Dependence of the Wind Profile Power Law on Stability for Various Locations

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Recent environmental regulations have increased the need for construction of meteorological towers at power generation facilities. Due to practical and economic considerations, tower heights are usually lower than effluent release heights. At heights where wind speed data are not available, the wind speed is usually estimated from the measured wind speed using the $\frac{1}{7}$th wind profile power law and assuming neutral stability conditions. This study examines published data for many locations and shows that the $\frac{1}{7}$th wind profile power law is often unrepresentative of actual conditions because the degree of variation of wind speed with height depends greatly on atmospheric stability. The frequency of neutral stability conditions also varies appreciably by site. These two considerations are especially important in dispersion models which extrapolate wind speed at stack height from low level wind speed data.

There is a variety of ways to express the variation of wind speed with height. Among these are the logarithmic law, Deacon's law, and the power law. Recently, the power law has been used widely in many engineering practices. In particular, the $\frac{1}{7}$th power law is often applied when the wind speed at one level is to be extrapolated from the wind speed at another level. The general power law may be expressed as:

$$V_1/V_2 = (Z/_1/Z_2)^p \qquad (1)$$

Where:

V_1 = Wind speed at height Z_1
V_2 = Wind speed at height Z_2
p = Exponential parameter ($0 \leq p \leq 1$)

$$p = \frac{\ln(V_1/V_2)}{\ln(Z_1/Z_2)}$$

The wind profile is dependent on the depth of the layer at which the wind speed is being extrapolated, surface roughness length (Z_0), and atmospheric stability. This study examines published field data measured from 10–90 meters and atmospheric stabilities that are based on the vertical temperature gradient criteria. The uniformity with which the data were measured and analyzed allows the direct comparison of numerous sites.

The purpose of this paper is to show that the $\frac{1}{7}$th power law is often unrepresentative of actual on-site conditions. Local terrain may give extreme wind speed and direction variations, even when the local wind structure is free from disturbances such as fronts, thunderstorms and squall lines. Also, the terrain surrounding the anemometers quite frequently becomes heavily built up and internal boundary layers may develop in which the velocity profiles are poorly described by the $\frac{1}{7}$th power law. It is, therefore, advantageous for the practicing engineer or meteorologist to obtain the applicable local meteorological record whenever possible and develop a site-specific power law relationship.

Data Sources

Meteorological data are increasingly being collected in diffusion and air pollution studies that use instrumented towers. In this study, tower data collected by uniform methods from several nuclear power plant sites are analyzed. These methods are defined by the Nuclear Regulatory Commission (NRC) in Regulatory Guide 1.23.[1] As applicable to this study, the guidelines require that wind speed be measured at the 10 and 60 m levels to a time averaged accuracy of ±0.5 mile per hour and anemometer starting speed of less than 1 mph. Temperature differences (ΔT) between the 10 and 60 m levels are measured to an accuracy of ±0.1°C.

Table I lists the Pasquill stability classes used in this study. They are based on the vertical temperature gradient criteria given in Regulatory Guide 1.23. Although there are several other methods to compute stability,[2] the NRC requires that stability should be based on direct measurement of temperature differences between two levels.

Table I. Pasquill stability classes in terms of vertical temperature differences (ΔT).

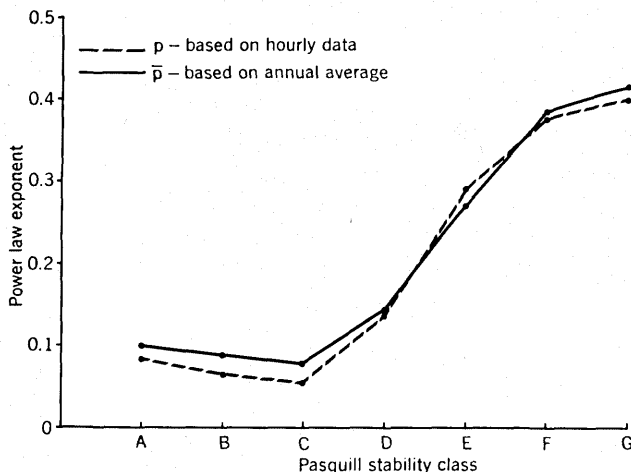| Stability class	Range of vertical temperature gradient		Turbulence
	°F/1000 ft	°C/100 m	
A = Very unstable	ΔT < −10.4	ΔT < −1.9	High
B = Moderately unstable	−10.4 ⩽ ΔT < −9.3	−1.9 ⩽ ΔT < −1.7	
C = Slightly unstable	−9.3 ⩽ ΔT < −8.2	−1.7 ⩽ ΔT < −1.5	
D = Neutral	−8.2 ⩽ ΔT < −2.7	−1.5 ⩽ ΔT < −0.5	Moderate
E = Slightly stable	−2.7 ⩽ ΔT < 8.2	−0.5 ⩽ ΔT < 1.5	Low
F = Moderately stable	8.2 ⩽ ΔT < 22.0	1.5 ⩽ ΔT < 4.0	
G = Very stable	22.0 ⩽ ΔT	4.0 ⩽ ΔT	

Figure 1. Comparison of power law exponent by using hourly data (p) vs. annual average wind speed data $(\bar{p})$—10 to 60 mΔT, Missouri.

