A little book about waves

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Lesson 6: Light (part II)

Concept #1: Huygen's principle

Newton was convinced that light consisted of fast-moving, small particles and because Newton was a very influential thinker[1], most other scientists chose to believe him. There were however, a few people who pointed out that the properties of light could be explained by assuming it was a wave. The Dutch scientist Christiaan Huygens was among those people, and in 1690 he published the first mathematical wave theory of light. In this work he successfully explained the laws of reflection and refraction by introducing what is known as **Huygen's principle** and assuming that it holds true for light: *Every point on a wavefront is a source of a secondary wave that spreads out in all directions with a speed equal to the speed of the original wave*. Other wavefronts are found by constructing surface tangents to the secondary waves as illustrated in figure 1 and shown in this animation.[2]

[1] He had, after all, been right about motion and gravity. Not a small feat!

[2] Mechanical waves already satisfy this property and it's is easy to experimentally verify this phenomenon, for example for surface waves on water of different depths: As the depth gets more shallow, the wave speed decreases and the wavefronts turn towards the normal (which is the reason waves often come in parallel to the shore).

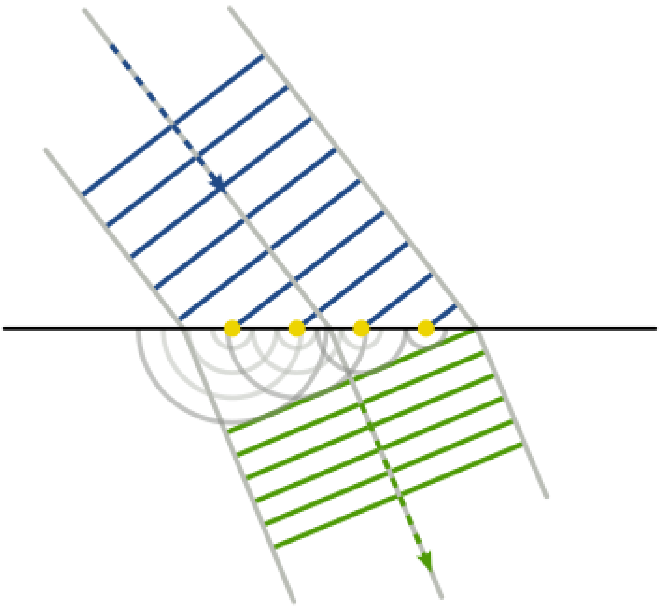


Figure 1: The law of refraction can be explained using Huygen's wave theory of light.

We won't cover the details of Huygen's wave theory since it is not part of the IB DP syllabus anymore, but know that the bending of waves (including light) can be explained very well in this model: The wavefronts travel with different speeds[3] in the two media (here slower in the second, green medium), and this decrease in speed turns the angle of the ray and decreases the wavelength. The common analogy is to imagine a wavefront as a row of soldiers marching in phase. As they march into a different medium

[3] Problem 13 contains a link to a great video explaining why light actually slows down in a transparent medium. Watch it after you have read this whole set of lesson notes.

in which they travel slower (e.g. mud), the rows will automatically turn and bunch up. One of the necessary boundary conditions in this wave explanation is that *the frequency doesn't change at the boundary*. You can visualise that in this desmos simulation (in the soldier analogy, this corresponds to the soldiers marching at the same frequency).

1. We have previously seen that the law of refraction leads to the equations

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Show that when light is assumed to be a wave, then the wavelengths can be included as follows:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Notice how speed is directly proportional to the wavelength. This is a very useful equation to keep in mind!

Concept #2: There is more to light than meets the eye!

Things started to change dramatically right at the beginning of the 1800s (it's no coincidence this period is called The Age of Enlightenment). The first discovery of light other than visible light came in 1800, when the German-Britsh astronomer William Herschel discovered **infrared light**. He was studying the temperature of different colours by moving a thermometer through light split by a prism. He noticed that the highest temperature was beyond red indicating that some sort of invisible beams – beyond the red colour – existed. The next year, Johann Ritter, working at the other end of the spectrum, noticed what he called "chemical rays" (invisible light rays that induced certain chemical reactions). These behaved similarly to visible violet light rays, but were beyond them in the spectrum. These rays were later renamed **ultraviolet light**. There was apparently more to light than met the eye!

