PROBABILITY & STATISTICS – LAB 3-1

B.Tech. Computer Science and Engineering (Cybersecurity)

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Batch: K2/A2	Date of performance: 11/01/2021

Aim: To work with probability distribution functions

1. PDF of random variable X is:

X	1	2	3	4	5	6	7
P(X)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

Find $k, P(X < 5), P(1 \le X \le 5)$

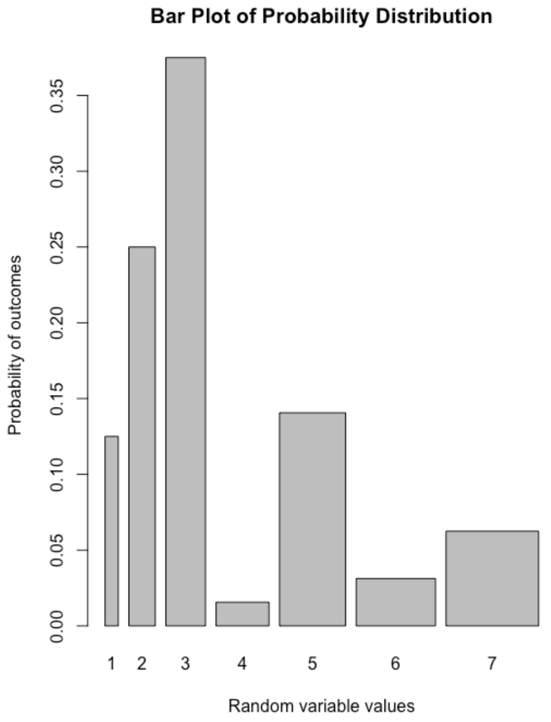
Write a R program for the above problem. Also write a R program to plot probability distribution

Code:

```
print("Question 1")
f<-function(k)(8*k^2+7*k-1)
k<-uniroot((8*k^2+7*k-1),lower=0,upper=1)$root
print("Value of k: ")
k
x<-c(1,2,3,4,5,6,7)
p<-c(k,2*k,3*k,k^2,k^2+k,2*k^2,4*k^2)
p
y<-data.frame(x,p)
y
barplot(p,x,names.arg=x,xlab="Random variable values",ylab="Probability of outcomes",main="Bar Plot of Probability Distribution")
print("P(x<5): ")
sum(p[1]+p[2]+p[3]+p[4])
print("P(1<=x<=5): ")
sum(p[1]+p[2]+p[3]+p[4]+p[5])
```

Output:

```
> print("Question 1")
[1] "Question 1"
> f<-function(k)(8*k^2+7*k-1)
> k<-uniroot((8*k^2+7*k-1),lower=0,upper=1)$root
> print("Value of k: ")
[1] "Value of k: "
> k
[1] 0.1249938
> x<-c(1,2,3,4,5,6,7)
> p<-c(k,2*k,3*k,k^2,k^2+k,2*k^2,4*k^2)
> p
[1] 0.12499385 0.24998769 0.37498154 0.01562346 0.14061731 0.03124692 0.06249385
> y<-data.frame(x,p)
> y
  x          p
1 1 0.12499385
2 2 0.24998769
3 3 0.37498154
4 4 0.01562346
5 5 0.14061731
6 6 0.03124692
7 7 0.06249385
> barplot(p,x,names.arg=x,xlab="Random variable values",ylab="Probability of outcomes",main="Bar Plot of Probability Distribution")
> print("P(x<5): ")
[1] "P(x<5): "
> sum(p[1]+p[2]+p[3]+p[4])
[1] 0.7655865
> print("P(1<=x<=5): ")
[1] "P(1<=x<=5): "
> sum(p[1]+p[2]+p[3]+p[4]+p[5])
[1] 0.9062038
```



2. A random variable X has the following pdf

X	-2	-1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	3k

Find k, p(X <2), c.d.f.