Data summaries as well as program documentation are published in Safety Analysis Reports (SAR). The NRC requires an applicant for a nuclear power plant construction permit to submit a SAR providing sufficient information to determine whether the nuclear power plant can be constructed and operated without undue risk to the health and safety of the public.

The Results and their Interpretation

Figures 1 and 2 compare at two locations the variation with stability class of the power law exponents computed by two methods. In one method the power law exponent (p) was calculated for each hourly average wind speed at each stability category and then averaged for the year. In the other, power law exponent $(\bar{p})$ was computed by averaging all the hourly

average wind speeds for the year at each stability class and then using the power law to find the exponent $(\bar{p})$. The values compare very well.

It is believed that p is more nearly accurate than $\bar{p}$ and should be used in actual cases. For the purposes of this study, however, $\bar{p}$ derived from annual wind speed data was used because these data are available in summaries published in the SAR while the hourly data are not.

Available SAR publications that contained annual average wind speeds at two levels were used to derive the power law exponent $(\bar{p})$ by stability class.

Because of the variability between the SAR in the amount of data presented, Tables II, III and IV of this paper list only the sites at which the applicable data were presented.

The derived $\bar{p}$ values are presented in Table II and are plotted in Figure 3. In general, it is apparent from Figure 3 that the power law exponent increases as the layer becomes
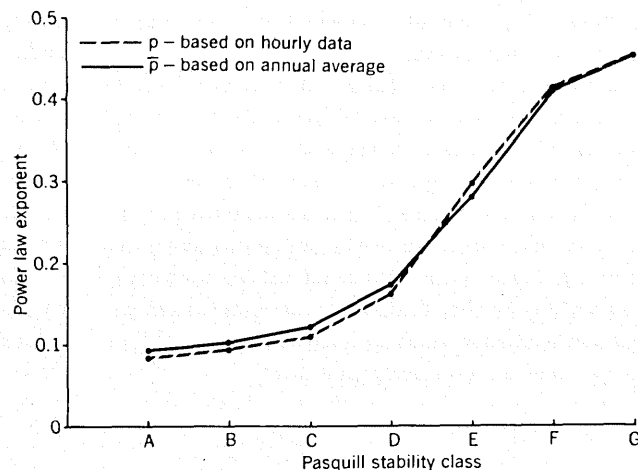

Figure 2. Comparison of power law exponent by using hourly data (p) vs. annual average wind speed data $(\bar{p})$—10 to 60 mΔT, Kansas.

Table II. Power law exponents for various sites computed from annual average wind speeds at two levels.

Stability class	Missouri[a] 1973−74	Missouri[a] 1974−75	Kansas[a] 1973−74	Kansas[a] 1974−75	Iowa[a] 1973−74	Texas[a] 1973−74	Michigan[a] 1975−76	Missouri[b] 1973−74
A	0.103	0.099	0.124	0.091	0.104	0.120	0.109	0.111
B	0.079	0.092	0.145	0.103	0.101	0.123	0.085	0.119
C	0.082	0.080	0.152	0.122	0.114	0.128	0.078	0.104
D	0.115	0.144[c]	0.199	0.172	0.188	0.174	0.116	0.136
E	0.271	0.273	0.341	0.282	0.313	0.330	0.261	0.272
F	0.423	0.385	0.480	0.412	0.466	0.562	0.425	0.424
G	0.504	0.417	0.506	0.452	0.444	0.624	0.516	0.447
Terrain	Rolling	Rolling	Rolling	Rolling	Rolling	Rolling	Hilly	Rolling

[a] Stability class based on a ΔT of 10 to 60 m.
[b] Stability class based on a ΔT of 10 to 90 m.
[c] For 1/7 power law p = 0.143.

Table III. Annual frequencies of each stability class for several sites (percent).

