001

Lab - 0

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001. Joint probabilities

$P(E_0 \cap F_0) = 0.10$ $P(F_0) = 0.10 + 0.05 = 0.15$

$P(E_1 \cap F_0) = 0.05$ $P(F_1) = 0.50 + 0.35$

$= 0.85$

$P(E_0 \cap F_1) = 0.50$

$P(E_1 \cap F_1) = 0.35$

a) $P(E_0 / F_0) = \frac{P(E_0 \cap F_0)}{P(F_0)} = \frac{0.10}{0.15} = 0.67$

$P(F_0) = P(E_0 \cap F_0) + P(E_1 \cap F_0) = 0.10 + 0.05 = 0.15$

b) $P(E_0 / F_1) = \frac{P(E_0 \cap F_1)}{P(F_1)}$

$= \frac{0.50}{0.85} = 0.5882$

c) $P(F_0 / E_0) = \frac{P(E_0 \cap F_0)}{P(E_0)} = \frac{0.10}{0.60} = 0.1667$

d) $P(F_1 / E_0) = \frac{P(E_0 \cap F_0)}{P(E_0)} = \frac{0.50}{0.6} = 0.8333$

~~e) $P(F_0) = P(E_0 \cap F_0) + P(E_1 \cap F_0)$~~

~~$= 0.10 + 0.05 = 0.15$~~

e) $P(E_0) = P(E_0 \cap F_0) + P(E_0 \cap F_1) = 0.10 + 0.5 = 0.6$

$P(E_1) = P(E_1 \cap F_0) + P(E_1 \cap F_1)$

$= 0.05 + 0.35 = 0.40$

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f) $P(F_0) = 0.15$ | $P(F_0) = P(E_0 \cap F_0) + P(E_1 \cap F_0)$

$= 0.10 + 0.05$

$= 0.15$

$P(F_1) = 0.85$ | $P(F_1) = P(E_0 \cap F_1) + P(E_1 \cap F_1)$

$= 0.50 + 0.35$

$= 0.85$

g) $P(E_0) \times P(F_0) = 0.60 \times 0.15 = 0.09$

But $P(E_0 \cap F_0) = 0.10 \neq 0.09$

E & F are not independent.

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$P(X=0, Y=1) = \frac{1}{4}$

$P(X=0, Y=-1) = \frac{1}{4}$

$P(X=1, Y=0) = \frac{1}{4}$

$P(X=-1, Y=0) = \frac{1}{4}$

$E(X) = 0 \cdot P(X=0) + (1) \cdot P(X=1) + (-1) \cdot P(X=-1)$

$P(X=0) = P(X=0, Y=1) + P(X=0, Y=-1)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(X=1) = \frac{1}{4}$

$P(X=-1) = \frac{1}{4}$

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$E(X) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} - 1 \cdot \frac{1}{4} = 0$

$E(Y) = 0$

~~$E(XY) = 0$~~

$E(XY) = (0 \times 1)\frac{1}{4} + (0 \times (-1))\frac{1}{4}$

$+ (1 \times 0)\frac{1}{4} + (-1 \times 0)\frac{1}{4} = 0$

$E(XY) = 0 = E(X)\,E(Y)$ the covariance $\mathrm{cov}(X, Y) = 0$ Therefore X & Y are uncorrelated.

$P(Y=0) = P(X=1, Y=0) + P(X=-1, Y=0)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(X=0, Y=0) = 0$

$P(X/Y) \neq P(X)$ which implies X & Y are dependent.

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Problem-03

Prior

30% of twins are identical $P(\text{Identical}) = 0.30$

70% of twins are fraternal $P(\text{Fraternal}) = 0.70$

Identical twins are always same gender

$P(\text{Girls} / \text{Identical}) = 0.5$

$P(\text{Girls} / \text{Fraternal}) = \frac{1}{4}$

$$P(\text{Identical}/\text{Girls}) = \frac{P(\text{Girls}/\text{Identical}) \times P(\text{Identical})}{P(\text{Girls})}$$

$$P(\text{Girls}) = P(\text{Girls}/\text{Identical}) \times P(\text{Identical}) + P(\text{Girls}/\text{Fraternal}) \cdot P(\text{Fraternal})$$

$$P(\text{Girls}) = 0.15 + 0.175 = 0.325$$

$$P(\text{Identical}/\text{Girls}) = \frac{P(\text{Girls}/\text{Identical}) \times P(\text{Identical})}{P(\text{Girls})}$$

$$= \frac{0.5 \times 0.30}{0.325} = \frac{0.15}{0.325}$$

$$= 0.4615$$

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The posterior prob that Elsa and Ella are identical twins, and girl is 0.4615