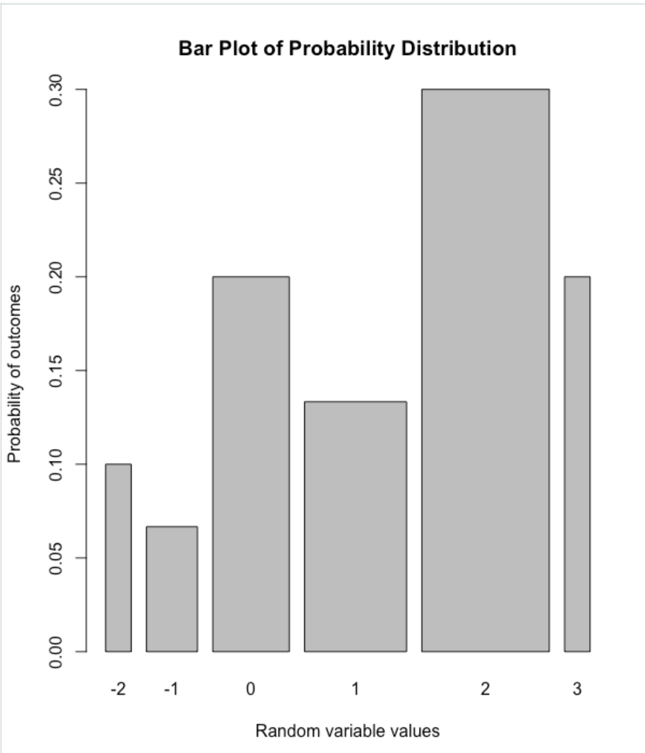
Write a R program for the above problem. Also write a R program to plot cumulative distribution function.

Code:

```
print("Question 2")
f<-function(k)(6*k-0.4)
k<-uniroot(f,lower=0,upper=1)$root
print("Value of k: ")
k
x<-c(-2,-1,0,1,2,3)
p<-c(0.1,k,0.2,2*k,0.3,3*k)
p
y<-data.frame(x,p)
y
x_bar<-c(1,2,3,4,5)
barplot(p,x_bar,names.arg=x,xlab="Random variable values", ylab="Probability of outcomes",
        main="Bar Plot of Probability Distribution")
print("P(x<2): ")
sum(p[1]+p[2]+p[3]+p[4])
cum_probability<-cumsum(p)
cframe<-data.frame(x,cum_probability)
print("CDF:")
cframe
```

Output:

```
> print("Question 2")
[1] "Question 2"
> f<-function(k)(6*k-0.4)
> k<-uniroot(f,lower=0,upper=1)$root
> print("Value of k: ")
[1] "Value of k: "
> k
[1] 0.06666667
> x<-c(-2,-1,0,1,2,3)
> p<-c(0.1,k,0.2,2*k,0.3,3*k)
> p
[1] 0.10000000 0.06666667 0.20000000 0.13333333 0.30000000 0.20000000
> y<-data.frame(x,p)
> y
   x          p
1 -2 0.10000000
2 -1 0.06666667
3  0 0.20000000
4  1 0.13333333
5  2 0.30000000
6  3 0.20000000
> x_bar<-c(1,2,3,4,5)
> barplot(p,x_bar,names.arg=x,xlab="Random variable values", ylab="Probability of outcomes",
+         main="Bar Plot of Probability Distribution")
> print("P(x<2): ")
[1] "P(x<2): "
> sum(p[1]+p[2]+p[3]+p[4])
[1] 0.5
> cum_probability<-cumsum(p)
> cframe<-data.frame(x,cum_probability)
> print("CDF:")
[1] "CDF:"
> cframe
   x cum_probability
1 -2       0.1000000
2 -1       0.1666667
3  0       0.3666667
4  1       0.5000000
5  2       0.8000000
6  3       1.0000000
```



3. A RV X has the following probability distribution:

X	-2	-1	0	1	2
P(X=x)	1/5	1/5	2/5	2/15	1/15

Find the probability distribution of $V = X^2 + 1$.

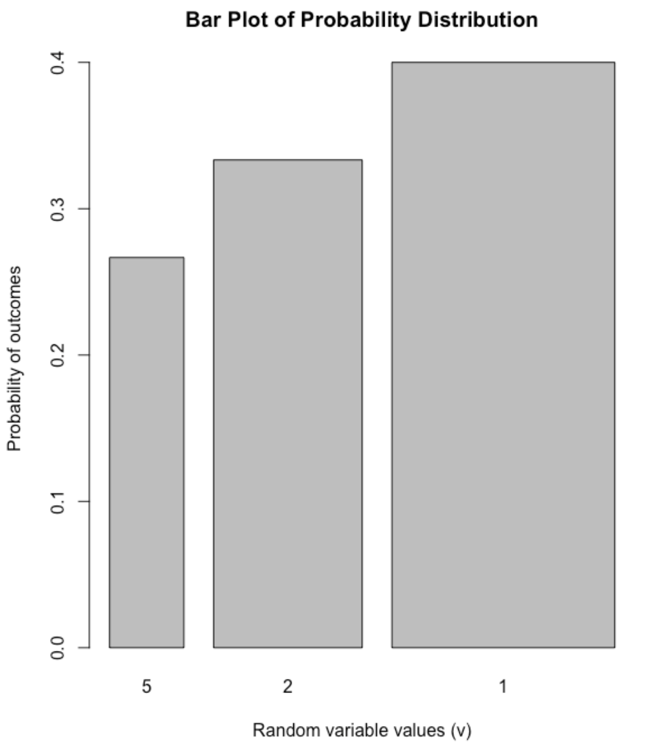
Write a R program for the above problem.

