2014-2015 TMSCA Middle School Mathematics State Championship Test

1. 56,389 – 44,879 = _____(nearest thousand)

A. 11,500 B. 11,000 C. 12,000 D. 10,000 E. 12,500

2. 17.8 + 22.094 = _____

A. 39.794 B. 40.74 C. 40.774 D. 39.894 E. 39.174

3. $1.54 \cdot 4.3 =$ _____(nearest hundredth)

A. 6.7 B. 6.6 C. 6.61 D. 6.62 E. 6.71

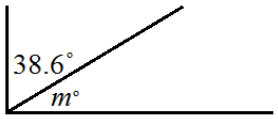
4. $15\frac{6}{7} \div 4\frac{1}{2} =$ _____

A. $3\frac{5}{21}$ B. $3\frac{11}{21}$ C. $3\frac{9}{14}$ D. $3\frac{11}{14}$ E. $3\frac{4}{7}$

5. One-dozen apples cost $5.52. What is the unit rate per apple?

A. $0.46 B. 0.46¢ C. $0.52 D. $0.92 E. 0.34¢

6. The supplement of $\angle m$ is equal to _____°.



A. 51.4 B. 141.4 C. 128.6 D. 77.2 E. 102.8

7. 1,386 cubic inches = _____ gallons

A. 5 B. 6 C. 7 D. 8 E. 9

8. What is the remainder when the number 432,651,117 is divided by 4?

A. 0 B. 1 C. 2 D. 3 E. 4

9. $\sqrt{1{,}558}$ is between which two integers?

A. 39 & 40 B. 40 & 41 C. 37 & 38 D. 36 & 37 E. 41 & 42

10. If the Roman numeral *MMMCDXLVIII* were changed into an Arabic number, what would be the sum of the digits?

A. 16 B. 19 C. 21 D. 22 E. 23

11. What is 14.5% of 640?

A. 94.6 B. 90.8 C. 96.2 D. 91.4 E. 92.8

12. Change the positive difference of 54,000,000 and 72,000,000 into scientific notation.

A. -1.8×10^6 B. -1.8×10^8 C. 18×10^7 D. 1.8×10^6 E. 1.8×10^7

13. A rectangular prism has dimensions of 10 feet high, 12 feet long and 6 feet wide. If the box is filled ¼ full with packing peanuts, how much of the prism does not have packing peanuts?

A. 540 ft^3 B. 180 ft^3 D. 360 ft^3 D. 240 ft^3 E. 480 ft^3

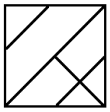
14. One card is picked from a standard deck of cards. What are the odds the card chosen will be a face card?

A. 7:10 B. 5:26 C. 2:13 D. 3:10 E. 3:13

15. Find the next term in the sequence 2, 3, 4, 9, 16, 29, 54, 99, …

A. 124 B. 132 C. 158 D. 174 E. 182

16. How many triangles can be found in the picture below?



A. 7 B. 8 C. 9 D. 10 E. 6

17. After slicing, one whole watermelon can feed one-baker's dozen people. How many people can a dozen watermelons feed?

A. 144 B. 156 C. 164 D. 148 E. 169

18. The length of a rectangle is one less than twice its width and the perimeter of the rectangle is 34 cm. If the dimensions of the rectangle are to be dilated by a scale factor of 3, what is the area of the enlarged rectangle?

A. 564 cm^2 B. 590 cm^2 C. 198 cm^2 D. 598 cm^2 E. 594 cm^2

19. Let L = lower limit for outliers and U = upper limit for outliers. What are the values of L and U for the set of data {16, 16, 24, 26, 30, 38, 42, 44, 44, 52}?

A. L = -6; U = 69 B. L = -6; U = 74 C. L = -10; U = 84 D. L = 0; U = 100 E. L = -10; U = 70

20. The sum of three consecutive even integers is 294. What is five less than the value of twice the smallest of these integers?

A. 182 B. 192 C. 187 D. 190 E. 186

21. $\sqrt[3]{8} + \sqrt[2]{4} + 2^0 =$ _____

A. 6 B. $8.\overline{6}$ C. $7.\overline{6}$ D. 6.5 E. 5

22. {2, 4, 6, 8} ∪ {1, 2, 3, 4, 5} ∪ {1, 3, 5, 7} has how many proper subsets?

A. 257 B. 256 C. 1 D. 255 E. 260

23. If you have a bag full of nine green marbles and fifteen red marbles and you make two draws, what is the probability of getting a red marble and then with replacement getting a green marble (in ratio form)?

A. 15:64 B. 13:32 C. 5:8 D. 3:8 E. 49:64

24. What is the sum of all the positive integral divisors of the number 42?

A. 96 B. 95 C. 47 D. 53 E. 101

25. What is the value of the discriminant of the quadratic equation $y = ½ x^2 – 5x + 3$?

A. 17 B. 9.5 C. 19 D. 16 E. 42

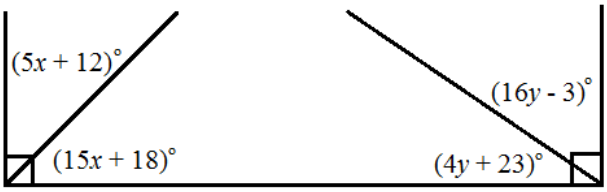
26. What is the linear equation $y = \frac{5}{8}x + 2$ written in standard form?

