Lab 05

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Q1 $N = 781$ $n = 150$

a) Beta $(0.01, 0.01)$

Posterior mean $\hat{\pi} \approx 0.20$
95% credible interval $[0.17, 0.23]$

Uniform $(0,1)$, Beta $(1,1)$:

Posterior mean $\hat{\pi} \approx 0.20$

95% credible interval $[0.17, 0.23]$

The prior has little impact. The data dominates the posterior distn.

b) $N = 8$, $n = 0$

i) Beta $(0.01, 0.01)$
Posterior mean $\hat{\pi} \approx 0.009$
95% Credible interval $[0.0001, 0.039]$
The prior pushes the posterior towards 0 & 1.

ii) Uniform $(0,1)$ = Beta $(1,1)$
Posterior mean $\hat{\pi} \approx 0.111$
95% C.I $[0.014, 0.307]$

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iii) Beta(0.5, 0.5)

Posterior mean $\hat{\pi} \simeq$ ~~0.111~~ 0.058

95% C.I = [0.1005, 0.206]

Prior is likely to follow extreme probabilities.

When the sample size is small, (N=8, n=0) the choice of prior strongly influences posterior.

c) • If $N_{new} = 100$, the predictive mean for n_{new} would be approximately 20.

• The predicted probability for the new survey group would remain near 0.2 for large sample size.

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#02

a) <u>Model Summary</u>

Likelihood $y_i \sim t(\mu, \psi, 4)$

Prior $\mu \sim N(0.1 \times 10^{-6})$

$\psi \sim Gamma(0.001, 0.001)$

variance $\sigma^2 = 1/\psi$

~~b)~~ posterior mean for μ:

Robust estimate ~~for~~ of the center

posterior variance: Accounts for outlier due to heavy tails

~~c) Normal vs t-distn~~

b) • Heavy tails reduce the outliers.

• Produces more reliable estimate for μ & σ^2,

t- distn provides robustness when outlier exist.

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c) Ynew ~ t(m, Φ, 4)

#3 a) Posterior Estimate for λ

fata[i] ~ Poisson (λ) with

* Prior λ ~ Gamma(0.1, 0.1)

Posterior mean of λ. Represents the average annual number of airline fatalities.

* 95% credible interval for λ, Shows uncertainity in the estimate.

b) use the posterior of λ to Stimulate fata[27] ~ Poisson(λ)

Summarize the predictive mean and 95% credible interval.

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a) Posterior Estimates for parameters

$\mu_i = \beta_0 + \beta_1 gen[i] + \beta_2 b.length[i]$

Likelihood $b.mass[i] \sim N(\mu, \varphi)$

Priors:

$\beta_0, \beta_1, \beta_2 \sim N(0, 1 \times 10^{-4})$

$\varphi \sim uniform(0, 100)$

Posterior

Estimates $\beta_0, \beta_1, \beta_2$ show the relationship between gender, body length and body mass.

variance $\sigma^2 = 1/\varphi$.

b) Convergence Diagnostics

The model estimates the effect of gender and body length on the body mass.

Posterior predictions for new ~~obsepv~~ observations and Convergence diagnostic ensuring reliable results.