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Hybrid Cost-Tolerance Allocation and Production Strategy Selection for Complex Mechanisms: Simulation and Surrogate Built-In Optimization Models

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In manufacturing companies, assembly is an essential process to obtain the final product. The life cycle of an assembly product depends on various production strategies, e.g., resource allocation, rework decision, selection strategy, etc. In this regard, achieving a reliable assembly product commence with engineering a comprehensive design plan which can mitigate various uncertainties a company can face. The counteraction of uncertainties can be altered by introducing a set of tolerances into the design of the components. Tolerances define a practical margin on components design without downgrading the required performance of products. Thus, producers are confronted with high-quality requirements, cost pressure, and a rising number of demands. On these bases, this paper aims at modeling a statistical framework for a set of production strategies, including resource allocation (as a decision to assign practical resources to components) and reworking decision (as a decision to improve components' conformity rate). Moreover, a generic simulation and surrogate approach are established to evaluate the performance of the assembled product. Within this approach, simulation and surrogate models can be used to investigate a variety of deviations over components' geometries within the process deviation domain and deploy reworking decision. Ultimately, a modular costing system is developed, and a genetic algorithm is adapted to locate optimal solutions. In addition, the applicability of the statistical model is studied on an assembly product. [DOI: 10.1115/1.4056687]

Keywords: adaptive tolerancing, cost-tolerance optimization, production strategies, design optimization, multidisciplinary optimization, multiscale modeling and simulation

1 Introduction

The need for highly reliable products has broadened the scope of manufacturing technologies. Despite the introduction of new technologies in the manufacturing environment, geometrical deviations are inevitable. The geometrical deviations are consequences of uncertainties occurring during the product life cycle. The life cycle of an assembly product includes various activities, e.g., processing, inspection, rework, assembly, inventory, etc. Due to the interrelationship between manufacturing line activities, uncertainties and risks propagate through the whole life cycle. Therefore, a comprehensive engineering plan which includes key functions of the product and mitigates the uncertainties to reduce their effects and ensure product functioning is a necessity for the manufacturer. Consequently, the need for a reliable engineering plan and more precise components has impacted the development of tolerancing. Tolerance is an essential part of the design, and the ubiquitousness of tolerances entails the various stages of a product's life cycle. Since the role of tolerances in a life cycle varies from stage to stage, depending on their design objectives, it is a crucial task for designers to determine a tolerance that meets the design objectives. Thus, the tolerancing decision should meet the functionality and/or assemblability constraints as well as respect the limited capabilities of the required manufacturing processes [1].

The effects of tolerance and the contributions of tolerances on the system functionality can be determined by classifying tolerancing activity into two distinct categories, tolerance analysis, and tolerance allocation, respectively. Tolerance analysis is a method to verify the functionality of a design after tolerances have been specified on each component. To analyze the functionality, tolerance analysis can be distinguished by three main issues where first, the geometrical deviations of the product should be modeled. Second, a behavior model which allows the designer to know how features of a mechanism interact requires to be built. Finally, tolerance analysis techniques such as worst-case or statistical methods are expected to involve all the characteristics of the behavior model and estimate the quality level. On the other hand, tolerance allocation involves the assignment and the distribution of the values of adequate tolerances and, therefore, is the inverse problem of tolerance analysis. Commonly used tolerance allocation methods are based on specific rules of thumb for the distribution of tolerances such as equal tolerances assumption, same influence, proportional scaling, and constant precision factor.

The limitations within the tolerance allocation lie in the fact, that no approach takes production strategies into account as a complement to the existing approaches. A few approaches include an assessment of functional fulfillment using components with manufacturing deviations under the worst-case deviation scenario [2]. The necessity for an appropriate holistic methodology for tolerance allocation, taking production strategies and the functional

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fulfillment degree of components with manufacturing and assembly deviations emerges.

On these bases, this research aims at developing a methodology for simulation-based optimization under uncertainties of product tolerances and production strategies such as resource allocation, and reworking. Within the research, a new holistic approach to tolerance allocation and production strategies will be developed, evaluating both technical and economic assessments of the product. The rest of this research is structured as follows: Section 2 provides a state-of-the-art concurrent tolerance allocation problem considering resource allocation and reworking decisions. Section 3 explains the uncertainty flow propagated within the manufacturing system and a coherent cost-tolerance model, and supports a simulation-based optimization approach. Section 4, an extension of the proposed optimization approach using a built-in surrogate model is proposed. In Sec. 5, an electric motor is illustrated, and the results of the proposed model are analyzed. Finally, Sec. 6 wraps up this paper and provides an outlook on prospects.

2 Concurrent Tolerance Allocation Optimization Review

2.1 Tolerance Allocation. The allocation of design and manufacturing tolerances has a significant effect on both manufacturing cost and quality. Designers prefer tight tolerances to assure product performance; manufacturers prefer loose tolerances to cut production costs and ease the manufacturing process. Indeed, tolerances are allocated to ensure the respect of geometrical product requirements and to achieve optimal manufacturing costs. Three tolerance synthesis techniques are used: rules-based synthesis, knowledge-based synthesis, and optimization synthesis [3]. The optimization approach is commonly based on parametric models of the tolerance cost [4–6,2,7,8].

The relationship between tolerance and the associated cost is investigated in the literature. This relationship is mainly based on mathematical functions such as power, exponential, and polynomial functions, which only express the manufacturing cost considering the tolerance interval to produce. Chase et al. [5] summarized several tolerance cost models, and Yeo et al. [9] compared them with empirical data (Fig. 1). The authors underlined that "Quantitative manufacturing knowledge is an abstraction of empirical production data. One important abstraction is the relation between design accuracy and production cost. A survey of various models has been used to carry out modeling of empirical cost-tolerance relations for optimal tolerance design" [9] (Fig. 1). This comparison concludes that several models have limited accuracy in cost prediction. Moreover, the identification of the model parameters depends on the company context, the product, the geometric specification, and so on. Since this identification is only relevant in the case of routine manufacturing, it requires quantified data on similar products.

In this regard, Hong and Chang [1] said that "Most of the tolerance allocation approaches that have been published are based on

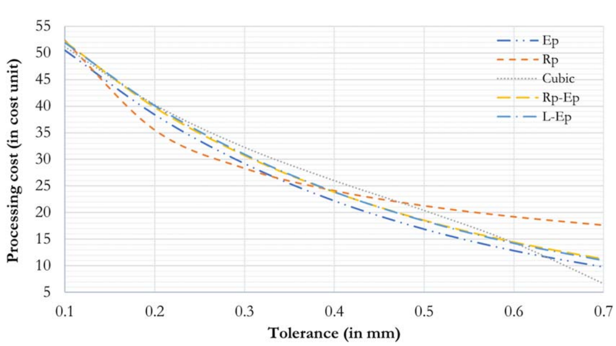

Fig. 1 Comparison of available cost-tolerance models

the optimization of the cost-tolerance function. They usually try to get optimal tolerance "values" while the tolerance "types" are assumed to be fixed. Unfortunately, however, the usage of these models in the industry is fairly limited. One of the major reasons for this is that they usually try to take advantage of the superficial knowledge of processes, which is usually obtained from the machinist handbook or the company manual." Furthermore, several researches have been done to expand the tolerance allocation problem by exploring more relevant objective functions. Zhang and Li [10] presented a new robust optimization strategy to cope with the tolerance optimization problem for internal combustion engines under parameter and model uncertainties, also including Gaussian process metamodeling uncertainty into account. Khodaygan [11] proposed an allocation of tolerances based on maximizing product quality and minimizing manufacturing cost while satisfying functional requirements.

Additionally, the context of tolerancing can be explored where it is more focused on complex systems such as mechanical assembly products. Singh et al. [12] studied mechanical assemblies with alternative manufacturing processes and the impact of resulted tolerances regarding various processes. An adapted genetic algorithm (GA) is proposed which can satisfy constrained assembly tolerancing problems such as those involving interrelated dimension chains, complex cost function, etc. Sivakumar et al. [13] developed a concurrent tolerancing model which optimizes over-constrained process tolerance involving dimensional and geometrical tolerances. The authors proposed a multi-objective optimization model and two evolutionary algorithms, namely, the non-dominated sorting genetic algorithm and multi-objective differential evolution are proposed to locate the optimum solution.