A. $5x – 8y = -16$ B. $5x + 8y = 16$ C. $5x + 8y = -16$ D. $5x – 8y = 16$ E. $-5x – 8y = 16$

27. $121_5 + 34_6 + 52_7 = 231_8 -$ _____$_9$

A. 31 B. 238 C. 18 D. 76 E. 64

28. Using the picture below, what is the sum of x and y?



A. 9.5 B. 7 C. 8.5 D. 6.5 E. 5.5

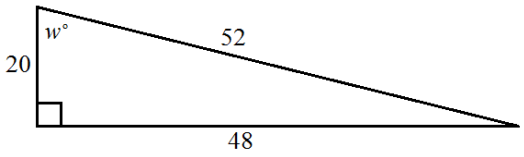
29. Maria deposited $500 into a simple interest bank account at 4.2% 3 years ago. Maria's friend needs to borrow some money. Maria decides to give her friend half of the interest she has accumulated in her bank account. How much money will Maria be giving her friend?

A. $34.84 B. $34.25 C. $31.50 D. $27.00 E. $29.50

30. If $f(x) = \frac{2}{3}x^3 + 4$ and $g(x) = \frac{1}{8}x^2 - 7$, find the value of $\frac{-5f(3)-g(4)}{2}$.

A. -52.5 B. 27.5 C. 17.5 D. -110 E. -60

31. Which expression below could be used to find the measure of $\angle w$?



A. $cos^{-1}\left(\frac{48}{52}\right)$ B. $tan^{-1}\left(\frac{52}{48}\right)$ C. $tan^{-1}\left(\frac{20}{48}\right)$ D. $cos^{-1}\left(\frac{20}{48}\right)$ E. $cos^{-1}\left(\frac{20}{52}\right)$

32. Letting $\pi = 3$, what is the volume of a hemisphere with a radius of 3 meters?

A. 222 m^3 B. 108 m^3 C. 36 m^3 D. 48 m^3 E. 54 m^3

33. Find n, if $\left(\left(\sqrt[4]{3}\right)^{-8}\right)^2 = 9^n$

A. -1 B. -½ C. -3 D. -4 E. -2

34. Thirteen people are in a room and must shake hands with everyone else exactly once. How many handshakes will occur?

A. 78 B. 82 C. 91 D. 85 E. 86

35. If $2a - 3b = 3$, the area of a square with a side length of $12a^2 - 36ab + 27b^2$ is equal to _____ $units^2$.

A. 676 B. 529 C. 841 D. 729 E. 1,369

36. Which of the following exponential decay functions below has a rate of decay of 2.4%?

A. $y = 23(0.24)^x$ B. $y = 13(0.76)^x$ C. $y = 1(0.98)^x$ D. $y = 2(0.076)^x$ E. $y = 76(0.976)^x$

37. Let A = the mean absolute deviation of the data set 120, 131, 117, 152 and 210. Find the value of $2A - 6$.

A. 50 B. 30 C. 62 D. 140 E. 286

38. What is the area of a polygon with its vertices located at (-3, -1), (-1, -1), (1, -3), (4, -3), (5, 0), (5, 2), (2, 3) and (2, 1)?

A. 22 $units^2$ B. 22.5 $units^2$ C. 24 $units^2$ D. 36 $units^2$ E. 28.5 $units^2$

39. $0.6\overline{39}$ = _____ (fraction)

A. $\frac{71}{990}$ B. $\frac{211}{330}$ C. $\frac{71}{330}$ D. $\frac{71}{100}$ E. $\frac{106}{165}$

40. Point *C* is the midpoint of $\overline{AB}$. If the coordinates of *A* are (15, -14) and the coordinates of *C* are (12.5, -20), then what is the distance of $\overline{AB}$?

A. $\sqrt{42.25}$ units B. $\sqrt{72.25}$ units C. 16 units D. 13 units E. 14 units

41. How many zeroes are at the end of 10!?

A. 12 B. 6 C. 25 D. 2 E. 20

42. What is the sum of all the integral solutions of the following equation? $|6x - 2| \leq 10$

A. 2 B. 1 C. 4 D. 6 E. 5

43. If the point (-7, 14) is reflected over the line $y = x$ and then rotated 270° counterclockwise about the origin, what are its new coordinates?