Code:

```
print("Question 3")
x<-c(-2,-1,0,1,2)
p_x<-c(1/5,1/5,2/5,2/15,1/15)
v<-(x^2+1)
p_v<-double()
final_v<-double()
seq<-1:length(x)
counter<-numeric(length(x))
#loop logic to add up probabilities of same values
for (i in seq) {
  for (j in seq) {
    if ((v[i]==v[j])&&(i!=j)&&(counter[j]==0)&&(counter[i]==0)){
      p_v<-c(p_v,(p_x[i]+p_x[j]))
      final_v<-c(final_v,v[i])
      counter[j]<-1
      counter[i]<-1
    }
  }
}
#loop logic to include probabilities of distinct values
for (k in seq) {
  for (l in seq) {
    if ((v[k]==v[l])&&(k==l)&&(counter[k]==0)&&(counter[l]==0))
      {
      p_v<-c(p_v,p_x[k])
      final_v<-c(final_v,v[k])
    }
  }
}
print("Probability distribution of v")
data.frame(final_v,p_v)
v_bar<-c(1,2,3)
barplot(p_v,v_bar,names.arg=final_v,xlab="Random variable values (v)", ylab="Probability of outcomes",
         main="Bar Plot of Probability Distribution")
```

Output:

```
> print("Question 3")
[1] "Question 3"
> x<-c(-2,-1,0,1,2)
> p_x<-c(1/5,1/5,2/5,2/15,1/15)
> v<-(x^2+1)
> p_v<-double()
> final_v<-double()
> seq<-1:length(x)
> counter<-numeric(length(x))
> #loop logic to add up probabilities of same values
> for (i in seq) {
+   for (j in seq) {
+     if ((v[i]==v[j])&&(i!=j)&&(counter[j]==0)&&(counter[i]==0)){
+       p_v<-c(p_v,(p_x[i]+p_x[j]))
+       final_v<-c(final_v,v[i])
+       counter[j]<-1
+       counter[i]<-1
+     }
+   }
+ }
> #loop logic to include probabilities of distinct values
> for (k in seq) {
+   for (l in seq) {
+     if ((v[k]==v[l])&&(k==l)&&(counter[k]==0)&&(counter[l]==0))
+       {
+       p_v<-c(p_v,p_x[k])
+       final_v<-c(final_v,v[k])
+     }
+   }
+ }
> print("Probability distribution of v")
[1] "Probability distribution of v"
> data.frame(final_v,p_v)
  final_v       p_v
1       5 0.2666667
2       2 0.3333333
3       1 0.4000000
> v_bar<-c(1,2,3)
> barplot(p_v,v_bar,names.arg=final_v,xlab="Random variable values (v)", ylab="Probability of outcomes",
+         main="Bar Plot of Probability Distribution")
> |
```



4. Given the following distribution:

x	-3	-2	-1	0	1	2
P(X =x)	0.05	0.1	0.2	0.3	0.2	0.15

Find Mean and Variance.

