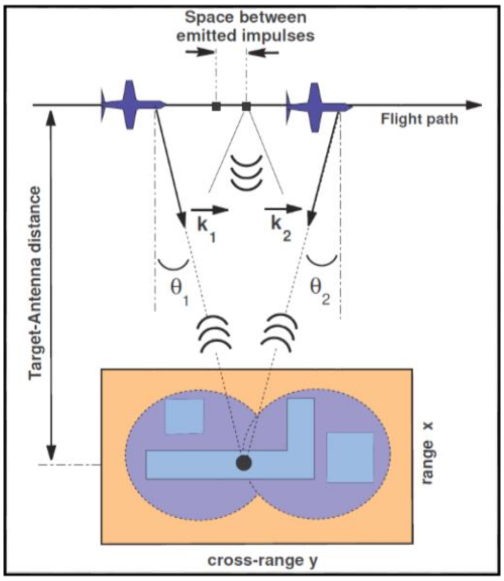
# Time-Frequency Analysis of High Resolution SAR Images

Synthetic Aperture Radar (SAR) imaging is used to create two-dimensional or three-dimensional images of objects. This form of radar imaging uses a moving antenna to continuously capture images with fine spatial information. These high resolution images are helpful in remote sensing where several images can be analysed to detect and/or measure changes in the region of interest. Some SAR systems produce multivariate images, where each pixel is represented by a data vector so as to provide diverse information. One such information is the spectral and angular diversity of various scatterers present at the location which respond to electromagnetic wave differently.




SAR Imaging is done by emitting an electromagnetic wave through a moving radar and analysing the backscattered signal to obtain a map of the scatterers in the region. The radar moves along the azimuth axis observing the scene at different angles. The signal emitted is located in a certain range of frequencies and its bandwidth determines the range resolution of the radar. Coherent detection enables capturing of the signal phase information in addition to the signal amplitude information.

One of the many possible methods, to retrieve the angular and spectral information, is using a linear time-frequency analysis based on SAR geometry. It consists of computing Radar cross-section H(**k**) and going from Cartesian to Polar coordinates and then taking IFT2 to reconstruct the image.However,IFT2 on coloured scatters leads to loss of information about the specific variables. 2-D wavelet transform by decimating the image according to range of frequencies and angle proves to be a better option. This can be simplified by using Short Time Fourier Transform. We collected the SAR image of static aircraft from “Sandia National Laboratories”. The image that we have selected for simulation has a spectrum with **k** ϵ [1, 2510] and **θ** ϵ [1, 1638]. A moving window on H(**ϕ**(***l,m***)) is constructed to select a range of frequencies and angles. The window size being determined by number of sub-bands ($N_k$) and sub-angles ($N_t$).

**<u>Frequency Analysis in 2-Dimensions:</u>**

A signal varying in frequencies in two directions, say horizontal and vertical, both in the range of digital frequency (0,2π), represents an image.

<u>2-D DFT:</u>

$$F(k1,k2)=\sum_{n2=0}^{N2-1}\sum_{n1=0}^{N1-1} f(k,\theta)e^{-\frac{2j\pi}{N1}(n1)(k1)}e^{-\frac{2j\pi}{N2}(n2)(k2)}$$

<u>2-D IDFT:</u>

$$f(k,\theta)=\frac{1}{N2}\sum_{n2=0}^{N2-1}\frac{1}{N1}\sum_{n1=0}^{N1-1} F(k1,k2)e^{\frac{2j\pi}{N1}(n1)(k1)}e^{\frac{2j\pi}{N2}(n2)(k2)}$$

We interpreted the above equation in terms of two matrices of $S_k$ corresponding to two different values of **k**=k1 and k2. The $S_k$ matrix was computed in the same way as done in the lab assignment for 1-D FT. The 2-D FT was calculated by using the reordering property of integration. We can use multiple passes of 1-D DFT, first rows then columns, of $S_k$ matrix in any one dimension and calculate its respective DFT to implement a DFT in 2-D, which is analogous to a double integration in continuous domain.

The paper proposes to make a domain **Δ** (l,m) of specific range of values which will be the basis for the formation of windows to implement STFT for time-frequency analysis of SAR images. 2-D FFT for each sub image is calculated by multiplying **ϕ** (l,m) and **H.** Inverse DFT is done on each of the above obtained images. The function of inverse DFT has been defined with respect to the DFT calculation that we did with S(k1,k2) matrix.

**<u>RESULT:</u>**

On element-wise multiplication of **ϕ** (l,m) and **H** yielded a windowed section of **H** whose inverse DFT was calculated. It is observed that the sub image which corresponds to highest magnitude in colour map of DFT corresponds to portion of densest land cover. This approach allows us to exploit angular and spectral diversity. Thus, allowing for an accurate change detection, with less complexity as compared to that which was done by construction of monovariate images as it worked on amplitude only.

**<u>Deviations from Actual Implementation (as shown in paper):</u>**

Taking $N_k$=$N_t$=3, the last row of sub-images that we got were zeros, which didn't have any mappings in the image. So the output image of those windows was an image of constant colour. As SAR are high resolution images, we approached the DFT calculations using the decimation in time algorithm but as the algorithm only works in the powers of 2 we were getting extra coefficients in the frequency domain. So we decided to stick with the general definition of DFT by translating in matrix form. We were also not able to implement the steering vector, which is the change detection, which is the final aim of this paper.

**<u>Further Scope:</u>**

After the calculation of the windowed images, we can choose specific targets on random locations and compare the images to get an accurate idea of how the land cover is changing in the traverse of the plain. This can easily be accomplished using the signal processing toolbox in MATLAB. The algorithm defined in this paper has been classified as a major breakthrough for time-frequency analysis of SAR imaging.

**<u>Contributing Members:</u>**

Shubham Kumar (1510110384) – 2D DFT using matrix method

Sarthak Sharma (1510110342) – SAR, Report compilation

Yatin Malhotra (1510110466) – 2D DFT equation form