#09 Problem = 009

$P(D/X=3)$

$P(D/X=1)$

$P(D/X=2)$

$$P(D/X=x) = \frac{P(X=x/D)\,P(D)}{P(X=x)}$$

Prior probability of the Die being defective

$P(D) = \frac{1}{4} = 0.25$

If Defective

$P(X=x/D)$ & $P(X=x/\text{Not Defective})$

$P(X=1/D) = \frac{2}{6} = \frac{1}{3}$

$P(X=3/D) = \frac{1}{6}$

$P(X=2/D) = 0$

If Not Defective

$P(X=x/\text{Not Defective}) = \frac{3}{4}$

$P(\text{Not Defective}) = 3/4$

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$P(X=x)$ Total prob

$P(X=x) = P(X=x/D) \cdot P(D)$
$+ P(X=x \mid \text{Not Defective})$
$\cdot P(\text{Not Defective})$

a) $P(D/X=3)$

1. Calculate $P(X=3)$

$P(X=3) = \frac{1}{6} \times 0.25 + \frac{1}{6} \cdot 0.75 = \frac{1}{6}$

$$P(D/X=3) = \frac{P(X=3/D) \cdot P(D)}{P(X=3)}$$

$$= \frac{\left(\frac{1}{6}\right) \times 0.25}{1/6} = 0.25$$

b) ~~$P(D/X)$~~ $P(D/X=1)$

$P(X=1)$

$P(X=1) = \left(\frac{1}{3}\right) 0.25 + \left(\frac{1}{6}\right) 0.75$

$P(X=1) = \frac{1}{12} + \frac{1}{8} = \frac{5}{24} = 0.2083$

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~~P(D/X=1)~~

$$P(D/x=1) = \frac{P(x=1/D)\,P(D)}{P(x=1)}$$

$$= \frac{(1/3)\times 0.25}{5/24} = 2/5$$

$$= 0.4$$

c) $p(D/x=2)$

$P(X=2) = p(x=2/$ Not Detective$)$
$\times p($Not Detective$) = \frac{1}{6}\times 0.75$

$= \frac{1}{8} \approx 0.125$

Apply Bayes theorem

$$P(D/X=2) = \frac{P(x=2/D)\times P(D)}{P(x=2)}$$

$$= \frac{0.025}{0.125} = 0$$

$p(D/x=2) = 0$

Sum up $P(D/X=3) = 0.25$

$P(D/X=1) = 0.4$

$P(D/X=2) = 0$

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Problem 05

1. Genetics of the Black / Brown

1. BB with prob 1/4
2. Bb with prob $\frac{1}{2}$
3. bb with prob $\frac{1}{4}$

Since offspring is black, we only consider genotypes BB & Bb

or

a) $P(\text{Black}) = P(BB) + P(Bb) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

b) $P(BB \mid \text{Black}) = \frac{P(BB)}{P(\text{Black})} = \frac{1/4}{3/4} = \frac{1}{3}$

c) $P(Bb \mid \text{Black}) = \frac{P(Bb)}{P(\text{Black})} = \frac{1/2}{3/4} = \frac{2}{3}$

$P(BB \mid \text{Black}) = \frac{1}{3}$

$P(Bb \mid \text{Black}) = \frac{2}{3}$

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a) If the black mouse is BB

prob of all seven being black = 1.

b) If the black mouse is Bb

$$= \left(\frac{1}{2}\right)^7 = \frac{1}{128}$$

- $P(B/A) = 1$
- $P(B/A') = \frac{1}{128}$
- $P(A)$ & $P(A')$

$$P(A) = \frac{1}{3} \quad P(A') = \frac{2}{3}$$

- $P(B) = P(B/A)\,P(A) + P(B/A')\,P(A')$

$$= 1\left(\frac{1}{3}\right) + \frac{1}{128} \cdot \frac{2}{3} = \frac{1}{3} + \frac{1}{192}$$

0.3368

$$P(A/B) = \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{1}{3} = 0.995.$$

The Black mouse is homozygous BB given ~~that all~~ seven offspring are black is approximately 99.5% or 0.995