GATE-2024

Questions with Detailed Solutions

ELECTRONICS & COMMUNICATION ENGINEERING

SUBJECTWISE WEIGHTAGE

S.No.	Name of the Subject	One Mark Questions	Two Marks Questions	No. Of Questions
1	Verbal Ability	1	1	2
2	Numerical Ability	4	4	8
3	Engineering Mathematics	3	4	7
4	Networks	3	2	5
5	Signals & Systems	2	4	6
6	Electronic Devices	2	5	7
7	Analog Circuits	3	3	6
8	Digital Circuits and Computer Organization	2+1	4	7
9	Control Systems	2	3	5
10	Communications	5	3	8
11	Electromagnetics	2	2	4
Total No. Of Questions		30	35	65

(GENERAL APTITUDE)

Questions-01 to 05 Carry ONE mark each.

01. Sequence the following sentences (P. Q. R. S) in a coherent passage:

P: Shifu's student exclaimed. "Why do you run since the bull is an illusion?"

Q: Shifu said, "Surely my running away from the bull is also an illusion."

R: Shifu once proclaimed that all life is illusion.

S: One day, when a bull gave him chase, Shifu began running for his life.

(a) RSPQ (b) RPQS
(c) SPRQ (d) SRPQ

01. Ans: (a)

02. If '→' denotes increasing order of intensity, then the meaning of the words [charm→ enamor→ bewitch] is analogous to [bored→ _____ → weary]. Which one of the given options is appropriate to fill the blank?

(a) baffled (b) jaded
(c) worsted (d) dead

02. Ans: (b)

Sol: Jaded means fatigued by over work

03. Two identical sheets A and B, of dimensions 24 cm × 16 cm, can be folded into half using two distinct operations. FO1 or FO2.

In FO1, the axis of folding remains parallel to the initial long edge, and in FO2, the axis of folding remains parallel to the initial short edge.

If sheet A is folded twice using FO1, and sheet B is folded twice using FO2, the ratio of the perimeters of the final shapes of A and B is

(a) 18:11 (b) 11:18
(c) 14:11 (d) 11:14

03. Ans: (c)

Sol: After folding sheet A using F01 twice the dimensions of final shape 24×4

After folding sheet B using F02 twice the dimensions of final shape are 6×16

The ratio of perimeters of final shapes is

$2\,(24 + 4) : 2\,(6 + 16)$

$28 : 22$

$14 : 11$

04. For a real number $x > 1$,

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = 1$$

The value of x is

(a) 12 (b) 36 (c) 24 (d) 4

04. Ans: (c)

Sol: $\log_x 2 + \log_x 3 + \log_x 4 = 1$

$\log_x(2\times3\times4) = 1$

$\log_x(24) = 1$

$\Rightarrow x^1 = 24$

$\Rightarrow x = 24$

05. Five years ago, the ratio of Aman's age to his father's age was 1:4, and five years from now, the ratio will be 2:5. What was his father's age when Aman was born?

(a) 35 years (b) 28 years
(c) 30 years (d) 32 years

05. Ans: (c)

Sol: $\dfrac{A-5}{f-5} = \dfrac{1}{4} \Rightarrow 4A - 20 = F - 5$

$4A - 15 = F \Rightarrow (1)$

$\dfrac{A+5}{F+5} = \dfrac{2}{5} \Rightarrow 5A + 25 = 2F + 10$

$5A + 15 = 2F \Rightarrow (2)$

from (1) and (2)

$5A + 15 = 2(4A - 15)$

$5A + 15 = 8A - 30$

$3A = 45$

$A = 15$

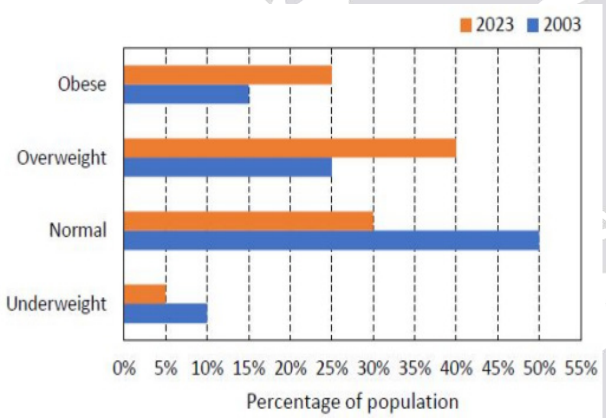
$F = 4A - 15 = 45$

Aman was born 15 years ago.

Hence age of father 15 years ago was 45 – 15 = 30 years.

Questions-06 to 10 Carry TWO marks each.

06. The bar chart shows the data for the percentage of population falling into different categories based on Body Mass Index (BMI) in 2003 and 2023.



Based on the data provided, which one of the following options is INCORRECT?

(a) The ratio of the percentage of population falling into underweight category to the percentage of population falling into normal category has decreased in 20 years.

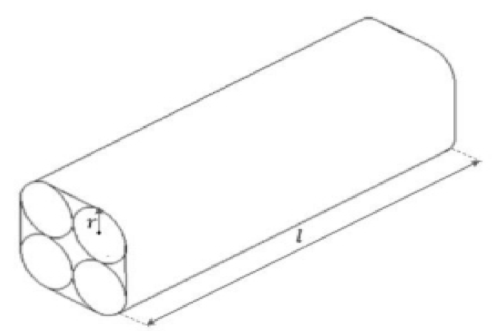
(b) The ratio of the percentage of population falling into obese category to the percentage of population falling into normal category has decreased in 20 years.

(c) The percentage of population falling into normal category has decreased in 20 years.

(d) The ratio of the percentage of population falling into overweight category to the percentage of population falling into normal category has increased in 20 years.

06. Ans: (b)

07. Four identical cylindrical chalk-sticks, each of radius r = 0.5 cm and length l = 10 cm. are bound tightly together using a duct tape as shown in the following figure.



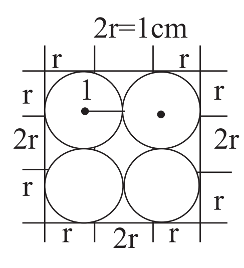
The width of the duct tape is equal to the length of the chalk-stick. The area (in cm^2) of the duct tape required to wrap the bundle of chalk-sticks once, is

(a) $20(4+\pi)$ (b) $10(8+\pi)$

(c) $20(8+\pi)$ (d) $10(4+\pi)$

07. Ans: (d)

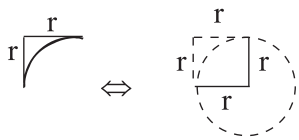
Sol: Square = 16r = 16(0.5) = 8cm



Remove edges length square = 8 – (8r)

= 8 – 4 = 4 - - - (1)

Add the length of arc in edges



length of arc $= \frac{90}{360} \times 2\pi r = \frac{1}{4} \times 2 \times \pi \times 0.5 = \frac{\pi}{4}$

four such arcs need to be added

$= 4 \times \frac{\pi}{4} = \pi$ - - - (2)

Total length of tape required = (1) + (2) = 4 + π

Area of tape = 10 (4 + π) cm^2

08. P. Q, R, S, and T have launched a new startup. Two of them are siblings. The office of the startup has just three rooms. All of them agree that the siblings should not share the same room.
If S and Q are single children, and the room allocations shown below are acceptable to all.

PR	TS	Q

PQ	RT	S

then, which one of the given options is the siblings?

(a) P and T (b) T and Q
(c) P and S (d) T and R

08. Ans: (a)

Sol: QS S and Q are single children, they do not have siblings.

out of P, R and T two of them must be siblings.

In first given allocation [PR | TS | Q] as P is sharing room with R, so P and R can not be siblings

Either PT or RT are siblings.

In second given allocation [PQ | RT | S] as R is sharing room with T, so R and T can not be siblings

So P and T must be siblings.

hence option (a).

09. The greatest prime factor of $(3^{199} - 3^{196})$ is

(a) 11 (b) 13 (c) 3 (d) 17

09. Ans: (b)

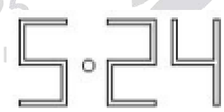
Sol: $(3^{199} - 3^{196}) = 3^{196}(3^3 - 1) = 3^{196} \times 26 = 3^{196} \times 2 \times 13$

greatest prime factor of above product is 13.

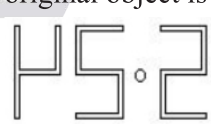
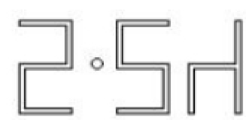
10. Examples of mirror and water reflections are shown in the figures below:

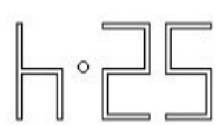
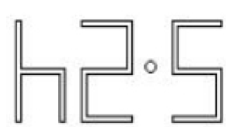
MIRROR REFLECTION WATER REFLECTION

An object appears as the following image after first reflecting in a mirror and then reflecting on water.



The original object is

(a)  (b) 

(c)  (d) 

10. Ans: (d)

(TECHNICAL)

Questions-01 to 25 Carry ONE marks each.

01. For a causal discrete-time LTI system with transfer function: $H(z) = \dfrac{2z^2+3}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)}$

Which of the following statements is/are true?

(a) The initial value of the impulse response is 2.

(b) The system is stable.

(c) The system is a minimum phase system.

(d) The final value of the impulse response is 0.

01. Ans: (a, b & d)

Sol: $H(z) = \dfrac{2z^2+3}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{3}\right)}$

(a) $h(0) = \underset{z\to\infty}{\mathrm{Lt}}\, H(z) = \underset{z\to\infty}{\mathrm{Lt}} \dfrac{z^2(2+3z^{-2})}{z^2\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} = 2$

(b) For causal & stable system |poles| < 1

poles are at $z = \frac{1}{3}$ & $-\frac{1}{3}$

(c) zeros at $z^2 = -\frac{3}{2} \Rightarrow z = \pm j\sqrt{\frac{3}{2}} = \pm j\sqrt{1.5}$

$2z^2+3 = 0$

$2z^2 = -3$

not minimum phase system

(d) $h(\infty) = \underset{z\to 1}{\mathrm{Lt}}(1-z^{-1})H(z) = 0$

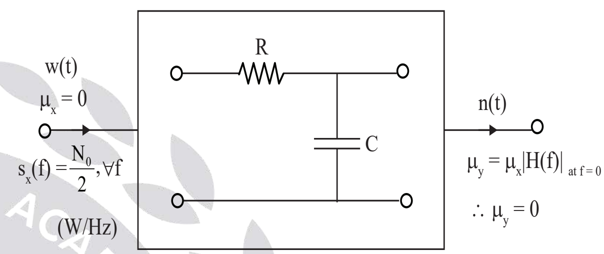
02. A white Gaussian noise w(t) with zero mean and power spectral density $\frac{N_0}{2}$, when applied to a first order RC low pass filter produces an output n(t). At a particular time $t = t_k$, the variance of the random variable $n(t_k)$ is _______ .

(a) $\dfrac{N_0}{2RC}$ (b) $\dfrac{2N_0}{RC}$

(c) $\dfrac{N_0}{4RC}$ (d) $\dfrac{N_0}{RC}$

02. Ans: (c)

Sol:



$H(f) = \dfrac{1}{1+j\left(\frac{f}{f_c}\right)}$

$|H(f)|^2 = \dfrac{1}{1+\left(\frac{f}{f_c}\right)^2}, \forall f$

where $f_c = \dfrac{1}{2\pi RC}$ Hz

Variance of $n(t_k)$ = power of n(t)

$= \int S_y(f)\,df$

$= \int S_x(f)\,|H(f)|^2 df$

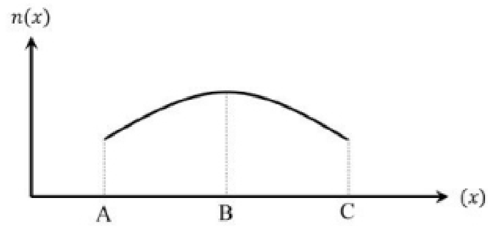
$= \int_{-\infty}^{\infty} \dfrac{N_0}{2} \dfrac{1}{1+\left(\frac{f}{f_c}\right)^2} df$

$= \dfrac{N_0}{2} \times f_c \times \pi$

$= \dfrac{N_0}{2} \times \dfrac{1}{2\pi RC} \times \pi$

$= \dfrac{N_0}{4RC}$ Watts

03. The free electron concentration profile n(x) in a doped semiconductor at equilibrium is shown in the figure, where the points A, B, and C mark three different positions. Which of the following statements is/are true?



(a) For x between B and A, the electric field is directed from A to B.
(b) For x between B and C, the electric field is directed from B to C.
(c) For x between B and A, the electron drift current is directed from B to A.
(d) For x between B and C, the electron diffusion current is directed from C to B.

