Stochastic programming Advanced Modelling Techniques

Plan

- Important metrics in stochastic programming
- Chance constraint:
  - Concept
  - Exercices VRP
- Multi-stage Stochastic Programs
  - Concept
  - Famer’s example
  - Exercices

Learning Objectives

- Be able to compute import metric in stochastic programming (EVPI and VSS).
- Understand the concept of chance constraints and its links with robust optimization
- Understand the concept of multi-stage stochastic program
- Be able to formulate a multi-stage stochastic program with the scenario tree approach.

EVPI and VSS

Consider a European farmer who specializes in raising wheat, corn, and sugar beets on his 500 acres of land. During the winter, he wants to decide how much land to devote to each crop. (We refer to the farmer as “he” for convenience and not to imply anything about the gender of European farmers.)

The farmer knows that at least 200 tons (T) of wheat and 240 T of corn are needed for cattle feed. These amounts can be raised on the farm or bought from a wholesaler. Any production in excess of the feeding requirement would be sold. Over the last decade, mean selling prices have been \$170 and \$150 per ton of wheat and corn, respectively. The purchase prices are 40% more than this due to the wholesaler’s margin and transportation costs.

Another profitable crop is sugar beet, which he expects to sell at \$36/T; however, the European Commission imposes a quota on sugar beet production. Any amount in excess of the quota can be sold only at \$10/T. The farmer’s quota for next year is 6000 T.

Based on past experience, the farmer knows that the mean yield on his land is roughly 2.5 T, 3 T, and 20 T per acre for wheat, corn, and sugar beets, respectively. Table 1 summarizes these data and the planting costs for these crops.

Data of the farmer's problem

	Wheat	Corn	Sugar Beets
Yield (T/acre)	2.5	3	20
Planting cost ($/acre)	150	230	260
Selling price ($/T)	170	150	36 under 6000 T 10 above 6000 T
Purchase price ($/T)	238	210	–
Minimum requirement (T)	200	240	–
Total available land: 500 acres			

Formulate this problem as a linear program.

Expected Value Problem

Deterministic version of the problem, where the uncertain parameter is replaced by its expected value.

Expected Value Problem

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	120	80	300
Yield (T)	300	240	6000
Sales (T)	100	–	6000
Purchase (T)	–	–	–
Overall profit: $118,600			

Expectation of the Expected Value Problem

You can forget about uncertainty, but uncertainty does not forget you

To compute the EEV:

- Solve the deterministic problem: Optimize decisions $\hat{x}$ using the expected value of uncertain parameter $\mathbb{E}[\xi]$.
- Evaluate the performance: Compute the expected objective value using $\hat{x}$ under the full distribution of ξ.

$$\boxed{\text{EEV} = \mathbb{E}[f(\hat{x}, \xi)]}$$

Expectation of the Expected Value Problem

In practice:

(1) Solve the Expected value problem, and get solution $\hat{x}$

(1) Solve the stochastic model, with the additional constraint:

$$x = \hat{x}$$

```
FarmerProbStoch += quantiry_vars["wheat"] == 120
FarmerProbStoch += quantiry_vars["corn"] == 80
FarmerProbStoch += quantiry_vars["beets"] == 300
```

	Wheat	Corn	Sugar Beet
Surface (acres)	120	80	300
Scenario 1 (normal) Yield (T) Sales (T) Purchase (T)	 2.5 100 0	 3 0 0	 20 6000
Scenario 1 (above) Yield (T) Sales (T) Purchase (T)	 3 160 0	 3.6 48 0	 24 6000
Scenario 1 (under) Yield (T) Sales (T) Purchase (T)	 2 40 0	 3.4 0 48	 16 4800

Profit : 107 240

Perfect Information solution

The Wait-and-See (WS) value measures the outcome when decisions are made with full knowledge of the realization of uncertain parameters.