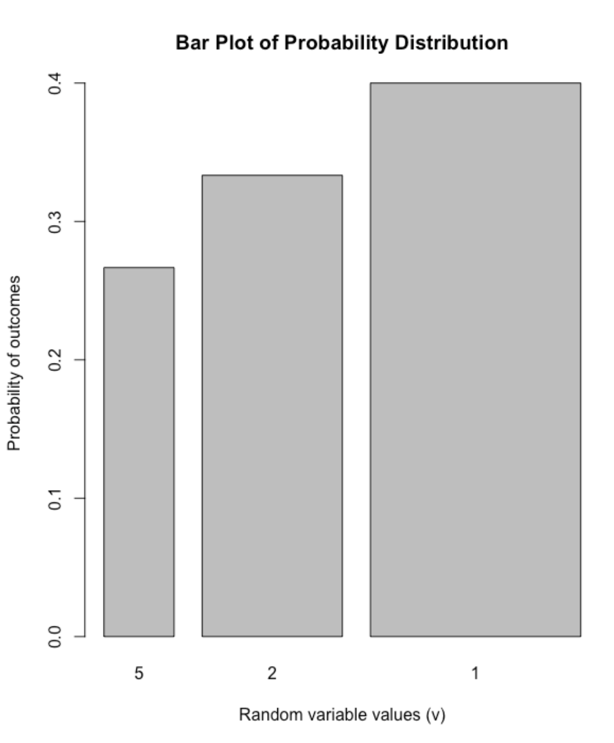
Write a R program for the above problem.

Code:

```
print("Question 4")
x<-c(-3,-2,-1,0,1,2)
p<-c(0.05,0.1,0.2,0.3,0.2,0.15)
x_Px<-x*p
x2<-x*x
x2_Px<-x2*p
data.frame(x,p,x_Px,x2_Px)
E_x<-sum(x_Px)
E_x2<-sum(x2Px)
Var=E_x2-(E_x^2)
print("Mean:")
E_x
print("Variance:")
Var
```

Output:

```
> print("Question 4")
[1] "Question 4"
> x<-c(-3,-2,-1,0,1,2)
> p<-c(0.05,0.1,0.2,0.3,0.2,0.15)
> x_Px<-x*p
> x2<-x*x
> x2_Px<-x2*p
> data.frame(x,p,x_Px,x2_Px)
   x    p  x_Px x2_Px
1 -3 0.05 -0.15  0.45
2 -2 0.10 -0.20  0.40
3 -1 0.20 -0.20  0.20
4  0 0.30  0.00  0.00
5  1 0.20  0.20  0.20
6  2 0.15  0.30  0.60
> E_x<-sum(x_Px)
> E_x2<-sum(x2Px)
> Var=E_x2-(E_x^2)
> print("Mean:")
[1] "Mean:"
> E_x
[1] -0.05
> print("Variance:")
[1] "Variance:"
> Var
[1] 1.8475
> -
```



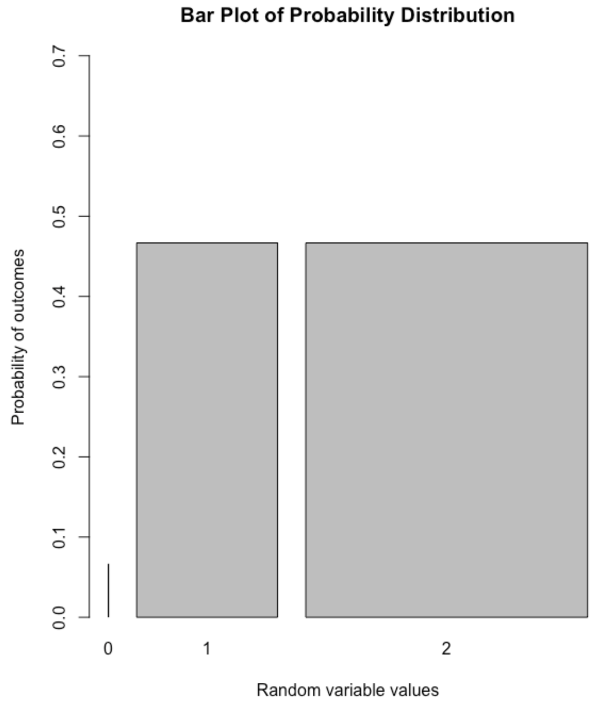
5. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the expected number of white balls drawn
Write a R program for above problem. Also write a program for to plot probability distribution and cumulative probability distribution.