03. Ans: (b, c & d)

Sol: Since electron concentration gradient $\Rightarrow$ Diffusion of carrier exist from B $\to$ A & B $\to$ C

$\Rightarrow$ Diffusion current exist between
A $\to$ B & C $\to$ B

Since at equilibrium balance diffusion
$\Rightarrow$ Drift current exist from
B $\to$ A & B $\to$ C

As Drift is due to electron field $\Rightarrow$ $\overline{E}$ directs from B $\to$ A & B $\to$ C.

04. A machine has a 32-bit architecture with 1-word long instructions. It has 24 registers and supports an instruction set of size 40. Each instruction has five distinct fields, namely opcode, two source register identifiers, one destination register identifier, and an immediate value. Assuming that the immediate operand is an unsigned integer, its maximum value is ____ .

04. Ans: 2047

Sol: Machine has 32 bit architecture
$\Rightarrow$ i.e 1 word = 32 bits
* No of registers = 24
$\Rightarrow$ Thus no of bits required to select one register is $2^n \geq 24$ n = 5bits
* Instruction set size = 40
$\Rightarrow$ No of bits required for opcode is $2^m \geq 40$ m = 6bits
* Instruction size = 1 word i.e 32bits
Given instruction format is

opcode	Source reg 1	Source reg 2	Destination reg	Immediate value

Given opcode requires 6bits and source and destination register requires 5bits each.
Thus, 32 = 6 + 5 + 5 + 5 + immediate value, N
i.e. immediate value, N = 32 – 21
N = 11bits
* We know, using N bits, the range of unsigned numbers is 0 to $2^N - 1$
* Hence max value of immediate operand having 11 bits = $2^{11} - 1$
= 2048 – 1 = 2047

05. In the context of Bode magnitude plots, 40 dB/decade is the same as _______.
(a) 20 dB/octave (b) 12 dB/octave
(c) 6 dB/octave (d) 10 dB/octave

05. Ans: (b)

Sol: ±20ndB/decade equal to ±6ndB/octave
+40dB/decade is same as +12dB/octave

06. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x and y axes, respectively and let A be a positive constant. Which one of the following statements is true for the vector fields $\vec{F}_1 = A(\hat{i}y + \hat{j}x)$ and $\vec{F}_2 = A(\hat{i}y - \hat{j}x)$?

(a) Both $\vec{F}_1$ and $\vec{F}_2$ are electrostatic fields.

(b) Only $\vec{F}_1$ is an electrostatic field.

(c) Neither $\vec{F}_1$ nor $\vec{F}_2$ is an electrostatic field.

(d) Only $\vec{F}_2$ is an electrostatic field.

06. Ans: (b)

Sol: $\overline{F}_1 = A(\hat{i}y + \hat{j}x), \overline{F}_2 = A(\hat{i}y - \hat{j}x)$

Electrostatic field is conservative field

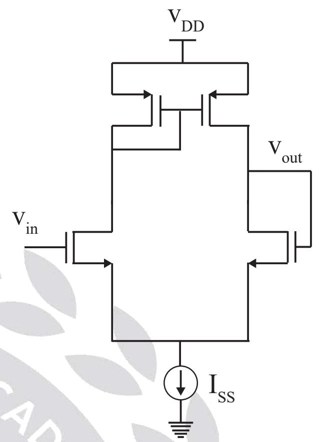
i.e, $\nabla \times \overline{E} = 0$

$$\nabla \times \overline{F}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & Ax & 0 \end{vmatrix} = \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(A-A) = 0$$

$$\nabla \times \overline{F}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & -Ax & 0 \end{vmatrix} = \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(-A-A) \neq 0$$

So only $\overline{F}_1$ is an Electrostatic field.

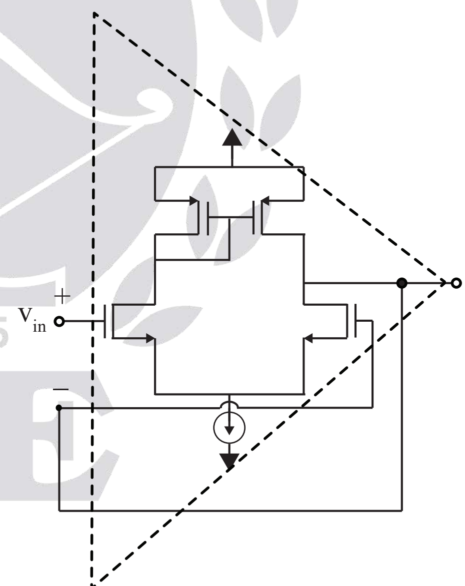
07. For the closed loop amplifier circuit shown below, the magnitude of open loop low frequency small signal voltage gain is 40. All the transistors are biased in saturation. The current source I_{SS} is ideal. Neglect body effect, channel length modulation and intrinsic device capacitances. The closed loop low frequency small signal voltage gain $\frac{V_{out}}{V_{in}}$ (rounded off to three decimal places) is ____

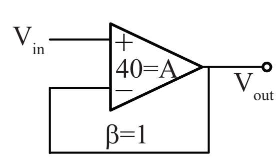


(a) 1.025 (b) 1.000 (c) 0.976 (d) 0.488

07. Ans: (c)

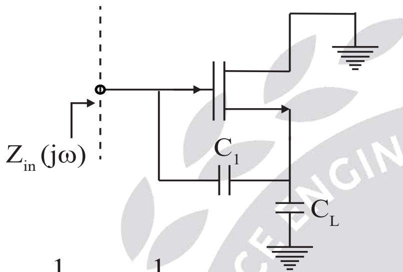
Sol:





$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + A\beta} = \frac{40}{1+40} = \frac{40}{41} = 0.976$$

08. In the circuit below, assume that the long channel NMOS transistor is biased in saturation. The small signal trans-conductance of the transistor is g_m. Neglect body effect. channel length modulation and intrinsic device capacitances. The small signal input impedance $Z_{in}(j\omega)$ is _____ .



(a) $\frac{g_m}{C_1C_L\omega^2}+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_L}$

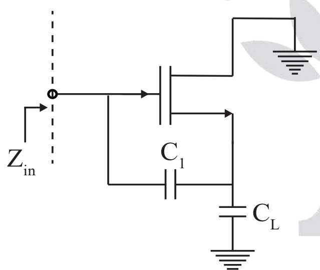
(b) $\frac{-g_m}{C_1C_L\omega^2}+\frac{1}{j\omega C_1+j\omega C_L}$

(c) $\frac{1}{j\omega C_1}+\frac{1}{j\omega C_L}$

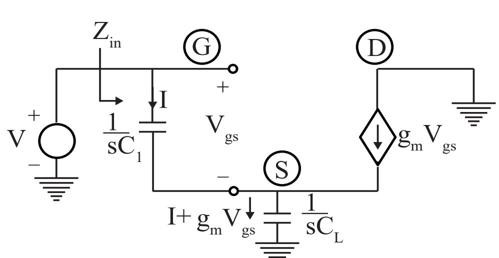
(d) $\frac{-g_m}{C_1C_L\omega^2}+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_L}$

08. Ans: (d)

Sol:



Replace the MOSFET with the small signal model



KVL

$$-V+I\left(\frac{1}{sC_1}\right)+(I+g_mV_{gs})\left(\frac{1}{sC_L}\right)=0 \text{ - - - - (1)}$$

$$\text{But } V_{gs}= I\left(\frac{1}{sC_1}\right) \text{ - - - - (2)}$$

Sub (2) in (1)

$$-V+I\left(\frac{1}{sC_1}\right)+I\left(\frac{1}{sC_L}\right)+g_m\left(\frac{1}{sC_L}\right)I\left(\frac{1}{sC_1}\right)=0\text{- - - (2)}$$

$$\frac{V}{I}=\frac{1}{sC_1}+\frac{1}{sC_L}+\frac{g_m}{s^2C_1C_L}$$

put $s=j\omega$

$s^2=-\omega^2$

$$\therefore Z_{in}=\frac{V}{I}=\frac{1}{j\omega C_1}+\frac{1}{j\omega C_L}-\frac{g_m}{\omega^2C_1C_L}$$

09. For the Boolean function

$F(A, B, C, D)=\sum m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$

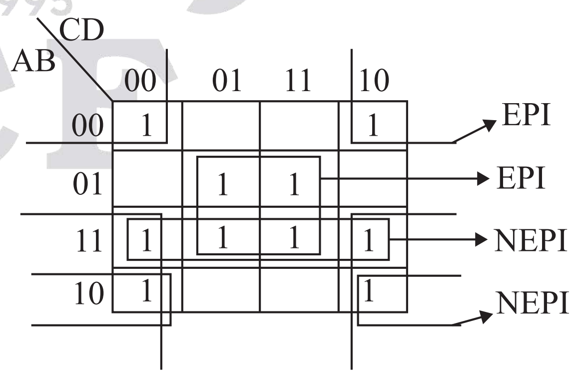
the essential prime implicants are _______

(a) BD, AB (b) BD, $\overline{B}\,\overline{D}$

(c) AB, $\overline{B}\,\overline{D}$ (d) BD, $\overline{B}\,\overline{D}$, AB

09. Ans: (b)

Sol: $F(A,B,C,D)=\sum m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$



The essential prime implicants are BD & $\overline{B}\,\overline{D}$

10. A digital communication system transmits through a noiseless band-limited channel [–W, W]. The received signal z(t) at the output of the receiving filter is given by $z(t) = \sum_{n} b[n]\, x[t-nT]$, where b[n] are the symbols and x(t) is the overall system response to a single symbol. The received signal is sampled at t = mT. The Fourier transform of x(t) is X(f). The Nyquist condition that X(f) must satisfy for zero intersymbol interference at the receiver is ______.

(a) $\sum_{m=-\infty}^{\infty} X\left(f+\frac{m}{T}\right) = \frac{1}{T}$ (b) $\sum_{m=-\infty}^{\infty} X(f+mT) = \frac{1}{T}$

(c) $\sum_{m=-\infty}^{\infty} X\left(f+\frac{m}{T}\right) = T$ (d) $\sum_{m=-\infty}^{\infty} X(f+mT) = T$

10. Ans: (c)

Sol: $z(t) = \sum_{n=-\infty}^{\infty} b[n]\, x[t-nT]$

$x(t) \rightarrow X(f)$

For ZERO ISI

$\frac{1}{T}\sum_{m=-\infty}^{\infty} X[f-m/T] = x(t)_{\text{at } t=0}$

(or) $\sum_{m=-\infty}^{\infty} X[f+m/T] = T\,x(0)$ $_{\text{Assuming } x(0)=1}$

$\sum_{m=-\infty}^{\infty} X[f+m/T] = T$

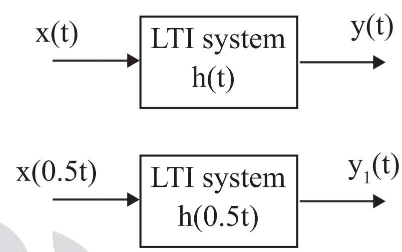
11. A causal and stable LTI system with impulse response h(t) produces an output y(t) for an input signal x(t). A signal x(0.5t) is applied to another causal and stable LTI system with impulse response h(0.5t). The resulting output is ______ .

(a) 4y(0.5t) (b) 0.25y(2t)

(c) 0.25y(0.25t) (d) 2y(0.5t)

11. Ans: (d)

Sol:



$x(\alpha t) * h(\alpha t) = \frac{1}{|\alpha|} y(\alpha t)$ $\alpha = 0.5$

$x(0.5t) * h(0.5t) = \frac{1}{|0.5|} y(0.5t) = 2y(0.5t)$

12. A source transmits symbols from an alphabet of size 16. The value of maximum achievable entropy (in bits) is ________ .

12. Ans: 4

Sol: $H_{max} = \log_2(16) = 4$ bits

13. An amplitude modulator has output (in Volts)
$s(t) = A\cos(400\pi t) + B\cos(360\pi t) + B\cos(440\pi t)$.
The carrier power normalized to 1 Ω resistance is 50 Watts. The ratio of the total sideband power to the total power is 1/9. The value of B (in Volts, rounded off to two decimal places) is ______ .

13. Ans: 2.5

Sol: $s(t) = A\cos(400\pi t) + B\cos(360\pi t) + B\cos(440\pi t)$

$P_C = \frac{A_C^2}{2} = \frac{A^2}{2} = 50$

$\therefore A = 10$ Volts

$\frac{\mu^2}{2+\mu^2} = \frac{1}{9} \Rightarrow 9\mu^2 = 2+\mu^2 \Rightarrow 8\mu^2 = 2$

$\mu^2 = \frac{1}{4} \Rightarrow \therefore \mu = \frac{1}{2}$

$B = \frac{A_C\mu}{2} = \frac{10\times 1}{2\times 2} = 2.5$ Volts

14. The general form of the complementary function of a differential equation is given by $y(t) = (At + B)e^{-2t}$, where A and B are real constants determined by the initial condition. The corresponding differential equation is _____.