Rezaei Aderiani et al. [14] developed a selective assembly technique using a variation simulation tool for sheet metal assemblies. The authors studied the impacts of batch-size manufacturing and adapted an optimization algorithm to obtain the best-suited mating combinations. Hallmann et al. [15] investigated the impact of an over-constrained assembly system with gaps in tolerance optimization. A cost-tolerance optimization model was established to ensure model accuracy in time-consuming applications while providing cost-optimum tolerance values. Zhao et al. [16] developed a multiple attributes decision-making algorithm, assisted with the rule-based algorithm and axiomatic design algorithm applied in computer-aided tolerance specification. The model generates the specification which is adhered to two categories of data, that of static factors from the rules including that of dynamic factors based on the manufacturing site's information. Franz et al. [17] proposed a surrogated-based optimization model that calculates tolerance values for laminate design parameters. A Gaussian process regression model has been employed to reduce the computational effort. The regression model verifies whether the allocated tolerances satisfy the required functionality or not. Armillotta [18] extended the form of the reciprocal power cost-tolerance function, in which its parameters are expressed based on the features of the product (material, type of feature, area, and nominal dimension). The properties of the extended function allow allocating optimal tolerances by locating the initial solution which satisfies the optimal rates between tolerances. Tabar et al. [19] proposed a clustering approach for geometric variation analysis and optimization of non-rigid assemblies with stochastic part inputs. The proposed method significantly reduces the computation time required for optimization. Tlija et al. [20] established an integrated decision support tool to examine the sources of multiple defects in the tolerancing process, such as tolerances and external mechanical stresses. The worst-case method and the small displacement torsor are utilized to simulate rigid components with orientation and positioning errors. And, the Gaussian perturbation technique is used to model the component with form defects.

Two conclusions can be expressed: the first one of the overviews of the cost-tolerance function is that the accuracy of this predictive model is uncertain. The parameters of this function depend on several factors. All proposed cost models in the literature are

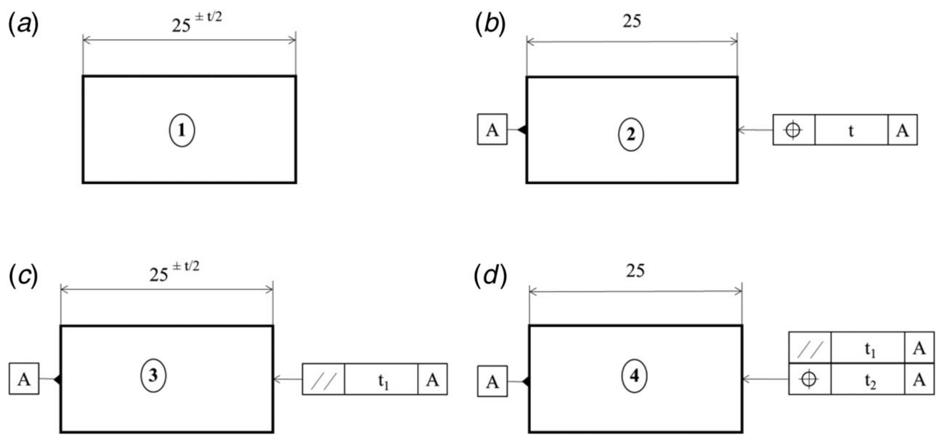

Fig. 2 Different types of tolerances on an engineering design: (*a*) a tolerance allocation problem on part's dimension, (*b*) a tolerance allocation problem on part's position, (c) a tolerance allocation problem on part's rotation and dimension, and (*d*) a tolerance allocation problem on part's rotation and position

following a common hypothesis where cost parameters are predefined and extracted from the company data. Nevertheless, in the real-world industry providing much information regarding the cost model is complicated and might not be easy to modify afterward. Moreover, parametric cost models would not consider different types of tolerances in a common cost model. As in Eq. (1), the cost model depends on a specified tolerance and does not apply to responding to several tolerances that may be required in an engineering design. More in detail, for detailing geometric characteristics and other dimensional requirements of an engineering design, several types of tolerances can be specified. For example, Fig. 2 illustrates allocated tolerances on form, orientation, and location of a designed part.

The second conclusion of this overview is that cost is a relevant indicator of tolerance allocation, but it is necessary to add other components like the impact of tolerance on the non-conformance rate. This is the goal of the next section.

2.2 Resource Allocation. The integration of resource capability and the introduction of machine selection into tolerance allocation can be found in Ref. [21]. The author studied the interdependency between machine selection and tolerance allocation and suggested a simultaneous continuous linear cost-optimization model. In extension, Irani et al. [22] developed a graph-based optimization model representation of tolerance chains to find optimal processing sequences. Moreover, Zhang and Wang [23] in complementary research, proposed a robust approach to appropriately allocating assembly and machining tolerances while maximizing a product's robustness.

The impact of resource variation has also been studied in the context of quality loss. Feng and Kusiak [24] addressed quality loss in tolerance allocation and resource allocation where the resource variation was applied on a multidimensional tolerance chain. Afterward, Vasseur et al. [25] expanded the concept of quality loss into manufacturing cost and presented a method for the selection of resources to manufacture various parts of an assembly product.

The extensive research on the topic has led to several formulations. The integration of tolerance allocation and resource allocation on one hand brings two essential challenges in advanced tolerance design, however, on the other hand, arouses an incontrovertible challenge. The common cost-tolerance models in the literature are parametric models whose structures vary from linear to non-linear [26–29]. For instance, several types of manufacturing cost models respectively, exponential function (Ep) [26], reciprocal power function [28], cubic polynomial (Cubic-P) [30], also, hybrid models have been proposed are adopted from conventional cost models [30,31]. The cost model development relies on an extensive individual study of existing manufacturing resources and tolerance variation-sensitive analysis to yield an appropriate cost-tolerance model [32–36]. An alternative to parametric modeling can be seen in Etienne et al. [7] where authors proposed an activity-based cost modeling that rationally provides an accurate indicator of the relevance of tolerances values fixed by designers. This model associates the impacts of tolerance allocation on the manufacturing process and so the production cost. Moreover, Wu et al. [37] introduced machine tool static geometric accuracy into tolerance modeling. The small displacement torsor is applied to map the relationship between the geometric error of machine tools and tolerance design. Afterward, the Monte-Carlo simulation method is established to determine the response model of the torsor parameters and the tolerance variation bandwidths. Moreover, Khezri et al. [38] and Khezri et al. [39] extended the cost-tolerance model by integrating resource allocation problem and tolerance analysis techniques such as the worst-case method and Monte-Carlo simulation to estimate the assembly conformity rate. The application was illustrated in a two-dimensional tolerancing case study. The study of resource allocation implies the impact of the process deviation on the conformity of the final product and the processing cost. As a common conclusion, the more precise the machine being used, the more costly will be the process. Therefore, an alternative to this solution can be found in applying a reworking process which can enhance the conformity of the product with the available machine resources.

2.3 Reworking. The impact of reworking on the economy was studied in Ref. [40] which was introduced as a process to repair or substitute components that are worn out or obsolete. The observations reported a significant reduction in the level of inter-industry transactions, as well as an improvement in the manufacturing cost. The traces of reworking in the context of tolerancing can be seen in Lee et al. [41] where authors proposed a cost-effective means for tolerance allocation. The authors compounded the probabilities of scrap and rework to obtain the expected loss cost. Additionally, Shin and Cho [42] addressed the reworking concept providing a mean to balance the quality and manufacturing costs. Moskowitz [43] developed a cost model to determine appropriate tolerance allocations where a nonconforming product can be scrapped or reworked. This model is based on a partial information case, where design parameter distributions are not identified. Moreover, the authors studied both parametric and non-parametric rework models. Mustajib and Irianto [44] modeled an optimization model for quality improvement in multi-stage processes. The model integrated alternatives process selection to determine the unit of production and associated manufacturing costs. Mustajib [45] proposed a concurrent tolerance-cost-optimization model considering process capability and costs of non-conformance. The nonconformity is the failure to meet the designed requirement due to process variances. Costs of non-conformance include rework and scrap costs. Sofiana et al. [46] considered the impact of rework on the quality of product, and the impact of a profit-sharing policy, which may stimulate the commitment of suppliers to quality improvement. Liu et al. [47] proposed a novel double tolerance scheme for determining tolerance sets on a production line with processing and rework stations, as well as instantaneous inspection and scrap operations. The production line comprises a rework station which handles non-conformed products, and a queuing system is applied.

Therefore, the contribution of this work on the tolerance allocation problem is to find adequate solutions to the following questions:

(1) How can integrate different types of tolerances into the cost-tolerance optimization model?
(2) How can designers ease off cost dependencies and have an adaptable cost model?
(3) How accurate and manipulable is the proposed cost model to cope with different production strategies?

3 Cost-Tolerance Model: Description, Concurrent Model, and Approach

At all stages of product development and throughout the product life cycle, uncertainty is ubiquitous and incurs. The risk can impact product performance(s), process scheduling, market acceptance, or the whole business. Therefore, a comprehensive engineering design plan which includes key functions of the product using tolerance analysis techniques and mitigating the uncertainties within manufacturing activities to reduce their effects and ensure product functioning is a necessity to the manufacturer risk (as illustrated in Fig. 3). To explain more in detail, each production strategy is associated with consequences in the life cycle of the product and can be clarified as follows:

(1) Resource allocation: a tool to assign available practical resources to components to increase manufacturing line efficiency.
(2) Reworking decision: a decision to improve components conformity rate and decrease the number of scraps.
(3) An allocated exclusive manufacturing resource processes the component and the reworking.