Best possible outcome achievable under perfect information.

$$\boxed{\mathrm{WS} = \mathbb{E}\left[\min_{x} f(x, \xi)\right]}$$

Perfect Information solution

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	183.33	66.67	250
Yield (T)	550	240	6000
Sales (T)	350	–	6000
Purchase (T)	–	–	–
Overall profit: $167,667			

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	100	25	375
Yield (T)	200	60	6000
Sales (T)	–	–	6000
Purchase (T)	–	180	–
Overall profit: $59,950			

WS= $\frac{1}{3}118{,}600 + \frac{1}{3}167{,}667 + \frac{1}{3}59{,}950 = 115{,}406$

Expected Value of Perfect Information (EVPI)

Measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future.

$$\boxed{EVPI = RP - WS}$$

Example: farmer's problem

Recourse problem solution:

		Wheat	Corn	Sugar Beets
First Stage	Area (acres)	170	80	250
$s = 1$ Above	Yield (T)	510	288	6000
	Sales (T)	310	48	6000 (favor. price)
	Purchase (T)	–	–	–
$s = 2$ Average	Yield (T)	425	240	5000
	Sales (T)	225	–	5000 (favor. price)
	Purchase (T)	–	–	–
$s = 3$ Below	Yield (T)	340	192	4000
	Sales (T)	140	–	4000 (favor. price)
	Purchase (T)	–	48	–
Overall profit: $108,390				

EVPI = RP - WS = 115,406 – 108,390 = 7016

Value of the stochastic solution

How much is it worth to use the *SP* approach versus the *EV* approach?

$$\boxed{VSS = RP - EEV}$$

Example: farmer's problem

VSS = RP - EV = 108,390 – 107,240 = 1150

Chance constraints

Chance constraint example

Consider two six-sided dice:

- Dice 1 gives a result $a_1$when thrown

  $a_1= I$ (i= 1, …, 6) with probability 1/6
- Dice 2 yields a2

  $a_2= j$ (j= 1, …, 6) with probability 1/6

Chance constraint example

$$\begin{aligned} \min \quad & 5x + 6y \\ \text{s.t.} \quad & a_1 x + a_2 y \geq 3 \\ & x, y \geq 0 \end{aligned}$$

What does this LP mean?
If a_1 and a_2 were known numbers we would know exactly, but they are not.

Chance constraint example

<u>Interpretation 1</u>: we wish the constraint $a_1 x + a_2 y \geq 3$ to hold for all possible values of a_1 and a_2

$$Min\ 5x + 6y$$

s.t.

$$ix + jy \geq 3 \qquad \forall\, i \in \{1, \dots, 6\}, j \in \{1, \dots, 6\}$$

$$x, y \geq 0$$

→ **Robust optimization**

Chance constraint example

Interpretation 2: Constraint. $a_1 x + a_2 y \geq 3$ holds with a specified probability 1–*α*. (where 0 < *α* < 1)

- e.g., $\alpha = 0.05$ would mean that we want the constraint to hold with $p = 0.95$
- A constraint need not always be true now, rather only 95% of the time

Chance constraint example

$$
\begin{aligned}
\min \quad & 5x + 6y \\
\text{s.t.} \quad & \Pr\left(a_1 x + a_2 y \geq 3\right) \geq 1 - \alpha \\
& x, y \geq 0
\end{aligned}
$$

Chance constraint example

Each pair of values (a_1, a_2) has joint probability 1/36

Given values for $x \geq 0$ and $y \geq 0$, we can easily check whether the constrain is true with probability $1 - \alpha$

Chance constraint example

Let us enumerate possibilities for x= 0, y= 1, and α= 0.05

a_1	a_2	$xa_1 + ya_2 >= 3$?	Probability?
1	1	False	1/36
2	1	False	

- x= 0 and y=1 is not feasible

- –Probability of 2/36 = 0.0555 that the constraint is infeasible
- –1 –0.0555 = 0.9455 < 0.95

