2.3.

(i) $B_1 A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ a & b \end{pmatrix}$.

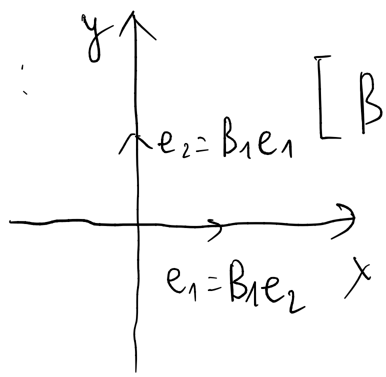
$$B_2 A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a\cos\theta - c\sin\theta & b\cos\theta - d\sin\theta \\ a\sin\theta + c\cos\theta & b\sin\theta + d\cos\theta \end{pmatrix}$$

$$B_3 A = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 5a+c & 5b+d \end{pmatrix}$$

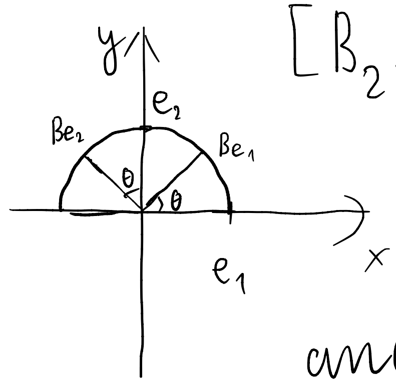
$$B_4 A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 3c & 3d \end{pmatrix}$$

ii) B_1:



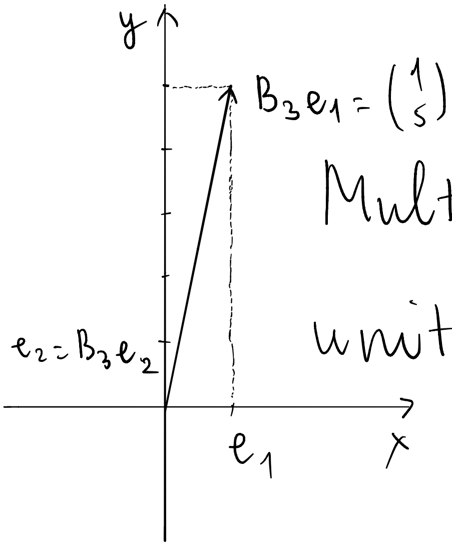
$[B_1 e_1, B_1 e_2] = [e_2, e_1]$, or e_1 and e_2 swaps place after multiplying B_1.

B_2:



$$[B_2 e_1, B_2 e_2] = \left[\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right]$$

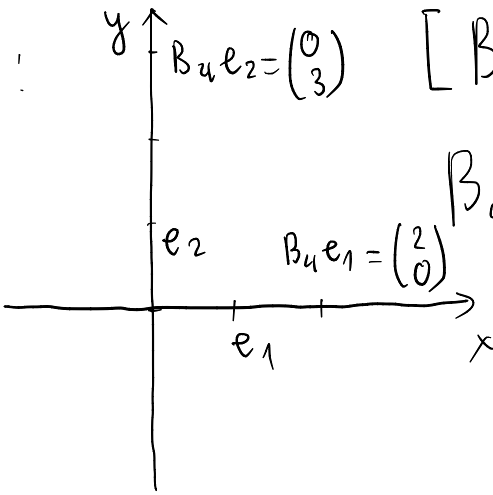
Description: Multiplying B_2 to e_1 and e_2 rotates the vectors by θ radians anti-clockwise through the origin.



$[B_3e_1, B_3e_2] = [\binom{1}{5}, e_2]$.

Multiplying B_3 to e_1 moves e_1 up by 5 units while e_2 remains the same.

B_4:



$[B_4e_1, B_4e_2] = [2e_1, 3e_2]$. Multiplying B_4 to e_1, e_2 scales e_1, e_2 by 2 and 3 times, respectively.

iii). $\det(B_1A) = \begin{vmatrix} c & a \\ d & b \end{vmatrix} = bc - ad = -\det(A)$

$\det(B_2A) = \begin{vmatrix} a\cos\theta - c\sin\theta & b\cos\theta - d\sin\theta \\ a\sin\theta + c\cos\theta & b\sin\theta + d\cos\theta \end{vmatrix}$

$= (a\cos\theta - c\sin\theta)(b\sin\theta + d\cos\theta) - (a\sin\theta + c\cos\theta)(b\cos\theta - d\sin\theta) = ad(\sin^2\theta + \cos^2\theta) - bc(\sin^2\theta + \cos^2\theta) = \det(A)$

$\det(B_3A) = \begin{vmatrix} a & b \\ 5a+c & 5b+d \end{vmatrix} = ad - bc = \det(A)$

$\det(B_4A) = \begin{vmatrix} 2a & 2b \\ 3c & 3d \end{vmatrix} = 6(ad - bc) = 6\det(A)$