Therefore, the info array which connects the adaptive tolerancing box to the manufacturing process includes optimal and practical resources and tolerances to be allocated to each of which individual component.

In the following section, a concurrent tolerance allocation problem concerning resource allocation and reworking decisions is studied. The section is divided into three main sub-sections. The first section explains statistical definitions of the problem linking production strategies in the context of conformity probabilities. Next, the conformity probabilities are used to formulate manufacturing cost. The last part represents a simulation-based genetic algorithm minimizing manufacturing cost developed. The nomenclatures used in this paper are given as follows:

Parameters	
N	Set of components of the assembly
O	Set of manufacturing operations
NMC	Number of Monte-Carlo simulation
N_d	Number of design constraints on characteristics
N_r	Number of rework constraints
N_f	Number of functional constraints
N_c^i	Number of characteristics on component i
C_d	Set of design constraints
C_r	Set of rework constraints
C_f	Set of functional constraints
LSL_i	Lower specification limit for component i
USL_i	Upper specification limit for component i
μ_i	The nominal value of dimension for component i
$\sigma_{i,j}$	Process deviation of operation j for component i
σ_y	Assembled product deviation
t_y	Assembled product tolerance
α	Inspection Type I failure rate
β	Inspection Type II failure rate
Decision variables	
$a_{i,j}$	1 if resource j is allocated to component i, otherwise, 0
rw	1 if reworking decision is taken, otherwise, 0
t_i	Allocated tolerance to component i and characteristic j
γ_i	Component i conformity rate
γ_i^{rw}	Component i reworking rate
γ_i'	Component i conformity rate after reworking
λ	Assembled product conformity rate
C_{Total}	Manufacturing cost

3.1 Statistical Definition of the Concurrent Problem. The tolerance of a component can be defined as the permissible variation in measurements deriving from the nominal value. It can be expressed as follows:

$$t_i^j = \mathrm{USL}_i^j - \mu_i^j = \mu_i^j - \mathrm{LSL}_i^j, \quad \forall i \in N, \ \forall j \in N_c^i \tag{1}$$

where USL and LSL express upper and lower specification limits and μ denotes nominal value. This paper supports a statistical-based approach integrating resource allocation and reworking decisions into the tolerance allocation problem. Within this approach, the consequences of the decisions are associated with probability rates. Therefore, to go further, the model follows several assumptions:

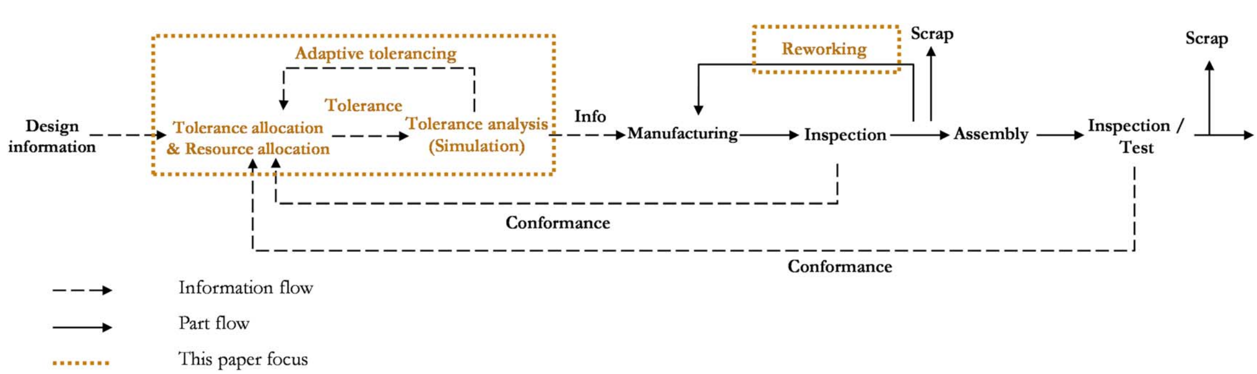

Fig. 3 Assembled product life cycle

(1) A generic form of conformity rate estimator is developed based on the normal distribution.
(2) Dimensions are independent, therefore, the sole dependency in this model is between parts tolerances and functional requirements.

Using the assumptions aforementioned, they lead us to develop an estimation model predicting the conformity rate of the manufactured components considering resource deviations and reworking impact. The conformity rate can be separated into two states of the manufacturing system: the state without reworking ability, and the one with reworking ability. On these bases, the conformity rate can be estimated function of three decision variables, namely: allocated tolerance (t_i), process variation associated with the assigned resource ($\sigma_{i,j}$), and reworking decision (rw).

Consequently, the conformity rate without reworking is formulated in Eq. (2). Afterward, the decision on integrating reworking into the manufacturing scheme can be seen in Eqs. (3) and (4) investigating the conformity rate with the reworking decision. Equation (4) investigates the conformity rate of the engineering design with the allocated tolerances lies in the admissible rework domain. This domain represents the capability of the resource to perform the reworking, as well as the designers' preference on how it should be performed.

$$\gamma_i = \underset{t\in\mathbb{R},\,\sigma\in\Sigma}{Prob}\left(\bigcap_{i=1}^{N_d} C_d^{(i)}(Dev_i)\in[Deviation\ domain]\right),\quad \forall i\in N \tag{2}$$

$$Dev_i = Rand(0,\sigma_i),\quad \forall i\in N \tag{3}$$

$$\gamma_i^{RW} = \underset{t\in\mathbb{R}}{Prob}\left(\bigcap_{i=1}^{N_r} C_r^{(i)}(Dev_i)\in[Admissible\ rework\ domain]\right),\quad \forall i\in N \tag{4}$$

$$\gamma_i' = \underbrace{\gamma_i}_{Conformed\ without\ reworking} + \underbrace{rw\times\gamma_i^{RW}\times\gamma_i}_{Reworked\ and\ conformed},\quad \forall i\in N \tag{5}$$

In Eq. (5), the conformity rate of the components is evaluated after reworking is considered as a production strategy in the manufacturing system. Therefore, this equation sums up the conformity of the component accepted in the first place and the conformity of the rejected component which was reworked and conformed. Moreover, the process deviations associated with allocated resources can be used to approximate assembled product deviation (σ_y) and estimating assembly conformity rate (Eq. (6)). However, this equation lacks precision when it comes to estimating the assembly conformity rate of a complex mechanism with more than a few unbiased functional requirements. Therefore, this equation can be altered by developing a simulation-based technique to calculate the rate.

$$\lambda = \underset{\sigma\in\Sigma}{Prob}\left(\bigcap_{i=1}^{N_f} C_f^{(i)}(Dev_i)\in[Functional\ domain]\right) \tag{6}$$

$$\begin{aligned}
&\min\ Cost_{Total}(t,a,s)\\
&Subject\ to\\
&\textstyle\sum_{j\in O} a_{i,j}=1\\
&t\in[Tolerance\ domain] \qquad ,\forall i\in N\\
&s\in\{Production\ strategies\}\\
&\gamma_i\ge Quality\ rate\ requirement\\
&\lambda\ge Quality\ rate\ requirement
\end{aligned} \tag{7}$$

The abstract model of the proposed cost-tolerance optimization method is presented in Eq. (7). This method integrates production strategies such as resource allocation and reworking into the tolerance allocation. The application of the strategies is associated with their performance on the components and assembly conformity. The evaluation of the application and allocated tolerances are analyzed with the simulation-based technique. This technique is detailed in Sec. 3.3.

3.2 Manufacturing Cost Model. In Sec. 2.1, the existing manufacturing cost models were discussed. In summary, an appropriate cost model which properly represents the manufacturing capabilities relies on extensive study of variation-sensitive analysis. Hence, activity-based modeling provides an accurate cost assessment tool [7], consequently, a manufacturing cost is developed. The cost model is structured in the relevance of the impact of the decisions taken on the conformity rate of the components pre-assembly and assembled product. Therefore, the following definitions are brought explaining the rate of conformed and non-conformed components/products after inspection. Afterward, Eq. (8) represents the developed cost model where each activity is associated with the relevant decision impacts.