Chance constraint example

Because it contains just two variables, this problem can be easily solved by a simple numeric search procedure

- for $\alpha = 0.01$, the solution is $x = 3$, $y = 0$

- for $\alpha = 0.05$, the solution is $x = 1$, $y = 1$

Chance constraint example

This problem is an example of a *stochastic (linear) program with probabilistic constraints*

- a.k.a. *chance-constrained linear program*

This simple example can be extended to:

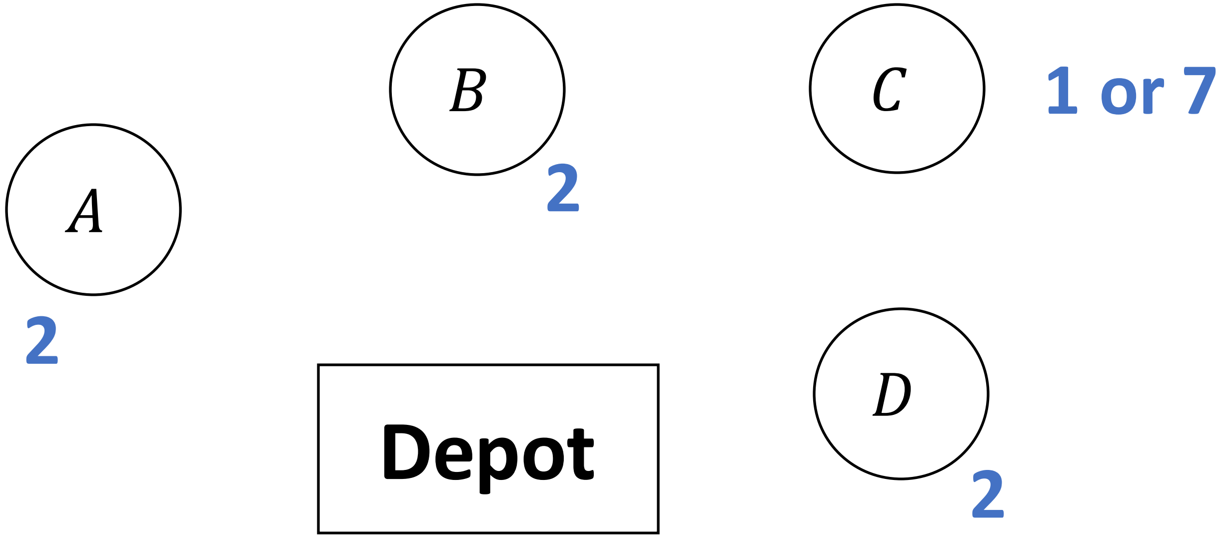
- Mix of probabilistic and deterministic coefficients or constraints in the same problem

Chance constraint based on scenario

- Efficient when all scenarios can be generated.
- Sampling work as well.
- However, the constraint is approximated, and we cannot ensure the constraint trully respect the given probability limit.

Exercice

A single vehicle, starting and ending at a depot, must visit 4 customers (A,B,C,D), where each customer has a specified quantity of items to be picked up. *The demand of clients A , B and D are known and equal to 2. Demand of client C is random. To put things to the extreme, assume that the demand of C is either 1 or 7 with equal probability 1/2. The vehicle has a capacity of 10, and it must serve all demands.*



	0	A	B	C	D
0	0	2	4	4	1
A	2	0	3	4	2
B	4	3	0	1	3
C	4	2	1	0	3
D	1	1	3	3	0

Distances matrix

Exercice

(1) Compute the wait and see solution

(2) Compute the expected value solution

(3) Compute the expectation of the expected value problem

(4) Compute the expected cost of the recourse problem. We consider, the travellers can change the route after visiting client C.

(5) Compute EVPI, VSS, and verifies that $WS \leq RP \leq$ EEP

(6) Compute the optimal solution for the chance constrain version of the problem where the route must be feasible with probability 95%.

