3) a)
H=X^T WX, where W is the weight matrix.
Compute the inverse of the Hessian matrix

H^{-1}

```
beta_hat <- c(1, 0.5)
phi_hat <- 0.05


X <- matrix(c(rep(1, 3), rep(1, 3), rep(1, 3), log(c(3, 6, 9))), ncol = 2, byrow =
TRUE)


W <- diag(1 / phi_hat, nrow(X))


H <- t(X) %*% W %*% X


H_inv <- solve(H)


covariance_matrix <- H_inv


print(covariance_matrix)
```

```
           [,1]       [,2]
[1,]  0.6700160 -0.6032975
[2,] -0.6032975  0.5482054
```

3) b)

Given that Yi ~ Poisson(μi), the probability density function of the random variable Yi is:

$f(yi|\mu i) = e^{-\mu i} * (\mu i)^{yi} / yi!$

$f(yi|\mu i) = \exp\{yi * \log(\mu i) - \log(yi!) - \mu i\} = \exp\{ T(yi) \bullet \alpha - \beta(\mu i) + \gamma(yi) \}$

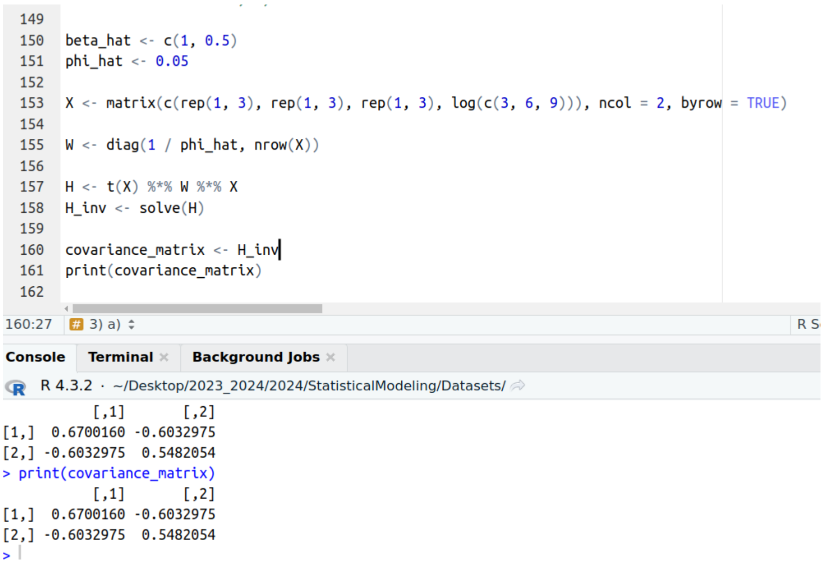
where:
$T(yi) = yi$
$\alpha = \log(\mu i)$
$\beta(\mu i) = \mu i$
$\gamma(yi) = -\log(yi!)$

Therefore, Yi indeed belongs to the exponential family of distributions.

To show that $E(Yi) = \mu i$ and $Var(Yi) = \mu i$, we need to use the properties of the Poisson distribution. The mean and variance of a Poisson random variable are both equal to the parameter λ of the Poisson distribution:

$E(Yi) = \lambda = \mu i$
$Var(Yi) = \lambda = \mu i$

Thus, $E(Yi) = \mu i$ and $Var(Yi) = \mu i$ for the Poisson distribution with parameter μi.



```
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W <- diag(1 / phi_hat, nrow(X))

H <- t(X) %*% W %*% X
H_inv <- solve(H)

covariance_matrix <- H_inv
print(covariance_matrix)
```

```
160:27   3) a)
Console   Terminal   Background Jobs
R 4.3.2 · ~/Desktop/2023_2024/2024/StatisticalModeling/Datasets/
          [,1]       [,2]
[1,]  0.6700160 -0.6032975
[2,] -0.6032975  0.5482054
> print(covariance_matrix)
          [,1]       [,2]
[1,]  0.6700160 -0.6032975
[2,] -0.6032975  0.5482054
>
```

3) b)
Given that $Yi \sim Poisson(\mu i)$, the probability density function of the random variable Yi is:

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$f(yi|\mu i) = \exp\{yi * \log(\mu i) - \log(yi!) - \mu i\} = \exp\{ T(yi) \bullet \alpha - \beta(\mu i) + \gamma(yi) \}$

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Thus, $E(Yi) = \mu i$ and $Var(Yi) = \mu i$ for the Poisson distribution with parameter μi.

3) c)
Following steps can be followed to construct a prediction interval for a new observation:

1. Fit the Gamma Model $\beta 0$ and ϕ
2. Calculate the point estimate of the response variable Yf using the estimated parameters.
3. Compute the standard error of prediction (σ_pred) for Yf.
4. Determine the quantiles of the Gamma distribution to construct the prediction interval.