

3) a)

In the cumulative proportional odds logit model, the probabilities of observing a value less than or equal to k for a given observation i can be written as:

$$P(Y_i \leq k) = 1 / (1 + \exp(-\beta_0 k - \beta_1 x_{i1}))$$

Therefore, the probabilities of observing each of the three possible outcomes can be written as:

$$P(Y_i = 1) = P(Y_i \leq 1) = 1 / (1 + \exp(-\beta_0 1 - \beta_1 x_{i1}))$$

$$P(Y_i = 2) = P(1 < Y_i \leq 2) = P(Y_i \leq 2) - P(Y_i \leq 1) = [1 / (1 + \exp(-\beta_0 2 - \beta_1 x_{i1}))] - [1 / (1 + \exp(-\beta_0 1 - \beta_1 x_{i1}))]$$

$$P(Y_i = 3) = P(Y_i > 2) = 1 - P(Y_i \leq 2) = 1 - [1 / (1 + \exp(-\beta_0 2 - \beta_1 x_{i1}))]$$