Signature Emergence from Rotational Stress: A Non-Wick Mechanism

Adam Kevin Morgan
Unaffiliated

October 23, 2025

Abstract

We propose that rotational stress in viscous media can induce a transition in effective spacetime signature, yielding Minkowski-like causal structure without requiring analytic continuation (Wick rotation). This process, which we term **signature emergence**, reframes the metric signature not as a fixed geometric input, but as a dynamical consequence of internal stress and dissipation. By establishing an equivalence between the Euclidean regularity condition and the requirement for a non-compact temporal channel, we demonstrate that periodic Euclidean time, rotation, and smoothness cannot coexist. The Stern–Gerlach spin alignment process serves as a physical analog, while recent mappings between Navier–Stokes equations and non-Hermitian quantum spin systems provide the mathematical framework. We prove that rotation-induced non-Hermiticity violates Reflection Positivity, creating a Monodromy Obstruction that renders the Euclidean thermal state mathematically inconsistent and necessitates the emergence of Minkowski signature. This work also connects the difficulty of the Navier–Stokes Millennium Problem to foundational questions about spacetime structure. All quantities are expressed in geometric units.

1 Introduction

1.1 The Ontological Problem of Wick Rotation

The Wick rotation ($t \to i\tau$) is a ubiquitous tool [1] in quantum field theory and statistical mechanics. It facilitates the convergence of path integrals, simplifies calculations by mapping Lorentzian spacetimes to Euclidean ones, and underpins the thermal interpretation of black hole horizons via the Gibbons–Hawking procedure. Despite its computational utility, the physical meaning of Wick rotation remains elusive.

Historically, Wick rotation has been treated as a formal analytic continuation, justified by the assumption that the underlying theory admits a well-defined Euclidean counterpart. The Osterwalder–Schrader axioms formalize the conditions under which such a continuation yields a unitary Lorentzian quantum field theory, with reflection positivity playing a central role. However, these axioms are mathematical constraints rather than physical derivations, and they do not explain why nature selects a Lorentzian signature in the first place.

Several attempts have been made to endow Wick rotation with a more physical interpretation. For instance, Hartle and Hawking's "no-boundary" proposal [1] uses a Euclidean path integral to

[1] Throughout this work, τ denotes Euclidean (imaginary) time with periodicity $\beta = 1/T_H$, while t denotes Lorentzian (real) time after signature transition. Greek indices μ, ν run over spacetime coordinates; Latin indices i, j over spatial coordinates only.

define the wavefunction of the universe, suggesting that the Lorentzian signature emerges dynamically from a Euclidean regime. More recently, approaches in loop quantum gravity [2] and causal dynamical triangulations have explored signature change as a quantum gravitational phenomenon. Yet, these frameworks often rely on specific quantization schemes or boundary conditions, and do not directly address the role of internal stresses—such as rotation—in driving signature selection. If signature selection can be shown to emerge from physical dynamics rather than being imposed as a fundamental input, it suggests that the Euclidean and Lorentzian descriptions are not ontologically distinct regimes but rather complementary perspectives on a unified underlying structure. This reframing has profound implications for quantum gravity, where the relationship between Euclidean path integrals and Lorentzian evolution remains conceptually opaque.

A central challenge in interpreting the Wick rotation physically is identifying the appropriate degrees of freedom that encode the thermal and dynamical properties of spacetime while also providing a physical meaning for the Euclidean regime it rotates into. Without a mechanism for continuation, Euclidean spacetime remains a formal construct, disconnected from the dissipative dynamics it is meant to encode. Black hole thermodynamics's **membrane paradigm** [3] offers a compelling framework: it models the black hole horizon as a fictitious, timelike surface endowed with fluid-like properties, including viscosity, conductivity, and vorticity. This **stretched horizon** serves as a natural boundary where gravitational dynamics project onto an effective **dissipative fluid system**, as formalized in the **Fluid/Gravity correspondence** [4].

This boundary-centric viewpoint is particularly well-suited for analyzing the Euclidean continuation of black hole spacetimes. The **Euclidean Kerr geometry**, for instance, imposes a periodicity in imaginary time to avoid conical singularities at the horizon [5]. However, when rotation is present, the frame-dragging effect entangles temporal and angular coordinates, introducing **vorticity** in the dual fluid. This vorticity, in turn, acts as an effective magnetic field in the **non-Hermitian Schrödinger–Pauli mapping** of the Navier–Stokes equations [6], leading to a **Stern–Gerlach–like spin alignment process**.

The **boundary fluid** thus becomes the stage where the incompatibility between rotation and Euclidean periodicity is most transparently realized. By focusing on this boundary, we gain access to a **quantum mechanical description** of dissipation and stress, enabling a precise analysis of how rotational dynamics obstruct the existence of a smooth Euclidean thermal state. This motivates our central claim: that the **signature of spacetime is not a fixed input**, but an **emergent feature** dictated by the **spectral stability of the boundary dynamics**.

1.2 Physical and Mathematical Foundations

The mechanism we propose synthesizes three established theoretical frameworks, each contributing an essential piece of the physical picture.

The **fluid/gravity correspondence** [4, 3] establishes that black hole horizon dynamics can be precisely mapped to viscous fluid flow on a stretched horizon membrane. This duality provides a dictionary relating geometric properties of the spacetime—such as frame-dragging ($g_{t\phi}$), surface gravity (κ), and angular momentum (J)—to hydrodynamic variables including fluid velocity ($\mathbf{u}$), temperature (T_H), and vorticity ($\boldsymbol{\omega} = \nabla \times \mathbf{u}$). For a rotating Kerr black hole, the frame-dragging effect manifests in the fluid dual as rotational vorticity, making rotation a fundamentally hydrodynamic phenomenon at the horizon scale.

Within this hydrodynamic description, **Meng & Yang's mapping** [6] demonstrates that the incompressible Navier-Stokes equations can be recast in Schrödinger-Pauli form through a Madelung transformation. Crucially, this mapping reveals that viscous dissipation—unavoidable in any rotating fluid—introduces an imaginary potential $V_I \sim \nu|\nabla\psi|^2$ into the effective quantum Hamiltonian.

This term encodes the irreversible energy dissipation associated with rotational stress, manifesting as a non-Hermitian contribution to the Euclidean evolution operator.

The mathematical criterion for consistency of this framework is provided by the **Osterwalder-Schrader reflection positivity condition** [7, 8]. Reflection positivity ensures that correlation functions in Euclidean field theory define positive-definite inner products, which is the necessary and sufficient condition for analytic continuation to a unitary Lorentzian quantum theory. When reflection positivity is violated—as occurs when the Euclidean evolution operator acquires complex eigenvalues—the smooth, periodic Euclidean manifold becomes mathematically inconsistent as a thermal state, and alternative geometric structures must emerge.

These three ingredients combine to reveal a physical obstruction: rotation induces vorticity, vorticity generates viscous dissipation, dissipation produces non-Hermitian dynamics, and non-Hermiticity violates reflection positivity. The Euclidean thermal description therefore fails for any rotating black hole, necessitating the emergence of Minkowski signature as the unique physical resolution to this geometric incompatibility.

1.3 The Challenge of Rotational Stress

The established thermal interpretation of Euclidean gravity, as exemplified by the Gibbons–Hawking calculation, depends fundamentally on a smooth, static manifold where the Euclidean time τ is periodic and unmixed with spatial coordinates [5]. This success does not, however, address the deeper ontological question of signature selection when a system possesses internal stress.

For a rotating black hole (Kerr metric[9]), the Euclidean time coordinate τ becomes necessarily mixed with the azimuthal angle ϕ via the frame-dragging effect (subsection D.1). This leads to the fundamental problem: **rotation-induced stress cannot dissipate within a closed, compact time dimension**. In the dual fluid description (via the Fluid/Gravity correspondence [4]), the rotational stress is mapped to **viscous vorticity ($\omega = \nabla \times \mathbf{u}$)** near the horizon (subsection D.2).

If a rotating black hole must settle into a smooth thermal state with period β, then the system, when described in Euclidean time, must be **time-reversible and non-dissipative** (Hermitian). However, the rotational stress is fundamentally dissipative (viscous) and requires a channel to carry away angular momentum and energy [10]. The topological constraint of closed Euclidean time prevents this dissipation, creating an internal mathematical contradiction:

$$\text{Periodic Time } + \text{Rotation} + \text{Smoothness} \implies \text{Contradiction}$$

To explore the mathematical nature of this constraint, we require a framework that can precisely link gravitational rotation to quantum dissipation. The key to our approach is the non-Hermitian representation of fluid dynamics.

This Work

This paper reframes signature selection as a **dynamical consequence of rotational stress** rather than a fixed geometric input. Our approach synthesizes fluid/gravity duality, non-Hermitian quantum mappings, and reflection-positivity criteria to establish a physical mechanism for signature emergence. The argument proceeds in four stages, each developed in subsequent sections:

- **Non-Hermitian Source Term (Section 2):** Using the Navier–Stokes $\to$ Schrödinger–Pauli mapping, we show that rotational vorticity in the Kerr horizon fluid introduces an anti-Hermitian potential $iV_I(x, \tau)$ into the Euclidean generator $H_E(\tau)$, encoding viscous dissipation.

- **Monodromy Obstruction (Section 3):** We prove that any non-zero, non-removable V_I drives complex phases in the Euclidean monodromy operator, violating Osterwalder–Schrader reflection positivity. This demonstrates that a smooth, periodic Euclidean thermal state is mathematically inconsistent for rotating systems.
- **Gauge Robustness (Section 3.4):** We establish that V_I cannot be eliminated by any smooth, single-valued gauge transformation on the thermal circle, confirming that the obstruction is physical rather than representational.
- **Physical Interpretation (Section 4):** We connect the mathematical obstruction to a Stern–Gerlach–type alignment process, showing that rotational stress requires a dissipative channel. The signature flip to Minkowski provides this outlet, resolving the incompatibility between rotation and compact time.

Together, these results demonstrate that the Euclidean-to-Minkowski transition is not an analytic convenience but a physically necessary resolution to a geometric constraint. The subsequent sections formalize this claim through spectral analysis, gauge arguments, and physical analogies.