(a) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

(b) $\frac{d^2y}{dt^2} + 4y = f(t)$

(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$

(d) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t)$

14. Ans: (a)

Sol: The Auxiliary equation of the differential equation

$y'' + 4y' + 4y = f(t)$ is $m^2 + 4m + 4 = 0$

$\Rightarrow m = -2, -2$

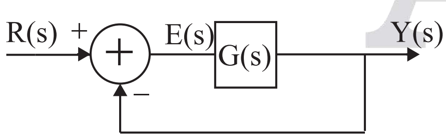
$\therefore$ The complementary function is

$y = y(t) = (A + Bt)e^{-2t}$

(or)

$y = y(t) = (At + B)e^{-2t}$

15. In the feedback control system shown in the figure below $G(s) = \frac{6}{s(s+1)(s+2)}$



R(s), Y(s) and E(s) are the Laplace transforms of r(t), y(t) and e(t), respectively.

If the input r(t) is a unit step function, then ___ .

(a) $\lim_{t\to\infty} e(t) = 0$

(b) $\lim_{t\to\infty} e(t)$ does not exist, e(t) is oscillatory

(c) $\lim_{t\to\infty} e(t) = \frac{1}{3}$

(d) $\lim_{t\to\infty} e(t) = \frac{1}{4}$

15. Ans: (b)

Sol: $G(s) = \frac{6}{s(s+1)(s+2)}$, $H(s) = 1$

Steady state errors are calculated for closed loop stable system.

R-H criteria:

CE $\rightarrow 1 + G(s) = 0$

CE $\rightarrow s^3 + 3s^2 + 2s + 6 = 0$

s^3	1	2
s^2	3	6
s^1	$\cancel{0}$ 6	0
s^0	6	

AE $\rightarrow 3s^2 + 6 = 0 \Rightarrow s^2 + 2 = 0$

$\Rightarrow s = \pm j\sqrt{2}$

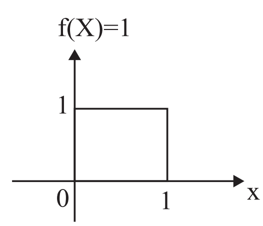
s^1 row: IROZ

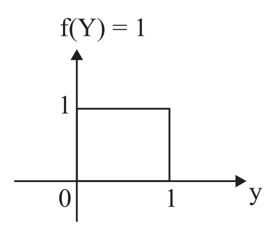
Given system marginal stable. Hence steady state error is doesnot exist. [$e_{ss} = \underset{t\to\infty}{Lt}\, e(t)$ does not exist] response is oscillatory.

16. Suppose X and Y are independent and identically distributed random variables that are distributed uniformly in the interval [0, 1]. The probability that $X \geq Y$ is _______ .

16. Ans: 0.5

Sol:





$f(X, Y) = f(X) \times f(Y)$

Since x and y are independent random variables

$$P[X \geq Y] = P[Y < X] = \int_{X=0}^{1}\int_{Y=0}^{X} f(X,Y)\,dxdy$$

$$= \int_{X=0}^{1}\int_{Y=0}^{X} 1 \times 1\,dydx$$

$$\therefore\ P[X \geq Y] = \int_{X=0}^{1} (Y)_{Y=0}^{X}\,dx = \int_{X=0}^{1} X\,dx = \frac{1}{2} = 0.5$$

17. Let $\rho(x, y, z, t)$ and $u(x, y, z, t)$ represent density and velocity, respectively, at a point (x, y, z) and time t. Assume $\frac{\partial \rho}{\partial t}$ is continuous. Let V be an arbitrary volume in space enclosed by the closed surface S and $\hat{n}$ be the outward unit normal of S Which of the following equations is/are equivalent to $\frac{\partial \rho}{\partial t}dv + \nabla.(\rho u) = 0$

(a) $\int_v \frac{\partial \rho}{\partial t}dv = \int_v \nabla.(\rho u)dv$

(b) $\int_v \frac{\partial \rho}{\partial t}dv = \oint_s \rho u.\hat{n}\,ds$

(c) $\int_v \frac{\partial \rho}{\partial t}dv = -\oint_s \rho u.\hat{n}\,ds$

(d) $\int_v \frac{\partial \rho}{\partial t}dv = -\int_v \nabla.(\rho u)dv$

17. Ans: (c & d)

Sol: Given $\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0$

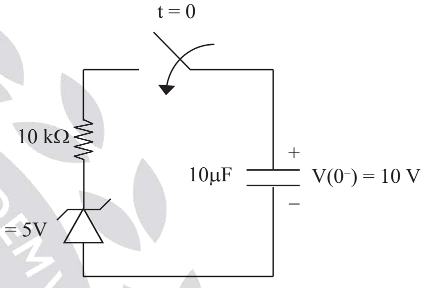
$$\Rightarrow \frac{\partial \rho}{\partial t} = -\nabla.(\rho u)$$

$$\Rightarrow \int_v \frac{\partial \rho}{\partial t}dv = \int_v -\nabla.(\rho u)dv$$

$$= \oint_s -(\rho u).\hat{n}\,ds \text{ (By divergence theorem)}$$

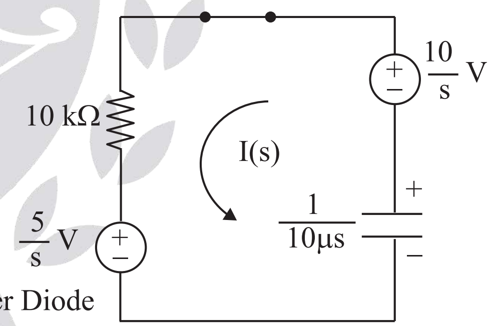
So, (c & d) are correct

18. As shown in the circuit, the initial voltage across the capacitor is 10V, with the switch being open. The switch is then closed at t = 0. The total energy dissipated in the ideal Zener diode (V_Z = 5V) after the switch is closed (in mJ, rounded off to 3 decimal places) is ________.



18. Ans: 0.25

Sol: Using Laplace Transform



By KVL

$$I(s)\left[\frac{1}{10\mu s} + 10000\right] = \frac{5}{s}$$

$$I(s)\left[\frac{1 + 10000 \times 10\mu s}{10\mu s}\right] = \frac{5}{s}$$

$$I(s) = \frac{50\mu}{0.1s + 1} = \frac{500\mu}{s + 10}$$

$$i(t) = L^{-1}[I(s)] = 500\mu \times e^{-10t}\ A$$

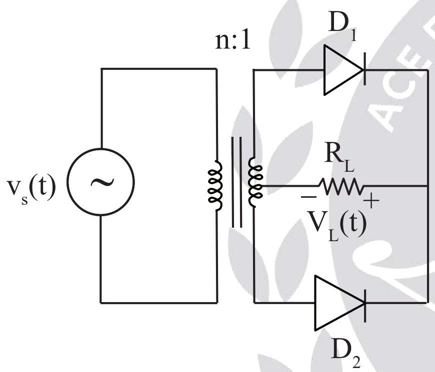
Energy dissipated in ideal zener diode

$$E_z = \int_0^{\infty} 5(500\mu \times e^{-10t})\,dt$$

$$E_z = 2500\mu \left.\frac{e^{-10t}}{-10}\right|_0^\infty = -250\mu[0-1] = 250\mu J$$

$= 0.25mJ$

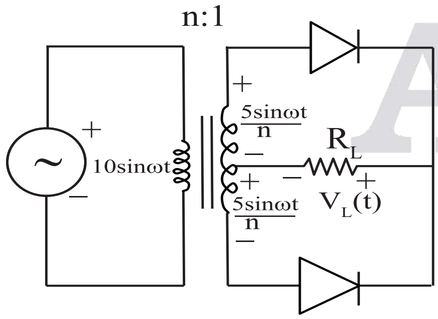
19. In the circuit shown, the n : 1 step down transformer and the diodes are ideal. The diodes have no voltage drop in forward biased condition. If the input voltage (in Volts) is $V_s(t) = 10\sin\omega t$ and the average value of load voltage $V_L(t)$ (in volts) is $2.5/\pi$, the value of n is ________ .



(a) 12 (b) 4 (c) 16 (d) 8

19. Ans: (b)

Sol:



Given $V_{L_{avg}} = \frac{2.5}{\pi}$

[Note: The average value of Full wave rectifier output is $\frac{2V_m}{\pi}$]

$$\rightarrow \frac{2V_m}{\pi} = \frac{2.5}{\pi}$$

$$\rightarrow \frac{2}{\pi}\left[\frac{5}{n}\right] = \frac{2.5}{\pi} \rightarrow n = 4$$

20. Let R and R^3 denote the set of real numbers and the three dimensional vector space over it, respectively. The value of α for which the set of vectors $\{[2\ \ -3\ \ \alpha], [3\ \ -1\ \ 3], [1\ \ -5\ \ 7]\}$ does not form a basis of R^3 is _______ .

20. Ans: 5

Sol: Let $X = [2\ \ -3\ \ \alpha]$, $Y = [3\ \ -1\ \ 3]$

and $Z = [1\ \ -5\ \ 7]$

Then $A = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{bmatrix}$

Given that X, Y and Z vectors do not form a basis for R^3.

$\Rightarrow$X, Y and Z are not linearly independent vectors

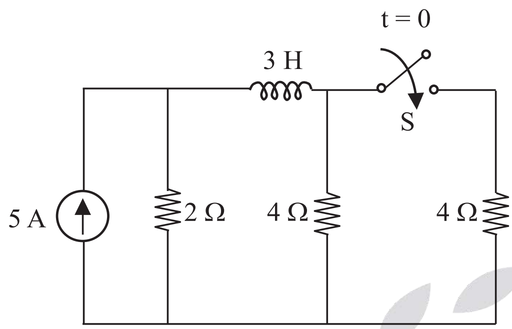
$\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{vmatrix} = 0$$

$\Rightarrow 2(-7+15) + 3(21-3) + \alpha(-15+1) = 0$

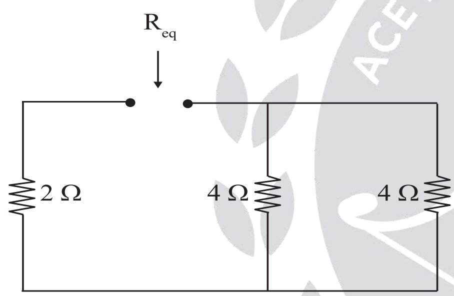
$\therefore\ \alpha = 5$

21. In the circuit given below, the switch S was kept open for a sufficiently long time and is closed at time t = 0. The time constant (in seconds) of the circuit for t > 0 is ______ .



21. Ans: 0.75

Sol: $\tau = \frac{L}{R_{eq}}$



$R_{eq} = 2 + [4//4] = 2 + 2 = 4\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{3}{4} = 0.75\ \text{s}$

22. In a number system of base r, the equation $x^2 - 12x + 37 = 0$ has $x = 8$ as one of its solutions. The value of r is ______ .

22. Ans: 11

Sol: Given equation, $x^2 - 12x + 37 = 0$

Given one of the roots is $x = 8$

Convert the coefficients of above equation into decimal.

Let the base = b

$x^2 - (b + 2)x + (3b + 7) = 0.$

As $x = 8$ is one of the root, substitute $x = 8$ in the above equation

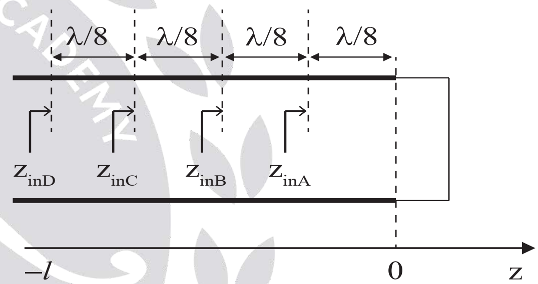
$8^2 - (b + 2)8 + (3b + 7) = 0$

$64 - 8b - 16 + 3b + 7 = 0$

$-5b + 55 = 0 \Rightarrow 5b = 55$

Base, b = 11

23. Consider a lossless transmission line terminated with a short circuit as shown in the figure below. As one moves towards the generator from the load, the normalized impedances z_{inA}, z_{inB}, z_{inC} and z_{inD} (indicated in the figure) are ______ .