	Site, Number of months of record									
Stability class	Missouri 24 Mo.	Kansas 24 Mo.	Iowa 12 Mo.	New York 12 Mo.	Texas 12 Mo.	Michigan 12 Mo.	Ohio 24 Mo.	Indiana 12 Mo.	Illinois 12 Mo.[a]	Illinois 12 Mo.[a]
A	4.1	15.1	6.1	7.7	2.4	2.0	8.4	20.2	16.2	8.9
B	3.9	5.7	4.4	3.6	3.1	3.9	4.0	7.8	4.6	5.1
C	5.7	6.4	5.9	5.5	7.2	8.3	6.1	8.9	5.9	6.0
D	36.3	28.0	41.1	45.0	34.9	54.4	38.8	32.3	25.7	24.3
E	29.8	22.7	28.0	26.1	29.1	20.9	31.1	20.2	21.3	22.1
F	14.6	13.3	9.4	8.5	14.1	7.3	8.7	7.5	19.3	24.7
G	5.6	8.8	5.0	3.5	8.9	3.2	2.8	3.3	6.5	8.9

[a] Two different sites.

more stable. The $\frac{1}{7}$th power law ($p = 0.143$), which is often used as being representative of neutral conditions, is closest to $(\bar{p})$ for stability Class D. It is also evident that $(\bar{p})$ for stability Class D varies from site to site. However, for all except one site, the variation with location is not appreciable.

The $\frac{1}{7}$th power law, therefore, appears to be a good approximation to use only during neutral conditions since it is not valid for other conditions. In fact, the exponent almost triples at very stable classes.

In practical considerations, this variability cannot be ignored. For example, if the wind speed at the 10 m level is 5 m·sec^{-1}, then the 60 m level wind speed using the exponents in the first column of Table II is 6.01, 5.76, 5.79, 6.14, 8.13, 10.66, 12.34 m·sec^{-1} for the seven stability classes. Using the

$\frac{1}{7}$th power law, it is equal to 6.45 m·sec^{-1}. For this example, the $\frac{1}{7}$th power law yields almost one half of the actual wind speed for the G stability class.

To obtain a good idea of how often the neutral stability classes occur in relation to other classes, available data were compared in Table III. The percentage of occurrence of the neutral stability Class D varies from site to site and ranges from 24 to 45% of the hourly observations. In other words, between 55 and 76% of the time, the $\frac{1}{7}$th power law exponent could be unrepresentative of true on-site wind conditions.

In some engineering design applications, high average wind speeds are of interest. It is also during conditions of high wind speeds that the atmosphere is assumed to be well mixed and neutral conditions are most likely to occur. In Table IV, the

Table IV. Annual frequencies of high hourly average wind speeds at two levels by stability class for various sites.

| Site:	Missouri		Kansas		Texas		Michigan		Illinois	
Height:	10 m	60 m	10 m	60 m	10 m	60 m	10 m	60 m	10 m	60 m
Wind speed:[a]	>7.5 ms^{-1}		>7.5 ms^{-1}		>7.5 ms^{-1}		>8.0 ms^{-1}		>5.6 ms^{-1}	
Length of record:	24 Mo.		24 Mo.		12 Mo.		12 Mo.		9 Mo.	
Stability class										
A	0.70	1.32	3.94	6.68	0.15	0.45	0.10	0.34	1.45	3.33
B	0.58	1.18	1.32	2.52	0.26	0.85	0.41	0.84	1.07	1.96
C	0.46	1.09	1.43	2.97	0.49	1.50	0.84	1.65	1.49	2.46
D	4.37	12.86	5.66	12.70	1.96	9.89	3.20	8.28	13.57	21.33
E	1.99	9.72	1.37	10.60	0.18	5.11	0.53	2.41	6.85	7.68
F	0.14	2.71	0.02	4.75	0.01	0.76	0.01	0.29	0.03	5.56
G	0.07	0.80	0	2.72	0	1.04	0	0.02	0.01	2.47

[a] Threshold speed applies to both instrument levels.
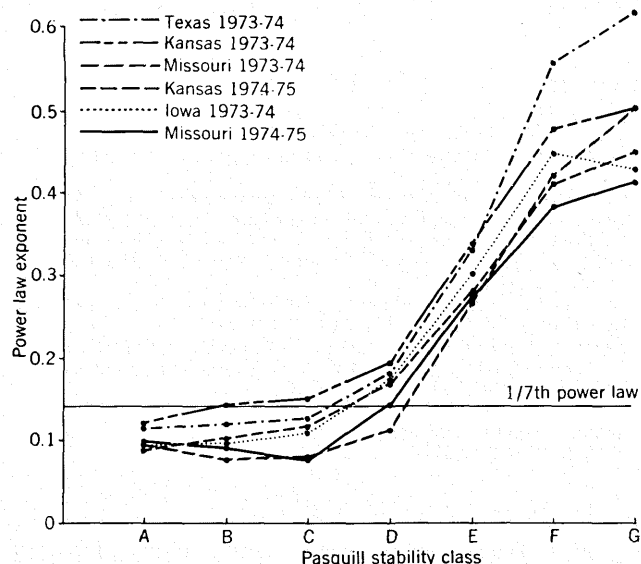
Figure 3. Comparison of site variability of power law exponent $(\bar{p})$ based on annual average wind speeds at two levels—10 to 60 mΔT.

sites are compared in terms of high wind speeds (in percent of all wind speed occurrences) and stability class. It appears from this table that a higher percentage of these wind speeds occurs in the D stability class. However, a large number occurs in all other stability categories.