$\gamma_i'(1-\alpha)$	Rate of conformed components pre-assembly
$(1-\gamma_i')\beta$	Rate of non-conformed components pre-assembly
$\gamma_i'\alpha$	Rate of undetected non-conformed components pre-assembly
$(1-\gamma_i')(1-\beta)$	Rate of detected non-conformed components pre-assembly
$\lambda(1-\alpha)$	Rate of marketable assembled conformed products
$(1-\lambda)\beta$	Rate of marketable assembled non-conformed product
$\lambda\alpha$	Rate of undetected assembled non-conformed product
$(1-\lambda)(1-\beta)$	Rate of detected assembled non-conformed product

$$\begin{aligned}
Cost_{Total} = &\sum_i^{Part}\sum_j^{Oper}\frac{Cost_{Proc\ i,j}\times a_{i,j}}{\gamma_i'(1-\alpha)+(1-\gamma_i')\beta} + \sum_i^{Part}\frac{Cost_{Inspec\ i}}{\gamma_i'(1-\alpha)+(1-\gamma_i')\beta} + \sum_i^{Part}\frac{Cost_{Scrap\ i}(\gamma_i'\alpha+(1-\gamma_i')(1-\beta))}{\gamma_i'(1-\alpha)+(1-\gamma_i')\beta}\\
&+\sum_i^{Part}\frac{Cost_{Rew\ i}*rw}{\gamma_i'(1-\alpha)+(1-\gamma_i')\beta} + \frac{Cost_{Assembly}}{\lambda(1-\alpha)+(1-\lambda)\beta} + \frac{Cost_{Product\ Scrap}(\lambda\alpha+(1-\lambda)(1-\beta))}{\lambda(1-\alpha)+(1-\lambda)\beta} + \frac{Cost_{Inspec}}{\lambda(1-\alpha)+(1-\lambda)\beta}
\end{aligned} \tag{8}$$

The model is constrained following technical and design constraints. The technical constraint takes into account that each component can be processed with only one resource (Eq. (9)).

$$\sum_{j\in O} a_{i,j} = 1,\quad \forall i\in N \tag{9}$$

3.3 A Simulation-Based Optimization Model. So far, a manufacturing cost model functions of tolerances, resources, and reworking decisions is presented. Accordingly, a practical optimization tool is required to yield an optimal solution. Since the model proposed is non-linear, therefore, a simulation-based optimization is developed (Fig. 4). The structure of this model lies in the fact that the optimization approach is deployed allocating the optimal tolerances and resources for individual compounds while the least cost is obtained. Afterward, due to the process deviation and imprecise machinery tools, consequently, fluctuation in components geometry is inevitable. Hence, the fluctuation is