(a) $z_{inA} = -1j\Omega$, $z_{inB} = 0$, $z_{inC} = +1j\Omega$, $z_{inD} = \infty$

(b) $z_{inA} = \infty$, $z_{inB} = +0.4j\Omega$, $z_{inC} = 0$, $z_{inD} = +0.4j\Omega$

(c) $z_{inA} = +1j\Omega$, $z_{inB} = \infty$, $z_{inC} = -1j\Omega$, $z_{inD} = 0$

(d) $z_{inA} = +0.4j\Omega$, $z_{inB} = \infty$, $z_{inC} = -0.4j\Omega$, $z_{inD} = 0$

23. Ans: (c)

Sol: For λ/8 line

$$Z(\ell) = Z_0\left[\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right]$$

$$Z'(\ell) = \frac{Z(\ell)}{Z_0} = \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right]$$

AT A:

$$Z_{inA} = \left[\frac{0 + jZ_0}{Z_0 + j0}\right] = j = +1j\Omega$$

AT B: λ/4 Length (inverter)

$Z_{inB} = \infty\ \Omega$

$$\text{AT C: } Z_{inC} = \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right] = \left[\frac{1 + j\frac{Z_0}{Z_L}}{\frac{Z_0}{Z_L} + j}\right] = \left[\frac{1 + j\frac{Z_0}{\infty}}{\frac{Z_0}{\infty} + j}\right] = -j$$

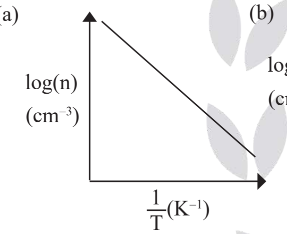
$$Z_{inC} = -1j$$

At D: $\frac{\lambda}{2}$ Length (Mirror)

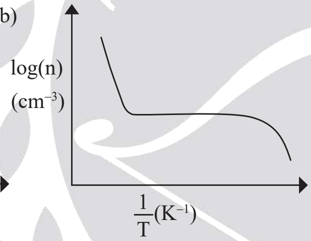
$$Z_{inD} = 0\Omega$$

24. For non-degenerating doped n-type silicon, which of the following plot represents the temperature (T) dependence of free electron concentration (n)?

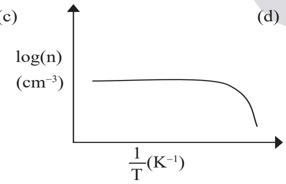
(a)



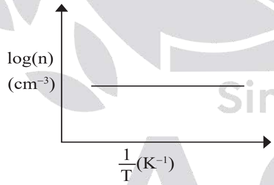
(b)



(c)



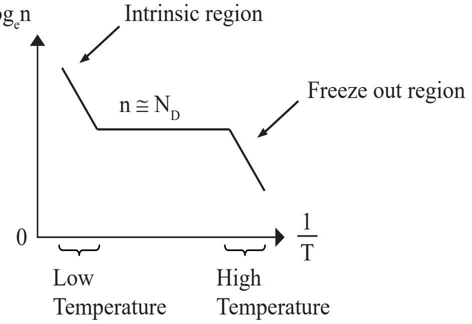
(d)



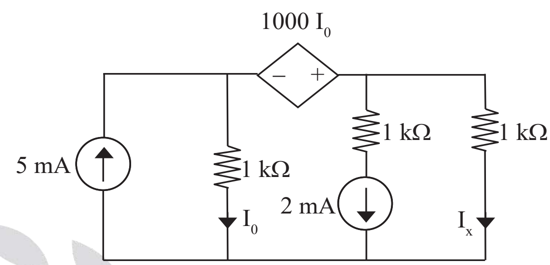
24. Ans: (b)

Sol: Given n-type, Si, non-degenerating doping

Then → $\log_e n$

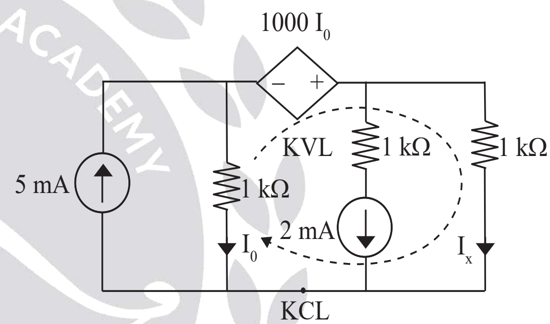


25. In the given circuit, the current I_x (in mA) is ____.



25. Ans: 2

Sol:



By KVL

$$-1000\, I_0 - 1000\, I_0 + 1000\, I_x = 0$$

$$I_0 = \frac{I_x}{2} \quad \text{---} \; (1)$$

By KCL

$$I_0 + 2m + I_x = 5m$$

$$\Rightarrow I_0 + I_x = 3m \quad \text{---} \; (2)$$

Solve (1) & (2)

$$3I_x = 6\text{ m}$$

$$I_x = 2\text{ mA}$$

Questions-26 to 55 Carry TWO marks each.

26. Which of the following statements is/are true for a BJT with respect to its DC current gain β?

(a) Under low-level injection condition in forward active mode, where the current at the emitter-base junction is dominated by recombination-generation process, β will decrease with increase in the magnitude of collector current.

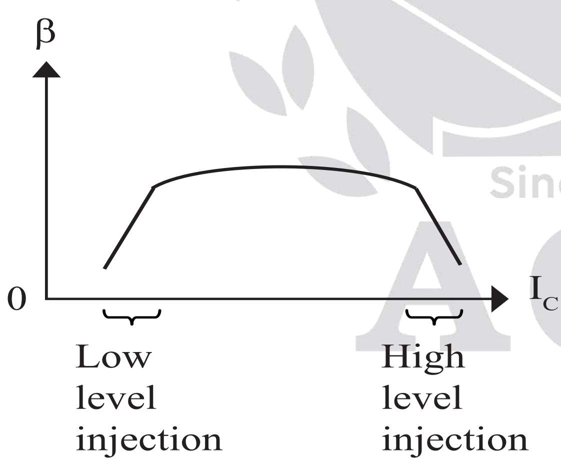
(b) β will be lower when the BJT is in saturation region compared to when it is in active region.

(c) Under high-level injection condition in forward active mode, β will decrease with increase in the magnitude of collector current.

(d) A higher value of β will lead to a lower value of the collector-to-emitter breakdown voltage.

26. Ans: (b, c & d)

Sol: Since we know $\rightarrow$



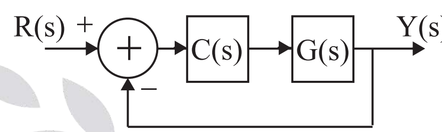
Also in saturation $\beta_{sat} < \beta_{F.\,Active}$ is true

Also as Dop.$\uparrow\uparrow \rightarrow I_C\uparrow\uparrow \rightarrow$ Break down voltage$\downarrow\downarrow$

$\therefore$ Higher $\beta \rightarrow$ Lesser Break down voltage.

$\therefore$ Options (b, c & d).

27. A satellite altitude control system, as shown below, has a plant with transfer function $G(s) = \frac{1}{s^2}$ cascaded with a compensator $C(s) = \frac{K(s+\alpha)}{s+4}$, where K and α are positive real constants.



In order for the closed-loop system to have poles at $-1 \pm \sqrt{3}$, the value of α must be ______ .

(a) 1 (b) 0 (c) 2 (d) 3

27. Ans: (a)

Sol: $G(s) = \frac{1}{s^2}$, Compensator $C(s) = \frac{K(s+\alpha)}{(s+4)}$

$$G(s)\big|_{\omega C} = \frac{K(s+\alpha)}{s^2(s+4)},\ H(s) = 1$$

CE $\rightarrow s^3 + 4s^2 + Ks + K\alpha = 0$ ---- (1)

for given data CE $\rightarrow (s+1)^2 + (\sqrt{3})^2 = 0$

CE $\rightarrow s^2 + 2s + 4 = 0$. It is a 2nd order system.

But required 3rd order system. Let assume 3rd pole at $s = -a$.

CE $\rightarrow (s+a)(s^2+2s+4) = 0$

CE $\rightarrow s^3 + 2s^2 + 4s + as^2 + 2as + 4a = 0$

CE $\rightarrow s^3 + s^2(a+2) + s(4+2a) + 4a = 0$ ---- (2)

Compare equation (1) & (2)

$\Rightarrow (a+2) = 4 \Rightarrow a = 2$

$\Rightarrow 4 + 2a = K$

$\Rightarrow K = 8$

$\Rightarrow 4a = K\alpha \Rightarrow 8 = 8\alpha \Rightarrow \alpha = 1$

28. The photocurrent of a PN - junction diode solar cell is 1mA. The voltage corresponding to its maximum power point is 0.3V. If the thermal voltage is 30mV, the reverse saturation current of the diode (in NA, rounded off to two decimal places) is ______ .

28. Ans: (*) Data insufficient

29. A non-degenerate n-type semiconductor has 5% neutral dopant atoms. Its Fermi level is located at 0.25 eV below the conduction band (E_C) and the donor energy level (E_D) has a degeneracy of 2. Assuming the thermal voltage to be 20 mV, the difference between E_C and E_D (in eV, rounded off to two decimal places) is _______ .

29. Ans: 0.177

Sol: Given Non-degenerative semiconductor, n-type

Given 5% neutral donor atoms

$\Rightarrow$ Donor atoms not yet ionized .

Let call them as N_D^O

If ionized donor $\rightarrow$ N_D^+ .

Given $N_D^O = 5\% \, N_D = 0.05N_D$.

$\therefore \; N_D^+ = 5\% \, N_D = 0.05N_D$.

Given $(E_C - E_F) = 0.25$ eV & $V_T = 20$mV

$$\text{Since, } a = \frac{N_D^+}{N_D} = \frac{1}{1 + 2\exp\left[\frac{E_F - E_D}{KT}\right]}$$

Degenerative Factor $\rightarrow$ 2

$$\Rightarrow 0.95 = \frac{1}{1 + 2\exp\left[\frac{E_F - E_D}{KT}\right]}$$

By solving $E_F - E_D = -72.7$ meV

$\therefore E_D - E_F = 72.7$ meV.

Given, $E_C - E_F = 0.25$ eV

and $E_D - E_F = 0.0727$ eV $\rightarrow$ we got.

$\therefore E_C - E_F - E_D + E_F = 0.25 - 0.0727$

$\therefore E_C - E_D = 0.177$

30. Consider a MOS capacitor made with p-type silicon. It has an oxide thickness of 100 nm. A fixed positive oxide charge of 10^{-8} C/cm^2 at the oxide-silicon interface, and a metal work function of 4.6 eV. Assume that the relative permittivity of the oxide is 4 and the absolute permittivity of free space is 8.85×10^{-14} F/cm. If the flatband voltage is 0 V, the work function of the p-type silicon (in eV, rounded off to two decimal places) is _____ .

30. Ans: 4.32

Sol: Given $t_{ox} = 100$nm

$Q'_{ox} = 10^{-8}$C/cm^2

N-MOS, [P-substrate]

Metal Work function

$\rightarrow q\phi_m = 4.6$eV

$(\varepsilon_r)_{Sio2} = 4$

$(\varepsilon_0) = 8.85 \times 10^{-14}$F/cm

Given $V_{FB} = 0$V

Need to find $q\phi_{Si}$ = __________?

$$\text{As } V_{FB} = \phi_{ms} - \frac{Q'_{ox}}{C_{ox'}}$$

$$\Rightarrow 0 = \phi_m - \phi_s - \frac{Q'_{ox}}{C_{ox'}}$$

$$\Rightarrow \phi_s = \phi_m - \frac{Q'_{ox}}{C_{ox'}} = 4.6 - \frac{10^{-8}}{\frac{\varepsilon_{ox}}{t_{ox}}}$$

$\therefore q\phi_s \cong 4.32$eV

31. A continuous time signal $x(t) = 2\cos\left(8\pi t + \frac{\pi}{3}\right)$ is sampled at a rate of 15 Hz. The sampled signal $x_s(t)$ when passed through an LTI system with impulse response
$h(t) = \left(\frac{\sin 2\pi t}{\pi t}\right)\cos\left(38\pi t - \frac{\pi}{2}\right)$
produces an output $x_0(t)$. The expression for $x_0(t)$ is ______ .

