

Assignment (Deep Learning)

Homework 0

A.

$$1. \quad 2A - 2B = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}$$

$$2. \quad \|A\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} = 3.7416$$

$$\text{Angle } A \text{ makes with } x \text{ axis} = \cos^{-1} \left(\frac{1}{\sqrt{14}} \right) = 74.49^\circ$$

3. A unit vector in the direction of A:

$$= \frac{A}{\|A\|} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.53 \\ 0.80 \end{bmatrix}$$

4. Direction cosines of A

$$\cos \alpha = \frac{1}{\sqrt{14}} \Rightarrow \alpha = 74.49^\circ$$

$$\cos \beta = \frac{2}{\sqrt{14}} \Rightarrow \beta = 51.68^\circ$$

$$\cos \gamma = \frac{3}{\sqrt{14}} \Rightarrow \gamma = 36.69^\circ$$

$$5. A \cdot B$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = (1 \cdot 4) + (2 \cdot 5) + (3 \cdot 6) = 4 + 10 + 18 = 32$$

$$B \cdot A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = (4 \cdot 1) + (5 \cdot 2) + (6 \cdot 3) = 32$$

$$6. \vec{A} \cdot \vec{B} = \|A\| \|B\| \cos \theta -$$

$$32 = \sqrt{14} \sqrt{77} \cos \theta$$

$$\theta = 12.93^\circ$$

$$7. \vec{A} \cdot x = 0$$

$$x_1 + 2x_2 + 3x_3 = 0$$

Let $y_1 = 1, z_1 = 1$ and $x_1 = -5$.

$$\Rightarrow \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} \text{ is a vector orthogonal to } A.$$

$$8. A \times B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 - 15 \\ 12 - 6 \\ 5 - 8 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \quad 9.$$

$$B \times A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 - 12 \\ 6 - 12 \\ 8 - 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

$$10. \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix} \quad |(15 - 6) - 4(6 - 3) - 1(12 - 15)| = 9 - 12 + 3 = 0$$

\therefore linearly dependent.

$$11. \quad A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$$

$$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

B.

$$1. \quad 2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

$$2. \quad A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+5-4 & 3+6-1 \\ 2+4 & 4-2-20 & 6+3+4 \\ 3-8 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

$$3. AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} (AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 4-4+9 & 10-3 \\ 2+2-6 & 8-2-6 & 5+2 \\ 1-8+3 & 4+8+3 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$4. |A| = 1(2-15) - 2(-4) + 3(20)$$

$$= -13 + 8 + 60 = 55$$

$$|C| = 1(15-6) - 2(12+6) + 3(4+5)$$

$$= 9 - 36 + 27 = 0$$

5. Matrix C.

$$6. A^{-1} = \frac{\text{Adj } A}{|A|}$$

co factor matrix of A:

$$\begin{vmatrix} (2-8) & (-4) & (20) \\ -(-2-15) & (-1) & -(5) \\ (12) & -(3-12) & (-2-8) \end{vmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -43 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -1/11 & -2/11 \end{bmatrix}$$

by $B^{-1} = \text{Adj}(B) / |B|$

cofactor matrix of B:

$$\begin{bmatrix} (1-8) & -(2+12) & (-4-3) \\ -(2+2) & (1-3) & -(-2-6) \\ (-8-1) & -(-4-2) & (1-4) \end{bmatrix}$$

$$\text{Adj } B = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} +7/42 & 4/42 & 9/42 \\ 14/42 & 2/42 & -6/42 \\ 7/42 & -8/42 & 3/42 \end{bmatrix}$$

C.

$$1. \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} (1-\lambda) & 2 \\ 3 & (2-\lambda) \end{bmatrix}.$$

$$|A - \lambda I| = (1-\lambda)(2-\lambda) - 6 = 0$$

$$\Rightarrow 2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + (\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 4 \text{ or } -1.$$

$$\lambda = 4 :$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} -3 & 2 & | & 0 \\ 3 & -2 & | & 0 \end{bmatrix}.$$

$$R_1 = R_1 + R_2 \rightarrow$$

$$\begin{bmatrix} 0 & 0 & | & 0 \\ 3 & -2 & | & 0 \end{bmatrix}$$

$$x_1, x_2$$

$$3x_1 - 2x_2 = 0 \Rightarrow 3x_1 = 2x_2$$

$$\text{Let } x_1 = 2 \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\lambda = -1 : \begin{bmatrix} x_1 & x_2 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 \rightarrow 3R_1 \Rightarrow \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 = R_1 - R_2$$

$$6x_1 + 6x_2 = 0$$

$$x_1 = -x_2 \quad x_2 = 1 \quad x_1 = -1$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$2. \quad V = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{aligned} V^{-1} A V &= \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 4 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 20 & 0 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}. \end{aligned}$$

3. Dot product of eigen vectors of A.

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -2 + 3 = 1.$$

$$4. \quad B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \quad B - \lambda I = \begin{bmatrix} (2-\lambda) & -2 \\ -2 & (5-\lambda) \end{bmatrix}.$$

$$|B - \lambda I| = (2-\lambda)(5-\lambda) - 4$$

$$= 6 - 7\lambda + \lambda^2$$

$$= 6 - 6\lambda - \lambda + \lambda^2$$

$$= 6(1-\lambda) - \lambda(1-\lambda)$$

$$(6-\lambda)(1-\lambda) = 0 \Rightarrow \lambda = 6 \text{ or } 1.$$

$$\text{i)} \quad \left[\begin{array}{cc|c} -4 & -2 & 0 \\ -2 & -1 & 0 \end{array} \right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 4x_2 \\ x_1 - 2x_2 \end{bmatrix} = \begin{bmatrix} +2 \\ -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$ii) \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 = 2x_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{Dot product: } \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} : 2 - 2 = 0.$$

5- The eigenvectors of B are orthogonal.
Hence their dot product is equal to zero

The eigenvectors of symmetric matrix is
always ~~equal to~~ orthogonal

D.

$$1. f'(u) = 2x \\ f''(u) = 2.$$

$$2. \frac{\partial g}{\partial u} = 2u + y^2 = 2u$$

$$\frac{\partial g}{\partial y} = x^2 + 2y = 2y$$

$$3. \nabla g(x, y) = \begin{bmatrix} \frac{\partial}{\partial x} (x^2 + y^2) \\ \frac{\partial}{\partial y} (x^2 + y^2) \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

4. pdf of a Gaussian distribution is equal to

$$p(x|\mu, \sigma^2) = N(x; \mu, \sigma^2) = \frac{1}{Z} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

$$Z = \sqrt{2\pi\sigma^2}$$