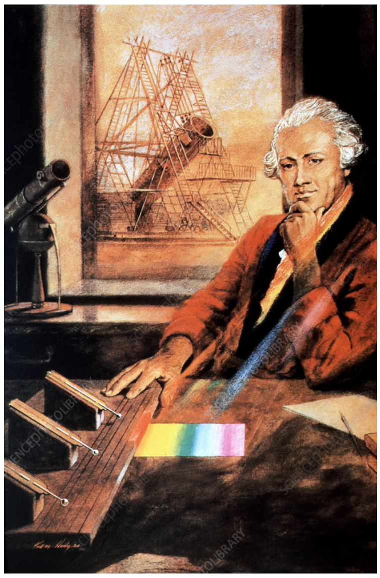


Figure 2: William Herschel and his discovery of infrared light.

In 1803 an experiment performed by the English scientist Thomas Young convinced many people that light definitely has wave-like properties. Young's famous **double-slit interference** experiment showed that light could combine in ways that was only possible if one assumed it was a wave: He shone light from a single source onto two thin vertical slits close apart and regions of constructive and destructive interference were created. There are a number of reasons why this hadn't been noticed earlier. First, light waves travel very fast (3.0×10^8 m/s in empty space), secondly, visible light waves have very small wavelengths (400 to 700 nanometers as we will soon see), and third, it's very hard to create two light sources that emit waves of *identical frequency and with a constant phase difference (= coherent)* (remember that was the condition to see an intereferece pattern). Luckily for us, we invented lasers in the 1960s and these light sources produce waves that have a single[4]

[4] More realistically, it is a very narrowly defined range of frequencies.

frequency (**monochromatic** light = "one color"), are very coherent, and stay narrow over great distances (= a "collimated" beam). With laser light it becomes relatively easy to demonstrate the interference of light and I will show you that in class. The interference pattern consists of bands of bright and dark **fringes**, which represent points of constructive interference (brightest spots) and destructive interference (darkest spots). Hence it is possible to *create darkness from light*!

1. In a class demonstration a source of red laser light is shone on two very thin slits that are separated by a distance $d = 0.25 \pm 0.01$ mm. The interference pattern is projected on a screen $D = 2.36 \pm 0.02$ m away from the slits. The interference pattern is only considered in directions that are at small angles away from the centerline. If the distance between the center of seven successive bright fringes is measured to be $s = 3.7 \pm 0.1$ cm, then what is the wavelength of this red light with uncertainty?.

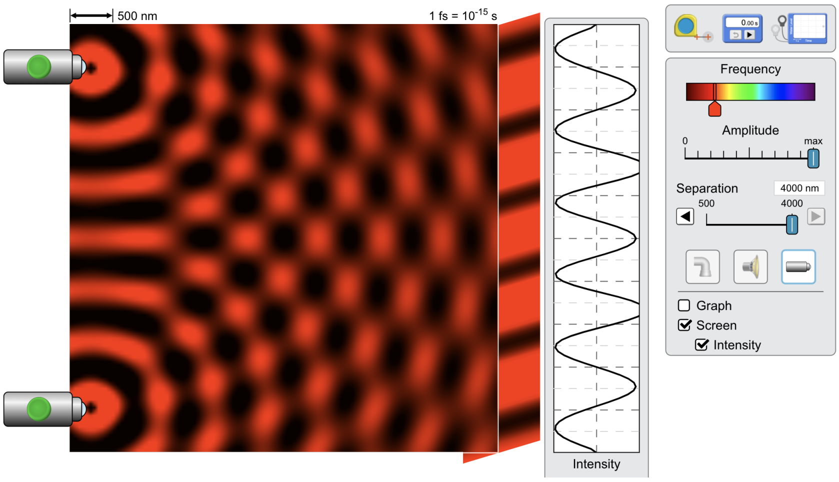
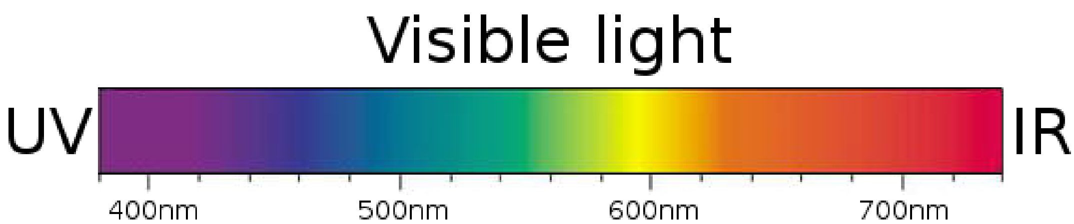