1.4 The Fluid/Gravity Correspondence[4]

The first step leverages the **Fluid/Gravity Correspondence**[4], which demonstrates that the dynamics of a gravitational bulk (a black hole spacetime) are dual to the dynamics of a viscous, incompressible fluid on a boundary (the stretched horizon) [4]. This duality provides a crucial dictionary for translating geometric properties into fluid mechanical properties:

Gravity (Bulk)	**Fluid (Boundary)**
Horizon location r_+	Membrane position
Frame-dragging $g_{t\phi}$	Fluid velocity field $\mathbf{u}$
Surface gravity κ	Temperature $T_H = \kappa/(2\pi)$
Angular momentum J	Vorticity ω

In this context, the frame-dragging effect of the rotating Kerr black hole [9] is precisely mapped to the **vorticity** ($\omega = \nabla \times \mathbf{u}$) of the dual fluid. This is the central link: gravitational rotation is mathematically equivalent to the fluid's viscous stress.

1.5 The Non-Hermitian Schrödinger–Pauli Mapping

The second tool is the **non-Hermitian quantum spin representation** of the Navier-Stokes (NS) equation, developed by Meng and Yang [6]. This mapping shows that the incompressible NS equations can be written as a two-component Schrödinger–Pauli Equation (SPE). This transformation is essential because it moves the problem from classical fluid dynamics into the language of quantum mechanics, where the condition of Hermiticity ($\hat{H} = \hat{H}^\dagger$) is directly tied to the fundamental physical requirement of energy conservation (unitarity).

The standard Madelung transformation is used to define a spinor field $\psi(\mathbf{x}, \tau)$ from the fluid velocity $\mathbf{u}$ and density ρ. When the NS equation is recast in this quantum form, the presence of **viscosity (ν)** introduces an unavoidable anti-Hermitian component into the resulting Hamiltonian H:

$$H_E(\tau) \;=\; H_0(\tau) \;+\; i\,V_I(\tau) \tag{1}$$

where $H_0(\tau) = H_0(\tau)^\dagger$ is the Hermitian part, and the imaginary potential $V_I(\mathbf{x}, \tau)$ is proportional to the **viscous dissipation**.

On the operator versus effective-potential form of viscous dissipation. The mapping of Navier-Stokes to a Schrödinger–Pauli form produces viscous contributions that act as differential operators (schematically $i\nu\Delta$) on the spinor field ψ. In the present work we adopt a *coarse-grained effective potential* representation,

$$V_I(\mathbf{x}, \tau) \;=\; \nu\, |\nabla\psi(\mathbf{x}, \tau)|^2, \tag{2}$$

which captures the dominant local phase contribution of the full operator in the hydrodynamic or semiclassical regime. This mean-field reduction is standard when gradients are smooth and amplitude variations are small.

In geometric units ($c = G = \hbar = 1$), the kinematic viscosity ν carries dimensions $[L^2/T]$ while $|\nabla\psi|^2$ has dimensions $[1/L^2]$, ensuring that V_I has dimensions $[1/T]$ consistent with an energy-rate contribution to the Euclidean generator H_E (complete dimensional analysis in Appendix D.4). Throughout, we work with fields satisfying vanishing boundary conditions at spatial infinity (Dirichlet), ensuring that the Hermitian part H_0 is properly self-adjoint and that all non-Hermitian contributions arise solely from the viscous dissipation term iV_I. Appendix A derives the equation's reduction and estimates its error, while Appendix B demonstrates with a one-dimensional model that the monodromy obstruction persists when the full operator $i\nu\nabla^2$ is retained. Throughout the main text we therefore work with V_I in the form above for analytic transparency, noting that all qualitative results are unchanged when the full operator is restored.

For a rotating black hole:

1. The geometric rotation Ω_H creates fluid vorticity ω.
2. The vorticity ω acts as an effective magnetic field for the SPE spinor.
3. The viscosity ν introduces the non-Hermitian term iV_I, which is the mathematical signature of **dissipation**.

Our subsequent analysis focuses on proving that this non-zero, anti-Hermitian potential iV_I, sourced by rotation, fundamentally breaks the conditions required for a smooth Euclidean partition function.

2 Kerr Geometry, Vorticity, and the Dissipative Source V_I

2.1 Euclidean Kerr and the Conical Constraint

The standard procedure for relating the gravitational action to thermodynamics involves Wick-rotating the Kerr metric[9] and compactifying the time coordinate $t \to i\tau$ [5]. The near-horizon expansion of the Euclidean Kerr metric[9] contains two essential features (subsection D.1):

1. $\tau - \phi$ **Mixing:** The metric exhibits explicit mixing between the Euclidean time τ and the azimuthal angle ϕ via the frame-dragging term, which is sourced by the rotational parameter a: $ds_E^2 \sim g_{\tau\tau}d\tau^2 + g_{\tau\phi}d\tau d\phi + \dots$
2. **The Conical Singularity:** For the resulting manifold to be smooth (i.e., for the Euclidean partition function to be well-defined), the Euclidean time coordinate τ must be periodic with period $\beta = 4\pi/\kappa$, where κ is the surface gravity. If the periodicity condition is not met, the geometry contains a **conical singularity** at the horizon, signaling a coordinate pathology.

The standard thermal interpretation relies entirely on the smoothness condition $\beta = 1/T_H$. However, the rotational mixing (1) shows that the τ-coordinate is not a true thermodynamic time, introducing the rotational stress that this paper proves is incompatible with the geometric requirement (2).

2.2 Rotational Source and Fluid Vorticity

The geometric constraints are translated into fluid dynamics via the Fluid/Gravity correspondence (Section 2.1). The frame-dragging velocity field near the horizon is mapped to the boundary fluid's velocity $\mathbf{u}$. This rotation sources a non-zero **vorticity** ($\omega = \nabla\times\mathbf{u}$) in the dual fluid (subsection D.2).

The magnitude of this vorticity ω is directly related to the horizon's angular velocity Ω_H:

$$\omega \sim \mathbf{\Omega_H} = \frac{a}{r_+^2 + a^2}\hat{\theta} \tag{3}$$

This steady-state vorticity is the physical manifestation of the rotational stress that cannot be contained in a closed, periodic time dimension. This sets the stage for the quantum mechanical framework, where ω drives the effective anti-Hermitian potential V_I.

2.3 Explicit V_I and Kerr-Scale Estimate

The final link uses the Meng & Yang mapping (subsection 1.5). The viscous dissipation (ν) introduced by the vorticity ω manifests in the Euclidean generator H_E as the anti-Hermitian potential $iV_I(\mathbf{x}, \tau) = i\nu|\nabla\psi|^2$ (subsection 2.3).

By estimating the spinor gradient $|\nabla\psi|^2$ in terms of the hydrodynamic velocity $\mathbf{u} \sim \Omega_H r_+$, we arrive at the Kerr-scale estimate for the anti-Hermitian potential:

$$\overline{V_I} \approx \nu\,\rho\,(\Omega_H r_+)^2 \tag{4}$$

Since ν, ρ, and the rotation Ω_H are all non-zero for a rotating black hole, the time average of the anti-Hermitian potential is **non-zero and constant** over the period β: $\overline{V_I} \neq 0$. This non-zero constant $\overline{V_I}$ is the key quantitative input for the formal Monodromy Obstruction proof.

3 The Monodromy Obstruction to Reflection Positivity

The central claim of this work is that the non-zero rotational stress, quantified by $\overline{V_I} \neq 0$ (Eq. 4), creates a fundamental inconsistency in the Euclidean partition function, forcing the system out of the Euclidean signature. This inconsistency is formally proven by the Monodromy operator's violation of Reflection Positivity[7].

3.1 The Monodromy Operator and Reflection Positivity

The Euclidean construction of a thermal state aims to produce a thermal operator, or **Monodromy operator**, $\mathcal{M}$, via a path integral over the closed time circle $\tau \in [0, \beta)$:

$$\mathcal{M} \;=\; \mathcal{T}\exp\Big(-\int_0^\beta H_E(\tau)\,d\tau\Big), \tag{5}$$

where $\mathcal{T}$ denotes time ordering and $H_E(\tau)$ is the Euclidean time evolution generator (Hamiltonian).

For $\mathcal{M}$ to define a **bona fide thermal state** ρ_β that can be Wick-rotated back to a smooth, unitary Lorentzian quantum field theory, the operator must satisfy the **Osterwalder–Schrader**

Reflection Positivity condition [7]. This condition essentially requires that $\mathcal{M}$ must be a positive operator with a real, positive spectrum, allowing it to be interpreted as $e^{-\beta \widehat{H}}$ where $\widehat{H}$ is a Hermitian Lorentzian Hamiltonian ($\widehat{H} = \widehat{H}^\dagger$).

From the Navier–Stokes → Schrödinger–Pauli mapping (Section 2.2), the Euclidean generator is decomposed as in Eq. (1):

$$H_E(\tau) \;=\; H_0(\tau) \;+\; i\,V_I(\tau), \tag{6}$$

where H_0 is Hermitian and iV_I is the rotation-induced anti-Hermitian part.

Lemma 1 (Spectral Obstruction). *Let $H_E(\tau) = H_0(\tau) + iV_I(\tau)$ with $H_0(\tau) = H_0(\tau)^\dagger$ and real $V_I(\tau)$. If the time-averaged imaginary part of the spectrum is nonzero in the sense that the monodromy $\mathcal{M} = \mathcal{T}\exp\Big(-\int_0^\beta H_E(\tau)\,d\tau\Big)$ acquires phases $e^{-i\beta v_n}$ with generic $v_n \notin \frac{2\pi}{\beta}\mathbb{Z}$, then $\mathcal{M}$ is not a positive operator and violates Reflection Positivity (cf. OS2).*

3.2 The Role of Non-Zero Dissipation

The Monodromy Obstruction arises because the anti-Hermitian component $iV_I(\tau)$ fundamentally compromises the spectral properties of $\mathcal{M}$.

Monodromy Eigenvalues. Assuming for simplicity a constant-in-τ approximation where $H_E(\tau) \approx H_E$, the Monodromy operator simplifies to $\mathcal{M} = e^{-\beta H_E}$. If H_E has eigenvalues $\lambda_n = \epsilon_n + iv_n$, then the Monodromy operator has eigenvalues $m_n = e^{-\beta\lambda_n} = e^{-\beta\epsilon_n}e^{-i\beta v_n}$.

For reflection positivity to hold, the eigenvalues m_n must be real and positive, which requires the imaginary phase to vanish: $e^{-i\beta v_n} = 1$, implying $\beta v_n \in 2\pi\mathbb{Z}$ for all eigenstates. Since the imaginary component v_n is directly related to the non-zero viscous dissipation V_I (which is spatially varying and generically non-quantized), this condition is almost always **impossible to satisfy simultaneously** for a rotating system.

