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Detecting GW memory with Compact Binary Coalescences[⋆,⋆⋆]

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ABSTRACT

Gravitational waves (GWs) incident on an interferometer causes permanent distortion on the order of 10^{-21}-m, the so-called memory effect. Linear and nonlinear components exist in GW memory, the latter appearing as a non-oscillatory, cumulative signal. Current GW detectors are unable to reliably detect and isolate this low-frequency, nonlinear component which skews the numerical inferences of GW source parameters. Because this effect is cumulative, it is non-negligible, and its non-oscillatory nature distinguishes it from the rest of the waveform, making it detectable, in theory. Though previous studies have quantified and suggested improvements for the detectability of nonlinear memory, more templates and new data are available than ever before. In this project, we apply Bayesian parameter estimation to simulated compact binary coalescences (CBCs) with injected memory to determine nonlinear memory detectability.

Key words. CBCs – GW memory – low-frequency, nonlinear memory – parameter estimation

1. Introduction

Although all accelerating masses radiate GWs, CBCs – binary systems consisting of black holes and/or neutron stars – are especially interesting because they emit the most detectable GW signals and many of their properties are unknown. Indeed, the amplitude and phase of a GW signal encodes source features such as mass, angular momentum, and location. The traditional waveform sourced from a CBC GW is an oscillatory traveling wave with increasing frequency and momentary amplitude spike corresponding to the merger phase. As it propagates through space, this waveform distorts surrounding mass arrangements in an oscillating pattern, but afterwards each arrangement returns to its original geometry. However, General Relativity (GR) predicts that after a GW passes a truly free-falling arrangement of masses, a memory effect occurs in which a permanent nonzero difference in deformation is observed. Further, all GWs possess some form of gravitational-wave memory, whether linear or nonlinear.

Linear memory arises from non-oscillating masses and, thus, usually appears only in systems with hyperbolic orbits, neutrino ejection, or gamma-ray bursts. Nonlinear memory arises from the signal contribution of secondary GWs sourced by the initial GW emission. Unlike non-oscillating masses, secondary GW production occurs in many CBCs, making nonlinear memory of especially prominence. Also, nonlinear memory accumulates over time because it is hereditary – depends on the entire past motion of the source. The non-oscillating and cumulative nature of nonlinear memory should, in theory, make it easy to distinguish from a normal GW signal (Favata 2010). In practice this is not the case.

There are two main reasons why nonlinear memory is, in fact, hard to extract from a GW signal. Current detectors are attuned to high-frequency input and are thus insensitive to nonlinear memory because the detector response is generally much shorter than the time over which the memory accumulates. Also, ground-based detectors are incapable of storing the memory over an extended period of time because electrostatic forces exist between adjoining particles throughout the detector, quickly eradicating the memory effect from early portions of the signal. This often lowers the observed memory effect below the resolution of the detectors.

Even without considering memory, overall detector data is very noisy. The primary goal of GW signal analysis is to distinguish actual signals from this background noise. All phases of CBC-sourced waveforms are well-modeled using GR numerical simulations, allowing a template library to be constructed over a broad range of binary component masses and spins. Matched filtering can then be used to compare these templates with the data and determine the best fit. When nonlinear GW memory enters the picture, this same process can also be used to determine the detectability of the memory contribution by comparing the template's memory with measured memory.

From here, we discuss the theoretical background behind GWs, nonlinear memory, and parameter estimation in Section 2. In Section 3, we summarize the procedure involved in obtaining memory detectability. In Section 4, we report our results and give a subsequent analysis. In Section 5, we make our conclusions and discuss future plans for the project. Finally, Section 6 features the work plan for the project.

2. Background

2.1. Gravitational wave theory

GWs are traveling waves which propagate at the speed of light, stretching and squeezing spacetime – and, thus, surrounding

⋆ No figures have yet been placed and the reference list/citations are incomplete.
⋆⋆ *A and A* template used for now. I am definitely willing to change the format if you would like.

masses – in an oscillatory manner. The oscillation frequency is related to the source's angular momentum and the amplitude is related to the source's mass. A Michelson-Morley interferometer may be used to record these variations in strain: two arms are set perpendicular to one another, and a laser and beamsplitter are arranged at the intersection point. The laser is fired through the beamsplitter, creating two beams which each travel along the length of an arm and return after reflecting from mirrors placed at the end of each arm. Both beam paths are aligned to recombine at a photodiode located at the intersection point. At this point, the only possible phase difference between beams comes from the difference in arm length, which is carefully adjusted to produce destructive interference at the photodiode.