Figure 3: If you don't have any equipment available, then play around with this PhET simulation

2. Visible light turns out to have wavelengths between roughly 400 and 700 nm in empty space. Calculate the range of frequencies of these light waves.



Concept #3: The wave model could explain much more

Because of Young's experiment, the wave theory of light was taken much more seriously and supporters started looking more carefully for other phenomena to explain using this model. The French scientist Fresnel contributed a lot to this research (e.g. the Fresnel equations that accurately predict how much light is reflected and refracted when incident on a boundary) and it became clear to everyone in the early 1800s that the nature of light was indeed *best* explained by a mathematical wave theory. Many observations were explained very well by this theory, and by 1830 almost no physicists doubted the model. One particular cool prediction of the wave theory of light is **Arago's spot**, a bright spot that appears at the center of a circular object's shadow. There's a great story behind this discovery that I recommend you read in your own time.

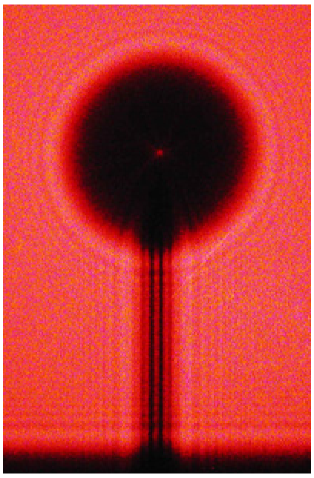


Figure 4: Arago's spot (the bright spot in the middle of the shadow) is an observation that can only be explained if you assume light has wave properties.

Concept #4: Maxwell makes the connection to E&M

As mentioned in an earlier lesson, during the 1860s James Clerk Maxwell developed four equations (Maxwell's Equations) that beautifully tie together all phenomena related to electricity and magnetism. Two of these equations predict the possibility and behaviour of certain **electromagnetic waves** consisting of a *transverse oscillation of an electric and magnetic field*. It follows from his equations[5], that *an oscillating electric field generates an oscillating magnetic field (perpendicular to it) and an oscillating magnetic field generates an oscillating electric field*, and so on ad infinitum, see figure 5. Here is a good animation of an electromagnetic wave in motion.

[5] And more importantly, it can be experimentally verified too! Heinrich Hertz was the first person to experimentally verify the existence of electromagnetic waves in 1887.

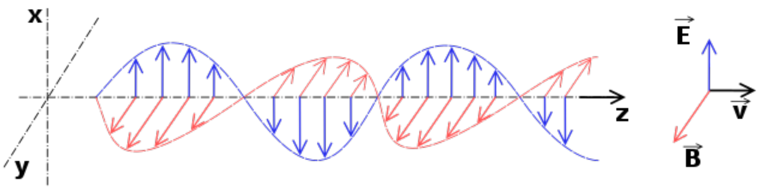


Figure 5: An electromagnetic wave is made of oscillating electric and magnetic fields that each generate the other.

One important property of electromagnetic waves is that they can exist in and travel through empty space. By analysing the speed of these waves, Maxwell realised that they travel at a speed that was equal to the known speed of light. This startling coincidence led Maxwell to draw the conclusion that *light must be such an electromagnetic wave*. His equations predicted an infinite number of frequencies of electromagnetic waves, all traveling at the speed of light and this was the first indication of the existence of the entire **electromagnetic spectrum** which we now know so well, see figure 6. Maxwell's connection between light and electromagnetism is considered one of the most amazing accomplishments in physics.