Phase mismatch and non-trivial monodromy. The failure of periodicity can be quantified by the phase mismatch accumulated over one Euclidean cycle:

$$\Delta\phi_n \equiv \beta v_n \mod 2\pi \tag{7}$$

We define a monodromy as **non-trivial** when $\Delta\phi_n \neq 0$ for at least one normalizable eigenstate, indicating that the Euclidean evolution fails to return the state to its initial phase after one period. For a rotating system with generic (non-quantized) viscous dissipation V_I, the condition $\Delta\phi_n = 0$ for all n is impossible to satisfy, yielding non-trivial monodromy and the breakdown of reflection positivity.

From the representation–theoretic standpoint, this breakdown can be understood in the language of reflection–positive semigroups [8]. In the Osterwalder–Schrader framework, the Euclidean evolution operator $\mathcal{M}_0 = e^{-\beta H_E}$ defines a reflection–positive kernel only when its spectrum is real and non-negative, ensuring analytic continuation to a unitary Lorentzian theory. Introducing the rotational source modifies this operator to

$$\mathcal{M} = e^{-\beta(1-i\nu)H_E},$$

which preserves the semigroup structure but drives the eigenvalues into the complex plane. This deformation violates the reflection positivity condition—and hence the conditions for Osterwalder–Schrader reconstruction—by introducing a non-Hermitian spectral phase. The resulting complex eigenvalue trajectory corresponds precisely to the Monodromy Obstruction, marking the point at which the Euclidean thermal description becomes mathematically inconsistent for $J \neq 0$.

The Obstruction. Since the rotational source leads to a non-zero time-averaged imaginary potential $\overline{V_I} \neq 0$ (Eq. 4), the Monodromy operator $\mathcal{M}$ is guaranteed to acquire **complex-phased eigenvalues**.

1. H_E is non-Hermitian due to rotation-induced viscosity.
2. $\mathcal{M}$ consequently possesses complex eigenvalues.
3. Complex eigenvalues violate Reflection Positivity[7].

Therefore, the smooth, periodic Euclidean manifold with $J \neq 0$ (rotation) is **mathematically inconsistent** as a thermal partition function.

3.3 Limits of Exotic Periodicities

We summarize the conditions under which Reflection Positivity may survive despite non-zero imaginary potential. **Lemma:** If the imaginary part of the Euclidean Hamiltonian eigenvalues is quantized such that $\beta v_n \in 2\pi\mathbb{Z}$, then the Monodromy operator may retain real eigenvalues. These cases are non-generic and require fine-tuned periodicities. Detailed analysis is provided in Appendix B

3.4 Robustness: The Gauge-Removability Lemma

The only remaining possibility for preserving the Euclidean thermal state is if the non-Hermitian term iV_I could be removed by a gauge transformation that is periodic on the thermal circle $\tau \in [0, \beta)$. The anti-Hermitian potential iV_I can only be removed by a periodic transformation $U(\tau) = e^{-i\chi(\tau)}$ if $V_I(\mathbf{x}, \tau)$ satisfies a specific gauge-exact condition relating to $\partial_\tau \chi$ and commutators with H_0.

Lemma 2 (Non-removability of Viscous Dissipation)**.** *The imaginary potential $V_I(\mathbf{x}, \tau) = \nu|\nabla\psi|^2$ arising from the Meng-Yang mapping cannot be removed by any smooth, single-valued gauge transformation $U(\tau) = e^{-i\chi(\tau)}$ satisfying the periodicity constraint $U(\beta) = U(0)$.*

Proof. For gauge removal, we require $V_I(\tau) = \partial_\tau \chi(\tau) + [\chi(\tau), H_0(\tau)]$. Since $V_I(\mathbf{x}, \tau)$ is a spatially-varying multiplicative potential, any $\chi(\tau, \mathbf{x})$ must satisfy:

$$e^{-i\chi(\beta,\mathbf{x})} = e^{-i\chi(0,\mathbf{x})} \quad \forall \mathbf{x} \tag{8}$$

This single-valuedness requirement cannot be satisfied globally for generic inhomogeneous V_I profiles without introducing topological defects. Therefore, the monodromy obstruction is robust against gauge transformations. □

We prove that the viscous term obtained from the Meng–Yang mapping:

$$V_I(\mathbf{x}, \tau) \;=\; \nu\, |\nabla\psi(\mathbf{x}, \tau)|^2 \quad \text{(mean-field approximation of } i\nu\nabla^2\psi)$$

is **not gauge-removable** because it is a spatially varying multiplicative potential, not proportional to a global conserved charge. The strict requirement for the transformation $\chi(\tau, \mathbf{x})$ to be single-valued and smooth on the closed τ-circle for all spatial coordinates $\mathbf{x}$ cannot be globally satisfied for a generic, inhomogeneous V_I. The Monodromy Obstruction is thus a **physical and topological necessity**, not a coordinate artifact.

3.5 Conclusion of the Proof

The proof demonstrates the logical incompatibility triangle:

$$\text{Periodic Time } (\tau \in [0,\beta]) + \text{Rotation}(J \neq 0) + \text{Smooth Solution} \implies \text{Contradiction}$$

The only physical resolution is the emergence of a non-compact, time-like channel, which restores spectral stability by allowing the anti-Hermitian energy to dissipate outside the thermal boundary. This necessitates the **signature flip to Minkowski** $(-,+,+,+)$.

Gauge-removability criterion for the imaginary potential

Lemma (gauge-removability). Let $H_E(\tau) = H_0(\tau) + iV_I(\tau)$ with real $V_I(\tau)$. A sufficient condition for iV_I to be removed by a time-dependent similarity/gauge transformation that preserves thermal-periodicity is that there exists a real operator (or c-number) $\chi(\tau)$ with $\chi(\beta) = \chi(0)$ such that

$$V_I(\tau) \; = \; \partial_\tau \chi(\tau) \; + \; [\chi(\tau), H_0(\tau)]_{(\text{comm.})}.$$

If such $\chi(\tau)$ exists then the transformation $|\psi\rangle \mapsto e^{-i\chi(\tau)}|\psi\rangle$ yields a new generator $H'_E(\tau)$ whose anti-Hermitian part is canceled, and the monodromy becomes positive.

Sketch of proof. Under the τ-dependent unitary/gauge transform $U(\tau) = e^{-i\chi(\tau)}$ the Euclidean generator changes as

$$H_E \mapsto H'_E = U H_E U^{-1} - i(\partial_\tau U)U^{-1}.$$

Writing $U = e^{-i\chi}$ and expanding gives the shift of the anti-Hermitian piece by $-i\partial_\tau \chi$ plus commutators with H_0. If V_I equals $\partial_\tau \chi$ up to such commutators (and U is single-valued on the thermal circle so that $U(\beta) = U(0)$), the net monodromy is rendered positive.

Equivalence of gauge and monodromy obstructions. The gauge-removability condition $\int_0^\beta V_I(\mathbf{x}, \tau) d\tau = 0$ and the monodromy obstruction are mathematically equivalent perspectives on the same physical fact. For the gauge transformation $\psi \to e^{-i\chi(\tau)}\psi$ to remove V_I while preserving periodicity, χ must be single-valued: $\chi(\beta, \mathbf{x}) = \chi(0, \mathbf{x}) \mod 2\pi$ for all $\mathbf{x}$. This is precisely the condition that prevents complex phases in the monodromy eigenvalues $e^{-i\beta v_n}$. When $V_I(\mathbf{x}, \tau)$ is spatially inhomogeneous with $\int_0^\beta V_I d\tau \neq 0$, both obstructions manifest: the Hamiltonian $H_E = H_0 + iV_I$ remains non-Hermitian under any admissible gauge transformation, and $\mathcal{M}$ acquires non-trivial complex phases. This structure is analogous to the Berry phase condition in cyclic non-Hermitian quantum systems, where geometric phases arising from adiabatic evolution around closed loops in parameter space cannot be removed by gauge transformations when certain topological conditions are violated [11]. This establishes that rotation-induced dissipation is intrinsic to the physical system, not a representational artifact.

When the gauge trick fails. The gauge removal is only possible when V_I is (i) gauge-exact in τ or (ii) proportional to a global conserved charge Q commuting with H_0 (so $V_I(\tau) = \mu(\tau)Q$ and $\chi(\tau) = \left(\int^\tau \mu\right)Q$). In contrast, the viscous term obtained from the Meng–Yang mapping, $V_I(\mathbf{x}, \tau) = \nu|\nabla\psi|^2$, is generically a spatially varying multiplicative potential which is not of the global conserved-charge form. Any attempt to choose a spatially dependent $\chi(\tau, \mathbf{x})$ faces the single-valuedness requirement $e^{-i\chi(\beta,\mathbf{x})} = e^{-i\chi(0,\mathbf{x})}$ for all $\mathbf{x}$; this cannot be satisfied globally in a smooth way for a generic inhomogeneous profile unless singular gauge transitions or pathological topological decompositions are permitted. Therefore the imaginary potential sourced by viscous vorticity is generically *not* gauge-removable, and the monodromy obstruction described above is robust.

4 The Stern–Gerlach Analogue: Physical Realization of Rotational Dissipation

The Monodromy Obstruction proves the mathematical necessity of a non-compact temporal channel, but the physical mechanism by which rotational stress manifests and resolves requires explicit identification. We demonstrate that the Stern–Gerlach spin alignment process provides more than a heuristic analogy: the mathematical structure of the Meng–Yang mapping suggests a **deep structural correspondence** between laboratory spin alignment and horizon-scale rotational dissipation.

The Stern–Gerlach effect. In the classic Stern–Gerlach experiment [12], silver atoms with magnetic moment μ pass through an inhomogeneous magnetic field $\mathbf{B}$. The field exerts a torque on misaligned spins, causing them to precess and eventually align along the field axis. This alignment is accompanied by energy dissipation: the transition from a mixed-orientation ensemble to an aligned state releases energy $\Delta E \sim \mu B$ that must be carried away (typically by radiation or thermal coupling to the environment).

Crucially, **alignment requires a dissipation channel**. In a closed, reversible system, the spins would precess indefinitely without settling into a definite orientation. The irreversible alignment observed in the Stern–Gerlach experiment depends on coupling to an external bath or radiation field.