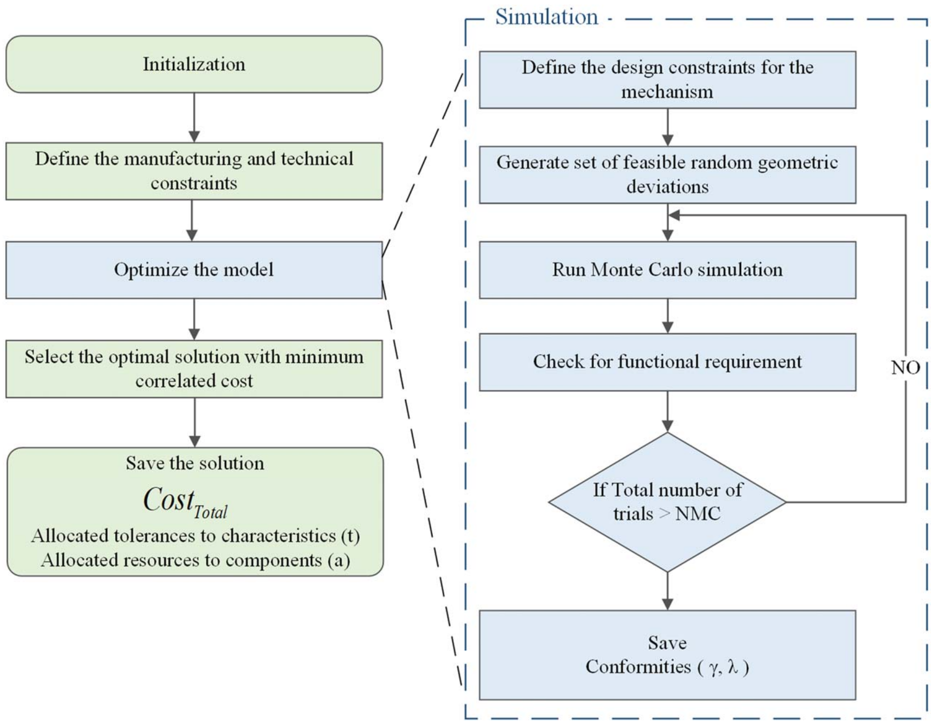

Fig. 4 Simulation-based optimization approach

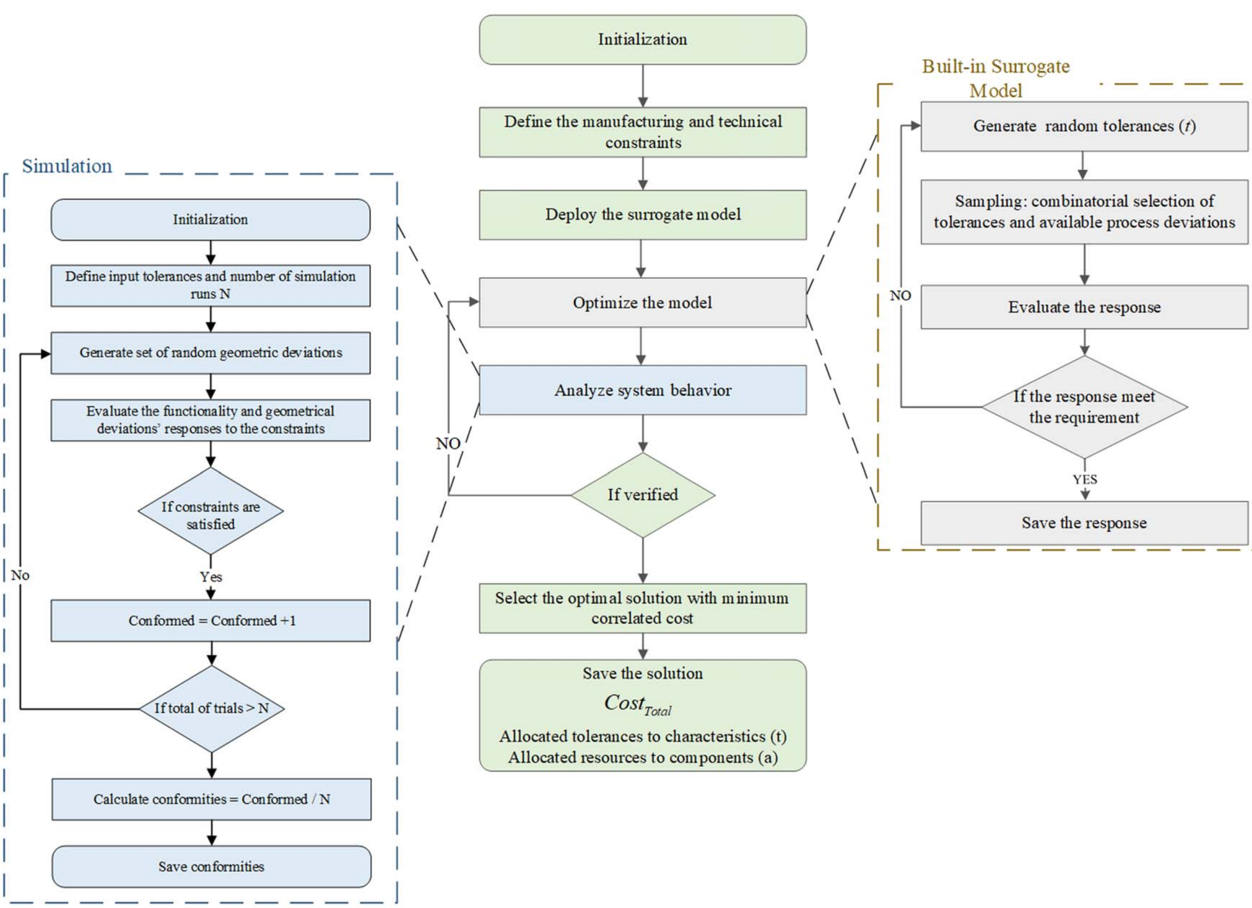

Fig. 5 Modified optimization model

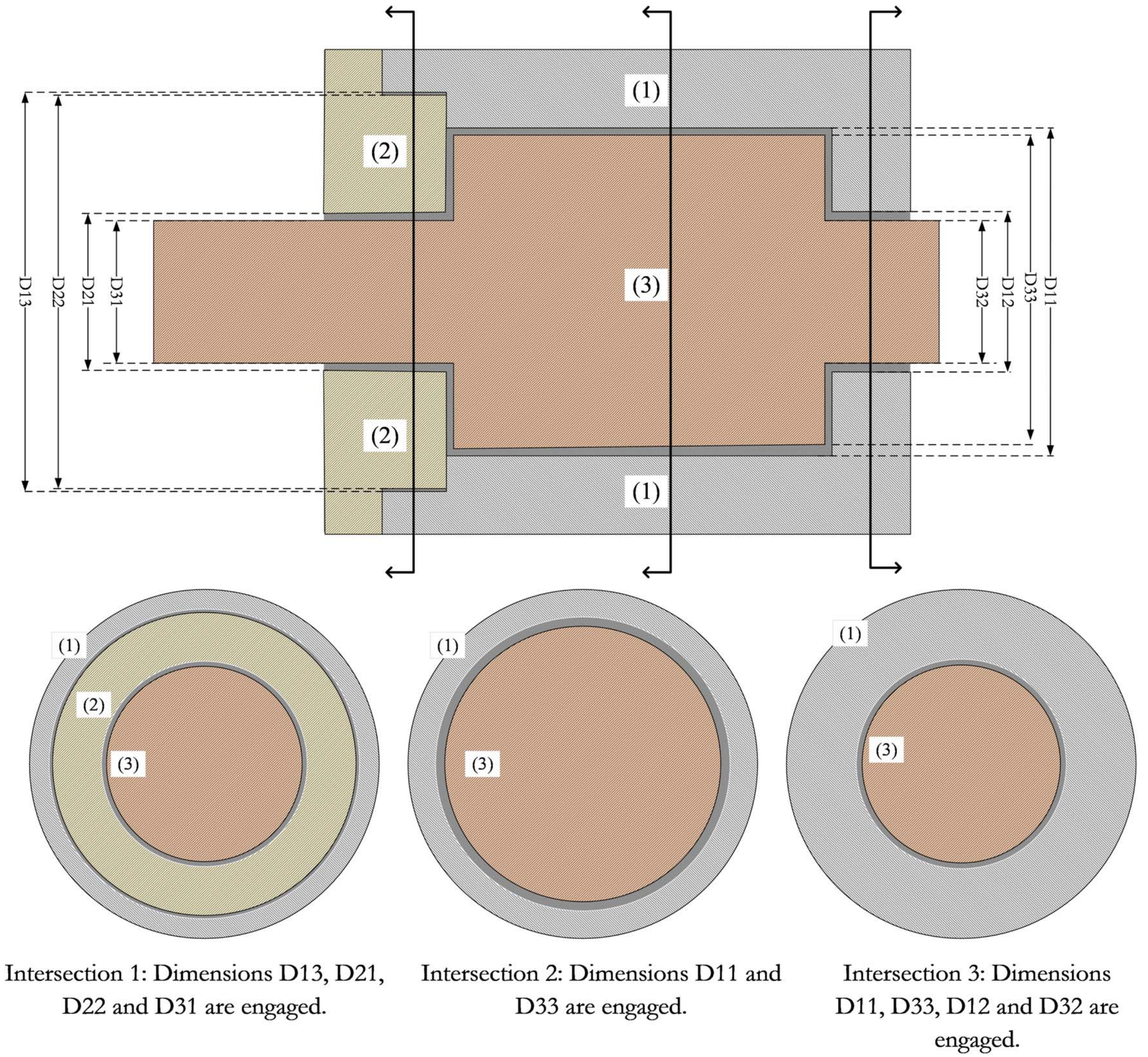

Fig. 6 An electrical motor mechanism

conclusive, then, simulation can be used to investigate a variety of deviations over components geometries within the process deviation domain and deploy reworking decision. The process is designed to verify whether the functional requirement is satisfied or not.

However, simulation is a practical tool to analyze system response corresponding to allocated tolerances and resources, it consumes time to simulate and study the response. An alternative tool is to modify the optimization model replacing the simulation model with a surrogate model.

4 Enhanced Optimization Model: Built-In Surrogate Model

In Sec. 3.3, a simulation-based optimization was established, a decision-making tool to locate optimal tolerance and resource sets. This model brings complex system behavior into tolerance allocation optimization which explores all the viable and practical solutions while simulations verify the solution's applicability and conformity. Therefore, the optimization problem merits the simulation tool which analyzes the allocated tolerance's reliability. Still, simulation implementation is based on the generation of several deviations in design characteristics, analyzing the behavior within the system of equations, and reporting back the conformity. On the one hand, an abundant number of generations of deviations provides a more accurate analysis of system behavior, on the other hand, optimization and simulation processing time depending on the number of the characteristics, generations, and system behavior complexity soar up, consequently.

