Computer Vision CS512: Assignment0
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A.

1. $2A - \cancel{2}B = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2. $\|A\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} = 3.7416$

Angle A makes with x axis $= \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) = 74.49^\circ$

3. A unit vector in the direction of A:

$= \frac{A}{\|A\|}$

$= \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.53 \\ 0.80 \end{bmatrix}$

4. Direction cosines of A

$\cos\alpha = \frac{1}{\sqrt{14}} \Rightarrow \alpha = 74.49^\circ$

$\cos\beta = \frac{2}{\sqrt{14}} \Rightarrow \beta = 57.68^\circ$

$\cos\gamma = \frac{3}{\sqrt{14}} \Rightarrow \gamma = 36.69^\circ$

5. $A \cdot B$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = (1 \cdot 4) + (2 \cdot 5) + (3 \cdot 6)$$

$$= 4 + 10 + 18 = 32.$$

$$B \cdot A \quad \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = (4 \cdot 1) + (5 \cdot 2) + (6 \cdot 3) = 32$$

6. $\vec{A} \cdot \vec{B} = \|A\| \, \|B\| \cos\theta$.

$$32 = \sqrt{14}\sqrt{77} \cos\theta$$

$$\theta = 12.93^\circ$$

7. $\vec{A} \cdot x = 0$.

$$\text{Let } x_1 \quad x_1 + 2y_1 + 3z = 0$$

let $y_1 = 1$ $z_1 = 1$ and $x_1 = -5$

$$\Rightarrow \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} \text{ is a vector orthogonal to } A.$$

8. $$A \times B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 12-15 \\ 12-6 \\ 5-8 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$$ 9.

$$B \times A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15-12 \\ 6-12 \\ 8-5 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

10. $$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix} \quad 1(15-6) - 4(6-3) - 1(12-15$$

$$9 - 12 + 3 = 0$$

$\therefore$ linearly dependent.

11. $A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$

$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

B.

1. $2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$

2. $A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+5-4 & 3+6-1 \\ 2+4 & 4-2-20 & 6+3+4 \\ 3-8 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 4-4+9 & 10-3 \\ 2+2-6 & 8-2-6 & 5+2 \\ 1-8+3 & 4+8+3 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4. $|A| = 1(2-15) - 2(-4) + 3(20)$

$= -13 + 8 + 60 = 55$

$|C| = 1(15-6) - 2(12+6) + 3(4+5)$

$= 9 - 36 + 27 = 0$

5. Matrix C.

6. $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

co factor matrix of A: $\begin{bmatrix} (2-15) & -(-4) & (20) \\ -(-2-15) & (-1) & -(5) \\ (12) & -(3-12) & (-2-8) \end{bmatrix}$

$$\text{Adj } A = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -1/11 & -2/11 \end{bmatrix}$$

lly $B^{-1} = \text{Adj}(B) / |B|$

cofactor matrix of B: $\begin{bmatrix} (1-8) & -(2+12) & (-4-3) \\ -(2+2) & (1-3) & -(-2-6) \\ (-8-1) & -(-4-2) & (1-4) \end{bmatrix}$

$$\text{Adj } B = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 7/42 & 4/42 & 9/42 \\ 14/42 & 2/42 & -6/42 \\ 7/42 & -8/42 & 3/42 \end{bmatrix}.$$

C.

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

$A - \lambda I = \begin{bmatrix} (1-\lambda) & 2 \\ 3 & (2-\lambda) \end{bmatrix}$.

$|A - \lambda I| = (1-\lambda)(2-\lambda) - 6 = 0$

$\Rightarrow 2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$

$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda - 4) + (\lambda - 4) = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\Rightarrow \lambda = 4$ or -1.

$\lambda = 4$:

$\Rightarrow A - \lambda I = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} -3 & 2 & 0 \\ 3 & -2 & 0 \end{array}\right]$.

$R_1 = R_1 + R_2 \rightarrow$

$\begin{array}{c} \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 3 & -2 & 0 \end{array}\right] \\ x_1 \quad x_2 \end{array}$

$3x_1 - 2x_2 = 0 \Rightarrow 3x_1 = 2x_2$.

let $x_1 = 2$

$x_2 = 3$. $\Rightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

$\lambda = -1$: $x_1 \quad x_2$.

$A - \lambda I = \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 3 & 3 & 0 \end{array}\right]$ $R_1 \rightarrow 3R_1$, $R_2 \rightarrow 2R_2$ $\Rightarrow \left[\begin{array}{cc|c} 6 & 6 & 0 \\ 6 & 6 & 0 \end{array}\right]$ $R_1 = R_1 - R_2$

$6x_1 + 6x_2 = 0$

$x_1 = -x_2$. $x_2 = 1$ $x_1 = -1$ $\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}}$

2. $V = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

$$V^{-1} A V = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 20 & 0 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}.$$

3. Dot product of eigen vectors of A.

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -2 + 3 = 1.$$

4. $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$ $\qquad B - \lambda I = \begin{bmatrix} (2-\lambda) & -2 \\ -2 & (5-\lambda) \end{bmatrix}.$

$$|B - \lambda I| = (2-\lambda)(5-\lambda) - 4$$
$$= 6 - 7\lambda + \lambda^2$$
$$= 6 - 6\lambda - \lambda + \lambda^2$$
$$= 6(1-\lambda) - \lambda(1-\lambda)$$
$$(6-\lambda)(1-\lambda) = 0 \Rightarrow \lambda = 6 \text{ or } 1.$$

i) $\left[\begin{array}{cc|c} -4 & -2 & 0 \\ -2 & -1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2x_1 = -1x_2$ $\qquad \begin{bmatrix} +1 \\ -2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x_1 = 1 \quad x_2 = -2$

ii) $\left[\begin{array}{cc|c} 1 & -2 & 0 \\ -2 & 4 & 0 \end{array}\right] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 = 2x_2 \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Dot product: $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 2 - 2 = 0.$

5. The eigenvectors of B are orthogonal. Hence their dot product is equal to zero

The eigenvectors of symmetric matrix is always ~~equal to zero~~. orthogonal.

D.

1. $f'(x) = 2x$
$f''(x) = 2.$

2. $\frac{\partial g}{\partial x} = 2x + \cancel{y^2}^{\,0} = 2x$
$\frac{\partial g}{\partial y} = \cancel{x^2}^{\,0} + 2y = 2y$

3. $\nabla g(x, y) = \begin{bmatrix} \frac{\partial}{\partial x}(x^2 + y^2) \\ \frac{\partial}{\partial y}(x^2 + y^2) \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

4. pdf of a Gaussian distribution is equal to

$$p(x|\mu, \sigma^2) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{Z} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

$$Z = \sqrt{2\pi\sigma^2}$$