3) a) The survival function $S(ti)$ gives the probability that the event (failure) has not occurred by time ti. We can derive it from the Weibull density function $f(ti)$ as follows:

$S(ti) = P(T > ti) =$ integral from ti to infinity of $f(t)$ dt

Substituting the Weibull density function, we get:

$S(ti) =$ integral from ti to infinity of $[p/\lambda\ (t/\lambda)^p-1\ .\exp[-(t/\lambda)^p]]$ dt

Using integration by substitution, we can simplify the integral as follows:

Let $u = (t/\lambda)^p$

$du/dt = p/\lambda\ (t/\lambda)^{(p-1)}$

$dt = (\lambda/p)\ u^{(1/p-1)}\ du$

Substituting u and dt in the integral, we get:

$S(ti) =$ integral from $(ti/\lambda)^p$ to infinity of $e^{(-u)}$ du

Using the relationship between the gamma and incomplete gamma functions, we can express $S(ti)$ in terms of the incomplete gamma function as follows:

$S(ti) = \text{gamma}(1/p, (ti/\lambda)^p)$

where $\text{gamma}(a, x) =$ integral from x to infinity of $t^{(a-1)}\ e^{(-t)}$ dt is the incomplete gamma function.

3) b)

The hazard function is defined as the rate of failure at time t, given that the individual has survived up to time t. It can be derived from the survival function as follows:

$h(ti) = \lim \Delta t \to 0\ P(ti \leq T < ti+\Delta t \mid T \geq ti) / \Delta t$
$= -d/dt \log(S(ti))$

Starting from the survival function $S(ti)$:

$S(ti) = P(T > ti) = \int ti^\infty f(t)\ dt$

Taking the derivative of both sides with respect to ti:

$d/dti\ S(ti) = -f(ti)$

Substituting $f(ti)$ from the Weibull density function:

$d/dti\ S(ti) = -p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p)$

Dividing both sides by S(ti) and simplifying:

$d/dti\ S(ti) / S(ti) = -p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p) / (1 - \int_0^{ti} f(u)\, du)$
$= -p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p) / S(ti)$

Taking the negative logarithmic derivative of both sides:

$-h(ti) = d/dti\ \log(S(ti))$
$= p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p) / S(ti)$
$= p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p)\ (1 / S(ti))$

Therefore, the hazard function for the Weibull distribution is:

$h(ti) = p/\lambda\ (ti/\lambda)^{(p-1)} \exp(-(ti/\lambda)^p) / S(ti)$