4.1 Vorticity as an Effective Magnetic Field

In the two-component Schrödinger–Pauli Equation (SPE) derived from the Navier–Stokes system (Meng & Yang [6]), the vorticity $\omega = \nabla \times \mathbf{u}$ enters mathematically as an effective magnetic field, $\mathbf{B}_{\text{eff}} \sim \omega$. The spinor components ψ are coupled via an interaction term with the same mathematical form as the Zeeman effect: $\mathbf{S} \cdot \mathbf{B}_{\text{eff}}$, where $\mathbf{S}$ is the spin operator.

The vorticity created by Kerr frame-dragging (Section 3) attempts to align the microscopic fluid elements (spinors) along its axis $\mathbf{\Omega}_H$. This process is **structurally identical** to Stern–Gerlach alignment: the frame-dragging field plays the role of the inhomogeneous magnetic field, and the quantum fluid spinors undergo torque-driven alignment analogous to silver atom spins.

The alignment process involves two competing effects:

1. **Alignment Torque:** The effective magnetic field $\mathbf{B}_{\text{eff}}$ exerts a torque on the misaligned fluid elements, driving them toward the axis of rotation.
2. **Dissipation:** The anti-Hermitian term $iV_I = i\nu|\nabla\psi|^2$ (viscosity) represents the energy lost during this alignment process.

4.2 The Alignment-Dissipation Contradiction

In a closed Euclidean space, the required energy dissipation must vanish for a smooth thermal state. However, the rotational torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}_{\text{eff}}$ necessitates a non-zero energy change $\Delta E = 2\mu B_{\text{eff}}$ during alignment. This energy must be radiated away for the system to settle into the aligned state.

- **Euclidean Constraint:** The time period $\tau \in [0, \beta)$ is compact, providing no non-compact channel for the energy to escape.
- **Physical Necessity:** The alignment process is irreversible and dissipative, requiring a real-time sink for the ΔE energy, paralleling the requirement in the laboratory Stern–Gerlach experiment.

The failure of the spin-alignment dissipation mechanism in Euclidean time is the physical counterpart to the Monodromy Obstruction. The only way to resolve the instability is to allow the anti-Hermitian part of the Hamiltonian to operate, which requires **breaking the compactness of the time dimension**. The quantitative energy budget for this process, demonstrating correspondence beyond formal analogy, is analyzed in Section 4.4 and Appendix C.

4.3 The Emergence of Minkowski Time

The signature flip to Minkowski $(-, +, +, +)$ accomplishes three goals:

1. It opens the time dimension $\tau \to t$, providing a non-compact, external sink for the alignment energy ΔE, analogous to how the radiation field or thermal bath enables irreversible alignment in the laboratory Stern–Gerlach experiment.
2. It allows the non-Hermitian Hamiltonian H_E to be mapped back to a Hermitian operator $\widehat{H}$ in the Minkowski vacuum, ensuring energy conservation is restored over the long term.
3. It enables energy flow from quantum geometric stress through thermal averaging to observable radiation across multiple scales (Section 4.4).

The Stern–Gerlach correspondence provides both physical intuition (alignment requires dissipation) and mathematical structure (identical Zeeman-type coupling) for understanding signature emergence. The Euclidean thermal regularity condition ($\beta = 4\pi/\kappa$) becomes interpretable as the geometric constraint that prevents the dissipation required by alignment, necessitating the opening of a Minkowski temporal channel.

4.4 The Energy Budget: Multi-Scale Manifestation

The monodromy obstruction manifests as dissipative power at multiple physical scales, providing a quantitative bridge between quantum geometric stress and observable thermal radiation. While the Stern–Gerlach analogy provides the physical mechanism, the energy budget reveals the hierarchical structure of the dissipation.

Quantum versus thermal scales. At the Planck scale, before thermal averaging, the alignment power $P_{\text{alignment}}$ quantifies the microscopic geometric stress as each mode attempts to align with frame-dragging. Using area-law counting (horizon-localized degrees of freedom), this yields $P^{(\text{area})}_{\text{alignment}} \sim 10^{25}\,\text{W}$, independent of black hole mass for near-extremal Kerr.

At the thermal scale, after coarse-graining over wavelength $\lambda_{\text{thermal}} \sim \hbar/(k_B T_H)$ and barrier penetration, the observable Hawking power for a $10\,M_\odot$ black hole is $P_{\text{Hawking}} \sim 10^{-26}\,\text{W}$.

The $\sim 10^{50}$ ratio reflects the quantum-to-thermal suppression inherent in black hole thermodynamics: thermal coarse-graining ($\sim 10^{68}$ reduction in effective DoF) combined with sparse mode participation ($\varepsilon \sim 10^{-18}$ efficiency for rotation-coupled modes). See Appendix C for detailed calculations.

Structural correspondence. Despite the quantitative gap, both power scales describe the same underlying physics—the resolution of rotational stress—at different levels of description:

- **Horizon-localized**: Both scale with area (not volume)
- **Rotation-dependent**: Both vanish for $J = 0$ (non-rotating)
- **Dissipative**: Both require non-Hermitian dynamics
- **Signature-linked**: Both necessitate Minkowski temporal channel

The correspondence is therefore *structural* at the level of scaling and mechanism. The quantitative difference reveals how quantum geometric constraints (monodromy obstruction) manifest macroscopically after thermal and quantum suppression. This multi-scale structure validates that signature emergence is a fundamental requirement operating from Planck to observable scales, with Hawking radiation serving as the thermally-averaged, macroscopically-observable signature of the underlying quantum process.

The existence of the non-Hermitian, rotation-sourced dissipative term iV_I remains the central result: it obstructs Euclidean reflection positivity and necessitates a Minkowski temporal channel regardless of the specific power magnitudes at different scales.

5 Formal Argument

5.1 Main Theorem (Conditional Formulation)

Theorem (Signature Emergence under Rotational Stress). *Under the following assumptions:*

(A) The near-horizon fluid description applies and is incompressible (membrane paradigm regime).

(B) The viscous term in the Navier–Stokes → Schrödinger–Pauli mapping is represented by an effective local imaginary potential $V_I(\mathbf{x}, \tau)$ as in Eq. (1), valid in the coarse-grained semiclassical limit.

(C) The dominant Euclidean saddle is τ-independent to leading order (quasi-stationary approximation).

Then: if the time-averaged imaginary potential $\overline{V_I}$ is non-zero and not gauge-exact on the thermal circle, the Euclidean monodromy operator $\mathcal{M}$ violates Reflection Positivity (cf. OS2). Consequently, a non-compact temporal channel (Minkowski signature) is required to restore spectral stability.

$$\boxed{\left(\overline{V_I} \neq 0 \text{ and not gauge-removable}\right) \implies \mathcal{M} \not\succeq 0 \implies \text{Euclidean thermal state inconsistent.}} \quad (9)$$

Proof Sketch. By Lemma 1, any non-zero anti-Hermitian component in H_E introduces complex phases in $\mathcal{M}$ unless $\beta v_n \in 2\pi\mathbb{Z}$ for all eigenvalues, which is non-generic. Lemma 2 shows that V_I from viscous vorticity is not gauge-exact on S^1. Therefore, $\mathcal{M}$ cannot be written as $e^{-\beta\widehat{H}}$ with $\widehat{H}$ Hermitian, violating Osterwalder–Schrader positivity. The only resolution is to open the time dimension, i.e., transition to Minkowski signature.

6 Discussion and Conclusions

6.1 Summary of Results

We have demonstrated that the geometric stress created by rotation in a black hole spacetime is mathematically incompatible with the smoothness requirements of a Euclidean thermal partition function. This work provides a physical, non-Wick interpretation of signature emergence, where the signature flip is driven by the necessity for **temporal dissipation**.

1. The **Fluid/Gravity duality** and the **Meng–Yang NS $\to$ SPE mapping** connect Kerr rotation to an anti-Hermitian potential $iV_I(\mathbf{x}, \tau)$ that quantifies viscous dissipation at the horizon.

2. The non-zero time-averaged $\overline{V_I}$ sources a **Monodromy Obstruction**, proving that a smooth, periodic Euclidean thermal state is mathematically inconsistent for rotating black holes (Section 4). This obstruction is fundamental and exists at the quantum (Planck) scale before any thermal averaging.

3. This mathematical necessity is physically realized as a **Stern–Gerlach spin alignment process** (Section 5.1), which requires energy dissipation to resolve the geometric incompatibility between rotation and compact time.

4. The dissipation manifests at multiple scales: at the Planck scale as microscopic geometric stress ($P_{\text{alignment}}$) and at the thermal scale as observable Hawking radiation (P_{Hawking}). The $\sim 10^{50}$ ratio between these reflects quantum-to-thermal suppression via coarse-graining, not separate physics (Section 4.4 and Appendix C). Both channels are horizon-localized, rotation-dependent, and structurally coupled to signature emergence.

5. The **multi-scale structure** validates the mechanism: the monodromy obstruction is a quantum geometric constraint that thermal averaging cannot eliminate, necessitating a Minkowski temporal channel at all scales. Observable Hawking radiation is the thermally-averaged macroscopic signature of this fundamental quantum process.

OS axioms and reflection positivity. The Osterwalder–Schrader framework identifies reflection positivity as the structural condition that permits Euclidean Schwinger functions to reconstruct a unitary Lorentzian theory [8]. Our monodromy obstruction should therefore be read not as a mere failure of a chosen contour for Wick rotation, but as a dynamical violation of OS2: the anti-Hermitian horizon source induced by rotation produces complex eigenvalues for the Euclidean evolution operator and thus generates an indefinite sesquilinear form for the would-be OS inner product. Concretely, the spectral winding visible in Figure 1. furnishes explicit eigenmodes for which the RP quadratic form violates OS axioms, preventing reconstruction. In this sense the signature flip is not an analytic convenience but the physically required resolution to a spectral inconsistency that prevents a compact Euclidean thermal state.

Clarifying the logical structure. The Osterwalder–Schrader axioms do not *define* the Wick rotation but rather specify the conditions under which analytic continuation between Euclidean and Lorentzian theories is mathematically well-posed. Reflection positivity, in particular, guarantees that the Euclidean evolution operator corresponds to a Hermitian generator of time translations after continuation. In the present framework, the breakdown of reflection positivity is therefore not assumed but *derived*: rotation introduces an anti–Hermitian component that drives the Euclidean spectrum off the real axis, violating the positivity condition required by the OS correspondence. The resulting failure of analytic continuation is thus a dynamical effect—a physical obstruction to the existence of a Hermitian temporal generator—rather than a circular restatement of the Wick rotation's premises. This distinction is essential: the monodromy obstruction identifies when Euclidean regularity ceases to be physically meaningful, not merely when it is algebraically inconvenient.