Polarised light

The electromagnetic wave shown in figure 5 is said to be **linearly polarised** because the electric field vector is *oscillating along the same direction* (the x-direction in this example). The idea of polarisation applies to all transverse waves (e.g. also mechanical string waves)

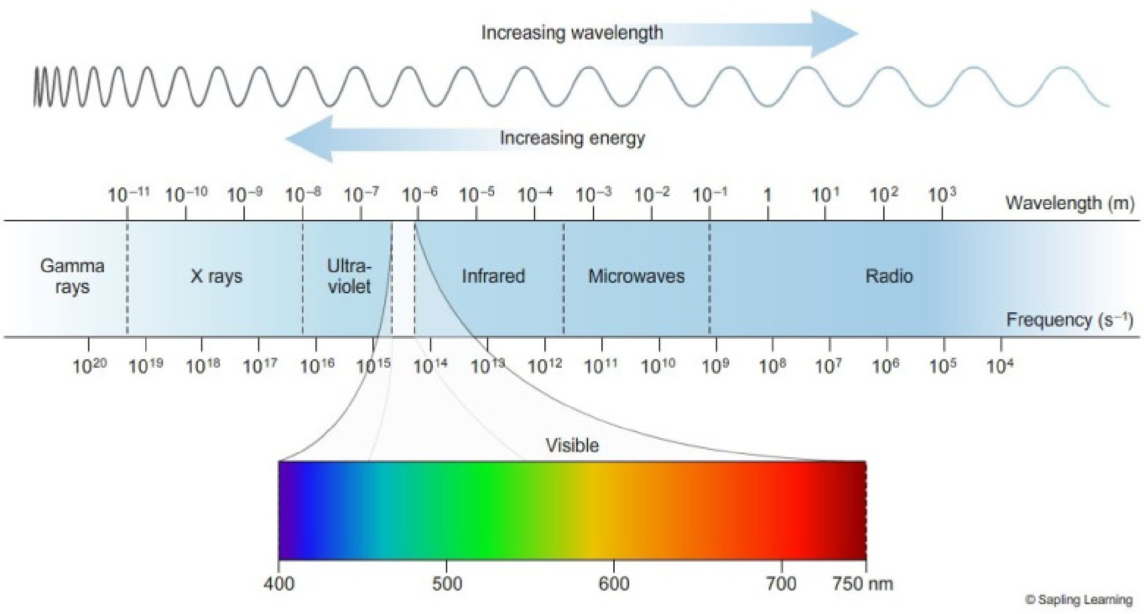


Figure 6: Electromagnetic waves of all wavelengths and corresponding frequencies are predicted by Maxwell's equations. They define what we call the **electromagnetic spectrum**.

but *not to longitudinal waves* (because there is no perpendicular direction to oscillate in).

The polarisation of light has quite a few well-known applications, e.g. polarised sunglasses, radio transmission and reception, LCD technology, 3D movies, etc. A lot of these applications are based on **polarisers**, which are materials that *only let light polarised along their axis pass through*. If incident light is polarised at an angle θ relative to the axis of the polariser, then **Malus' law** states that the intensity of the transmitted wave is

$$I_{\text{transmitted}} = I_{\text{incident}} \cos^2 \theta$$

We'll justify this law in the problem section. It follows from Malus' law (also justified in the problem section) that when **unpolarised** light (= light containing a random mix of polarisations) is incident on a polariser then *the overall intensity halves*. Figure 8 is an overview of these properties of a polariser.

Figure 7: For some materials, the refractive index depends on the polarisation of light. This is called **birefringence**. Only the wave theory of light could explain this.