An alternative solution to improve simulation-based optimization performance is to substitute simulation for a surrogate model. The surrogate model is constructed using data drawn from a high-fidelity simulated model and mimics approximately the behavior of the simulation model as fast as possible [48]. In this paper, the system's behavior response (f) is supported by set of constraints (Φ) who analyzes system according to associated allocated tolerances (t) and resources ($a_{i,j}$). The estimation of the system's behavior response ($\hat{f}$) can be approximated by assessing the simplified system's behavior constraints. Therefore, the response ($\hat{f}$) can be approximated based on the new set of constraints. On this basis, a modified optimization model is developed substituting the simulation model for the surrogate model proposed and depicted in Fig. 5.

Figure 4 illustrates the simulation-based optimization approach developed in the literature, this approach is time-consuming, therefore, Fig. 5 substitutes for the surrogate model. Hence, the surrogate

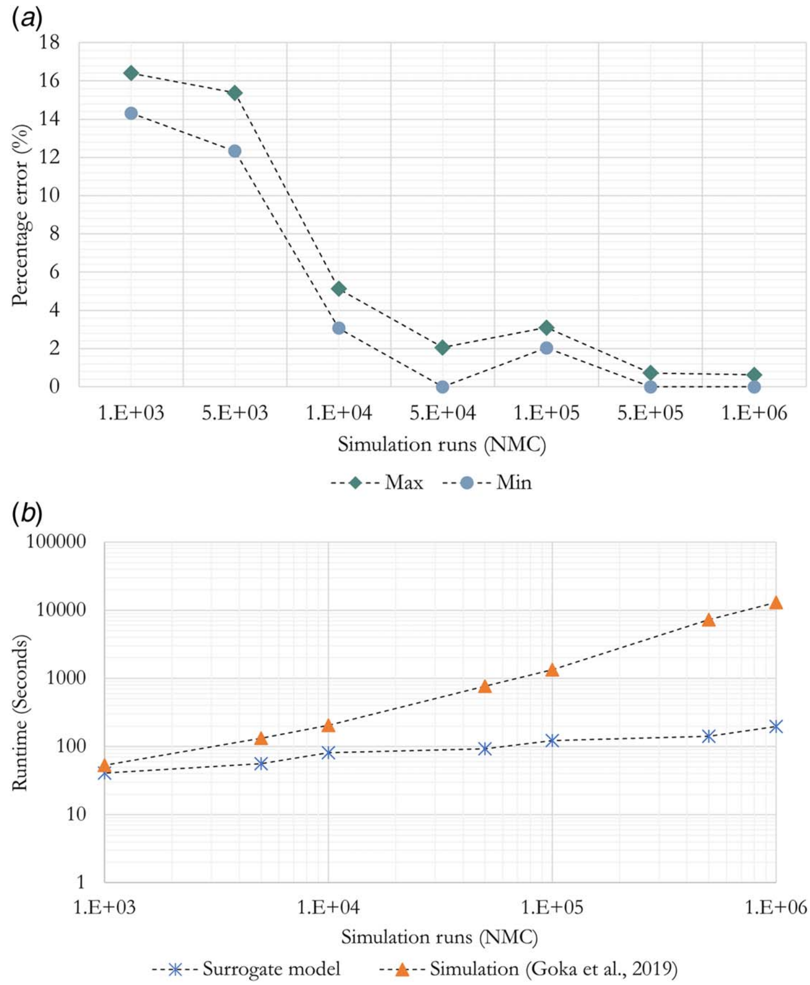

Fig. 7 Surrogate model efficiency evaluation in comparison to the simulation: (*a*) surrogate percentage error references to the simulation and (*b*) surrogate model and simulation runtime comparison

Table 1 Manufacturing data for the electric motor

Components	House			Body			Shaft		
Resources	M11	M12	M13	M21	M22	M23	M31	M32	M33
Processing cost (cu)	5	8	10	3	2.5	2.95	2.95	3.15	4
Resource deviation (mm)	0.05	0.013	0.01	0.016	0.03	0.021	0.021	0.013	0.01
Inspection cost (cu)	1			1.5			1		
Scrap cost (cu)	0.5			0.5			0.5		
Reworking cost (cu)	1			1			1		
Product assembly cost (cu)	3								
Product scrap cost (cu)	10								
Inspection cost (cu)	0.5								

Note: cu = cost unit

model is a simplified modification of the simulation approach, still, the simulation tool is deployed as a control to assure that the assembly response meets the needs. In this regard, once the optimization is terminated, the solutions are verified by using the simulation approach. If the solution is not verified, it will be eliminated and the optimization algorithm will be run again to locate the optimal solution.