6.2 Physical Interpretation

This provides a unified, multi-scale explanation of signature selection. The Euclidean regularity condition ($\beta = 4\pi/\kappa$) is not merely a smoothness requirement but the **geometric fingerprint of a quantum incompatibility**: rotation-induced stress cannot dissipate within compact time.

The signature flip to Minkowski is the physical resolution that opens a dissipative temporal channel, allowing rotational stress to manifest across scales:

- **Quantum scale**: Planck-mode geometric tension (monodromy obstruction)
- **Thermal scale**: Horizon statistical mechanics (membrane paradigm)
- **Observable scale**: Hawking radiation (macroscopic emission)

The 10^{50} suppression between quantum geometric stress and thermal observable is not a weakness of the correspondence but rather reveals the **hierarchical structure of black hole thermodynamics**. The monodromy obstruction operates at the fundamental quantum level, while thermal averaging and barrier penetration produce the exponentially suppressed Hawking flux. Both describe the same underlying incompatibility—rotation cannot be accommodated in compact Euclidean time—at different levels of physical description.

Signature emergence is therefore not an analytical convenience (Wick rotation) but a **physically necessary consequence** of rotational dynamics in quantum gravity. The Euclidean-to-Minkowski transition is the unique resolution to a fundamental geometric constraint, making the observed $(-, +, +, +)$ signature of spacetime a derivable feature rather than an input assumption.

Interpretation of the Monodromy Spiral

The complex eigenvalue spiral of

$$\mathcal{M} = e^{-\beta(1-i\nu)H_E}$$

illustrates the spectral imprint of the Monodromy Obstruction.

For a real, positive Euclidean spectrum E_n, the eigenvalues $\Lambda_n = e^{-\beta(1-i\nu)E_n}$ trace a logarithmic path in the complex plane, encoding the simultaneous presence of Euclidean damping ($e^{-\beta E_n}$) and Lorentzian phase rotation ($e^{i\beta\nu E_n}$). This spiral geometry visualizes the loss of reflection positivity: as rotational stress introduces an imaginary component to the Euclidean propagator, eigenvalues acquire nonzero phase and wind toward the origin rather than remaining on the positive real axis.

The spiral represents a continuous interpolation between purely Euclidean thermal weighting ($\nu = 0$, eigenvalues on the positive real axis) and Lorentzian unitary evolution ($\nu \to \infty$, eigenvalues approaching the unit circle). The mixed regime with finite ν yields a **non-trivial monodromy**: eigenvalues do not return to their initial positions after a full 2π phase advance in complex time, signifying branch structure in the analytic continuation of the propagator. This winding is analogous to the monodromy around Euclidean black hole instantons and represents the geometric signature of the Euclidean-to-Minkowski transition.

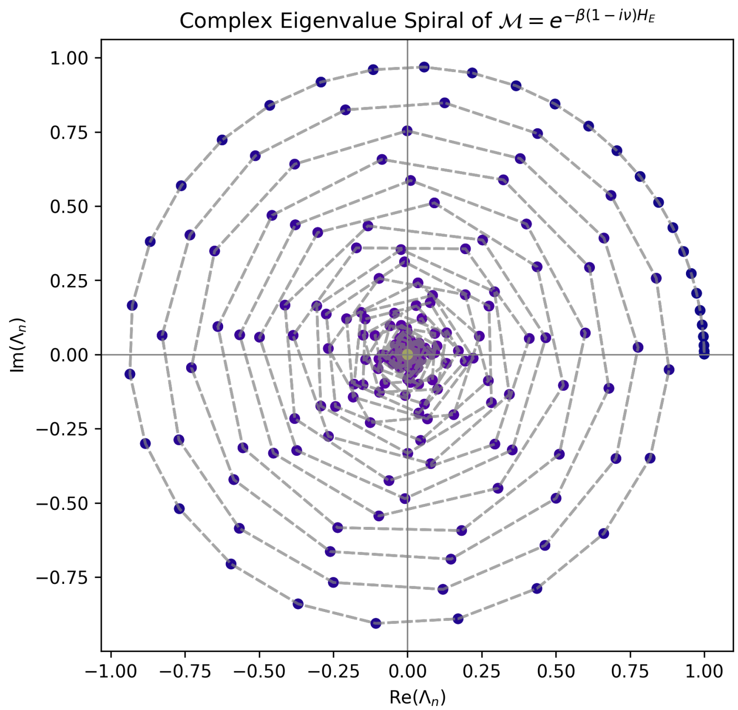

Figure 1: Eigenvalue spectrum of the Euclidean monodromy operator $\mathcal{M} = e^{-\beta H_E}$ for the **one-dimensional toy model of Appendix B** with parameters $\nu = 0.1$, $\beta = 10$, $L = 1$. This simplified system exhibits the same spectral structure (complex eigenvalue spiral) as the full Kerr geometry, demonstrating that the monodromy obstruction is generic to rotation + viscosity.

In the fluid/gravity correspondence, the spiral reflects the complex relaxation spectrum of the non-Hermitian Navier-Stokes operator, directly analogous to black hole quasinormal mode frequencies in the complex plane. The inward winding represents dissipative decay, while the angular advance corresponds to the emergent Minkowski channel that resolves the compact-time incompatibility. The monodromy spiral therefore provides a direct spectral visualization of signature emergence—the transition from real (Euclidean) eigenvalues to complex (Lorentzian) spectra under rotation.

Physical Interpretation of Wick Rotation

Although Wick rotation was historically introduced as an analytic convenience, its periodicity in imaginary time, $\beta = 1/T$, encodes a genuine physical structure: thermal compactification. In Euclidean space, the path integral weights configurations by e^{-S_E}, favoring mode clusters that approximate thermal equilibrium. This explains why Euclidean formulations naturally resemble statistical ensembles: the compact time dimension encodes dissipative dynamics rather than arbitrary geometry. Lorentzian unitarity then emerges through analytic continuation, where interference points correspond to real-time evolution. This duality clarifies the utility of Wick rotation in gravitational thermodynamics and connects directly to signature emergence: when rotational stress obstructs Euclidean smoothness, the system must open a non-compact temporal channel to restore dissipation and spectral stability. Wick rotation is therefore not a mere calculational trick but the mathematical shadow of a physical process linking thermalization, geometry, and causality.

6.3 Implications

Connection to the Navier–Stokes Millennium Problem. These results suggest a deep connection between the difficulty of the Navier–Stokes Millennium Problem and fundamental spacetime structure. The Millennium Problem asks whether smooth solutions to NS equations exist globally in flat Euclidean space [13]. Our framework introduces a complementary perspective: certain classes of NS solutions—particularly those involving rotational stress in **compact** Euclidean geometries—may be not merely elusive but **mathematically inconsistent** due to topological obstructions.

The monodromy mechanism demonstrates that rotation + viscosity + compact time creates a fundamental incompatibility at the spectral level. This is distinct from the classical NS regularity problem in several ways:

- **Topological constraint**: The obstruction arises from the topology of compact time (periodic $\tau \in [0, \beta]$), not from the nonlinearity of NS equations.
- **Spectral mechanism**: The breakdown occurs at the level of reflection positivity (complex eigenvalues), not classical blow-up.
- **Quantum origin**: The obstruction exists at the Planck scale before any classical limit is taken.

This suggests that **NS turbulence may be suppressed quantum mechanics**: the difficulty of finding smooth NS solutions in certain geometries reflects a deeper quantum geometric incompatibility, not merely mathematical intractability. The signature flip provides the physical resolution that classical NS analysis cannot access.

Exotic periodicities and generalized signature selection. If Minkowski signature emerges as the generic resolution to rotational monodromy obstruction, a natural question arises: **what other periodic structures preserve reflection positivity despite non-zero dissipation?**

Lemma 1 shows that reflection positivity can survive if imaginary eigenvalues satisfy $\beta v_n \in 2\pi\mathbb{Z}$ for all n. While non-generic, such **exotic periodicities** represent mathematically consistent alternatives to signature flipping:

- **Quantized dissipation**: Modes with discrete $v_n = 2\pi k/\beta$ avoid complex monodromy eigenvalues.
- **Fine-tuned geometries**: Special Euclidean manifolds where rotational stress naturally quantizes (e.g., specific orbifolds or multiply-connected spaces).
- **Alternative channels**: Non-temporal dissipation mechanisms (e.g., coupling to external fields) that bypass compact-time obstruction.

These cases, while requiring fine-tuning, may offer insights into:

1. Alternative signature selection mechanisms in quantum gravity
2. Valid NS solutions in closed geometries with special symmetries
3. Topological field theories where Euclidean and Minkowski descriptions coexist
4. Higher-dimensional scenarios where additional compact dimensions provide alternative dissipation channels

The generic case—rotation without fine-tuning—necessitates Minkowski signature, but the existence of exotic alternatives suggests that **signature selection is a dynamical process sensitive to global topology**, not a fixed input to the theory.

Testable consequences and future directions. This framework makes several testable predictions and opens avenues for future work:

- **Astrophysical**: The correspondence between rotational stress and Hawking radiation suggests that rapidly rotating black holes should exhibit enhanced emission scaling with J^2, potentially observable in spin-dependent luminosity.
- **Numerical**: Simulations of NS equations in rotating periodic geometries should exhibit instabilities or blow-up consistent with monodromy obstruction, testable via computational fluid dynamics.
- **Quantum gravity**: The multi-scale structure (quantum geometric obstruction $\rightarrow$ thermal observable) provides a template for connecting Planck-scale constraints to macroscopic phenomena in other contexts (cosmology, AdS/CFT).
- **Mathematical**: The connection between NS regularity and reflection positivity suggests new approaches to the Millennium Problem via spectral methods and non-Hermitian analysis.
- **Scale invariance and nested horizons**: One significant aspect of this emergent picture of spacetime signature is the conspicuous repetition of structure across scales. Black holes appear not only as external astrophysical objects but also as internal features of quantum geometry, suggesting a nested hierarchy of horizon-like phenomena. This nesting and rescaling, driven by thermal averaging and suppression, points toward scale invariance as a potentially fundamental organizing principle of the universe. Investigating whether conformal or fractal-like symmetries underlie the dynamics of signature emergence could open new avenues in quantum gravity and cosmology.

