AGA KHAN UNIVERSITY EXAMINATION BOARD

SECONDARY SCHOOL CERTIFICATE

CLASS IX

ANNUAL EXAMINATIONS (THEORY) 2023

Mathematics Paper II

Time: 1 hour 40 minutes   Marks: 30

INSTRUCTIONS

Please read the following instructions carefully.

1.   Check your name and school information. Sign if it is accurate.

> **I agree that this is my name and school.**
> **Candidate's Signature**

RUBRIC

2.   There are EIGHT questions. Answer ALL questions. Choices are specified inside the paper.

3.   When answering the questions:

Read each question carefully.
Use a black pointer to write your answers. DO NOT write your answers in pencil.
Use a black pencil for diagrams. DO NOT use coloured pencils.
DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
Complete your answer in the allocated space only. DO NOT write outside the answer box.

4.   The marks for the questions are shown in brackets ( ).

5.   A formulae list is provided on page 2. You may refer to it during the paper, if you wish.

6.   You may use a simple calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Sets and Functions

$$A\Delta B = (A \cup B) - (A \cap B) \qquad (A \cap B)^c = A^c \cup B^c \qquad (A \cup B)^c = A^c \cap B^c$$

Real and Complex Numbers

$$x^m \times x^n = x^{m+n} \qquad (x \times y)^n = x^n \times y^n \qquad (x^m)^n = x^{mn}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \qquad \frac{x^m}{x^n} = x^{m-n} \qquad a^{-m} = \frac{1}{a^m}$$

Exponents and Logarithms

$$\log_a(m \times n) = \log_a m + \log_a n \qquad \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n \qquad \log_a b = n \Leftrightarrow a^n = b$$

$$\log_a(m)^n = n\log_a m \qquad \log_a n = \log_b n \times \log_a b \qquad \log_a n = \frac{\log_b n}{\log_b a}$$

Algebraic Formulae & Applications and Factorisation

$$(a-b)^2 = a^2 - 2ab + b^2 \qquad\qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \qquad\qquad a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \qquad\qquad (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \qquad\qquad (a+b)^2 - (a-b)^2 = 4ab$$

Matrices and Determinants

$$A^{-1} = \frac{1}{|A|} AdjA$$

Q.1. (Total 3 Marks)

If $U = \{a, e, i, o, u\}$, $M = \{a, i, u\}$, $N = \{u\}$ and $R = \{e, o, u\}$, then represent

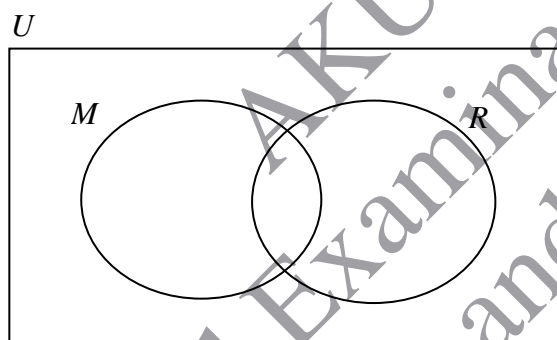i. M and N in the incomplete Venn diagram given below. (1 Mark)

U

ii. N^c on the Venn diagram completed in part (i) by shading. (1 Mark)

iii. $(M \cap R)^c$ in the Venn diagram given below, by shading. (1 Mark)

U

M   *R*

Q.2. (Total 3 Marks)

If the value of $\log 3 = 0.4771$, then find the value of $\log 81$.

_____

_____

_____

_____

_____

_____

_____

PLEASE TURN OVER THE PAGE

(ATTEMPT PART a OR PART b ONLY FOR Q.3.)

Q.3. (Total 4 Marks)

a. If $m = 2 - \sqrt{3}$ and $n = \dfrac{1}{2 + \sqrt{3}}$, then

   i. show that $m + n = \dfrac{2}{2 + \sqrt{3}}$. (2 Marks)

   ii. express $m + n$ in the form $a - b\sqrt{c}$ by rationalising the denominator. (2 Marks)

b. Prove that the simplest form of $r = \left\{ \dfrac{m-1}{2} \div \left(m^2 - 1 \right) \right\} \times \left(1 + \dfrac{1}{m} \right)$ is $r = \dfrac{1}{2m}$.