5 Case Study: An Electric Motor

In this section, an over-constrained mechanical assembly product is examined (illustrated in Fig. 6). The assembly product is an electric motor assembled of a body (part (2) in the figure), a shaft (part (3) in the figure), and a housing (part (1) in the figure). Anselmetti [49] investigated the electrical motor and established a tolerancing process that served by determining the specifications of

Table 2 Nominal designs values on dimensions

Dimension	D11	D12	D13	D21	D22	D31	D32	D33
Nominal value μ_D (mm)	100	60	130	60	129.96	59.9	59.9	99.8

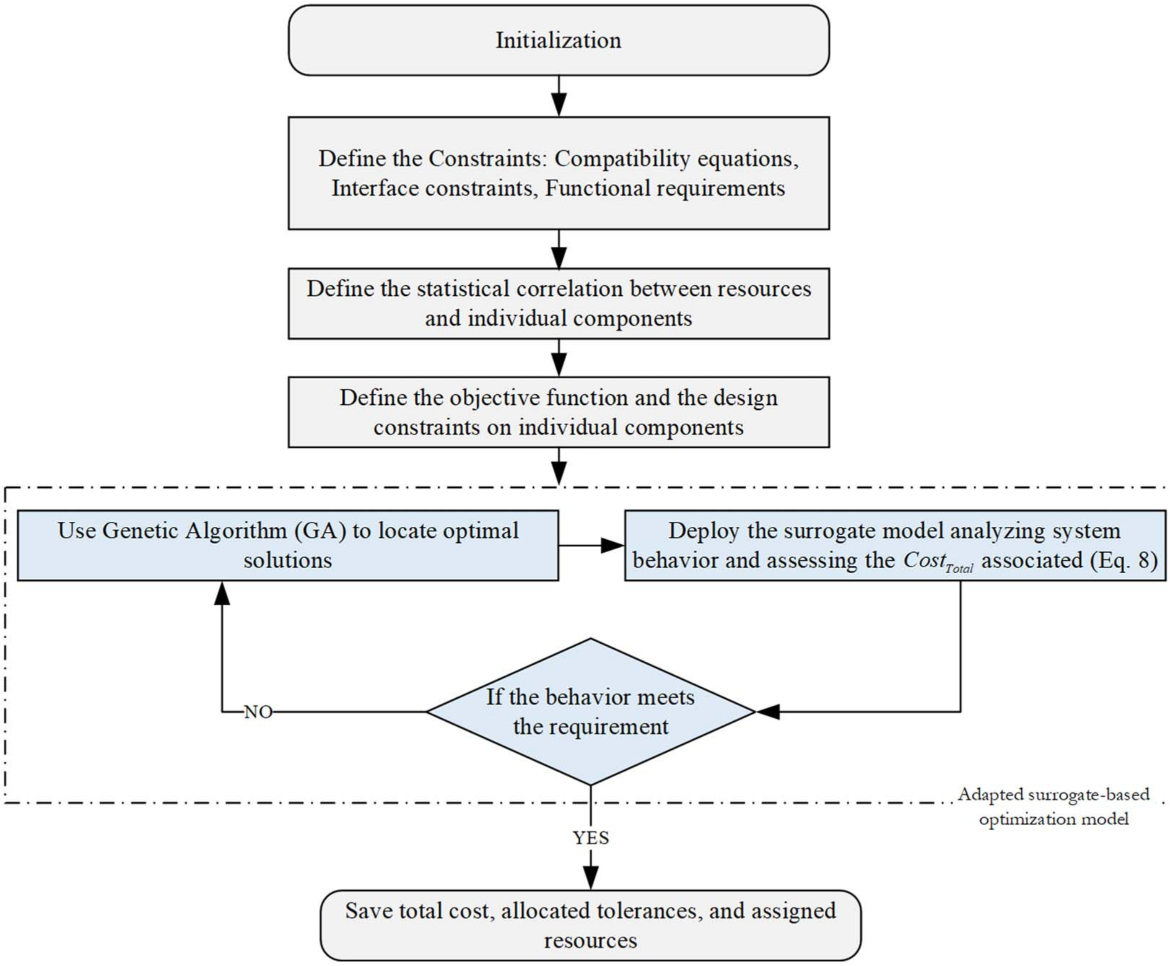

Fig. 8 Adapted optimization approach

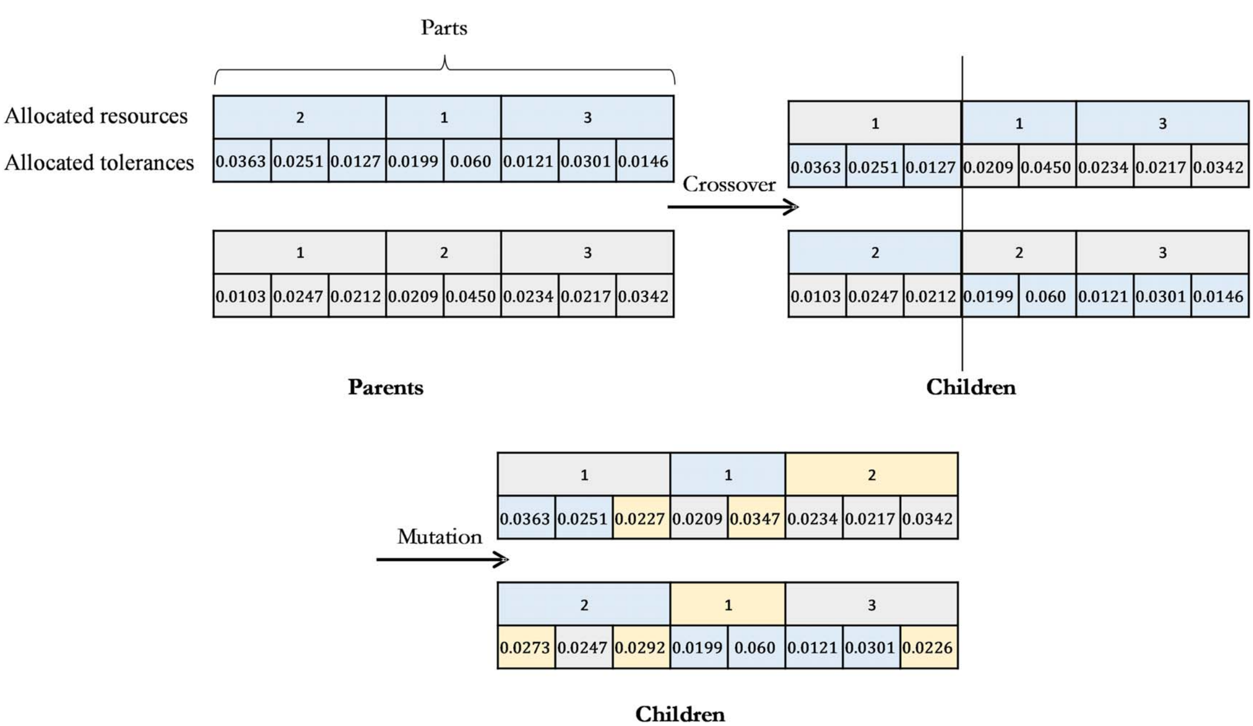

Fig. 9 Genetic algorithm chromosome, crossover, and mutation presentations

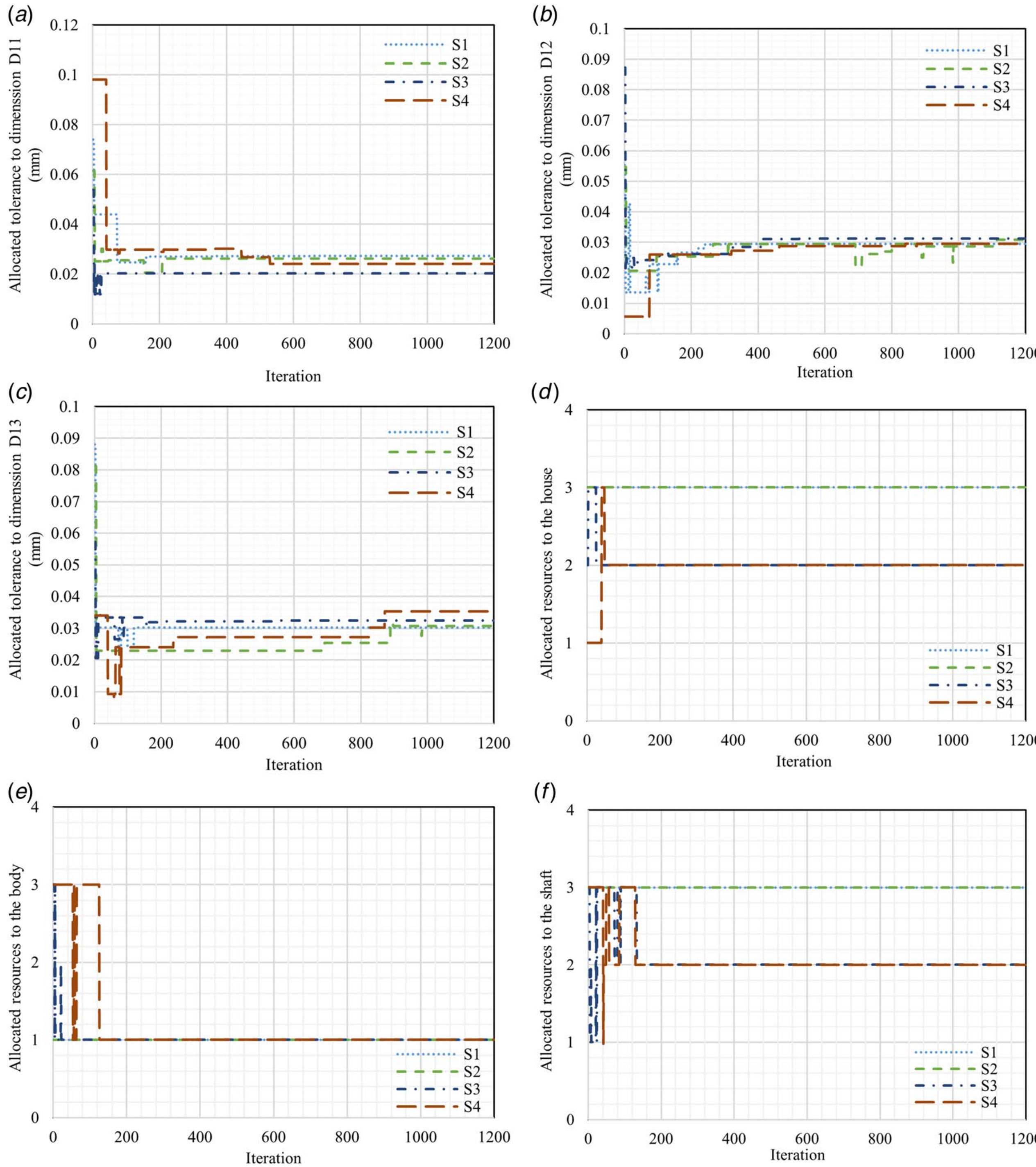

Fig. 10 Genetic algorithm optimization results. (*a*) Convergence of GA for the allocated tolerance to dimension D11, (*b*) Convergence of GA for the allocated tolerance to dimension D12, (*c*) Convergence of GA for the allocated tolerance to dimension D13, (*d*) Convergence of GA for the allocated resources to the house, (*e*) Convergence of GA for the allocated resources to the body, (*f*) Convergence of GA for the allocated resources to the shaft, (*g*) Convergence of GA for the house conformity rate, (*h*) Convergence of GA for the body conformity rate, (*i*) Convergence of GA for the shaft conformity rate, and (*j*) Convergence of GA for the manufactuirng cost.

influential parts without form defects. Afterward, Goka et al. [50] introduced form defects to contact surfaces of the system and developed a Mote-Carlo-based simulation approach to analyze the system behavior by assessing the assembly and functionality probabilities. Therefore, the application of the cost-tolerance optimization model is being studied using the tolerance analysis tool developed by Goka et al. [50].

In this mechanical design, diameters D11, D12, D13, D21, D22, D31, D32, and D33 represent the key characteristics of the design. Appropriate tolerances on diameters are required to be allocated with the given design (C_d), rework (C_r), and functional (C_f) constraints. These constraints prevent surfaces from inadvertent collisions by concerning designated gaps between surfaces and limit diameters exceeding practical value. Therefore, the gaps between the surfaces illustrate the functional requirement which assures the assembly functions perfectly. Subsequently, the set of equations is substituted for a surrogate model to improve the calculation time performance. The surrogate model in this case is a simplified