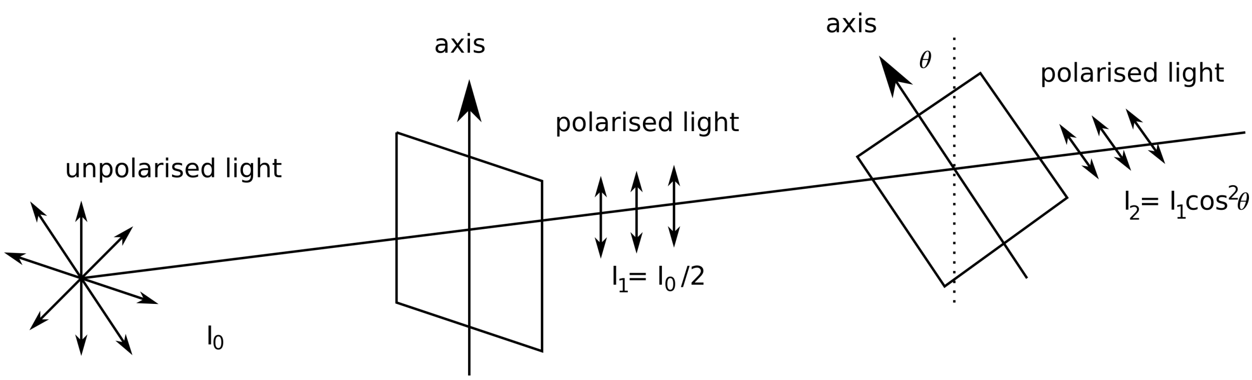


Figure 8: Polarisers polarising unpolarised light!

Time to go to the gym

If you want to learn physics, you have to work on solving problems. It's absolutely fine to make mistakes – it's actually the best way to learn! – but if you are completely stuck, then go back and read the relevant section again. If that doesn't help, then ask for help :)

People don't question the fact that in order to get a stronger body, you need to lift weights. Your muscles need to be pushed to (and beyond) their limit in order to grow back stronger. The same applies to physics, but instead of going to the gym, you need to sit down quietly and solve problems on a piece of paper! Making mistakes is the equivalent of breaking down your muscles. If you never exercise, your body suffers. If you never solve physics problems, your mind suffers.

The IB Data Booklet contains the equation(s) shown below. Find the equation(s) in the text and pay attention to any differences.

Sub-topic 4.3 – Wave characteristics

$$I = I_0 \cos^2 \theta$$

Problems first

1. The wavelength of some light in air is 500 nm. What is its wavelength in water ($n = 1.33$)?

2. Figure 9 shows three wavefronts, A, B and C, of a wave at a particular instant in time incident on a boundary between media X and Y. Wavefront B is also shown in medium Y.

   (a) Draw a line to show wavefront C in medium Y.

   (b) The refractive index in X is n_X and the refractive index in Y is n_Y. By making appropriate measurements, calculate n_X/n_Y.

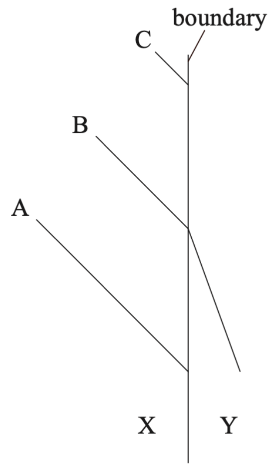


Figure 9: Problem 2.

3. In figure 10 a water wave moves from region A into a region B of shallower water. The waves move more slowly in region B. The diagram (not to scale) shows some of the wavefronts in region A.

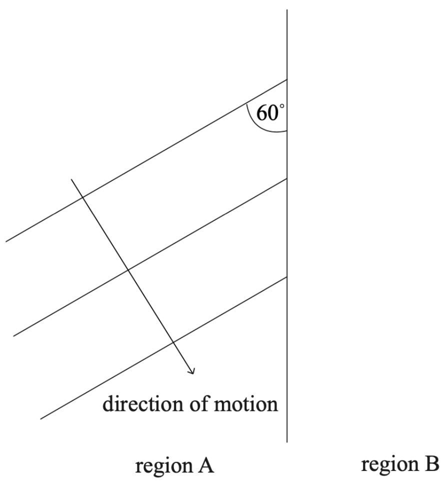


Figure 10: Problem 3.

   (a) On the diagram, draw three lines to complete the wavefronts in region B.

(b) Theory suggests that the wave speed v is related to the water depth d by

$$v = \sqrt{gd}$$

where g is a constant. The ratio of refractive indexes n_B/n_A is 1.4. Determine the following ratio

$$\frac{\text{water depth in A}}{\text{water depth in B}}$$

4. Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

5. If the entire apparatus of problem 4 (slits, screen and space in between) is immersed in water ($n = 1.33$), what then is the distance between the second and third dark lines?

6. Coherent light that contains two wavelengths, 660 nm (red) and 470 nm (blue), passes through two narrow slits separated by 0.300 mm, and the interference pattern is observed on a screen 5.00 m from the slits.