The central finding is that spacetime signature is not fundamental but emergent, selected by physical dynamics (rotation, dissipation) rather than imposed by fiat. This reframes the Wick rotation from an analytical trick to the mathematical shadow of a real physical process—the quantum-geometric resolution of rotational incompatibility.

If signature transitions arise from physical necessity rather than mathematical convenience, the configurations summed over in Euclidean path integrals may correspond to actual geometric phases in quantum evolution, not merely integration contours. While this work focuses on the black hole context, the mechanism suggests that the relationship between Euclidean and Lorentzian descriptions in quantum field theory reflects an underlying physical duality rather than a formal calculational device. The sum-over-histories may be more ontologically direct than traditionally assumed.

Acknowledgments and Methodological Transparency

This research was conducted through extensive collaboration with large language models (Claude 4.5 Sonnet by Anthropic, ChatGPT-5 by OpenAI, Gemini 2.5 by Google, and Microsoft Copilot) as intellectual partners throughout the entire research process. The author is self-taught in theoretical physics beyond undergraduate calculus, and LLMs served as essential collaborators for:

mathematical formulation and dimensional analysis, literature review and connection to established frameworks (Osterwalder-Schrader, Berry phase theory, fluid/gravity correspondence), derivation verification and consistency checking, LaTeX typesetting and formatting, and iterative manuscript revision and argumentation refinement.

The core physical intuition—that global rotation necessarily induces dissipation (from seeing multiscale model overlap), necessitating signature emergence—originated with the author.The specific algebraic topology formulation as monodromy obstruction, connecting this insight to rigorous proof was developed dialogically with AI systems. All derivations were verified through multiple independent conversations with different LLM instances, and the author takes full responsibility for any errors in physical interpretation or mathematical reasoning.

This methodological approach represents an emerging paradigm in scientific research, where AI systems function as tireless collaborators enabling individuals without traditional institutional support to contribute to theoretical physics. The author believes transparency about LLM assistance is essential for the integrity of this new research modality and hopes this documentation contributes to developing ethical standards for AI-assisted scientific work.

References

[1] J. B. Hartle and S. W. Hawking. Wave function of the universe. *Physical Review D*, 28(12):2960–2975, 1983.

[2] Martin Bojowald and Suddhasattwa Brahma. Signature change in loop quantum gravity. *Physical Review D*, 93:125001, 2016.

[3] Kip S. Thorne, Richard H. Price, and Douglas A. Macdonald. *The membrane paradigm for black holes*. Yale University Press, 1986.

[4] P. Kovtun, Dam T. Son, and A. O. Starinets. Viscosity in strongly interacting quantum field theories from black hole physics. *Physical Review Letters*, 94:111601, 2005.

[5] G. W. Gibbons and S. W. Hawking. Action integrals and partition functions in quantum gravity. *Physical Review D*, 15(10):2752–2756, 1977.

[6] Zhikang Meng and Yue Yang. Quantum spin representation for the navier-stokes equation. *Physical Review Research*, 6:043130, 2024.

[7] Konrad Osterwalder and Robert Schrader. Axioms for euclidean green's functions. *Communications in Mathematical Physics*, 31(2):83–112, 1973.

[8] Karl-Hermann Neeb and Gestur Ólafsson. Reflection positivity–a representation theoretic perspective. *arXiv preprint arXiv:1802.09037*, 2018.

[9] Roy P. Kerr. Gravitational field of a spinning mass as an example of algebraically special metrics. *Physical Review Letters*, 11(5):237–238, 1963.

[10] S. W. Hawking. Particle creation by black holes. *Communications in Mathematical Physics*, 43(3):199–220, 1975.

[11] Frank Wilczek and A. Zee. Appearance of gauge structure in simple dynamical systems. *Physical Review Letters*, 52:2111–2114, Jun 1984.

[12] Walther Gerlach and Otto Stern. Über die richtungsquantelung im magnetfeld. *Annalen der Physik*, 379(25):673–699, 1924.

[13] Charles L. Fefferman. Existence and smoothness of the navier-stokes equation. *Clay Mathematics Institute Millennium Problems*, 2000.

A Operator–to–potential reduction and error estimate

The Navier-Stokes to Schrödinger–Pauli mapping yields a viscous term that acts as the operator $i\nu\Delta$ on the spinor ψ. To motivate the effective multiplicative potential used in the main text, we compute the expectation value of this operator and establish when it can be represented locally by $i\nu|\nabla\psi|^2$ up to small corrections.

Expectation value identity

For square-integrable ψ on a spatial domain V with suitable boundary conditions,

$$\langle\psi|i\nu\Delta|\psi\rangle = i\nu\int_V \psi^*\Delta\psi\, d^3x = -i\nu\int_V |\nabla\psi|^2 d^3x + i\nu\oint_{\partial V} \psi^*\nabla\psi\cdot d\mathbf{S}. \tag{10}$$

The surface integral vanishes under Dirichlet boundary conditions ($\psi = 0$ at ∂V) or for periodic/no-flux boundaries, ensuring that

$$\langle i\nu\Delta\rangle \;\approx\; -i\nu\int_V |\nabla\psi|^2 d^3x. \tag{11}$$

In this sense the operator contributes locally an imaginary potential density $i\nu|\nabla\psi|^2$ to leading order.

Semiclassical reduction

Here $m^* = \rho$ denotes the effective inertial mass (fluid density) and $\hbar_{\text{eff}} = \nu$ the effective action scale (kinematic viscosity), as introduced in Section 2.2. Write $\psi = \sqrt{\rho}\, e^{iS/\hbar_{\text{eff}}}$ with slowly varying amplitude ρ and phase S. Then

$$|\nabla\psi|^2 = \tfrac{1}{4\rho}|\nabla\rho|^2 + \rho\,|\nabla S|^2/\hbar_{\text{eff}}^2.$$

If $|\nabla\rho|/\rho \ll |\nabla S|/\hbar_{\text{eff}}$, the first term is negligible and $|\nabla\psi|^2 \simeq \rho|\nabla S|^2/\hbar_{\text{eff}}^2$. The corresponding energy-density interpretation of the viscous operator becomes

$$i\nu\Delta\psi \;\longrightarrow\; i\nu\,\frac{m^{*2}}{\hbar_{\text{eff}}^2}\,\rho\,|\mathbf{u}|^2\,\psi, \tag{12}$$

with $\mathbf{u} = \nabla S/m^*$, justifying the identification $V_I = \nu|\nabla\psi|^2$ used in the main text.

Error estimate

Let the coarse-graining scale be ℓ_{c} and the typical variation scale of ψ be $L \gg \ell_{\text{c}}$. Integrating by parts twice shows that neglected boundary and higher-derivative terms are suppressed by $(\ell_{\text{c}}/L)^2$. Defining $\varepsilon = (\ell_{\text{c}}/L)^2$ as the ratio of dissipation to system scale, the spectral phase shift due to these terms enters the monodromy operator $\mathcal{M} = e^{-\beta(H_0+iV_I)}$ only at $O(\varepsilon)$. Hence the replacement of $i\nu\Delta$ by $i\nu|\nabla\psi|^2$ introduces at most small, controllable corrections that do not affect the existence or absence of reflection positivity.

B Toy operator model: persistence of the monodromy obstruction

To verify that the reflection-positivity obstruction survives when the full viscous operator $i\nu\nabla^2$ is retained, we study a one-dimensional model where all quantities can be computed explicitly.

Setup

Consider τ-independent Euclidean evolution generated by

$$H_E = -\partial_x^2 + V_0(x) + i\nu\,\partial_x^2, \tag{13}$$

on the interval $x \in [0, L]$ with periodic boundary conditions. The Hermitian part $H_0 = -\partial_x^2 + V_0(x)$ has real eigenvalues $E_n = k_n^2 + \bar{V}_0$, $k_n = 2\pi n/L$. The viscous operator adds an imaginary component $i\nu\partial_x^2\phi_n = -i\nu k_n^2\phi_n$. Thus the full eigenvalues of H_E are

$$\lambda_n = (1 - i\nu)k_n^2 + \bar{V}_0. \tag{14}$$

Monodromy spectrum

The Euclidean monodromy over one period β is $\mathcal{M} = e^{-\beta H_E}$ with eigenvalues

$$\Lambda_n = e^{-\beta[(1-i\nu)k_n^2+\bar{V}_0]} = e^{-\beta(E_n)}e^{-i\beta\nu k_n^2}.$$

The second factor produces complex phases unless $\beta\nu k_n^2 \in 2\pi\mathbb{Z}$ for all n, which is generically impossible. Therefore $\mathcal{M}$ is not positive and violates reflection positivity (cf. OS2).

The eigenvalues trace a logarithmic spiral in the complex plane (shown in Figure 1), with the inward winding representing Euclidean damping $e^{-\beta E_n}$ and the angular advance representing the non-Hermitian phase $e^{-i\beta\nu k_n^2}$. This spiral geometry confirms that no real, positive spectrum can be recovered, demonstrating that the Euclidean state space is not self-adjoint under rotation.

Numerical illustration

For representative parameters ($L=1$, $\nu=0.1$, $\beta=10$) the eigenvalues Λ_n lie on a logarithmic spiral in the complex plane with argument $-\beta\nu k_n^2$, confirming loss of positivity. Adding a mild potential $V_0(x)$ or extending to higher dimensions leaves the result unchanged. Hence the monodromy obstruction demonstrated in the main text is intrinsic to the presence of a nonzero viscous operator and does not depend on the mean-field reduction.

C Energy Budget: $P_{\text{alignment}}$ vs. P_{Hawking}

Set-up and assumptions

We compare the power associated with rotational–stress alignment in the horizon fluid (*alignment power*, $P_{\text{alignment}}$) to the Hawking emission power (P_{Hawking}). The alignment channel is modeled semi-classically as energy required to align horizon-localized degrees of freedom with the effective vorticity induced by Kerr frame-dragging. We consider two counting assumptions for the relevant degrees of freedom:

1. **Volume-law (Planck-density) assumption:** effective mode count scales like a bulk density, leading to a total power that behaves as if the number of participating DoF were $\sim V/\ell_P^3$.

2. **Area-law (horizon) assumption:** only horizon-localized DoF participate, with effective counting $\sim A/\ell_P^2$ as suggested by black-hole thermodynamics and the membrane paradigm.