_____

_____

_____

_____

_____

_____

_____

_____

_____

(ATTEMPT PART a OR PART b ONLY FOR Q.4.)

Q.4. (Total 4 Marks)

a.   Factorise $p^6 - q^6$ completely.

b.   For the polynomial $p(x) = x^3 - 3x - 2$,

   i.   using factor theorem, prove that $x + 1$ is a factor of $p(x)$. (2 Marks)

   ii.   find the remaining two factors of $p(x)$. (2 Marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

PLEASE TURN OVER THE PAGE

Q.5. (Total 3 Marks)

The quantity *m* varies directly as n^2. If $m = 4$ and $n = 6$, then find the relation between *m* and *n* in terms of an equation.

_____

_____

_____

_____

_____

_____

_____

Q.6. (Total 4 Marks)

For the matrices $S = \begin{bmatrix} 2 & -1 \\ s & 4 \end{bmatrix}$ and $T = \begin{bmatrix} 0 & 1 \\ -s & -1 \end{bmatrix}$, find the

i.   matrix $2(S - T)$. (2 Marks)

_____

_____

_____

_____

_____

ii.   value of *s*, if $S - T$ is a singular matrix. (2 Marks)
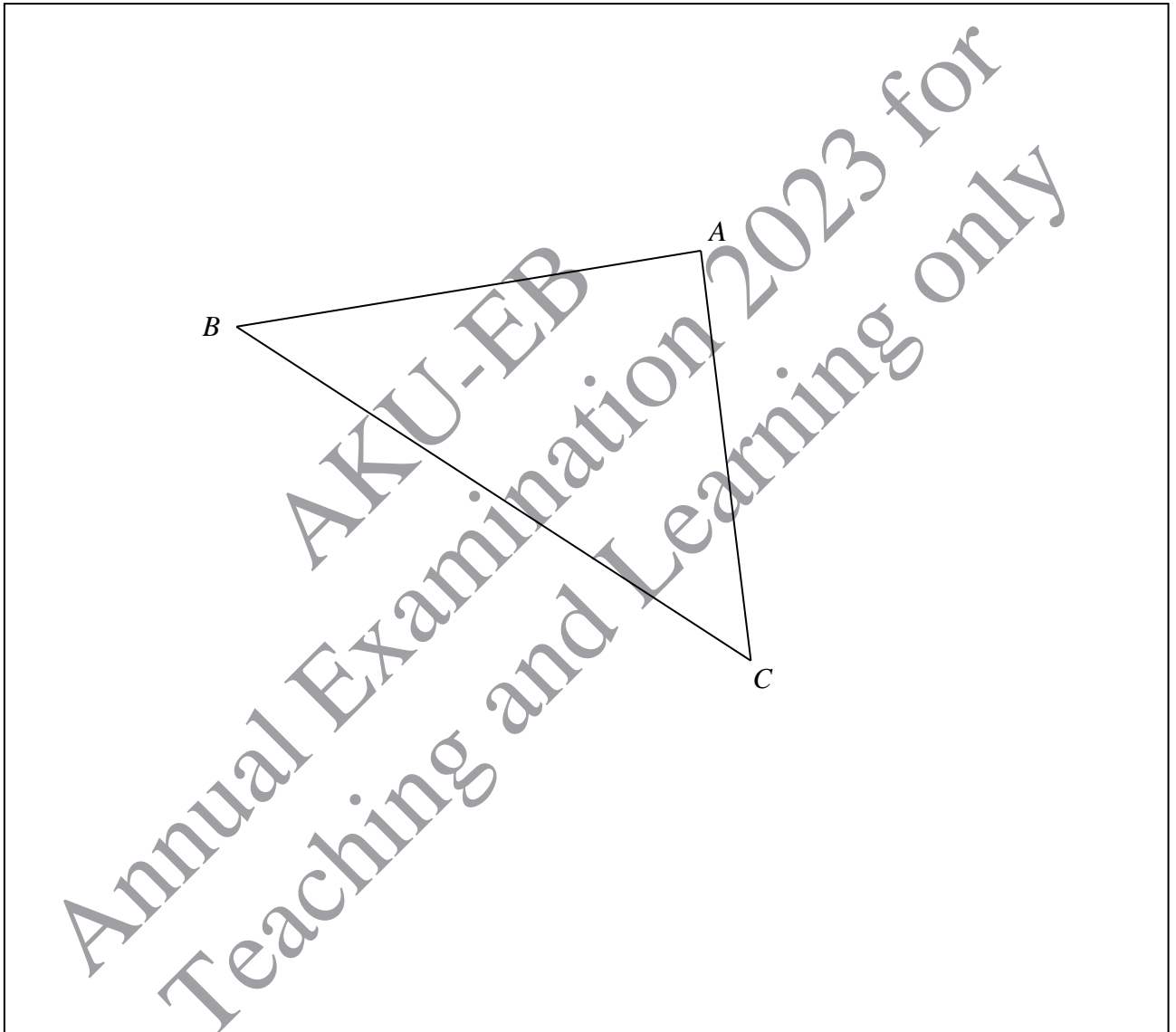
_____