The news vendor problem

A news vendor must decide the amount x of new paper to order for the next day. Each new paper cost to the vendor $\alpha = 1.00$, and he sells them at the price $\alpha + \beta = 1.50$. The unsold papers can be sent to recycling with a refund $\gamma = 0.1$. The new vendor aims to minimize the cost:

$$C(x,\xi) = \begin{cases} -0.5x, & \text{if } x < \xi \\ 0.9x - 1.4\xi, & \text{otherwise.} \end{cases}$$

Where ξ denotes the number of paper sold

Extension of the news vendor

The cost vary a lot for different demand outcome.

To protect against the risk the news vendor may want to impose a constraint: The cost must be lower than τ with probability α

We assume the demand follows a normal distribution

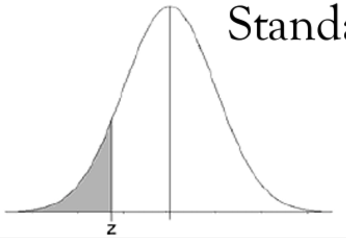
$$\xi \sim \mathcal{N}(\mu, \sigma^2)$$

Extension of the news vendor

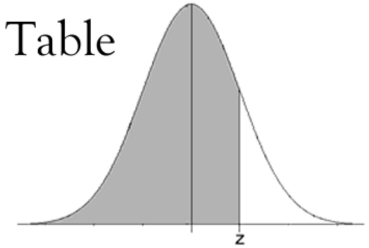
That is, the new vendor impose the following constraint :

$$\mathbb{P}(C(x,\xi) \leq \tau) \geq 1 - \alpha$$

We can reformulate this constraint



Standard Normal Distribution Probabilities Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Exercice : Passenger Seat Allocation

You are managing seat allocation for an airline on a flight with a maximum seating capacity of $C=180$ seats. The number of passengers who actually show up for the flight is uncertain and follows a normal distribution with a mean equal to the number of seat sold and a standard deviation of $\sigma=15$.

The airline can overbook seats to maximize revenue, but if more passengers show up than the available seats, the airline must pay a penalty of \$300 for each passenger who cannot be accommodated. Each seat sold generates a revenue of \$200.

Your goal is to determine the number of tickets to sell to maximize the airline's expected profit while ensuring that the probability of accommodating all passengers is at least 90%. Additionally, the airline has a policy that it cannot sell more than 10% over the flight's seating capacity.

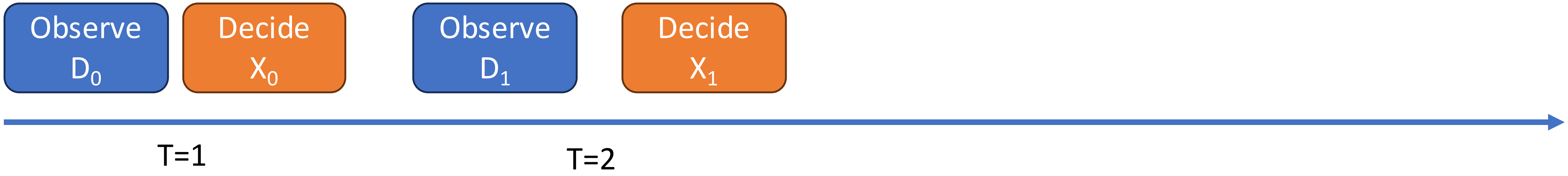
Write the Optimization Problem: objective function, the reformulated chance constraint, and the non-probabilistic constraint