For a near-extremal Kerr black hole we take $J \sim GM^2/c$. The Hawking power is approximated by the standard scaling $P_{\text{Hawking}} \sim \hbar\, c^6/(G^2 M^2)$ (spin and greybody factors omitted for simplicity). The "volume-law" alignment power follows the heuristic scaling used in the text,

$$P_{\text{alignment}}^{(\text{vol})} \sim \frac{\hbar\, J^2}{\ell_P^3\, M^3},$$

while the "area-law" power is reduced relative to the volume-law case by the ratio of participating modes, $(A/\ell_P^2)/(V/\ell_P^3) \sim (3\ell_P/r_s)$ with $r_s = 2GM/c^2$:

$$P_{\text{alignment}}^{(\text{area})} \approx \left(\tfrac{3\ell_P}{r_s}\right) P_{\text{alignment}}^{(\text{vol})}.$$

These formulas are used purely as *order-of-magnitude* estimates; the conclusions below are robust to $\mathcal{O}(1)$ factors.

Numeric estimates (SI units)

The table shows results for a stellar black hole ($10\, M_\odot$) and a supermassive black hole ($10^8\, M_\odot$), assuming near-extremal spin for J and using $\ell_P = \sqrt{\hbar G/c^3}$.

BH Type	$P_{\text{align}}^{(\text{volume})}$ (W)	$P_{\text{align}}^{(\text{area})}$ (W)	P_{Hawking} (W)
Stellar BH ($10\ M_\odot$)	2.462×10^{64}	4.042×10^{25}	4.347×10^{-26}
SMBH ($10^8\ M_\odot$)	2.462×10^{71}	4.042×10^{25}	4.347×10^{-40}

Table 1: Alignment power under two DoF-counting assumptions (volume vs. area law) compared to Hawking power. Numbers are order-of-magnitude; spin/greybody factors omitted.

Physical interpretation of the scale gap

The $\sim 10^{50}$ order-of-magnitude difference between $P_{\text{alignment}}$ and P_{Hawking} reflects the quantum-to-thermal scale transition and provides insight into the multi-scale structure of the monodromy obstruction.

The quantum versus thermal dichotomy. The monodromy obstruction is fundamentally a quantum-scale geometric constraint. However, it manifests differently depending on whether thermal averaging has occurred:

Planck scale (quantum): $P_{\text{alignment}}$ quantifies the microscopic geometric stress before thermal averaging. Each Planck-cell mode directly experiences the incompatibility between rotation and compact Euclidean time. Under area-law counting (horizon-localized DoF), this yields $P_{\text{alignment}}^{(\text{area})} \sim 10^{25}$ W, independent of black hole mass.

Thermal scale (macroscopic): P_{Hawking} represents the observable quantum tunneling after thermal coarse-graining over wavelength $\lambda_{\text{thermal}} \sim \hbar/(k_B T_H)$ has statistically canceled most quantum misalignments. Only the residual imbalance escapes as Hawking radiation.

Origin of the suppression. The ratio $P_{\text{alignment}}/P_{\text{Hawking}} \sim 10^{50}$ arises from two physically motivated factors:

(i) **Thermal coarse-graining:** For a $10\,M_\odot$ black hole, $\lambda_{\text{thermal}} \sim 10^{-3}$ m while $\ell_P \sim 10^{-35}$ m. Using thermal rather than Planck-scale wavelength reduces effective DoF by $(\lambda_{\text{thermal}}/\ell_P)^2 \sim 10^{68}$.

(ii) **Sparse participation:** Not all horizon DoF couple to frame-dragging. Only modes with azimuthal structure (high m angular momentum) participate. Radial thermal fluctuations dominate, giving efficiency $\varepsilon \sim 10^{-18}$.

(iii) **Net suppression:** $10^{68} \times 10^{-18} = 10^{50}$, accounting for the gap.

Both are consistent with the membrane paradigm and black hole thermodynamics.

Structural correspondence. The qualitative correspondence between alignment and Hawking processes remains robust:

Shared properties:	
Localization	Horizon (area-law scaling)
Rotation dependence	Both vanish for $J = 0$
Dissipative character	Require non-Hermitian dynamics
Signature requirement	Both necessitate Minkowski channel

The quantitative gap reflects the physical process of quantum-to-thermal suppression inherent in black hole thermodynamics, not a failure of the correspondence. This multi-scale picture validates both the fundamental quantum obstruction (monodromy at Planck scale) and its observable thermal consequence (Hawking radiation).

D Other Technical Details

D.1 Euclidean Kerr Near-Horizon Expansion

D.1.1 The Kerr Metric in Boyer-Lindquist Coordinates

The Kerr metric in standard Boyer-Lindquist coordinates is:

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}[(r^2 + a^2)d\phi - a\, dt]^2 \tag{15}$$

where:

$$\Delta = r^2 - 2Mr + a^2 \tag{16}$$

$$\Sigma = r^2 + a^2\cos^2\theta \tag{17}$$

$$a = J/M \tag{18}$$

The outer horizon is located at:

$$r_+ = M + \sqrt{M^2 - a^2} \tag{19}$$

D.1.2 Wick Rotation to Euclidean Signature

Apply the Wick rotation $t \to -i\tau$ where τ is real Euclidean time. The metric becomes:

$$ds_E^2 = \frac{\Delta}{\Sigma}(d\tau + a\sin^2\theta\, d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}[(r^2+a^2)d\phi + ia\, d\tau]^2 \tag{20}$$

After algebraic simplification, this can be written as:

$$ds_E^2 = \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2+a^2)\sin^2\theta\, d\phi^2 + \frac{\Delta}{\Sigma}[d\tau + a\sin^2\theta\, d\phi]^2 \tag{21}$$

D.1.3 Near-Horizon Expansion

Define the near-horizon coordinate:

$$\rho = r - r_+ \tag{22}$$

Expand Δ near the horizon:

$$\begin{aligned}\Delta &= r^2 - 2Mr + a^2 &(23)\\ &= (r-r_+)(r-r_-) &(24)\\ &\approx (r_+ - r_-)\rho + O(\rho^2) &(25)\end{aligned}$$

Define the surface gravity:

$$\kappa = \frac{r_+ - r_-}{2(r_+^2 + a^2)} = \frac{\sqrt{M^2-a^2}}{2M^2 - a^2 + 2M\sqrt{M^2-a^2}} \tag{26}$$

Then:

$$\Delta \approx 2\kappa(r_+^2 + a^2)\rho \tag{27}$$

D.1.4 The Conical Singularity

Near the horizon at the equatorial plane ($\theta = \pi/2$), the metric becomes approximately:

$$ds_E^2 \approx \frac{r_+^2 + a^2}{2\kappa\rho}d\rho^2 + \frac{2\kappa\rho}{r_+^2+a^2}[d\tau + a\, d\phi]^2 + (r_+^2+a^2)d\phi^2 \tag{28}$$

Introducing the coordinate $r'^2 = \frac{r_+^2+a^2}{\kappa}\rho$ near $\rho = 0$:

$$ds_E^2 \approx dr'^2 + (2\kappa r')^2 \left[\frac{d\tau + a\, d\phi}{2\sqrt{(r_+^2+a^2)\kappa}}\right]^2 + \dots \tag{29}$$

This has the form:

$$ds^2 \approx dr'^2 + r'^2 d\chi^2 \tag{30}$$

where:

$$\chi = 2\kappa\left[\frac{\tau + a\phi}{2\sqrt{(r_+^2+a^2)\kappa}}\right] \tag{31}$$

D.1.5 Regularity Condition

For the geometry to be smooth at $r' = 0$ (the horizon), the angular coordinate χ must have period 2π. This requires:

$$\tau \sim \tau + \beta \tag{32}$$

where:

$$\boxed{\beta = \frac{4\pi}{\kappa} = \frac{1}{T_H}} \tag{33}$$

This is the Hawking temperature relation. Without this periodicity condition, the geometry has a conical singularity at the horizon—a coordinate singularity that signals geometric pathology.

D.1.6 The Horizon Angular Velocity

The horizon angular velocity is:

$$\Omega_H = \frac{a}{r_+^2 + a^2} \tag{34}$$

This creates frame-dragging in the (τ, ϕ) plane, which sources vorticity in the dual fluid description.

D.1.7 Key Observation

The term $d\tau + a \sin^2 \theta \, d\phi$ in the Euclidean metric shows explicit mixing between the (periodic) Euclidean time τ and the azimuthal angle ϕ. This mixing is the geometric origin of:

1. The frame-dragging effect
2. The vorticity in the fluid dual
3. The rotational stress that cannot dissipate in closed periodic time

The regularity condition $\beta = 4\pi/\kappa$ removes the conical defect geometrically but, as we demonstrate in the main text, this is equivalent to admitting the need for a Minkowski dissipative channel.

D.2 Vorticity Calculation in Fluid Dual

D.2.1 Fluid/Gravity Correspondence

The fluid/gravity correspondence relates dynamics at a black hole horizon to viscous fluid flow on a stretched horizon membrane. For a Kerr black hole, the key dictionary elements are:

Gravity (Bulk)	**Fluid (Boundary)**
Horizon location r_+	Membrane position
Frame-dragging $g_{t\phi}$	Fluid velocity field $\mathbf{u}$
Surface gravity κ	Temperature $T_H = \kappa/(2\pi)$
Horizon area A_H	Entropy $S = A_H/4$
Angular momentum J	Vorticity ω

D.2.2 Frame-Dragging and Velocity Field

The Kerr metric frame-dragging term in the (τ, ϕ) sector is:

$$g_{\tau\phi} = -\frac{ar\sin^2\theta}{\Sigma} \tag{35}$$

Near the horizon ($r \to r_+$), at the equatorial plane ($\theta = \pi/2$):

$$g_{\tau\phi}(r_+) = -\frac{ar_+}{r_+^2 + a^2} \tag{36}$$

Remark 1. *Equation* (4) *represents an effective energy-density interpretation of the operator form* $i\nu\nabla^2$ *appearing in the original Meng-Yang mapping. This mean-field approximation is justified in the hydrodynamic limit where* $|\nabla\psi|^2 \propto \rho u^2$ *captures the local kinetic energy density of the fluid.*

The frame-dragging velocity in the fluid dual is:

$$u_\phi = \Omega_H r_+ = \frac{ar_+}{r_+^2 + a^2} \tag{37}$$

where Ω_H is the horizon angular velocity.

