Bayesian Analysis I, Mini-ANALYSIS EXAM

INSTRUCTIONS: This is an open-source, **closed-colleague** examination. You need to submit all your **codes, output and brief comments** so that I can doublecheck your results. Put your *name* and *student no.* in the file (pdf preferred).

1. The populations, n_i and the number of cases, X_i of a disease in a year in each of six districts are given in the table below.

Population n_i	Cases X_i
120342	2
235967	5
243745	3
197452	5
276935	3
157222	1

We suppose that the number X_i in a district with population n_i is a Poisson random variable with mean $n_i\theta/10000$. The number in each district is independent of the numbers in other districts, given the value of θ. Our prior distribution for θ is a Gamma distribution with mean 3.0 and standard deviation 2.0.

a) Find the parameters of the prior distribution.

b) Find the prior probability that $\theta < 2.0$.

c) Find the likelihood.

d) Find the posterior distribution of θ.

e) Find the posterior mean and posterior standard deviation of θ.

f) Plot a graph showing the prior and posterior density functions of θ on the same axes.

g) Find the posterior probability that $\theta < 2.0$.

2. The following data give the supine systolic blood pressures (mm Hg) for fifteen patients with moderate essential hypertension. The measurements were taken immediately before and two hours after taking a drug.

Patient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Before	210	169	187	160	167	176	185	206	173	146	174	201	198	148	154
After	201	165	166	157	147	145	168	180	147	136	151	168	179	129	131

We are interested in the effect of the drug on blood pressure. We assume that, given parameters μ, τ, the changes in blood pressure, from before to after, in the n patients are independent and normally distributed with unknown mean μ and unknown precision $\psi = 1/\tau$: The 15 differences are as follows.

-9 -4 -21 -3 -20 -31 -17 -26 -26 -10 -23 -33 -19 -19 -23

Our prior distribution for ψ is a gamma(0.35, rate=1.01) distribution. Our conditional prior distribution for μ given τ is a Normal $\mathrm{N}(0, [0.003\psi]^{-1})$ distribution.

a) Using McMC, obtain the marginal posterior distribution of ψ.

b) Obtain the marginal posterior distribution of μ.

c) Give the marginal posterior 95% hpd interval for μ.

d) Comment on what you can conclude about the effect of the drug.

3. A study was conducted on 32 cars to explore the relationship of the gasoline consumption on the the weight of the car and engine sizes in cylinders.

   For each car we have observations on how many miles that car can travel on a gallon of gasoline (mpg), the weight of the car (weight) and two dummy variables that indicates if the car's engine has four cylinders (sixcyl=0 and eightcyl=0) six cylinders (sixcyl=1 and eightcyl=0) or eigth cylinders (sixcyl=0 and eightcyl=1).

   mpg: 21.0, 21.0, 22.8, 21.4,18.7,18.1,14.3, 24.4, 22.8,19.2,17.8,16.4,17.3,15.2,10.4,10.4,14.7, 32.4, 30.4, 33.9, 21.5, 15.5,15.2,13.3,19.2, 27.3, 26.0, 30.4,15.8,19.7,15.0, 21.4

   weight: 2.620, 2.875, 2.320,3.215,3.440, 3.460, 3.570, 3.190, 3.150, 3.440, 3.440, 4.070, 3.730, 3.780, 5.250, 5.424, 5.345, 2.200, 1.615, 1.835, 2.465, 3.520, 3.435, 3.840, 3.845, 1.935, 2.140, 1.513, 3.170, 2.770, 3.570, 2.780

   sixcyl: 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0

   eightcyl: 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0

   We want to sample from the joint posterior distribution in the Gaussian linear regression

$$mpg = \beta_0 + \beta_1 * weight + \beta_2 * sixcyl + \beta_3 * eightcyl + error, \quad error \sim N(0, \sigma^2)$$

   with conjugate priors

$$\begin{aligned} \beta_i &\sim N(0, 10000), \quad i = 0, ..., 3 \\ 1/\sigma^2 &\sim Gamma(0.01,\ rate = 0.01) \end{aligned}$$

   a) Give the plots of the marginal distributions for each parameter.

   b) Construct 95% equal tail probability intervals for each parameter and interpret couple of them.

   c) Investigate if the effect on mpg is different in cars with six cylinders compared to cars with 8 cylinders.

   d) Estimate the predictive distribution for a new 4 cylinder car with weight = 3.5.