(a) $15\sin\left(38\pi t - \frac{\pi}{3}\right)$ (b) $15\cos\left(38\pi t + \frac{\pi}{6}\right)$

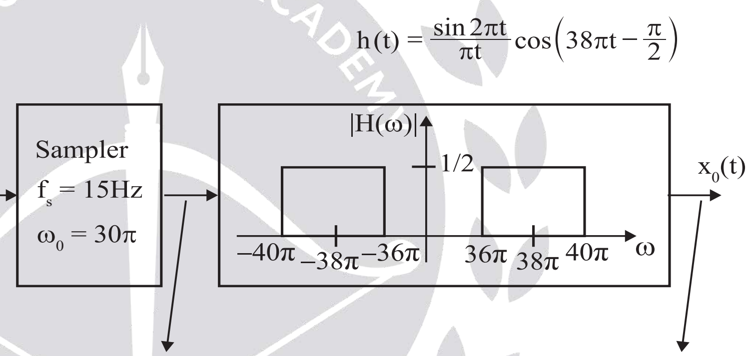
(c) $15\sin\left(38\pi t + \frac{\pi}{3}\right)$ (d) $15\cos\left(38\pi t - \frac{\pi}{6}\right)$

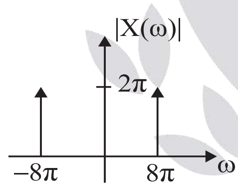
31. Ans: (d)

Sol:

$x(t) = 2\cos\left(8\pi t + \frac{\pi}{3}\right)$

$\cos\omega_c t \overset{FT}{\longleftrightarrow} \pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$





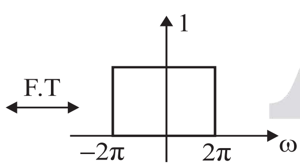
Spectrum of sampled signal is

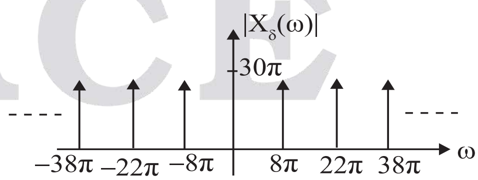
$$X_\delta(\omega) = \frac{1}{15}\sum X(\omega - n\omega_0)$$
$$= 15\sum X(\omega - 30\pi n)$$

Ouput of the filter

$$x_0(t) = 15(2)\left(\frac{1}{2}\right)\cos\left(8\pi t + \frac{\pi}{3} - \frac{\pi}{2}\right)$$
$$x_0(t) = 15\cos\left(38\pi t - \frac{\pi}{6}\right)$$

$\frac{\sin 2\pi t}{\pi t} \overset{F.T}{\longleftrightarrow}$





$$x(t)\cos\omega_0 t \overset{F.T}{\longleftrightarrow} \frac{X(\omega-\omega_0) + X(\omega+\omega_0)}{2}$$

32. A source transmits a symbol s, taken from {–4,0,4} with equal probability, over an additive white Gaussian noise channel. The received noisy symbol r is given by r = s+w, where the noise w is zero mean with variance 4 and is independent of s.

Using $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-t^2}{2}} dt$, the optimum symbol error probability is _______ .

(a) $\frac{2}{3}Q(1)$ (b) $\frac{4}{3}Q(2)$

(c) $\frac{2}{3}Q(2)$ (d) $\frac{4}{3}Q(1)$

32. Ans: (d)

Sol: $P(s_1 = -4) = \frac{1}{3}$

$P(s_2 = 0) = \frac{1}{3}$

$P(s_3 = 4) = \frac{1}{3}$

$r = s + w$

$\sigma_w^2 = 4$

$\mu_w = 0$

$\therefore$ Power of noise = 4

s and w are independent of each other.

The optimum threshold voltage

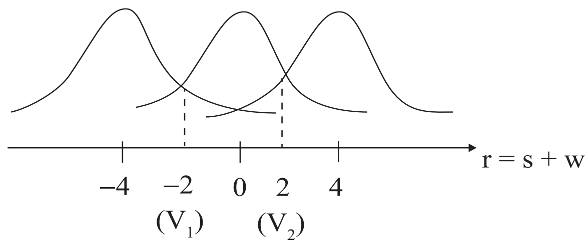
$$= \frac{V_1 + V_2}{2} + \frac{\sigma_0^2 \ln[P(s_2)/P(s_1)]}{V_1 - V_2}$$

Since all probabilities are same

$$V_{opt} = \frac{V_1 + V_2}{2}$$

$$V_1 = V_{opt} \text{ for } S_1 \text{ and } S_2 = \frac{-4+0}{2} = -2\ (V)$$

$$V_2 = V_{opt} \text{ for } S_2 \text{ and } S_3 = \frac{0+4}{2} = 2\ (V)$$



$\rightarrow \therefore P_e\left(\frac{s_2}{s_1}\right)$

$s_1 + w > -2$

$-4 + w > -2$

$w > 2$

$$\therefore P_e\left(\frac{s_2}{s_1}\right) = P(w > 2) = 1 - P[w \le 2]$$
$$= 1 - F[w = 2]$$
$$= 1 - \left[1 - Q\left(\frac{2-0}{2}\right)\right]$$
$$= Q(1)$$

$\rightarrow P_e\left(\frac{s_2}{s_3}\right)$

$s_3 + w < 2$

$4 + w < 2$

$w < -2$

$$\therefore P_e\left(\frac{s_2}{s_3}\right) = P[w < -2]$$
$$= F[w < -2]$$
$$= 1 - Q\left(\frac{-2-0}{2}\right)$$
$$= 1 - Q(-1)$$
$$= 1 - [1 - Q(1)]$$
$$= Q(1)$$

$\rightarrow \quad P_e\left(\frac{s_1}{s_2}\right)$

$s_2 + w < -2$

$w < -2$

$P_e\left(\frac{s_1}{s_2}\right) = P[w < -2]$

$= Q(1)$

$\rightarrow \quad P_e\left(\frac{s_3}{s_2}\right)$

$s_2 + w > 2$

$w > 2$

$P_e\left(\frac{s_3}{s_2}\right) = P(w > 2)$

$= Q(1)$

$\therefore P_e = P(s_2)P_e\left(\frac{s_2}{s_1}\right) + P(s_2)P_e\left(\frac{s_1}{s_2}\right)P(s_2)P_e\left(\frac{s_3}{s_2}\right) + P(s_3)P_e\left(\frac{s_2}{s_3}\right)$

$P_e = \frac{4}{3}Q(1)$

33. A uniform plane wave with electric field $\vec{E}(x) = A_y\hat{a}_y e^{-j\frac{2\pi x}{3}}$ V/m is travelling in the air (relative permittivity, $\varepsilon_r = 1$ and relative permeability, $\mu_r = 1$) in the +x direction (A_y is a positive constant, $\hat{a}_y$ is the unit vector along the y axis). It is incident normally on an ideal electric conductor (conductivity, $\sigma = \infty$) at x = 0. The position of the first null of the total magnetic field in the air (measured from x = 0, in metres) is _____.

(a) $-\frac{3}{2}$ (b) −3 (c) $-\frac{3}{4}$ (d) −6

33. Ans: (c)

Sol: $\bar{E}(x) = A_y\,\hat{y}e^{-j\frac{2\pi}{3}x}$

$\bar{E}(x,t) = A_y\,\hat{y}e^{j\left(\omega t - \frac{2\pi}{3}x\right)}$

$\bar{E}_i(x,t) = A_y\cos\left(\omega t - \frac{2\pi}{3}x\right)\hat{y}$

Free space (1) | Perfect conductor (2)

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \qquad \eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = 0$

x = 0

$\xrightarrow{\quad} +x$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \eta_1}{0 + \eta_1} = -1 = \frac{E_r}{E_i}$

$\bar{E}_i = A_y\cos\left(\omega t - \frac{2\pi}{3}x\right)\hat{y}$

$\bar{E}_r = -A_y\cos\left(\omega t + \frac{2\pi}{3}x\right)\hat{y}$

$\bar{E}_T = \bar{E}_i + \bar{E}_r = A_y\left[\cos\left(\omega t - \frac{2\pi}{3}x\right) - \cos\left(\omega t + \frac{2\pi}{3}\right)\right]\hat{y}$

$\bar{E}_T = A_y\hat{y}2\sin\omega t\sin\left(\frac{2\pi}{3}x\right)$

For ‘ E ’ Null

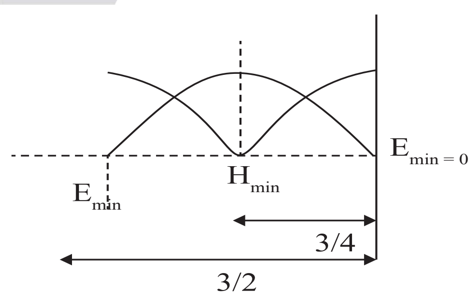
$E_T = 0$

$\Rightarrow \sin\left(\frac{2\pi}{3}x\right) = 0 \qquad \Rightarrow \frac{2\pi}{3}x = \pm m\pi$

$x = \pm\frac{3m}{2}$

If m = 0, x = 0 $\Rightarrow$ 1st E_{min} (null)

If m = 1, $x = -\frac{3}{2}$ $\Rightarrow$ 2nd E_{min} (null)



For magnetic field H_{min} occurs at $x = -\frac{3}{4}$ m

34. The radian frequency value (s) for which the discrete time sinusoidal signal $x[n] = A\cos\left(\Omega n + \frac{\pi}{3}\right)$ has a period of 40 is /are ______ .

(a) 0.225π (b) 0.15π

(c) 0.3π (d) 0.45π

34. Ans: (b & d)

Sol: $x(n) = A\cos\left(\Omega n + \frac{\pi}{3}\right)$

has a period of 40 (discrete signal)

For periodicity of discrete sinusoid $\frac{\Omega}{2\pi} = \frac{m}{N}$

For option (a)

$$\frac{0.225\pi}{2\pi} = \frac{225}{2000} = \frac{9}{80} = \frac{m}{N}$$

option (a) is wrong

For option (b)

$$\frac{0.15\pi}{2\pi} = \frac{15}{200} = \frac{3}{40} = \frac{m}{N}$$

option (b) is correct

For option (c)

$$\frac{0.3\pi}{2\pi} = \frac{3}{20} = \frac{m}{N}$$

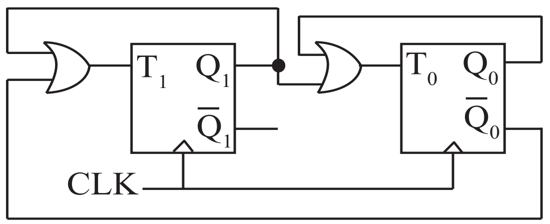
option (c) is wrong

For option (d)

$$\frac{0.45\pi}{2\pi} = \frac{45}{200} = \frac{9}{40} = \frac{m}{N}$$

option (d) is correct

35. The sequence of states (Q_1 Q_0) of the given synchronous sequential circuit is



(a) $01 \to 10 \to 11 \to 00 \to 01$

(b) $00 \to 01 \to 10 \to 00$

(c) $11 \to 00 \to 10 \to 01 \to 00$

(d) $00 \to 10 \to 11 \to 00$

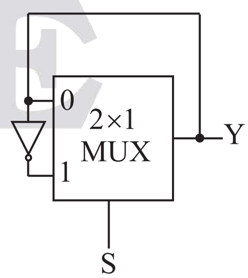
35. Ans: (c)

Sol:

PS		PI		NS	
Q_1	Q_0	T_1	T_0	Q_1	Q_0
		$Q_1+\overline{Q}_0$	Q_1+Q_0		
0	0	1	0	1	0
1	0	1	1	0	1
0	1	0	1	0	0
1	1	1	1	0	0

$11 \longrightarrow 00 \longrightarrow 10 \longrightarrow 01$ (01 returns to 00)

36. The propagation delay of the 2×1 MUX shown in the circuit is 10ns. Consider the propagation delay of the inverter as 0ns.



If S is set to 1, then output of Y is

(a) Square wave frequency is 50 MHz

(b) Square wave frequency is 100 MHz

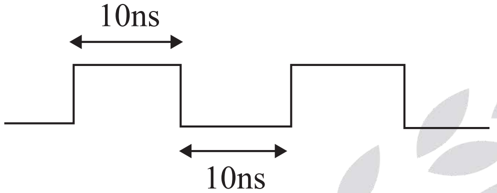
(c) constant at 1

(d) constant at 0

36. Ans: (a)

Sol: When select line S is 1, the output is complement of existing. And the delay of MUX is 10ns

So the output waveform is



The cycle duration is 20ns and the frequency is

$\frac{1}{20\text{ns}} = 0.05\text{GHz}$

$= 50\text{MHz}$

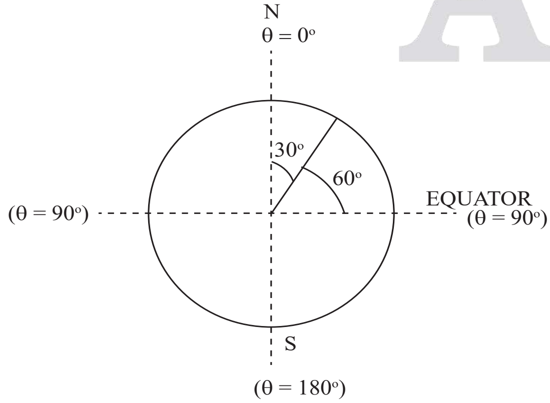
37. Consider the Earth to be a perfect sphere of radius R. Then the surface area of the region, enclosed by the 60°N latitude circle, that contains the north pole in its interior is ________.

