P_{00}^{(3)} + P_{00}^{(4)} + P_{00}^{(5)} + P_{00}^{(6)} = 0.5 + 0.25 + 0.125$

$= 0.875 + \cdots$

$\simeq 1$

For i = 1

$P_{11}^{(1)} = 0.5$.

$P_{11}^{(2)} = 1 \to 0, 0 \to 1 = 0.5 \times 0.5 = 0.25$.

$P_{11}^{(3)} = (1 \to 0)(0 \to 0)(0 \to 0)$

$= 0.5 \times 0.5 \times 0.5$

$= 0.125$

$P_{11}^{(4)} = 1 \to 0, 0 \to 0, 0 \to 1, 1 \to 1$

$= 0.5 \times 0.5 \times 0.5 \times 0.5$.

$= 0.0625$.

$P_{11}^{(5)} = 0.5 \times 0.5 \times 0.5 \times 0. \cdots \times 0. \times$

$\vdots$

$P_{11}^{(n)} =$

~~For i = 2.~~ $\therefore P_{11}^{(1)} + P_{11}^{(2)} + P_{11}^{(3)} + P_{11}^{(4)} = 0.5 + 0.25 + 0.125 + 0.0625 + 0.00 \cdots$

$= \cancel{0.9375}$

$\cancel{P_{22}^{(1)} = 0.25}$

P

$\therefore$ It is a recurrent state!

For $i = 2$

$P_{22}^{(1)} = 0.25$

$P_{22}^{(2)} = 0.$

$P_{22}^{(3)} = 0.$

Now, $P_{22}^{(1)} + P_{22}^{(2)} + P_{22}^{(3)} = 0.25 + 0 + 0 = 0.25$

For $i = 3$.

$P_{33}^{(1)} = 1$

$P_{(33)}^{(2)} = 0.$

$\therefore P_{33}^{(1)} + P_{33}^{(2)} = 0 + 1 =$
$= 1$

$\therefore$ It is a recurrent state.

Is it an irreducible markov chain?

A markov chain is irreducible when every state is reachable from every other state.

Here we can see that 2 is not reachable from 0.

Hence it is not an irreducible Markov chain

Ans [No. of recurrent states = 3.
Not an irreducible Markov chain.]

Recurrent states = P_{00}, P_{11}, P_{33}

Q4 Here given 3 state Markov chain with time Parameters a, b & c is as follows matrix.

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

Lets find the stationary distribution

$$[V_0 \; V_1 \; V_2] = [V_0 \; V_1 \; V_2] \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$[V_0 \; V_1 \; V_2] = \left[\frac{V_1}{2} + \frac{V_2}{2} \;\; \frac{V_0}{2} + \frac{V_2}{2} \;\; \frac{V_0}{2} + \frac{V_1}{2}\right]$$

that is,

$$V_0 = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V_1 = \frac{V_0}{2} + \frac{V_2}{2}$$

$$V_2 = \frac{V_0}{2} + \frac{V_1}{2}.$$

and $V_0 + V_1 + V_2 = 1$

From equation (2)

$$V_1 = 0.5(V_0 + V_2)$$

Equation (1) becomes

$$V_0 = 0.5 \times (0.5(V_0 + V_2)) + 0.5 \times V_2.$$

$$V_0 = 0.25 \times V_0 + 0.25 \times V_2 + 0.5 \times V_2$$

$$0.75 \times V_0 = 0.75 \times V_2$$

$$V_0 = V_2.$$

Therefore equation (4) becomes.

$$V_1 = V_0$$

$$V_0 + V_0 + V_0 = 1$$

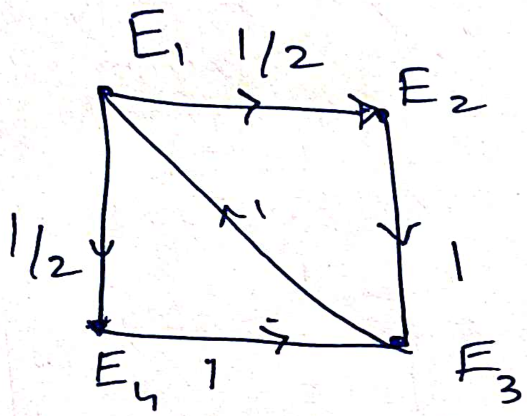
$$V_0 = 1/3$$

That is stationary distribution is as

~~$V_0 \; V_1 \; V_2$~~

$$\begin{bmatrix} V_0 & V_1 & V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Q3



The TPM is:

$$
\begin{array}{c|cccc}
 & E_1 & E_2 & E_3 & E_4 \\
\hline
E_1 & 0 & 0.5 & 0. & 0.5 \\
E_2 & 0 & 0 & 1 & 0. \\
E_3 & 1 & 0 & 0 & 0 \\
E_4 & 0 & 0 & 1 & 0
\end{array}
$$

From \ To	state 1	state 2
state 1	0.7	0.3
state 2	0.2	0.8
Initial state	1	0

The Probablity vector after 4 steps.

state 1	state 2
0.4375	0.5625

The probablity to be in state 1 after 4 steps is 0.4375

The probablity to be in state 2 after 4 steps is 0.5625

The steady state vector:

state 1	state 2
0.4	0.6

The Probablity vector in each step.

step	state 1	state 2	Formula.
S_0	1	0	Initial state.
S_1	0.7	0.3	$S_0 \times P = S_0 \times P^1$
S_2	0.55	0.45	$S_1 \times P = S_0 \times P^2$
S_3	0.475	0.525	$S_2 \times P = S_0 \times P^3$
S_4	0.4375	0.5625	$S_3 \times P = S_0 \times P^4$