PnS
Lab 3-1

Q.1.

n	1	2	3	4	5	6	7
$P(n)$	k	$2k$	$3k$	k^2	k^2+k	$2k^2$	$4k^2$

a. $\Sigma P(n) = 1$

$\therefore k + 2k + 3k + k^2 + k^2 + k + 2k^2 + 4k^2 = 1$

$7k + 8k^2 = 1$

$8k^2 + 7k - 1 = 0$

$8k^2 + 8k - k - 1 = 0$

$8k(k+1) - 1(k+1) = 0$

$(8k-1)(k+1) = 0$

$k = \dfrac{1}{8}$ or -1 ↖ neglected $\because P(n) \neq$ negative

$\therefore k = \dfrac{1}{8} = 0.125$

b. $P(n < 5) = P(n=1) + P(n=2) + P(n=3) + P(n=4)$

$= k + 2k + 3k + 4k^2$

$= \dfrac{6}{8} + \dfrac{1}{64} = \dfrac{49}{64} = 0.7656$

c. $P(1 \le n \le 5) = P(n < 5) + P(n=5)$

$= \dfrac{49}{64} + \dfrac{1}{64} + \dfrac{1}{8}$

$= \dfrac{49+1+8}{64} = \dfrac{58}{64} = 0.9062$

0.2.

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	$3k$

a. $\Sigma P(x) = 1$

$$\therefore 0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$
$$0.6 + 6k = 1$$
$$6k = 0.4$$
$$k = \frac{0.4}{6} = \frac{1}{15}$$
$$\therefore k = \frac{1}{15} = 0.067$$

b. $P(x < 2) = 1 - (P(x=2) + P(x=3))$
$$= 1 - \left(0.3 + \frac{3}{15}\right)$$
$$= 1 - \left(\frac{3}{10} + \frac{1}{5}\right)$$
$$= 1 - \left(\frac{5}{10}\right) = \frac{5}{10} = 0.5$$

c. CDF

x	$P(x)$	CDF
-2	0.1	0.1
-1	0.067	0.167
0	0.2	0.367
1	0.133	0.5
2	0.3	0.8
3	0.2	1

Q.3

x	-2	-1	0	1	2
$P(x)$	$1/5$	$1/5$	$2/5$	$2/15$	$1/15$

$$V = x^2 + 1$$

→

x	-2	-1	0	1	2
$P(x)$	$1/5$	$1/5$	$2/5$	$2/15$	$1/15$
V	5	2	1	2	5

V	5	2	1
$P(V)$	$4/15$	$5/15$	$6/15$

Q.4.

x	-3	-2	-1	0	1	2
$P(x)$	0.05	0.1	0.2	0.3	0.2	0.15

→

x	$P(x)$	$x\,P(x)$	$x^2\,P(x)$
-3	0.05	-0.15	0.45
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.3	0	$0.$
1	0.2	0.2	0.2
2	0.15	0.3	0.6

$$E(x) = \sum x\,P(x) = \underline{-0.05}$$

$$E(x^2) = \sum x^2\,P(x) = 1.85$$

$$Variance = E(x^2) - (E(x))^2 = 1.85 - (-0.05)^2$$
$$= 1.85 - 0.0025$$
$$= \underline{1.8475}$$

Q.5. Urn: 7W, 3R
n: white balls drawn
2 balls drawn together

→ | n | 0 | 1 | 2 |
|---|---|---|---|
| $P(n)$ | ${}^3C_2/{}^{10}C_2$ | ${}^7C_1\,{}^3C_1/{}^{10}C_2$ | ${}^7C_2/{}^{10}C_2$ |
| | $= 3/45$ | $= 21/45$ | $= 21/45$ |
| | $= 1/15$ | $= 7/15$ | $= 7/15$ |

n	$P(n)$	$n\,P(n)$	CDF
0	1/15	0	1/15
1	7/15	7/15	8/15
2	7/15	14/15	15/15

$$E(n) = \sum n\,P(n) = 21/15 = \underline{1.4}$$

Expectation $= 1.4 \approx 1$ white ball