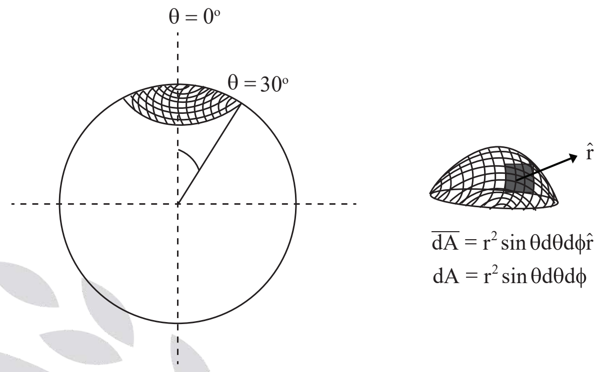
(a) $\frac{2\pi R^2}{3}$ (b) $\frac{(2+\sqrt{3})\pi R^2}{8\sqrt{2}}$

(c) $\frac{(\sqrt{2}-1)\pi R^2}{2}$ (d) $(2-\sqrt{3})\pi R^2$

37. Ans: (d)

Sol:





Area

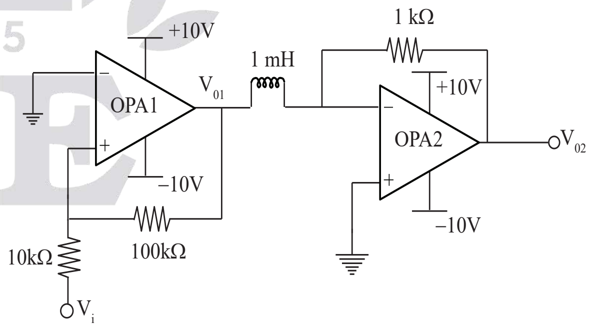
$= \int\int dA = \int\int r^2 \sin\theta \, d\theta d\phi = R^2 \int \sin\theta d\theta \int d\phi$

$= R^2[-\cos\theta]_0^{30} \; [\phi]_0^{2\pi}$

$= 2\pi R^2\left\{-\left(\frac{\sqrt{3}}{2}-1\right)\right\}$

Area $= \pi R^2(2-\sqrt{3})\ \text{m}^2$

38. The op-amps in the circuit shown are ideal, but have saturation voltages of ±10V



Assume that the initial inductor current is 0 A. The input voltage (V_i) is a triangular signal with peak voltages of ±2 V and time period of 8 µs. Which one of the following statements is true?

(a) V_{01} is not delayed relative to V_i. and V_{02} is a triangular waveform.

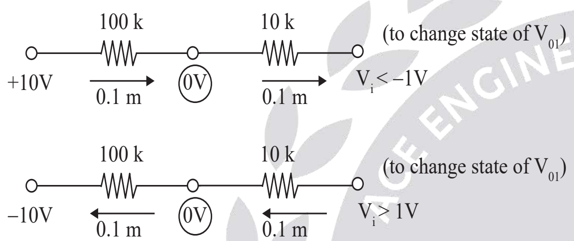
(b) V_{01} is not delayed relative to V_i, and V_{02} is a trapezoidal waveform.

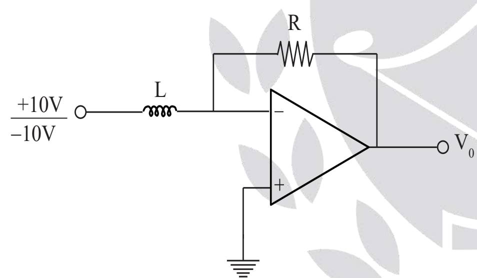
(c) V_{01} is delayed by 1 μs relative to V_i, and V_{02} is a trapezoidal waveform.

(d) V_{01} is delayed by 2 μs relative to V_i, and V_{02} is a triangular waveform.

38. Ans: (b)

Sol:

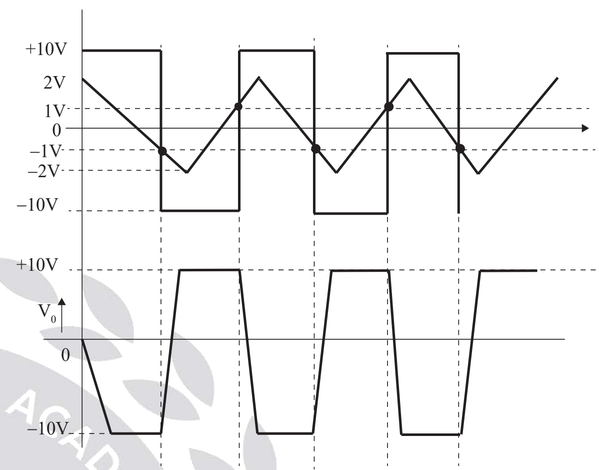




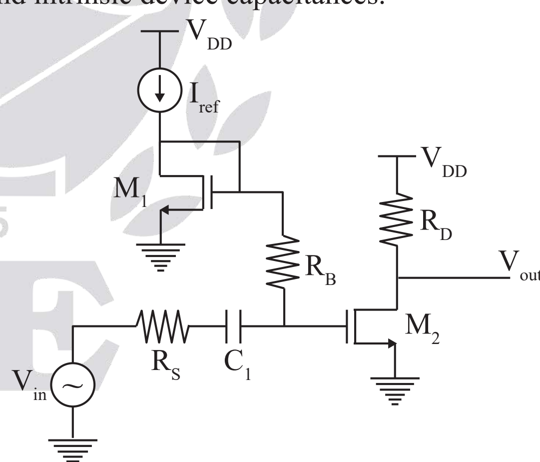
$$V_0 = -\frac{R}{L}\int V_{01}\,dt = \pm 10^7 t \quad [R = 1k\Omega, L = 1mH]$$

So $V_{01} = \pm 10$ V is integrated for just 1 μsec to

Saturate V_0 at ±10V as $V_0 = \pm 10^7 t$



39. In the circuit shown below, the transistors M_1 and M_2 are biased in saturation. Their small signal transconductances are g_{m1} and g_{m2} respectively. Neglect body effect, channel length modulation and intrinsic device capacitances.



Assuming that capacitor C_1 is a short circuit for AC analysis, the exact magnitude of small signal voltage gain $\left|\frac{V_{out}}{V_{in}}\right|$ is_______.

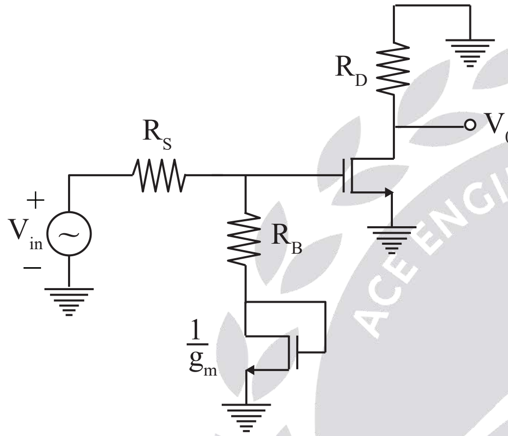
(a) $g_{m2}R_D$

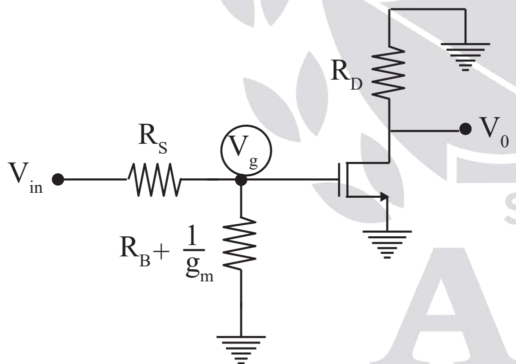
(b) $\dfrac{g_{m2}R_D\left(R_B + \frac{1}{g_{m1}}\right)}{R_B + \frac{1}{g_{m1}} + R_s}$

(c) $\dfrac{g_{m2}R_D\left(R_B+\dfrac{1}{g_{m1}}+R_s\right)}{R_B+\dfrac{1}{g_{m1}}}$ (d) $\dfrac{g_{m2}R_D\left(\dfrac{1}{g_{m1}}\right)}{\dfrac{1}{g_{m1}}+R_s}$

39. Ans: (b)

Sol: AC equivalent [Capacitor → short circuit]





$$\frac{V_0}{V_{in}}=\frac{V_0}{V_g}\cdot\frac{V_g}{V_{in}}$$

[Given amplifier is a common source amplifier]

$$=(-g_mR_D)\frac{\left(R_B+\dfrac{1}{g_{m1}}\right)}{R_B+\dfrac{1}{g_{m1}}+R_s}$$

40. Let z be a complex variable. If $f(z)=\dfrac{\sin(\pi z)}{z^2(z-2)}$ and C is the circle in the complex plane with $|z|=3$ then $\oint_C f(z)dz$ is _______.

(a) $j\pi\left(\frac{1}{2}-\pi\right)$ (b) $-\pi^2 j$

(c) $\pi^2 j$ (d) $j\pi\left(\frac{1}{2}+\pi\right)$

40. Ans: (b)

Sol: Given that $f(z)=\dfrac{\sin(\pi z)}{z^2(z-2)}$

⇒ the singular points of f(z) are z = 0 and z = 2

⇒ both singular points z = 0 and z = 2 lie in the given region | z | = 3

Now, $\oint_C f(z)dz=\oint_C \dfrac{\sin(\pi z)}{z^2(z-2)}dz$

$$\Rightarrow \oint_C f(z)dz=\oint_{C_1}\frac{\left[\dfrac{\sin(\pi z)}{z-2}\right]}{[z-0]^2}dz+\oint_{C_2}\frac{\left[\dfrac{\sin(\pi z)}{z^2}\right]}{[z-2]}dz,$$

where C is $|z|=3$, C_1 is $|z|=0$ and C_2 is $|z-2|=0$

$$\oint_C f(z)dz=\frac{2\pi j}{(2-1)!}\left[\frac{d}{dz}\left(\frac{\sin(\pi z)}{z-2}\right)\right]_{z=0}+2\pi j\left[\left(\frac{\sin(\pi z)}{z^2}\right)\right]_{z=2}$$

$$\Rightarrow \oint_C f(z)dz=2\pi j\left[\left(\frac{(z-2)\cos(\pi z)(\pi)-(1)\sin(\pi z)}{(z-2)^2}\right)\right]_{z=0}$$

$$\Rightarrow \oint_C f(z)dz=2\pi j\left[\left(\frac{(-2)\cos(0)(\pi)-\sin(0)}{(0-2)^2}\right)\right]$$

$$\Rightarrow \oint_C f(z)dz=2\pi j\left[\left(\frac{-2\pi}{4}\right)\right]$$

$$\therefore \oint_C f(z)dz=-\pi^2 j$$

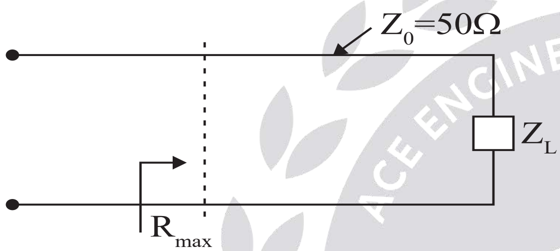
41. A lossless transmission line with characteristic impedance Z_0 = 50Ω is terminated with an

unknown load. The magnitude of the reflection co-efficient is $|\Gamma| = 0.6$. As one moves towards the generator from the load, the maximum value of the input impedance magnitude looking towards the load (in Ω) is ________

41. Ans: 200

Sol: Given $Z_0 = 50\ \Omega$

$|\Gamma| = 0.6$



$R_{max} = Z_0\rho$

$$\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.6}{1-0.6} = \frac{1.6}{0.4} = 4$$

$R_{max} = 50 \times 4 = 200\Omega$

42. Let F_1, F_2, and F_3 be functions of (x, y, z). Suppose that for every given pair of points A and B in space, the line integral $\int_C(F_1dx + F_2dy + F_3dz)$ evaluates to the same value along any path C that starts at A and ends at B. Then which of the following is/are true?

(a) $\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$

(b) There exists a differentiable scalar function f(x,y,z) such that $F_1 = \frac{\partial F}{\partial x}, F_2 = \frac{\partial F}{\partial y}, F_3 = \frac{\partial F}{\partial z}$

(c) For every closed path Γ, we have $\oint_\Gamma (F_1 dx + F_2 dy + F_3 dz) = 0$.

(d) $\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0$

42. Ans: (a, b & c)

Sol: Given that the value of the line integral $\int_C [F_1 dx + F_2 dy + F_3 dz]$ is same along any path 'C' that starts at A and ends at B.

⇒ the value of the integral is independent of the path 'C' by joining the points A & B.

⇒ the vector function $\overline{F} = F_1\,\overline{i} + F_2\,\overline{j} + F_3\,\overline{k}$ is irrotational vector (or) conservative vector function.

i.e curl of $\overline{F}$ is zero vector

⇒ curl $\overline{F} = \overline{0}$ (or) $= \nabla \times \overline{F} = \overline{0}$

$$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \overline{0}$$

$$\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\overline{i} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\overline{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\overline{k}$$

$$= 0\overline{i} + 0\overline{j} + 0\overline{k}$$

$$\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} = 0, \frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z} = 0, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0$$

$$\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \frac{\partial F_3}{\partial x} = \frac{\partial F_1}{\partial z}, \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

∴ option (a) is true

If $\overline{F} = F_1\,\overline{i} + F_2\,\overline{j} + F_3\,\overline{k}$ is an irrotational vector then there exists a scalar function f(x, y, z) such that $\overline{F} = \nabla f$

$$\Rightarrow \overline{F} = F_1\,\overline{i} + F_2\,\overline{j} + F_3\,\overline{k} = \frac{\partial f}{\partial x}\overline{i} + \frac{\partial f}{\partial y}\overline{j} + \frac{\partial f}{\partial z}\overline{k}$$

$$\Rightarrow F_1 = \frac{\partial f}{\partial x},\ F_2 = \frac{\partial f}{\partial y}, F_3 = \frac{\partial f}{\partial z}$$

∴ option (b) is true

If $\overline{F} = F_1\,\overline{i} + F_2\,\overline{j} + F_3\,\overline{k}$ is an irrotational vector then the value of the line integral along any closed path Γ is zero

$\Rightarrow \oint_{\Gamma} (F_1 dx + F_2 dy + F_3 dz) = 0$

$\therefore$ option (c) is true

If $\overline{F} = F_1\overline{i} + F_2\overline{j} + F_3\overline{k}$ is an irrotational vector function then the divergence of $\overline{F}$ may (or) may not be zero.