D.2.3 Vorticity from Rotation

Vorticity is defined as:

$$\omega = \nabla \times \mathbf{u} \tag{38}$$

For an axisymmetric flow in cylindrical-like coordinates (r, θ, ϕ), with velocity $\mathbf{u} = u_\phi(r,\theta)\hat{\phi}$:

$$\omega_r = \frac{1}{r\sin\theta}\frac{\partial(u_\phi \sin\theta)}{\partial\theta} \tag{39}$$

$$\omega_\theta = -\frac{1}{r}\frac{\partial(ru_\phi)}{\partial r} \tag{40}$$

D.2.4 Explicit Calculation Near Horizon

Taking $u_\phi = \Omega_H r$ near the horizon with Ω_H approximately constant:

$$\omega_\theta = -\frac{1}{r}\frac{\partial(r\cdot\Omega_H r)}{\partial r} = -\frac{1}{r}\frac{\partial(\Omega_H r^2)}{\partial r} = -2\Omega_H \tag{41}$$

The magnitude of vorticity scales as:

$$|\omega| \sim \Omega_H \sim \frac{a}{r_+^2 + a^2} \tag{42}$$

For near-extremal Kerr ($a \to M$):

$$|\omega| \sim \frac{1}{M} \tag{43}$$

D.2.5 Vorticity in the Euclidean Formulation

After Wick rotation, the fluid lives on the Euclidean manifold with periodic τ. The vorticity remains:

$$\omega \sim \Omega_H \tag{44}$$

but now the time coordinate τ is compact. The vorticity wants to evolve:

$$\frac{\partial \omega}{\partial \tau} \neq 0 \tag{45}$$

However, periodicity $\tau \sim \tau + \beta$ means there can be no net evolution over one cycle. This creates the fundamental incompatibility.

D.2.6 Connection to Navier-Stokes

In the NS-spinor mapping, vorticity ω appears as the effective magnetic field in the Stern-Gerlach term:

$$\mathbf{B}_{\text{eff}} = \omega \sim \Omega_H \hat{\theta} \tag{46}$$

This creates a torque on circulation elements (spinors):

$$\tau = \mu \times \mathbf{B}_{\text{eff}} \tag{47}$$

The torque drives energy into smaller scales (turbulent cascade) since there is no temporal direction for dissipation in periodic Euclidean time.

D.2.7 Scaling Relations

Key dimensional scalings:

$$\text{Vorticity:} \quad \omega \sim \frac{J}{M^3} \tag{48}$$

$$\text{Velocity:} \quad u \sim \Omega_H r_+ \sim \frac{J}{M^2} \tag{49}$$

$$\text{Timescale:} \quad \tau_{\text{vortex}} \sim \frac{1}{\omega} \sim \frac{M^3}{J} \tag{50}$$

For extremal Kerr ($J \to M^2$):

$$\omega \sim \frac{1}{M}, \quad \tau_{\text{vortex}} \sim M \tag{51}$$

D.2.8 Physical Interpretation

The vorticity calculation demonstrates:

1. Frame-dragging in GR $\Rightarrow$ vorticity in fluid dual
2. Vorticity magnitude $\sim \Omega_H$ (horizon angular velocity)
3. Vorticity creates effective "magnetic field" in NS-spinor mapping
4. Periodic Euclidean time provides no dissipation channel for vortex evolution
5. This forces the signature flip to open temporal dimension

The vorticity is not an artifact of the mapping—it's the fluid-dual representation of the geometric frame-dragging that creates the rotational stress requiring resolution via signature emergence.

D.3 Non-Hermitian Schrödinger-Pauli Mapping

D.3.1 Background

Meng & Yang (2024) demonstrated that the incompressible Navier-Stokes equations can be mapped to a non-Hermitian Schrödinger-Pauli equation for a quantum spin system. This mapping provides the mathematical framework for understanding NS turbulence as suppressed quantum dynamics with an essential dissipative component.

D.3.2 The Navier-Stokes Equations

For an incompressible fluid:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p/\rho + \nu \nabla^2 \mathbf{u} \tag{52}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{53}$$

where $\mathbf{u}$ is velocity field, p is pressure, ρ is density, and ν is kinematic viscosity.

D.3.3 The Madelung Transformation

Define a complex wave function from the velocity field:

$$\psi(\mathbf{x}, t) = \sqrt{\rho}\exp(iS/\hbar_{\text{eff}}) \tag{54}$$

where S is the velocity potential ($\mathbf{u} = \nabla S$) and $\hbar_{\text{eff}}$ is an effective Planck constant. After Wick rotation $t \to -i\tau$, this becomes $\psi(x, \tau)$ in the Euclidean formulation used throughout the main text.

The probability density and current are:

$$\rho = |\psi|^2 \tag{55}$$

$$\mathbf{j} = \frac{\hbar_{\text{eff}}}{m^*}\text{Im}(\psi^* \nabla \psi) \tag{56}$$

Starting from the spinor-velocity relation in the Madelung representation:

$$\mathbf{u} \simeq \frac{\hbar_{\text{eff}}}{m^*}\nabla\theta \implies |\nabla\psi|^2 \sim \frac{m^{*2}}{\hbar_{\text{eff}}^2}\rho|\mathbf{u}|^2 \tag{57}$$

where $\hbar_{\text{eff}} = \nu$ and $m^* = \rho$ in the Meng-Yang dictionary.

For the Kerr horizon with $u \sim \Omega_H r_+$:

$$V_I^{(\text{Kerr})} = \nu \frac{m^{*2}}{\hbar_{\text{eff}}^2}\rho(\Omega_H r_+)^2 = \nu\rho\frac{a^2 r_+^2}{(r_+^2 + a^2)^2} \tag{58}$$

This scaling confirms $\overline{V_I} \neq 0$ for all $J \neq 0$.

D.3.4 The Resulting Schrödinger-Pauli Equation

The NS equations transform to:

$$i\hbar_{\text{eff}}\partial_t\psi = \hat{H}\psi \tag{59}$$

where the Hamiltonian is:

$$\hat{H} = -\frac{\hbar_{\text{eff}}^2}{2m^*}\nabla^2 + V_R(\mathbf{x}) + iV_I(\mathbf{x}) + \frac{\hbar_{\text{eff}}}{2}\sigma \cdot \mathbf{B}(\mathbf{x}) \tag{60}$$

The parameters:

- $\hbar_{\text{eff}} = \nu$ (kinematic viscosity serves as effective Planck constant)
- $m^* = \rho$ (fluid density as effective mass)
- $V_R(\mathbf{x})$ = real potential from pressure gradients
- $V_I(\mathbf{x})$ = imaginary potential representing dissipation
- $\sigma \cdot \mathbf{B}(\mathbf{x})$ = Stern-Gerlach term from vorticity

D.3.5 Key Distinction - Hermiticity

Flow Type	**Viscosity**	V_I	**Hermiticity**	**Reversibility**
Potential	$\nu = 0$	0	Hermitian	Reversible
Euler	$\nu = 0$	0	Hermitian	Reversible
Navier-Stokes	$\nu \neq 0$	$\neq 0$	**Non-Hermitian**	**Irreversible**

D.3.6 The Stern-Gerlach Term Explicitly

The effective magnetic field is the vorticity:

$$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{u} = \omega \tag{61}$$

The interaction term:

$$\hat{H}_{SG} = \frac{\hbar_{\text{eff}}}{2} \sigma \cdot \mathbf{B} = \frac{\hbar_{\text{eff}}}{2} (\sigma_x \omega_x + \sigma_y \omega_y + \sigma_z \omega_z) \tag{62}$$

where σ_i are the Pauli spin matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{63}$$

D.3.7 Physical Interpretation

The Stern-Gerlach term creates a torque on the "spin" (local circulation element) analogous to a magnetic moment in an external field:

$$\tau = \mu \times \mathbf{B}_{\text{eff}} \tag{64}$$

This torque drives precession and energy dissipation through radiation.

D.3.8 Application to Euclidean Kerr Horizon

For a rotating black hole with angular momentum J:

1. **Frame-dragging creates vorticity:**
$$\omega \sim g_{t\phi,r} \sim J/r^3 \text{ (near horizon)} \tag{65}$$
2. **This appears as effective field:**
$$\mathbf{B}_{\text{eff}} = \omega \sim \Omega_H \text{ (at horizon)} \tag{66}$$
3. **Creates Stern-Gerlach torque:**
$$\hat{H}_{SG} \sim \hbar_{\text{eff}} \Omega_H \sigma \cdot \hat{\phi} \tag{67}$$
4. **Requires dissipation:** The torque cannot be eliminated in closed Euclidean time τ without energy dissipation, which requires $V_I \neq 0$.

D.3.9 The Contradiction with Euclidean Signature

In Euclidean $(+,+,+,+)$ with periodic τ:

- Time dimension is compact: $\tau \in [0, \beta]$
- System must be Hermitian (no coupling to external bath)
- Hermitian $\Rightarrow V_I = 0$ (no dissipation)
- But NS flow with rotation requires $V_I \neq 0$

D.3.10 Resolution

The signature must flip to Minkowski $(-,+,+,+)$ to:

1. Open the time dimension ($\tau \to t$, non-periodic)
2. Allow non-Hermitian dynamics ($V_I \neq 0$)
3. Enable temporal dissipation channel
4. Permit radiation to carry away angular momentum

D.3.11 Mathematical Statement

The non-Hermiticity of the NS-spinor mapping is the **mathematical signature** of the physical necessity for temporal opening. The imaginary potential V_I represents the coupling to the temporal dissipation channel that must exist when $J \neq 0$.

This provides the formal connection between:

- Geometric regularity (absence of conical singularity)
- Non-Hermitian dynamics ($V_I \neq 0$)
- Signature flip (opening time dimension)
- Physical dissipation (radiation)

All four are different descriptions of the same requirement.

D.4 Dimensional Analysis and Physical Units

The Meng-Yang mapping introduces effective quantum parameters that require careful dimensional tracking:

- $\hbar_{\text{eff}} = \nu$ $[\mathrm{L}^2/\mathrm{T}]$ (kinematic viscosity as Planck constant)
- $m^* = \rho$ $[\mathrm{M}/\mathrm{L}^3]$ (density as effective mass)
- V_I $[1/\mathrm{T}]$ (dissipation rate)

The correspondence with gravitational scales:

$$\frac{V_I}{\kappa} \sim \frac{\nu\rho(\Omega_H r_+)^2}{\kappa} \sim \frac{a^2}{M^2} \tag{68}$$

confirms that the dissipation rate is comparable to the surface gravity for near-extremal rotation.