_____

_____

_____

_____

Q.7. (Total 3 Marks)

For the given triangle *ABC*, draw the

i.   angle bisector of angle *A*. (1 Mark)

ii.   median passing through vertex *B*. (1 Mark)

iii.   altitude from vertex *A* to its opposite side. (1 Mark)

S2301-0711120

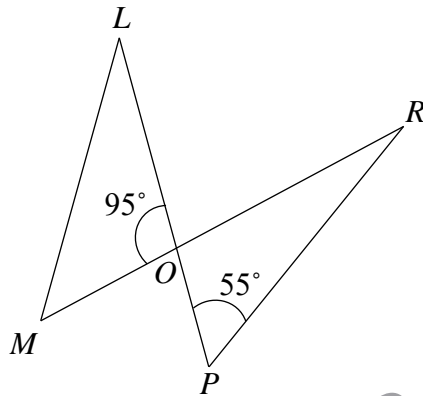**(ATTEMPT ANY TWO PARTS OF a, b AND c OF Q.8.)**

Q.8.   (Total 6 Marks)

a.   In the given diagram, $OM = OP$ and $OL = OR$

NOT TO SCALE

In the correspondence $\Delta MOL \leftrightarrow \Delta POR$,

i.   find the unknown angle R.   (2 Marks)

_____

_____

_____

_____

_____

ii.   find the unknown angle M.   (1 Mark)
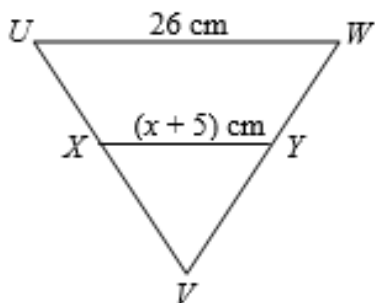
_____

_____

_____

(ATTEMPT ANY TWO PARTS OF a, b AND c OF Q.8.)

b. In the given figure, *X* and *Y* are the midpoints of *UV* and *WV* respectively.

NOT TO SCALE

If *UW* = 26 cm and *XY* = (*x* + 5) cm, then

i. find the value of *x*. (2 Marks)

_____

_____

_____

_____

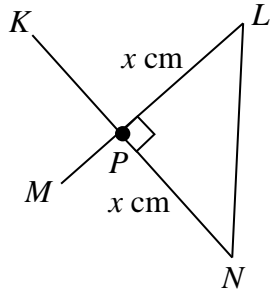ii. recalling the statement of a relevant theorem, complete the given sentence. (1 Mark)

XY and *UW* are lines_____ to each other.

(ATTEMPT ANY TWO PARTS OF a, b AND c OF Q.8.)

c.   In the given diagram, $PL = PN = x$ cm and KN is perpendicular to ML.

NOT TO SCALE

If $KN = y$ cm, then find an expression for the shortest distance from K to ML.

(**Note:** The expression shall be in terms of x, and y.)

_____

_____

_____

_____

_____

_____

END OF PAPER

Please use this page for rough work

Please use this page for rough work