Code:

```
print("Question 5")
x<-c(0,1,2)
p<-c(1/15,7/15,7/15)
x_Px<-x*p
data.frame(x,p,x_Px)
E_x<-sum(x_Px)
print("The Expectation:")
E_x
print("or")
as.integer(E_x)
barplot(p,x,names.arg=x,ylim=c(0,.70),xlab="Random variable values", ylab="Probability of outcomes",
        main="Bar Plot of Probability Distribution")
```

Output:

```
> print("Question 5")
[1] "Question 5"
> x<-c(0,1,2)
> p<-c(1/15,7/15,7/15)
> x_Px<-x*p
> data.frame(x,p,x_Px)
  x          p      x_Px
1 0 0.06666667 0.0000000
2 1 0.46666667 0.4666667
3 2 0.46666667 0.9333333
> E_x<-sum(x_Px)
> print("The Expectation:")
[1] "The Expectation:"
> E_x
[1] 1.4
> print("or")
[1] "or"
> as.integer(E_x)
[1] 1
> barplot(p,x,names.arg=x,ylim=c(0,.70),xlab="Random variable values", ylab="Probability of outcomes",
+         main="Bar Plot of Probability Distribution")
> 
```



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PnS
Lab 3-1

Q.1.

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$3k$	k^2	k^2+k	$2k^2$	$4k^2$

a. $\Sigma P(x) = 1$

$\therefore k + 2k + 3k + k^2 + k^2 + k + 2k^2 + 4k^2 = 1$

$7k + 8k^2 = 1$

$8k^2 + 7k - 1 = 0$

$8k^2 + 8k - k - 1 = 0$

$8k(k+1) - 1(k+1) = 0$

$(8k-1)(k+1) = 0$

$k = \frac{1}{8}$ or -1

neglected $\because P(x) \neq$ negative

$\therefore k = \frac{1}{8} = 0.125$

b. $P(x<5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= k + 2k + 3k + 4k^2$

$= \frac{6}{8} + \frac{1}{64} = \frac{49}{64} = 0.7656$

c. $P(1 \leq x \leq 5) = P(x<5) + P(x=5)$

$= \frac{49}{64} + \frac{1}{64} + \frac{1}{8}$

$= \frac{49+1+8}{64} = \frac{58}{64} = 0.9062$

Q.2.

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	$3k$

a. $\Sigma P(x) = 1$

$\therefore 0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$

$0.6 + 6k = 1$

$6k = 0.4$

$k = \frac{0.4}{6} = \frac{1}{15}$

$\therefore k = \frac{1}{15} = 0.067$

b. $P(x<2) = 1 - (P(x=2) + P(x=3))$

$= 1 - \left(0.3 + \frac{3}{15}\right)$

$= 1 - \left(\frac{3}{10} + \frac{1}{5}\right)$

$= 1 - \left(\frac{5}{10}\right) = \frac{5}{10} = 0.5$

c. CDF

x	$P(x)$	CDF
-2	0.1	0.1
-1	0.067	0.167
0	0.2	0.367
1	0.133	0.5
2	0.3	0.8
3	0.2	1

Q.3

x	-2	-1	0	1	2
$P(x)$	$1/5$	$1/5$	$2/5$	$2/15$	$1/15$

$V = x^2 + 1$

→

x	-2	-1	0	1	2
$P(x)$	$1/5$	$1/5$	$2/5$	$2/15$	$1/15$
V	5	2	1	2	5

V	5	2	1
$P(V)$	$4/15$	$5/15$	$6/15$

Q.4.

x	-3	-2	-1	0	1	2
$P(x)$	0.05	0.1	0.2	0.3	0.2	0.15

→

x	$P(x)$	$xP(x)$	$x^2 P(x)$
-3	0.05	-0.15	0.45
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.3	0	0
1	0.2	0.2	0.2
2	0.15	0.3	0.6

$$E(x) = \sum x P(x) = \underline{-0.05}$$

$$E(x^2) = \sum x^2 P(x) = 1.85$$

$$\text{Variance} = E(x^2) - (E(x))^2 = 1.85 - (-0.05)^2$$
$$= 1.85 - 0.0025$$
$$= \underline{1.8475}$$

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Q.5. Urn: 7W, 3R
x: white balls drawn
2 balls drawn together

x	0	1	2
$P(x)$	${}^3C_2/{}^{10}C_2$	${}^7C_1\,{}^3C_1/{}^{10}C_2$	${}^7C_2/{}^{10}C_2$
	$= 3/45$	$= 21/45$	$= 21/45$
	$= 1/15$	$= 7/15$	$= 7/15$

x	$P(x)$	$x\,P(x)$	CDF
0	1/15	0	1/15
1	7/15	7/15	8/15
2	7/15	14/15	15/15

$E(x) = \sum x\,P(x) = 21/15 = \underline{1.4}$

Expectation $= 1.4 \approx 1$ white ball