Multi-stage

Two-stage with simple recourse

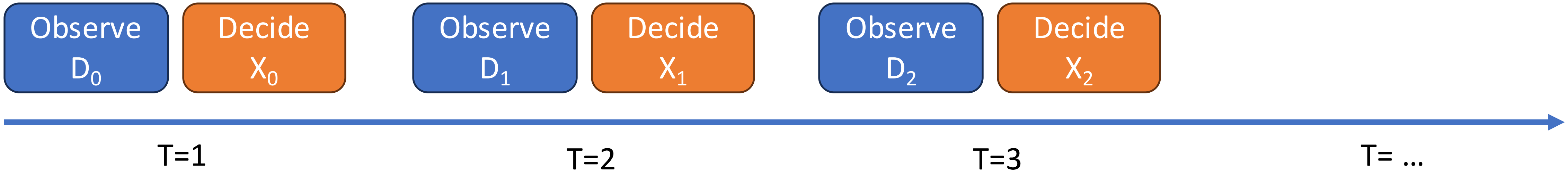
Example: inventory management

Two-stage with recourse



Multi-stage decision process

Sequential decision making

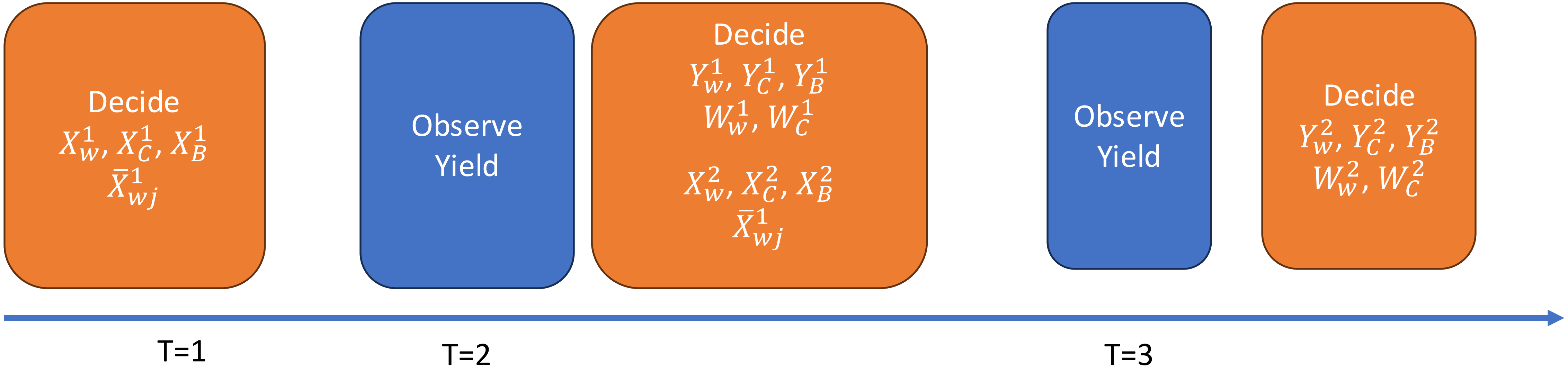


Extension of the farmer’s problem

The farmer uses crop rotation to maintain good soil quality. Sugar beets would, for example, appear in triennial crop rotation, which means they are planted on a given field only one out of three years. Formulate a multistage program to describe this situation. To keep things simple, describe the case when sugar beets cannot be planted two successive years on the same field, and assume no such rule applies for wheat and corn.

In terms of formulation, it is sufficient to consider a three-stage model. The first stage consists of first-year planting. The second stage consists of first year purchases and sales and second-year planting. The third-stage consists of second-year purchases and sales. Alternatively, a four-stage model can be built, separating first-year purchases and sales from second-year planting. Also discuss the question of discounting the revenues and expenses of the various stages.)

