Lab-2

Date: ....../....../..........

Q1 $f_0(H) = kH^2(1-H)^3 \quad 0 < H < 1$

a) To find k, we normalize $f(H)$ by ensuring that the integer over (0,1) is 1.

$$\int_0^1 f_0(H)\, dH = 1$$

The integral result is

k = normalizing constant of Beta(3,4) $= 60$

$$E(H) = \frac{3}{3+4} = \frac{3}{7}$$

$$V(H) = \frac{3\times4}{(3+4)^2(3+4+1)} = \frac{12}{245}$$

b) $L(H) \propto H^x(1-H)^{n-x}$

Posterior follows a Beta Distribution

$$H/x \sim \text{Beta}(x+3,\ n-x+4)$$

c) $$E(H/x) = \frac{x+3}{n+7}$$

$$V(H/x) = \frac{(x+3)(n-x+4)}{(n+7)^2(n+8)}$$

Date: ....../....../..........

d) $\theta > 0.3$

$n = 10, \; x = 4$

Prior prob $P(\theta > 0.3) = 0.74431$

Posterior prob $P(\theta > 0.3 / x) = 0.8247$

002

Y (Peating)	0	1	2	3	4
frequency	109	65	22	3	1

a) ~~$P(y/y$~~

$$P(y/m) = \prod_{i=1}^{n} \frac{m^{y_i} e^{-m}}{y_i!} \propto m^{\sum y_i} e^{-nm}$$

$$P(m/y) \propto m^{\sum y_i} e^{-nm} \sim \text{Gamma}\left(\sum y_i + 1, \frac{1}{n}\right)$$

with $n = 200$ & $\sum y_i = 122$

$$m/y \sim \text{Gamma}\left(123, \frac{1}{200}\right)$$

b) mean $E(m/y) = \frac{123}{200} = 0.615$

variance $V(m/y) = \frac{123}{200^2} = 0.003075$

002

Date: ..../......./...................

$$\text{mode} = \frac{123}{200} = 0.61$$

c) The uniform prior on $(0, \infty)$ is not conjugate prior

003 ~~D~~is regarding.

a) To find C,

1. ~~Integr~~ Integrate from 0 to 10 with density $C/100$

2. Integrate from 10 to ∞ with density $C\theta^{-2}$

$C = 5$

b)
$$p(x|\theta) = \frac{1}{\theta^{10}} \quad \text{for} \quad \theta \geq \max(x)$$

$x_{max} = 4.9$

i) $p(x/\theta) = \frac{1}{\theta}$ for $0 \leq x \leq \theta$

ii) $p(x/\theta) = \frac{1}{\theta^{10}}$ for $\theta \geq 4.9$

Date: ..../......./.................

iii) $\frac{1}{D} = \int_{4.9}^{10} \frac{1}{20\,\theta^{10}} d\theta + \int_{10}^{\infty} \frac{5}{\theta^{12}} d\theta$.

$D = \frac{1}{3.1406 \times 10^{-5}}$

004

a) $P(x|\theta) = \theta x^{\theta-1} \quad 0<x<1$

$P(x|\theta) = \theta^n \prod_{i=1}^{n} x_i^{\theta-1}$

$P(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}$

$P(\theta|x) \propto \theta^{n+\alpha-1} e^{-\theta(\beta - \sum_{i=1}^{n} \log x_i)}$

b) $\theta = 2$

$P(x|\delta) = 2\delta^2 x \quad 0<x<\delta^{-1}$

$P(x|\delta) = \prod_{i=1}^{n} 2\delta^2 x_i \propto \delta^{2n}$

5 $p(x_i|\theta) = 2\theta x_i e^{-\theta x_i^2}$

a) $P(x|\theta) = \prod_{i=1}^{n} p(x_i|\theta) = \prod_{i=1}^{n} 2\theta x_i e^{-\theta x_i^2}$

$= (2\theta)^n \left(\prod_{i=1}^{n} x_i\right) e^{-\theta \sum_{i=1}^{n} x_i^2}$

$P(\theta) = \frac{\beta^\alpha}{\Gamma\alpha} \theta^{\alpha-1} e^{-\beta\theta}$

$P(\theta|x) \propto \theta^{n+\alpha-1} e^{-\theta(\beta + \sum_{i=1}^{n} x_i^2)}$

b) The posterior mean of θ is

$E(\theta|x) = \frac{n+\alpha}{\sum_{i=1}^{n} x_i^2 + \beta}$

$V(\theta|x) = \frac{n+\alpha}{\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^2}$

006

$n = 12$

$\bar{x} = 16.35525$

$$p(\mu|x) \propto \exp\left(-\frac{n}{2}(\mu - \bar{x})^2\right)$$

Thus, the posterior distn is

$$\mu|x \sim \text{Normal}\left(\bar{x}, \frac{1}{n}\right) = \text{Normal}\left(16.35, \frac{1}{12}\right)$$

90% of highest density HDR is

approximately $\bar{x}$ then ± 1.645

$$= 16.35525 \pm 1.645 \frac{1}{\sqrt{12}}$$

$$\simeq 16.35525 \pm 0.4749$$