Correlation with Results Found by Other Investigations

These results do not seem unreasonable when compared with other studies. For example, Moses and Bogner[3] found that in January the most frequent values of p during the daytime and nighttime hours were 0.11 or 0.14 while in July the most frequent values were 0.33 or 0.45. These p values were calculated from hourly wind speed observations at the 19 and 75 ft levels using Eq. 1.

DeMarrais[4] used meteorological data from the 125 m tower at the Brookhaven National Laboratory to determine the power law profile. The power law exponents varied from 0.1 to 0.3 during the day when superadiabatic and neutral lapse rates prevail and from 0.2 to 0.8 during nighttime conditions when stable and isothermal conditions exist. These data are limited to a relatively short period of time and apply only to that site.

In another study[5] the calculated hourly values of the power law exponent (p) were plotted as a function of the corresponding Pasquill stability category as shown in Figure 4. At this site, which is located at the western shore of Lake Erie, the average value of p decreases with increasing temperature difference. For the annual period considered, the average value of p by stability class is given in Table V. The average values of p and the percentage of occurrence of the stability classes show remarkable similarity to data presented in Tables II and III.
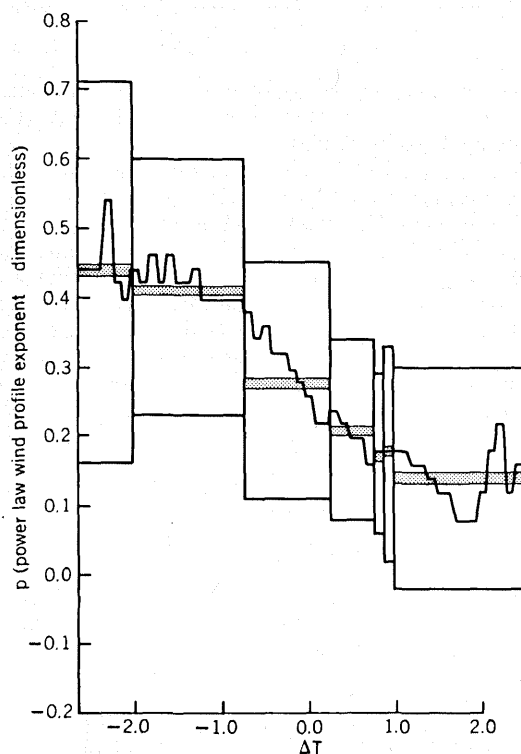

Figure 4. Hourly wind profile power law exponent as a function of temperature difference between 10 and 60 m levels. Center lines represent the mean value of p and its standard deviation within each Pasquill stability. (Data based on 12 months data, Fermi 2 Environmental Report, 1976.)

Table V. Average values of wind profile power law exponent by stability class.[a]

Pasquill stability class	Average value of exponent	Standard deviation of average	Average wind speed (mph)	Percentage of occurrence
A	0.141	0.157	8.95	9.17
B	0.176	0.154	9.94	2.08
C	0.174	0.117	10.08	2.40
D	0.209	0.131	10.04	30.29
E	0.277	0.172	8.79	40.46
F	0.414	0.186	6.82	10.31
G	0.435	0.274	5.41	5.30

[a] Data based on 12 months of data.[5]

Conclusion

It has been shown in this study that the 1/7th power law is generally a good approximation only under neutral conditions. The true power law relationship is highly variable and is dependent upon the stability class at each site. Thus, it is suggested that a site specific power law profile be developed and used when it is necessary to extrapolate wind speed data at another level.

Acknowledgments

Table VI. Wind profile power law exponents used in the climatological dispersion models.

CDM stability class	Pasquill stability class	Exponent (p)
1	A	0.1
2	B	0.15
3	C	0.20
4	D	0.25
5	E	0.25
6	F & G	0.30

Dispersion Modeling Applications

Many dispersion models such as the Climatological Dispersion Model (CDM)[6] and other models have been developed to predict concentrations of nonreactive pollutants due to emissions from area and point sources. To account for the increase of wind speed with height above anemometer height to the level of emission, a power law relationship, as in Eq. 1, is used in the computational program. The power law exponents currently used are shown in Table VI.

These models have attempted to improve accuracy by using a stability dependent wind speed profile rather than the conventionally used 1/7th power law. However, several problems remain. First, the stability classifications from which the exponents were derived are based on direct ΔT measurements while the classifications used in the CDM models generally are based on the STAR program which uses cloud cover, wind speed, etc. The frequency distributions of the two stability classifications vary appreciably. Second, these models use the exponents as constants in the computations. As has been shown earlier, these exponents differ from one site to another and may cause large errors in prediction of effluent concentrations.

References

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