We will show here how to construct the vectors $\vec{v}_g$ with an example. Recall the ciphertext given in the slide 19 of sec2.4.pdf (slide number is listed on the bottom right corner of the slide):

Recall the ciphered text:

```
DBMOKWWODJPAOPEPHGQAHWVUJNXDZA
IAQMRPNEMHGPERZOVKRVLWUGAEOPXE
HMXDVBXDDVKOYQHJJBLWKXIJDVUQDB
IPCMAWTBLATKSIZJEYFBSIZBSZVG
```

We noticed by computing the index of coincidences for $m = 1, 2, 3, 4, 5$ on the slides 20-23 of sec2.4.pdf that the key word length $m = 4$ is the correct guess. So we divide the given ciphertext into 4 substrings as follows:

Substring ($\mathbf{y}_1$):
`DKDOHHJZQNGZRUOHVDYJKDDCTTZFZV`

Substring ($\mathbf{y}_2$):
`BWJPGWNAMEPOVGPMBVQBXVBMBKJBBG`

Substring ($\mathbf{y}_3$):
`MWPEQVXIRMEVLAXXXKHLIUIALSESS`

Substring ($\mathbf{y}_4$):
`OOAPAUDAPHRKWEEDDOJWJQPWAIYIZ`

Substring ($\mathbf{y}_1$) contains every fourth letter of the ciphertext starting with first letter, that is, it has 1st letter, 5th letter, 9th letter and so on.

Similarly substring ($\mathbf{y}_2$) contains every fourth letter of the ciphertext starting with second letter, that is, it has 2nd letter, 6th letter, 10th letter and so on.

Substring ($\mathbf{y}_3$) contains every fourth letter of the ciphertext starting with third letter, that is, it has 3rd letter, 7th letter, 11th letter and so on.

Substring ($\mathbf{y}_4$) contains every fourth letter of the ciphertext starting with fourth letter, that is, it has 4th letter, 8th letter, 12th letter and so on.

Now let us compute the vectors $\vec{v}_g$ where $0 \leq g \leq 25$ for the substring $(\mathbf{y}_1)$:

First notice that there are 30 characters in the substring $(\mathbf{y}_1)$ and so the length of this substring $N = 30$.

By counting each alphabet in this substring, we get the following frequency table:

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	1	5	0	1	1	3	0	2	2	0	0

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	0	1	1	0	2	1	2	0	0	1	4

Therefore,

$$\vec{v}_0 = \frac{1}{30}(0,0,1,5,0,1,1,3,0,2,2,0,0,1,2,0,1,1,0,2,1,2,0,0,1,4)$$

Once we have $\vec{v}_0$, then $\vec{v}_1$ can be obtained by recycling the components. To obtain $\vec{v}_1$, simply move the first component to the last. So

$$\vec{v}_1 = \frac{1}{30}(0,1,5,0,1,1,3,0,2,2,0,0,1,2,0,1,1,0,2,1,2,0,0,1,4,0)$$

To obtain $\vec{v}_2$ from $\vec{v}_1$, move again the first component of $\vec{v}_1$ to the last. So

$$\vec{v}_2 = \frac{1}{30}(1,5,0,1,1,3,0,2,2,0,0,1,2,0,1,1,0,2,1,2,0,0,1,4,0,0)$$

By repeating this process, one gets $\vec{v}_g$ for $0 \leq g \leq 25$ as listed in the next page. Notice that if we continue this process, we will get $\vec{v}_{26} = \vec{v}_0$.

Now for the substring $(\mathbf{y}_1)$, we will have:

$$\vec{v}_0 = \frac{1}{30}(0, 0, 1, 5, 0, 1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4)$$

$$\vec{v}_1 = \frac{1}{30}(0, 1, 5, 0, 1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0)$$

$$\vec{v}_2 = \frac{1}{30}(1, 5, 0, 1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0)$$

$$\vec{v}_3 = \frac{1}{30}(5, 0, 1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1)$$

$$\vec{v}_4 = \frac{1}{30}(0, 1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1, 5)$$

$$\vec{v}_5 = \frac{1}{30}(1, 1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1, 5, 0)$$

$$\vec{v}_6 = \frac{1}{30}(1, 3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1, 5, 0, 1)$$

$$\vec{v}_7 = \frac{1}{30}(3, 0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1, 5, 0, 1, 1)$$

$$\vec{v}_8 = \frac{1}{30}(0, 2, 2, 0, 0, 1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 0, 1, 4, 0, 0, 1, 5, 0, 1, 1, 3)$$

and so on.

Recall that

$$\vec{p} = (p_0, p_1, \cdots, p_{25}) = (0.082, 0.015, 0.028, 0.043, \cdots, 0.001) \in \mathbb{R}^{26}$$

where p_i's are as given in the frequency table shown in alphabetfrequency.pdf.

Now if we compute the dot product

$$M_0 = \vec{p} \cdot \vec{v}_0 = \frac{1}{30}\left[0.082 \times 0 + 0.015 \times 0 + 0.028 \times 1 + 0.044 \times 5 + \cdots\right]$$

which is approximately 0.0341. I multiplied it by 100 to make it look nicer and entered as the top entry of the column 1 in the table given in the slide 26 of sec2.4.pdf.

Similarly we can compute the dot product $M_1 = \vec{p} \cdot \vec{v}_1$ to get the next entry of column 1 and so on.

More generally one computes the dot product $M_g = \vec{p} \cdot \vec{v}_g$ for $0 \leq g \leq 25$ to get all 26 entries in the column 1.

Now repeat this whole process with the substring $(\mathbf{y}_2)$ to get all the entries in column 2 and so on.

One needs to be careful that the length N is not necessarily constant for all the substrings. For the substrings given in our example, $N = 30$ for the first three substrings where as for the fourth substring we have $N = 28$.