$\Rightarrow \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0$ (or) $\nabla.\overline{F} = 0$ is a false statement.

$\therefore$ option (d) is not true

43. An NMOS transistor operating in the linear region has I_{DS} of 5μA at V_{DS} of 0.1V. Keeping V_{GS} constant, the V_{DS} is increased to 1.5V.

Given that $\mu_n C_{ox}\frac{W}{L} = 50\mu A/V^2$, the transconductance at the new operating point (in μA/V, rounded off to two decimal places) is ______.

43. Ans: 52.5

Sol: Given N-MOS

$I_{DS} = 5mA$

$V_{DS} = 0.1V$

V_{GS} = constant

Here NMOS operating in triode

Now new value of $V_{DS} = 1.5V$

$\mu_n C_{ox}\frac{W}{L} = 50\mu A/V^2$

Now (g_m) = _________?

at new V_{DS}

As at triode →

$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right]$

$5\mu = 50\mu\ [(V_{GS} - V_T)\ 0.1 - 0.005]$

$\therefore\ V_{GS} - V_{Th} = \frac{0.105}{0.1} = 1.05V$

$\therefore\ V_{0V} = 1.05V$

Now at new $V_{DS} = 1.5V \rightarrow V_{DS} > V_{GS} - V_{Th})$

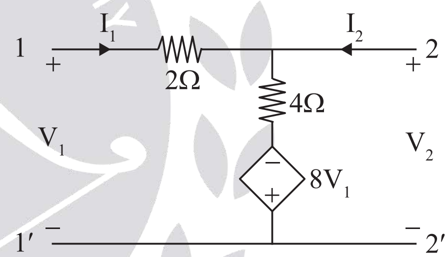
$\Rightarrow$ Q-point is in saturation

$\therefore\ (g_m)_{sat} = \mu_n C_{ox}\frac{W}{L}[V_{GS} - V_{Th}]$

$= 50\mu(1.05)$

$= 52.5\mu A/V$

44. For the two port network shown below, the value of the Y_{21} parameter (in Siemens) is _______.

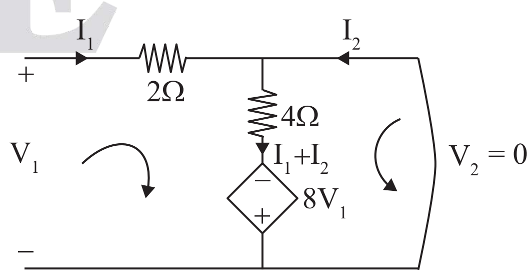


44. Ans: 1.5

Sol: $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$



KVL - 1

$-V_1 + 2I_1 + 4I_1 + 4I_2 - 8V_1 = 0$

$6I_1 + 4I_2 = 9V_1$ ---- (1)

KVL - 2

$4I_1 + 4I_2 - 8V_1 = 0$

$I_1 + I_2 = 2V_1$ ---- (2)

So, $I_1 = 2V_1 - I_2$ ---- (3)

Substitute equation (3) in (1)

$6[2V_1 - I_2] + 4I_2 = 9V_1$

$12V_1 - 6I_2 + 4I_2 = 9V_1$

$3V_1 = 2I_2$

$Y_{21} = \frac{I_2}{V_1} = \frac{3}{2} = 1.5$ mho

45. A full scale sinusoidal signal is applied to a 10-bit ADC. The fundamental signal component in the ADC output has a normalized power of 1W, and the total noise and distortion normalized power is 10µW. The effective number of bits (rounded off to the nearest integer) of the ADC is______.

(a) 9 (b) 7
(c) 8 (d) 10

45. Ans: (c)

Sol: We know, for sinusoidal input, the SNR of ADC is

SNR in dB = 1.76 + 6.02n. ________(1)

where n = no of bits

$SNR = \frac{1}{10 \times 10^{-6}} = 10^5$ Watts

SNR in dB = $10 \log_{10} 10^5$

$SNR_{dB} = 50dB$________(2)

Substitute (2) in (1)

$50 = 1.76 + 6.02n$

$48.24 = 6.02n$

$n = \frac{48.24}{6.02} = 8.01$

n = 8bits

46. The relationship between any N-length sequence x[n] and its corresponding N point discrete Fourier transform X[k] is defined as

$X[k] = \mathcal{F}\{x[n]\}$.

Another sequence y[n] is formed as below

$y[n] = \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{x[n]\}\}\}\}$.

For the sequence x[n] = {1, 2, 1, 3}, then the value of Y[0] is________.

46. Ans: 112

Sol: $x[n] \xleftrightarrow[\text{DFT}]{\text{Npt}} X[k]$

$\text{DFT}\{\text{DFT}\{x[n]\}\} = N\,x((-n))_N$

4 times $\text{DFT}\{x[n]\} = N^2\,x(n)$

Given $x(n) = \{1, 2, 1, 3\}$

$\downarrow$

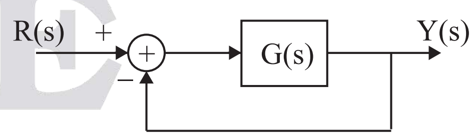
$N = 4$

$y(n) = F\{F\{F\{F\{x(n)\}\}\}\}$

$y(n) = N^2 x(n) = (4)^2[1\,2\,1\,3] = [16\;\;32\;\;16\;\;48]$

$Y(0) = \sum_{n=0}^{N-1=3} y(n) = 16 + 32 + 16 + 48 = 112$

47. Consider a unity negative feedback control system with forward path gain $G(s) = \frac{K}{(s+1)(s+2)(s+3)}$ as shown

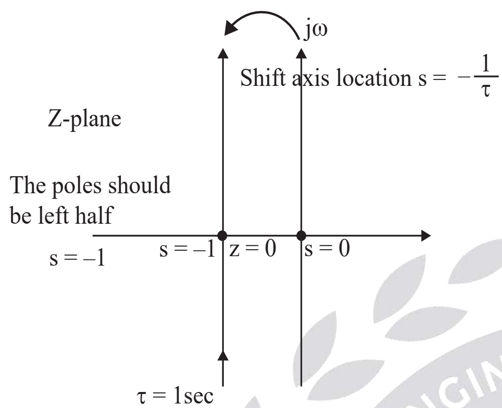


The impulse response of the closed-loop system decays faster than e^{-t} if ________.

(a) $7 \le K \le 21$ (b) $-4 \le K \le -1$
(c) $1 \le K \le 5$ (d) $-24 \le K \le -6$

47. Ans: (c)

Sol: Impulse response of the closed loop system faster than e^{-t}. That means time constant should be $\tau < 1$ sec



replace s = z – 1

$$G(z-1) = \frac{K}{(z-1+1)(z-1+2)(z-1+3)}$$

$$G(z-1) = \frac{K}{(z)(z+1)(z+2)},\ H(z) = 1$$

CE $\rightarrow 1 + G(z-1) = 0$

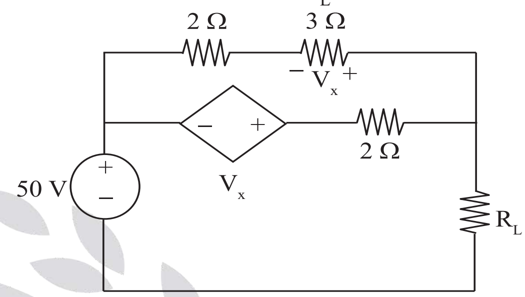
CE $\rightarrow z^3 + 3z^2 + 2z + K = 0$

$$\begin{array}{c|cc} z^3 & 1 & 2 \\ z^2 & 3 & K \\ z^1 & \frac{6-K}{3} > 0 \ \text{Ⓢ} \rightarrow K < 6 & \\ z^0 & K > 0 \ \text{Ⓢ} \rightarrow K > 0 & \end{array}$$

0 < K < 6 relatively closed loop system is stable and all the poles lies left half of s = –1 and time constant less than 1 sec.

correct answer is $1 \le K \le 5$

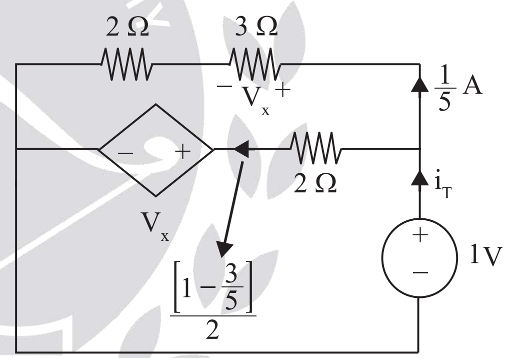
48. In the network shown below, maximum power is to be transferred to the load R_L.



The value of R_L (in Ω) is ________.

48. Ans: 2.5

Sol: 'R_L' by Ohm's Law



From Ohm's Law & voltage division rule

$$V_x = 1\left[\frac{3}{5}\right] \text{ - - - } (1)$$

$$\text{So, KCL } i_T = \frac{1}{5} + \frac{\left[1-\frac{3}{5}\right]}{2}$$

$$i_T = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$\text{So, } R_L = R_{Th} = \frac{1}{i_T} = \frac{1}{2/5} = \frac{5}{2} = 2.5\Omega$$

49. Consider the matrix $\begin{bmatrix}1 & k\\ 2 & 1\end{bmatrix}$, where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(a) $\begin{bmatrix}\sqrt{2k}\\ -1\end{bmatrix}$ (b) $\begin{bmatrix}1\\ \sqrt{2/k}\end{bmatrix}$

(c) $\begin{bmatrix}1\\ -\sqrt{2/k}\end{bmatrix}$ (d) $\begin{bmatrix}\sqrt{2k}\\ 1\end{bmatrix}$

49. Ans: (b & c)

Sol: Let $A = \begin{bmatrix}1 & k\\ 2 & 1\end{bmatrix}$, where $k > 0$.

Then the characteristic equation of a matrix $A_{2\times2}$ is

$\lambda^2 - (1+1)\lambda + (1-2k) = 0$

$$\Rightarrow \lambda = \frac{2 \pm \sqrt{4 - 4(1-2k)}}{(2)(1)}$$

$\therefore \lambda = 1 \pm \sqrt{2k}$ are the eigen values of a given matrix $A_{2\times2}$.

(i) $\lambda_1 = 1 + \sqrt{2k}$:

Consider $(A - \lambda I)X = 0$

$$\Rightarrow \begin{bmatrix}1-\lambda & k\\ 2 & 1-\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1-(1+\sqrt{2k}) & k\\ 2 & 1-(1+\sqrt{2k})\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-\sqrt{2k} & k\\ 2 & -\sqrt{2k}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$\Rightarrow (-\sqrt{2k})x_1 + kx_2 = 0$

Let $x_1 = C$, where 'C' is an arbitrary value

Then $x_2 = \dfrac{\sqrt{2k}\,C}{k} = C.\sqrt{\dfrac{2}{k}}$

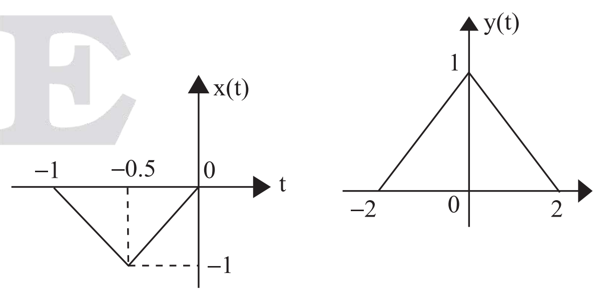
$$X = \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}C\\ C\sqrt{\frac{2}{k}}\end{bmatrix} = C\begin{bmatrix}1\\ \sqrt{\frac{2}{k}}\end{bmatrix}, \quad (C \neq 0)$$

$\therefore X_1 = \begin{bmatrix}1\\ \sqrt{\frac{2}{k}}\end{bmatrix}$ is an eigen vector of a matrix $A_{2\times2}$ corresponding to an eigen value $\lambda_1 = 1 + \sqrt{2k}$.