As a GW passes the interferometer, one leg is squeezed while the other is stretched, altering the phase difference and, thus, combined intensity of the light incident on the photodiode. This intensity information may be transferred to strain strain information which is given by Eqn. (1).

$$h_{ij}(t,\ \mathbf{r}) = \sum_{A=+,\times} e^{A}_{ij}(\hat{\mathbf{n}}) \int_{-\infty}^{+\infty} h_A(f) e^{-i2\pi f (t - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c})}\, dt \qquad (1)$$

In GR, the spacetime metric is TT gauge invariant, implying that free-falling objects are at rest in spacetime. So, if earth-based interferometers are in free-fall, Eqn. (1) is independent of $\mathbf{r}$, which may be set to 0. This yields Eqn. (2)

$$h_{ij}(t) = \sum_{A=+,\times} e^{A}_{ij}(\hat{\mathbf{n}}) h_A(t), \qquad (2)$$

which clearly expresses the total strain as a sum of two linear polarizations, plus and cross. Plus polarized GWs incident on a detector stretch and squeeze the arms directly along their length while cross polarized GWs alter each arm's length at a 45° angle. Generally, incident GWs are a combination of the two polarizations or even of nonlinear polarization.

2.2. Nonlinear Memory Theory

Not sure what to put here yet.

2.3. Bayesian Parameter Estimation

Let the hypothesis H be the statement, "nonlinear memory is present in the detector's data" and, further, let D be the detector's data. Then, $P(H \mid D)$ is the probability that nonlinear memory is present in the data given the data we have at hand, $P(D \mid H)$ is the likelihood that we will detect nonlinear memory given that nonlinear memory is, in fact, present, and $P(H)$ is the belief we have in the presence of nonlinear memory on the basis of prior information (or lack of information) alone. Bayes' Theorem relates these three quantities as shown in Eqn. (3).

$$P(H \mid D) \propto P(D \mid H)\ \times\ P(H) \qquad (3)$$

Although we would eventually like to satisfy Eqn. (3) by determining $P(H \mid D)$, our present concern is finding $P(D \mid H)$, the likelihood or memory detectability.

The first step in parameter estimation is to present H as in Eqn. (4)

$$h_{tot} = h + \lambda h_{mem}, \qquad (4)$$

where h_{tot} is the total signal, h is the non-memory portion of the signal, and h_{mem} is nonlinear memory. Then, for a template with known memory $\lambda = P(H)$ and is thus 1, but for the same template injected among noise, $\lambda = P(D \mid H)$ and can take on any value between 0 and 1. The value of this second λ is the memory detectability.

3. Procedure

The specifics of parameter estimation in determining memory detectability is covered by matched filtering. We will start by describing this process without memory and then bring in memory afterwards.

3.1. Matched filtering without memory

Data noisiness is given by the signal-to-noise ratio (SNR), which is typically quite low, and matched filtering is a process by which SNR may be increased. In matched filtering, templates are cross-correlated with observed data in frequency space to see if the resulting amplitude spikes occur at the known frequencies contained in a given template. If the frequencies match, the parameters, identity, and waveform of the template source is taken to be those of the signal source as well. However, the overwhelming presence of noise in the data makes a direct application of this process impossible. First, a filter must be matched with each template to maximize the SNR for all cross-correlations.

To accomplish this for a given template, the template signal is injected into typical noise and the result is cross-correlated with the initial template adjusted in some unknown way to maximize the SNR. The cross-correlation is given by:

$$\hat{s} \propto \int_0^T h(t) K(t)\, dt, \qquad (5)$$

where T is the observing time, $h(t)$ is the injected signal plus noise, and $K(t)$ is the adjusted template signal. The goal is to select a $K(f)$ in the frequency domain which maximizes Eqn. (5) at particular frequency values. A correctly chosen $K(f)$ is then known as the template's filter function and cross-correlated with the real data in frequency space as if it were the correct fit for the hidden signal. The extracted frequencies at which Eqn. (5) is at maximum are then the only signal frequencies: all other frequencies the data ranges over is inhabited by noise. These frequencies are removed from the data, thus increasing the data's SNR. If this template actually matches the signal, the peaks existing at each target frequency will now extend far above the noise.

3.2. Matched filtering with memory

Matched filtering can also be used to determine the detectability of the memory contribution. In this case, a template with known memory is injected into a typical noise distribution and extracted using matched filtering. The difference between the overall template signal and extracted signal determines the measured memory effect. Comparison between the known and measured values yields a given interferometer's memory detectability.

4. Analysis and Results

To be filled later.

5. Conclusion

To be filled later.

6. Work Plan

A work plan has been included in Table (1), listing weekly project goals for the Summer 2020 LIGO SURF project.[1]

References

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[1] Not yet edited because schedule is unclear to me.

Table 1. Work plan for the Summer 2020 LIGO SURF program. Schedule is broken down by week and target progress.

Date	Progress
May 15, 2020	1. Deadline for submitting project proposal
June 16, 2020	2. Start of LIGO SURF 2020.
July 5-11, 2020 (Week 4)	3. Submit interim report 1.
July 26- August 1, 2020 (Week 7)	3. Submit interim report 2. 4. Submit abstract.
August 19-21, 2020 (Week 10)	5. Final presentation.
September 28, 2020	6. Deadline for submitting final report.