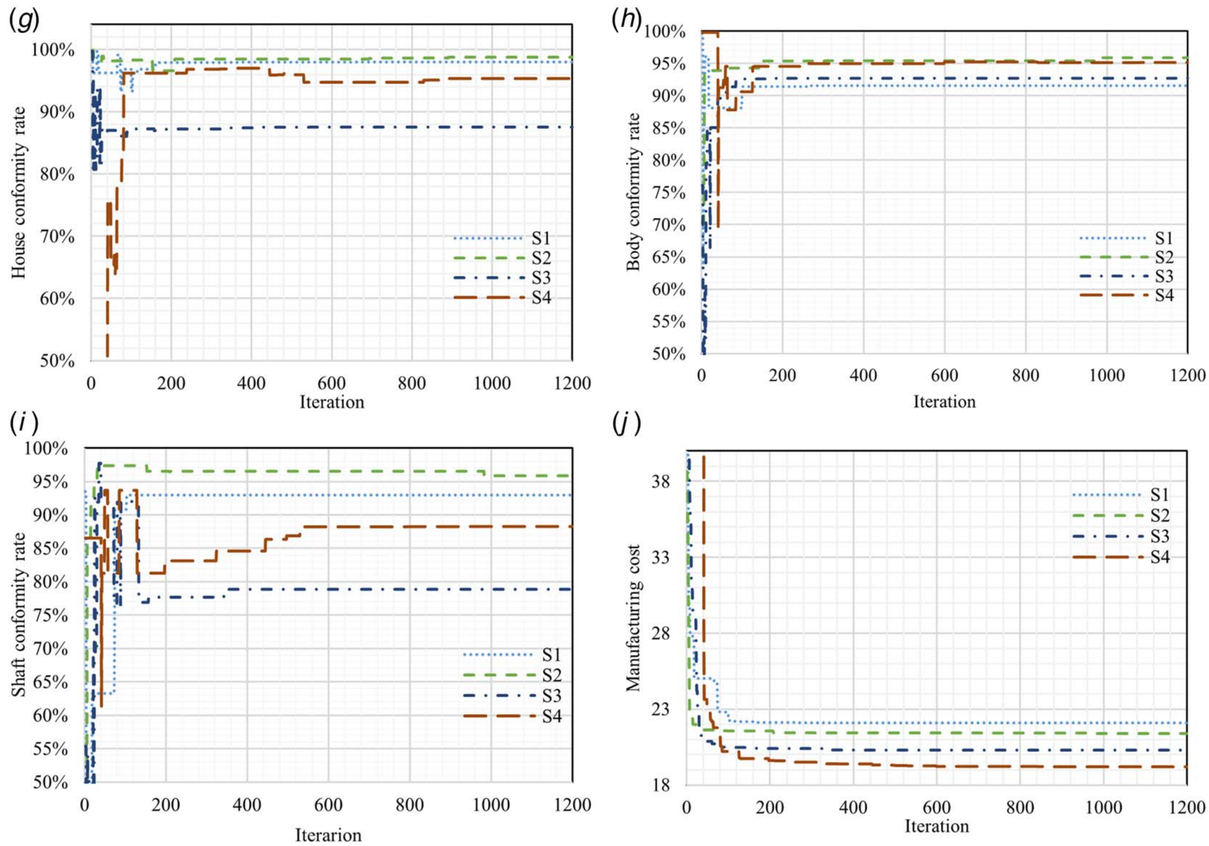

Fig. 10 ***Continued***

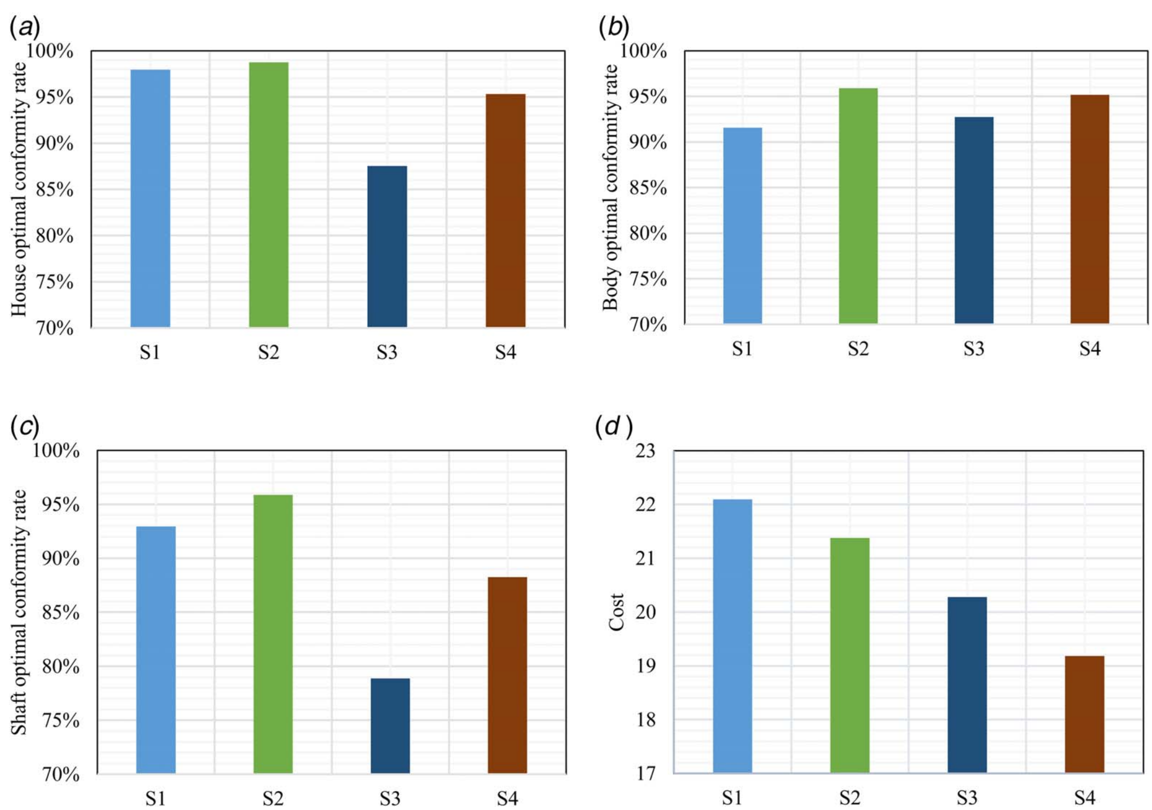

Fig. 11 Optimal cost and components conformity rates associated with each scenario. (*a*) House optimal conformity rate comparison associated with each scenario, (*b*) body optimal conformity rate comparison associated with each scenario, (c) shaft optimal conformity rate comparison associated with each scenario, and(*d*) optimal manufactuirng cost comparison associated with each scenario.

Table 3 Optimally allocated tolerances on design dimensions

	t_{D11}	t_{D12}	t_{D13}	t_{D21}	t_{D22}	t_{D31}	t_{D32}	t_{D33}
S1	±0.027	±0.030	±0.030	±0.030	±0.050	±0.020	±0.020	±0.023
S2	±0.026	±0.031	±0.029	±0.031	±0.051	±0.019	±0.019	±0.024
S3	±0.020	±0.031	±0.033	±0.032	±0.047	±0.018	±0.018	±0.030
S4	±0.024	±0.029	±0.035	±0.030	±0.044	±0.020	±0.020	±0.026

Table 4 Optimal allocated resources to the components

	House	Body	Shaft
S1	M13	M21	M33
S2	M13	M21	M33
S3	M12	M21	M32
S4	M12	M21	M32

Table 5 Simulation and surrogate models obtained cost assembly, and functionality rates relate to each scenario

	Simulation-based optimization			Surrogate-based optimization		
	λ	$Cost_{Total}$ (cu)	Calculation time	λ	$Cost_{Total}$ (cu)	Calculation time
S1	97.44%	25.83	1 h 18 min 17 s	98.35%	25.84	5 min 5 s
S2	97.44%	25.17	1 h 12 min 56 s	98.36%	25.11	5 min 33 s
S3	95.54%	24.28	1 h 10 min 48 s	95.90%	24.23	4 min 55 s
S4	95.54%	23.26	1 h 10 min 45 s	95.88%	23.35	5 min 8 s

representation of the mechanism in which the 3D model is replaced by 2D planes. The planes are the intersection of the key characteristics of the design. Three main intersections (shown in Fig. 6) are deemed which comprise all the key characteristics and functional requirements. Since the approach doesn't take into the account whole contact surface, it increases the uncertainty to assess requirement evaluation. However, the application of the Monte-Carlo helps to improve the accuracy (Fig. 7(*a*)), as well as tolerance analyzing time (Fig. 7(*b*)).

Moreover, the manufacturing data and nominal design values are given in Tables 1 and 2.

5.1 Cost-Tolerance Optimization Implementation. In this section, the tolerance analysis approach proposed in the literature embeds into the cost-optimization model developed. Thereby, the optimization and surrogate approaches enable finding optimal tolerances correlated to key characteristics of the components. The implemented surrogate-based optimization is illustrated in Fig. 8. The approach commences with the definition of the geometry design constraints, statistical relations, and objective function. Afterward, the tolerances and resource deviation be introduced over the key characteristics and analyzed via the surrogate model analyzing the system's behavior. In this step, the optimal solution is obtained where the least cost yields using a GA.

The algorithm is tuned with the following parameters: the number of iterations = 1200, population size = 150, mutation probability = 0.04, crossover probability = 0.5, and elite rate = 0.01. Moreover, the main contribution of this paper is to integrate the reworking and resource allocation decisions into tolerance allocation improving components conformity rates. The chromosome developed in this algorithm is structured into two sub-genes. The first sub-gene contains assigned resources' information to each part and the second sub-gene includes allocated tolerances' information, accordingly (Fig. 9).

5.2 Results and Discussion. In this section, to have a better comprehension of the problem, four different scenarios are intended. The first scenario (S1) takes available precise resources for each component to be processed. The second scenario (S2) takes available precise resources and applies reworking on components if it is required. The third scenario (S3) allocates optimal resources among available resources to each component. Lastly, the fourth scenario (S4) applies resource allocation and reworking for each component to be processed. Consequently, a comparison of resulted allocated tolerances to the house's main characteristics (as an illustration house is selected), allocated resources, cost, and conformity rates over iterated iterations for each scenario is illustrated in Figs. 10 and 11. Moreover, allocated optimal resources and tolerances to each component and correlated design dimensions are depicted in Tables 3 and 4.

The study of cost and conformity rates associated with different scenarios illustrates the impacts of resource allocation and reworking decisions. On one hand, it can be determined that applying resource allocation using available machine tools impacts associated costs diminishing the cost value. Tentatively, it influences conformity rates due to imprecise allocated resources. On the other hand, the rework decision is applied to cover up the drop-in conformity rates caused due to resources' imprecision and improve conformity rates (Figs. 10 and 11). Successively, the tolerances allocated on dimensions are influenced by the decisions, which resource to be assigned to the component, and whether reworking is necessary or not, the tolerances adapt, accordingly (Tables 3 and 4).

So far, the minimal manufacturing cost associated with each scenario is located and the optimal tolerances and resources are allocated. Subsequently, allocated resource deviations and tolerances feed into the tolerance analysis approach estimating assembled product functionality conformity rate and associated assembly cost. In Secs. 3.3 and 4, the simulation-based and the surrogate-based approaches were detailed. Consequently, the application of simulation and surrogate models estimates assembled product functionality conformity rate. Appropriately, the assembly cost is calculated, the minimal manufacturing cost is summed up and the total manufacturing cost is achieved. A comparison of the two models is detailed in Table 5.

Ultimately, the implementation of surrogate-based cost-tolerance optimization was demonstrated in this section. The integration of tolerance allocation taking into account available machine tools process deviations and the application of reworking to enhance component conformity rate were analyzed. Following the results achieved, resource allocation lesser manufacturing cost, however, it influences on component conformity rate. Therefore, rework is a practical decision to enhance the conformity of the component. Moreover, the simulation deploys verifying allocated tolerances and resource viability estimating assembly and assembled product functionality conformity rates.

6 Conclusions and Future Works

The need for a reliable engineering plan has broadened the scope of tolerancing. A comprehensive engineering plan which mitigates the uncertainties to reduce their effects and ensure product functioning is a necessity for the manufacturer. Therefore, in this paper, an appropriate holistic methodology for tolerance allocation, taking production strategies, and the functional fulfillment degree of components is developed. Initially, a simulation-based optimization approach to locate optimal tolerance and resource sets is established. Since calculation time is not convenient, the model is substituted for a surrogate model to improve the calculation time performance. Ultimately, a modular costing system is developed, and a genetic algorithm is adapted to locate optimal solutions.

The proposed approach supports a modular cost model and optimization approach which can be adjusted to comply with the case study. On one hand, it is a manipulable statistical model which can integrate different production strategies and analyze the consequences, on the other hand, analyzing complex mechanism behavior such as micro gears with numerous characteristics requires extensive study. Therefore, future work is mainly pursuing micro gears behavioral tolerance analysis from two aspects. First, an adapted efficient tolerance allocation tool that can examine gears behavior and distribute adequate tolerances of characteristics. Second, the impact of assembly strategies such as individual paring, selective assembly, etc., on the components and assembled parts are required to be studied.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Data Availability Statement

The authors attest that all data for this study are included in the paper.

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