Extension of the farmer’s problem : decision process



The plan is cyclic, so the third year rotation is the solution to the first year problem

Non-anticipiativity constraints

$$X_{i1}^{\omega} = X_{i1}^{\omega}, \quad \forall i \in \{W, C, B\}, \forall t \in T.$$

Non-anticipiativity constraints

Decisions in the current stage cannot depend on future realizations of uncertainties

$$X_{it}^{\omega} = X_{it}^{\omega}, \quad \forall \omega, \omega' \text{ with shared history}, \forall i \in \{W, C, B\}, \forall t \in T.$$

Non-anticipiativity constraints

Decisions in the current stage cannot depend on future realizations of uncertainties

$$X_{it}^{\omega} = X_{it}^{\omega}, \quad \forall \omega, \omega' | R_{i\tau}^{\omega} = R_{i\tau}^{\omega'}, \tau \in 1, \ldots, t-1,$$
$$\forall i \in \{W, C, B\}, \forall t \in T.$$

Must be added for all variables ($\bar{X}, Y, Z$)

Exercice: chemical process

A company operates a chemical processing network consisting of three production processes ($i=1,2,3$) and produces two chemical products ($j=1, 2$). The objective is to determine the optimal capacities to install in each process and their operating levels over a two-period planning horizon ($t=1,2$) to maximize the company's expected profit. The profit is the difference between revenue from selling chemical products and the costs of capacity installation and operation.

Each process has an initial installed capacity. In each period, the company may expand the capacity of the processes at a specified cost. The operating level of each process cannot exceed the available installed capacity in that period. The company incurs operating costs proportional to the operating level. Additionally, each process contributes to the production of one or both chemical products. The amount of each chemical produced depends on the operating level of the processes, as defined by fixed production coefficients.

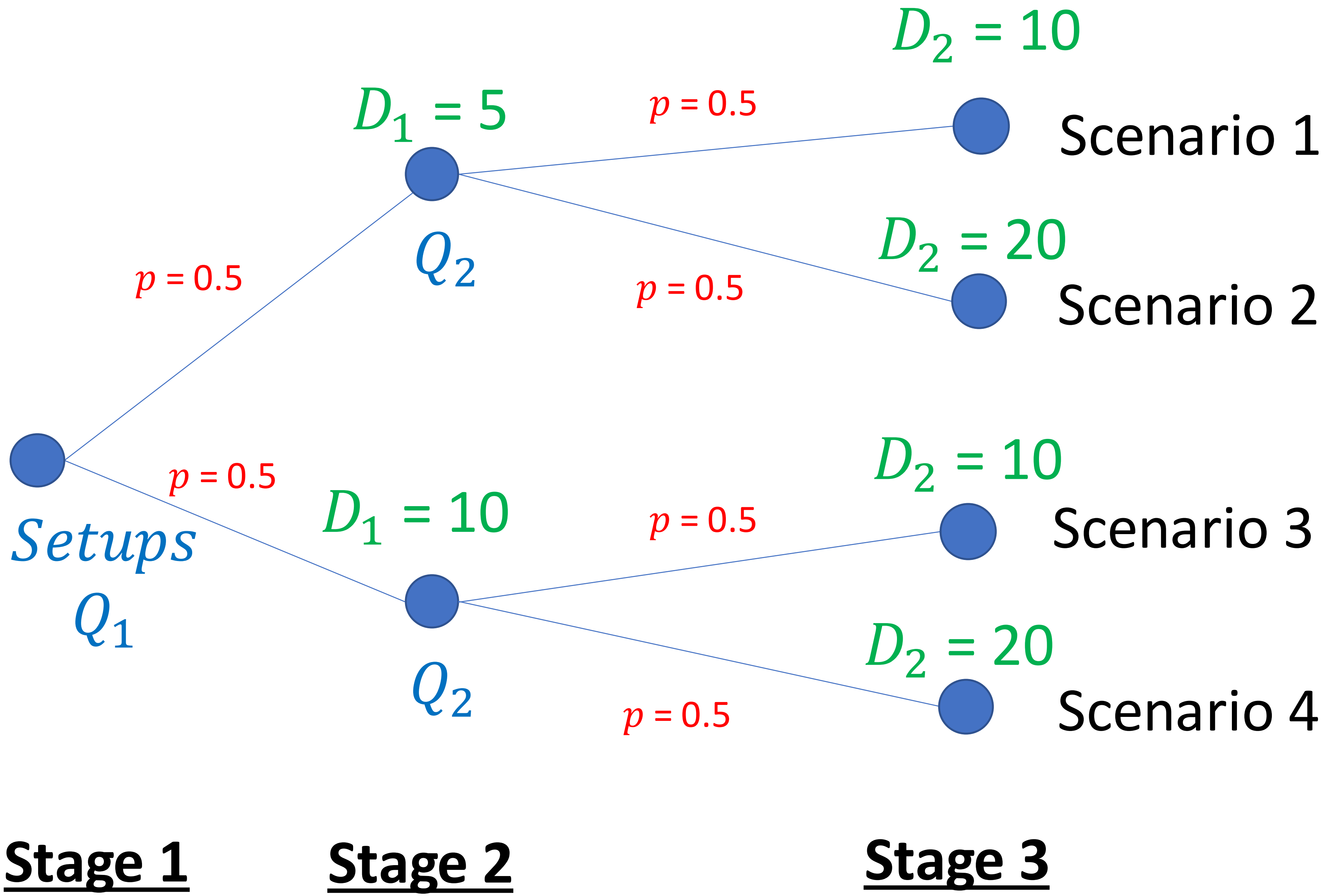
Chemical products are sold in the market at fixed prices in each period, but their production must meet demand constraints. Specifically, the production of each chemical product must fall within lower and upper demand bounds specified for each period. The company's goal is to decide: (1) the capacity expansions for each process in each period; (2) the operating levels of each process in each period.