Similarly, $X_2 = \begin{bmatrix}1\\ -\sqrt{\frac{2}{k}}\end{bmatrix}$ is an eigen vector of same matrix $A_{2\times2}$ corresponding to the eigen value $\lambda_2 = 1 - \sqrt{2k}$

$\therefore$ option (b & c) are true.

50. Consider two continuous time signals x(t) and y(t) as shown below



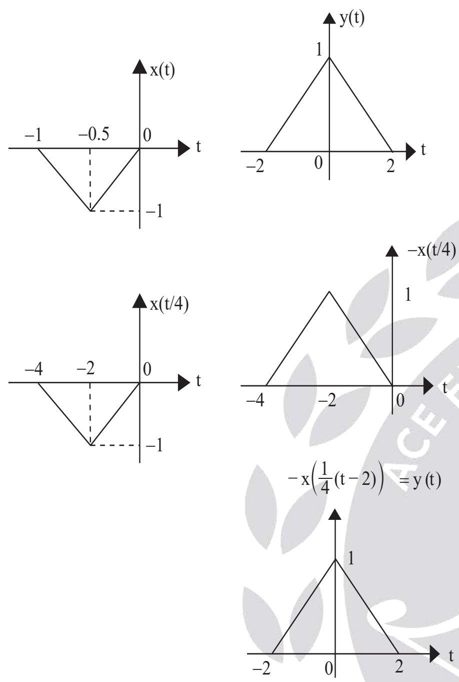
If X(f) denotes the Fourier transform of x(t) then the Fourier transform of y(t) is ________.

(a) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j\pi f}$ (b) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j4\pi f}$

(c) $-4X(4f)e^{-j4\pi f}$ (d) $-4X(4f)e^{-j\pi f}$

50. Ans: (c)

Sol:



$$y(t) = -x\left(\frac{1}{4}(t-2)\right)_{t_0=2} \qquad \alpha = \frac{1}{4}$$

By Applying F.T

$$Y(f) = -\frac{1}{|1/4|}X\left(\frac{f}{1/4}\right)e^{-j2\pi f(2)} \qquad x(\alpha t) \leftrightarrow \frac{1}{|\alpha|}X\left(\frac{f}{\alpha}\right)$$

$$= -4\,X(4f)e^{-j4\pi f} \qquad x(t-t_0) \leftrightarrow e^{-j2\pi f t_0}X(f)$$

51. A 4-bit priority encoder has inputs D_3, D_2, D_1, and D_0 in descending order of priority. The two-bit output AB is generated as 00, 01, 10 and 11 corresponding to inputs D_3, D_2, D_1, and D_0, respectively. The Boolean expression of the output bit B is ______.

(a) $\overline{D_3}D_2 + \overline{D_3}\,\overline{D_1}$ (b) $\overline{D_3}\,\overline{D_2}$

(c) $D_3\overline{D_2} + \overline{D_3}D_1$ (d) $\overline{D_3}\,\overline{D_1}$

51. Ans: (a)

Sol: According to given data about priority encoder

D_3	D_2	D_1	D_0	A	B
0	0	0	0	×	×
1	×	×	×	0	0
0	1	×	×	0	1
0	0	1	×	1	0
0	0	0	1	1	1

$B = \sum m(1,4,5,6,7) + d(0)$

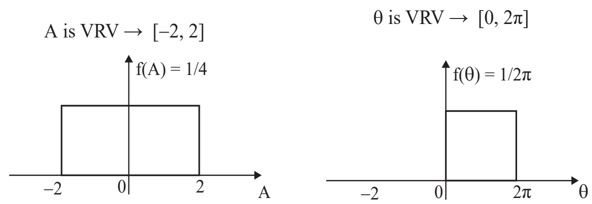
D_3D_2 \ D_1D_0	00	01	11	10
00	×	1		
01	1	1	1	1
11				
10				

$B = \overline{D}_3D_2 + \overline{D}_3\overline{D}_1$

52. Let $X(t) = A\cos(2\pi f_0 t + \theta)$ be a random process, where amplitude A and phase θ are independent of each other, and are uniformly distributed in the intervals [–2, 2] and [0, 2π], respectively. X(t) is fed to an 8-bit uniform mid-rise type quantizer. Given that the autocorrelation of X(t) is $R_x(\tau) = \frac{2}{3}\cos(2\pi f_0\tau)$, the signal to quantization noise ratio (in dB, rounded off to two decimal places) at the output of the quantizer is ______.

52. Ans: 45.15

Sol: $x(t) = A\cos(2\pi f_0 t + \theta)$, $R_x(\tau) = \frac{2}{3}\cos(2\pi f_0\tau)$



A and θ are independent random variables

8 -bit mid rise quantizer is used

Signal power = $R_x(0) = \frac{2}{3}$ Watts

Quantization Noise power $N_q = \frac{\Delta^2}{12}$

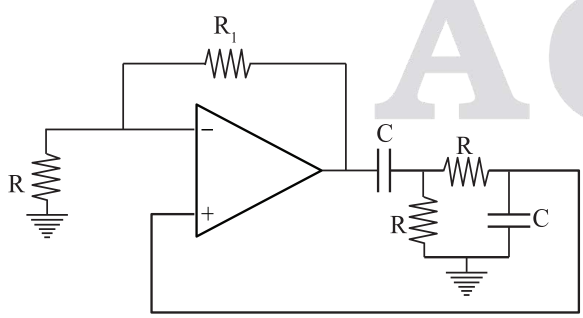
$$\Delta = \frac{2-(-2)}{2^8} = \frac{4}{256} = \frac{1}{64}$$

$$\therefore N_q = \frac{1}{64\times 64\times 12}$$

$$\frac{S}{N_q} = \frac{2\times 64\times 64\times 12}{3\times 1} = 32768$$

$$\left(\frac{S}{N_q}\right)_{\text{in dB}} = 10\log_{10}(32768) = 45.15 \text{ dB}$$

53. In the circuit below, the opamp is ideal.



If the circuit is to show sustained oscillations, the respective values of R_1 and the corresponding frequency of oscillations are

(a) 29R and 1/(2πRC)

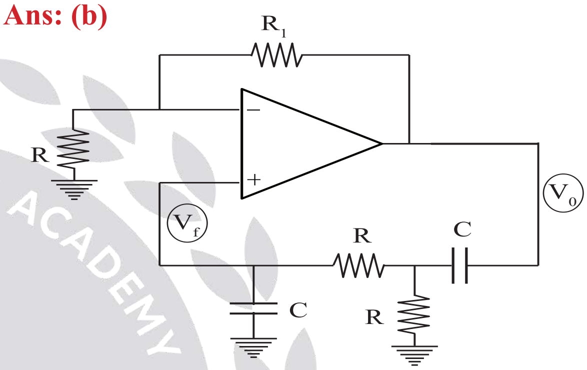
(b) 2R and 1/(2πRC)

(c) 29R and $1/(2\pi\sqrt{6}RC)$

(d) 2R and $1/(2\pi\sqrt{6}RC)$

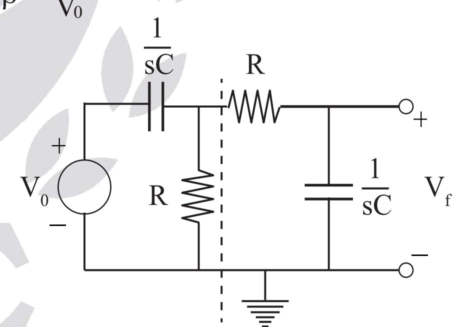
53. Ans: (b)

Sol:

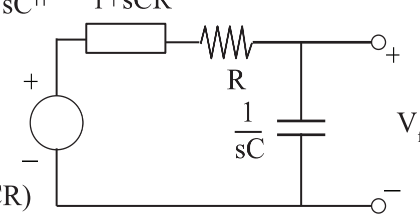


Step: 1 $A = \frac{V_0}{V_f} = 1 + \frac{R_1}{R}$ - - - - (1)

Step: 2 $\beta = \frac{V_f}{V_0}$



$$\frac{1}{sC}\Big|\Big|R = \frac{R}{1+sCR}$$



$$\frac{V_0 R}{R+\frac{1}{sC}} = \frac{V_0(sCR)}{1+sCR}$$

$$V_f = \frac{\dfrac{V_0(sCR)}{1+sCR}}{\dfrac{R}{1+sCR}+R+\dfrac{1}{sC}}$$

$$\frac{V_f}{V_0} = \frac{sCR}{s^2C^2R^2+3sCR+1} = \frac{1}{3+sCR+\dfrac{1}{sCR}}$$

$$\beta = \frac{V_f}{V_0} = \frac{1}{3+j\left(\omega RC-\dfrac{1}{\omega CR}\right)}$$

Step: 3

$$A\beta = 1 \rightarrow A = \frac{1}{\beta}$$

$$1+\frac{R_1}{R} = 3+j\left(j\omega RC-\frac{1}{\omega RC}\right)$$

$$1+\frac{R_1}{R} = 3 \text{ and } \omega RC-\frac{1}{\omega RC} = 0 \rightarrow \omega = \frac{1}{RC}$$

$$\Rightarrow R_1 = 2R$$

54. The information bit sequence {111010101} is to be transmitted by encoding th cyclic Redundancy Check 4(CRC-4) code, for which the generator polynomial is $C(x) = x^4 + x + 1$. The encoded sequence of bits is _______.

(a) (1110101011101)
(b) (1110101011110)
(c) (1110101011100)
(d) (1110101010100)

54. Ans: (c)

Sol: Message sequence = [1 1 1 0 1 0 1 0 1]

$C(x) = x^4 + 1 + 1$

in coded format this is [1 0 0 1 1]

Number of zeros added at end of message

= order of polynomial = 4

```
10011) 1110101010000 (111110100
       10011
       0)11100
         10011
         0)11111
           10011
           0)11000
             10011
             0)10111
               10011
               0)01000
                 00000
                 0)10000
                   10011
                   0)00110
                     00000
                     0)01100
                       00000
                       0)1100
```

Checksum/parity bits

∴ Transmitter sequence / Encoded sequence is

1 1 1 0 1 0 1 0 1   1 1 0 0

Message bits   Parity bits

55. Consider a system S represented in state space as

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}x + \begin{bmatrix} 1 \\ 0 \end{bmatrix}r,\ y = \begin{bmatrix} 2 & -5 \end{bmatrix}x$$

Which of the state space representations given below has/have the same transfer function as that of S?

(a) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r, y = [1 \;\; 2]x$

(b) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} r, y = [1 \;\; 1]x$

(c) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, y = [0 \;\; 2]x$

(d) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r, y = [1 \;\; 2]x$

55. Ans: (b & d)

Sol: $\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}[x] + \begin{bmatrix} 1 \\ 0 \end{bmatrix}[r], y = [2 \;\; -5]][x]$

$TF = C[sI - A]^{-1}B + D$

$TF = C\frac{Adj[sI - A]}{|sI - A|}B + D$

$$[sI - A] = \begin{bmatrix} s & +2 \\ -1 & s+3 \end{bmatrix}$$

$$TF = \frac{[2 \;\; -5]\begin{bmatrix} s+3 & -2 \\ +1 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+3s+2} + [0]$$

$$TF = \frac{[2 \;\; -5]\begin{bmatrix} s+3 \\ 1 \end{bmatrix}}{(s^2+3s+2)} = \frac{2s+6-5}{(s^2+3s+2)} = \frac{2s+1}{(s^2+3s+2)}$$

It is a transfer function of given system.

Here we require to check transfer function of for all the options.

(a) $[sI - A] = \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$

$$TF = \frac{[1 \;\; 2]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{(s+1)(s+2)}$$

$$TF = \frac{[1 \;\; 2]\begin{bmatrix} s+2 \\ s+1 \end{bmatrix}}{(s+1)(s+2)} = \frac{s+2+2s+2}{s^2+3s+2} = \frac{3s+4}{s^2+3s+2}$$

option (a) is wrong

(b) $[sI - A] = \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}$

$$TF = \frac{[1 \;\; 1]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}}{(s+1)(s+2)}$$

$$TF = \frac{[1 \;\; 1]\begin{bmatrix} -s-2 \\ 3s+3 \end{bmatrix}}{s^2+3s+2} = \frac{-s-2+3s+3}{s^2+3s+2} = \frac{2s+1}{s^2+3s+2}$$

option (b) is correct

(c) $[sI - A] = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$

$$TF = \frac{[0 \;\; 2]\begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+3s+2}$$

$$= \frac{[0 \;\; 2]\begin{bmatrix} s+3 \\ -2 \end{bmatrix}}{s^2+3s+2} = \frac{-4}{s^2+3s+2}$$

option (c) is wrong

(d) $[sI - A] = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$

$$TF = \frac{[1 \;\; 2]\begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2+3s+2}$$

$$= \frac{[1 \;\; 2]\begin{bmatrix} 1 \\ s \end{bmatrix}}{s^2+3s+2} = \left(\frac{1+2s}{s^2+3s+2}\right)$$

option (d) is correct