A. (14, -7) B. (7, -14) C. (-7, -7) D. (7, 14) E. (-7, -14)

44. Which of the following is equivalent to $\left(\sqrt{2} - 1\right)^2 + 3\sqrt{8}$?

A. $3 - 6\sqrt{2}$ B. $6 + \sqrt{2}$ C. $3 + 4\sqrt{2}$ D. $3 + 2\sqrt{6}$ E. $3 + 5\sqrt{2}$

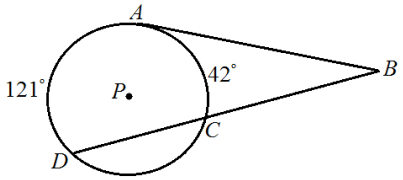
45. When solving the quadratic equation $6x^2 + 8x +$ _____= 11 + _____ by completing the square, what would you add to both sides of the equal sign?

A. $\frac{8}{3}$ B. $\frac{9}{16}$ C. $\frac{16}{9}$ D. $\frac{121}{64}$ E. $\frac{2}{3}$

46. What is the unit's digit of 12^6?

A. 2 B. 4 C. 6 D. 8 E. 1

47. Using the picture of $\odot P$ below, arc AC $= 42^\circ$ and arc AD $= 121^\circ$, what is the measure of $\angle ABC$?



A. 39.5° B. 21° C. 60.5° D. 18.5° E. 30.5°

48. Which of the following is equivalent to $\frac{6}{\sqrt{3}-\sqrt{5}}$?

A. $-6\left(\sqrt{3} - \sqrt{5}\right)$ B. $-6\left(\sqrt{3} + \sqrt{5}\right)$ C. -⅓$\left(\sqrt{3} - \sqrt{5}\right)$ D. -⅓$\left(\sqrt{3} + \sqrt{5}\right)$ E. $-3\left(\sqrt{3} + \sqrt{5}\right)$

49. Chris has 32 coins consisting of quarters and dimes. The total value of his coins is $6.35. If Chris's friend gives him 16 more quarters, what will be the value of all the quarters Chris will have then?

A. $10.35 B. $9.75 C. $9.25 D. $10.25 E. $9.50

50. After simplifying, what is the denominator of the following? $\frac{x^5-4x^3-x^2+4}{x^3-2x^2+x-2} \div \frac{3x^3+3x^2+3x}{x^2-1} \cdot \frac{6x}{x^2-2x+1}$

A. $x^2 + 1$ B. $x^2 + 7x + 12$ C. $x^2 + x + 1$ D. $2x^2 + 2$ E. $x - 1$

2014-2015 TMSCA Middle School Mathematics State Championship Test Answer Key

1. C
2. D
3. D
4. B
5. A
6. C
7. B
8. B
9. A
10. B
11. E
12. E
13. A
14. D
15. E
16. B
17. B
18. E
19. B
20. C
21. E
22. D
23. A
24. A
25. C
26. A
27. E
28. D
29. C
30. A
31. E
32. E
33. E
34. A
35. D
36. E
37. A
38. C
39. B
40. D
41. D
42. A
43. E
44. C
45. A
46. B
47. A
48. E
49. C
50. A

2014-2015 TMSCA Middle School Mathematics State Championship Test Selected Answers

7. 231 cubic inches = 1 gallon. Therefore, 1,386 cubic inches $\div 231 = 6$ gallons.

13. A rectangular prism has dimensions of 10 feet high, 12 feet long and 6 feet wide. If ¼ of the box is filled with packing peanuts, and we want to know how much is not, then we are trying to find ¾ of the volume of the box. The volume of the box is $10 \cdot 12 \cdot 6 = 720\ ft^3$. Now, $\frac{3}{4} \cdot 720 = 540\ ft^3$. The box has $540\ ft^3$ of empty space.

14. There are 52 cards in a standard deck of cards with 12 being face cards. Since 12 are face cards, that means 40 are not face cards. So, the odds of drawing a face card is 12:40, which reduces to 3:10.

39. We are given $0.6\overline{39}$ and let that equal *N*. $1000N = 639.\overline{39}$ and $10N = 6.\overline{39}$.
$1000N - 10N = 636 \rightarrow 990N = 633 \rightarrow N = \frac{633}{990} = \frac{211}{330}$.

41. 10! = $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 2 \cdot 5 \cdot 3^2 \cdot 2^3 \cdot 7 \cdot 2 \cdot 3 \cdot 5 \cdot 2^2 \cdot 3 \cdot 2 \cdot 1$. Now, simplify and 10! = $7 \cdot 5^2 \cdot 3^4 \cdot 2^8$. Since there are only two $(2 \cdot 5)$ pairs, 10! will end in 2 zeroes.

44. $\left(\sqrt{2}-1\right)^2 + 3\sqrt{8} = \left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right) + 3\sqrt{8} = \sqrt{4} - \sqrt{2} - \sqrt{2} + 1 + 3 \cdot 2\sqrt{2} =$
$3 - 2\sqrt{2} + 6\sqrt{2} = 3 + 4\sqrt{2}$.

46. $12^1 = 12$, $12^2 = 144$, $12^3 = 1{,}728$ and $12^4 = 20{,}736$. If you continued the pattern, you would see it repeats ending in a unit's digit of 2, 4, 6, 8. So, take the exponent 6 and divide it by 4 and get a remainder of 2. Since 12^2 ends in a four, 12^6 ends in a 4.