Exercice: chemical process

Formulate the mathematical model to determine the optimal decisions, given the following parameters:

- Initial installed capacity for each process (x_{i0}).
- Cost per unit of capacity installed for each process in each period (c_{it}).
- Operating cost per unit for each process in each period (q_{it}).
- Sales price per unit of each chemical product in each period (r_{jt}).
- Production coefficients (a_{ij}): the quantity of chemical j produced per unit operating level of process i.
- Demand bounds for each chemical product (D_{jt}^{min}, $D_{j,t}^{max}$).

Sampling Scenario tree.


D_2 = 10
D_1 = 5
p = 0.5
Scenario 1
D_2 = 20
Q_2
p = 0.5
p = 0.5
Scenario 2
D_2 = 10
p = 0.5
Scenario 3
D_1 = 10
p = 0.5
Setups
Q_1
D_2 = 20
Q_2
p = 0.5
Scenario 4
Stage 1
Stage 2
Stage 3

Sources:

- www.stoprog.org

- J. Birge & F. Louvaux (1997) Introduction to Stochastic Programming. *Springer*

Additional exercice.

Exercice: Technician Planning and Routing

- A company employs technicians who perform on-site maintenance tasks. At the start of the week, the company needs to decide where each technician will be assigned to begin their day (home bases) and their initial schedule. During the week, customer requests for maintenance arise (randomly), and the company needs to plan the routing of technicians to minimize total travel and waiting time.
- This problem has two stages:

1. **First Stage (Decisions Before Uncertainty):** Decide the initial home base locations for technicians.
2. **Second Stage (Recourse Decisions After Uncertainty):** Determine the which technicians attend customer requests.

Sets:

- T: Set of technicians ($t \in T$).
- H: Set of home bases ($h \in H$).
- C: Set of customer locations ($c \in C$).
- Ω: Set of scenarios ($\omega \in \Omega$).

Parameters:

- c_{th}: Cost of assigning technician t to home base h.
- r_{hc}^{ω}: Cost of routing a technician from base h to customer c in scenario s.
- Δ_{hc}^{ω}: Duration (in hours) of routing a technician from base h to customer c in scenario s.
- d_{c}^{ω}: The demand of customer c (in working hours) in scenario s.
- P_{ω}: Probability of scenario ω.