   (a) What is the distance on the screen between the first-order bright fringes for the two wavelengths?

   (b) Predict what you see when you shine white light on the two slits.

7. Radio waves are electromagnetic waves. A Sydney FM radio station has a listed frequency of 105.7 MHz (megahertz). What is the wavelength of these radio waves?

8. Of the various types of electromagnetic waves seen in figure 6, which type(s) would (a) have a wavelength of 500 nm? (b) have a wavelength of 1 cm? (c) have a frequency of 10^{18} Hz? (d) have a frequency of 10^6 Hz? (e) travel in air at 3.0×10^8 m/s?

9. A light wave emitted from a mercury street lamp has a wavelength of 577 nm in air. The cover of the lamp is made of glass which has a refractive index of 1.62 for this light.

   (a) When this light has left the lamp and is in the air, what is its frequency?

   (b) While the light is travelling through the glass cover, what is its

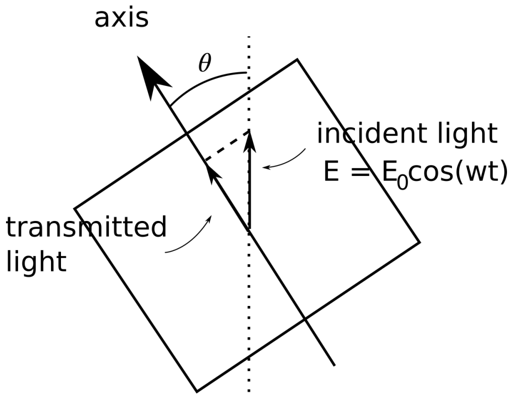
      i. frequency?

      ii. wavelength?

      iii. speed?

Figure 11: Mercury vapour street lamps.

10. **A derivation of Malus' law.** A vertically polarised light wave is incident on a polariser with a transmission axis at an angle θ relative to the vertical direction, see the figure below.



(a) The electric field strength vector (E) is performing simple harmonic oscillations according to $E = E_0 \cos(wt)$. What is the amplitude of this wave?

(b) Only the component E that is parallel to the transmission axis passes through the polariser. Show that this component is $E \cos \theta$.

(c) What is the amplitude of the component of E parallel to the axis?

(d) Recall that intensity is proportional to amplitude squared. Use this to derive Malus' law.

11. Unpolarised light is directed towards two polarisers, see figure 12. The dashed lines represent the transmission axis of the polarisers. The angle θ between the transmission axes of the polarisers is initially $0°$. Sketch a graph to show how the intensity I of the light emerging from the second polariser varies with θ from $0° < \theta < 180°$.

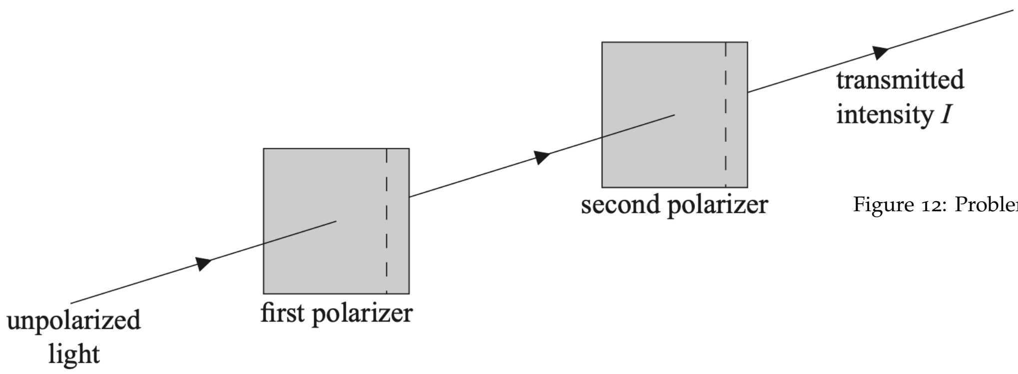


Figure 12: Problem 11.

12. Unpolarised light from a source is split, so that there is a path difference of half a wavelength between the two beams. After the split the light is still unpolarised and both beams are incident on a polariser. A lens brings the light to focus at point P on a screen, see figure 13. The lens does not introduce any additional path difference.

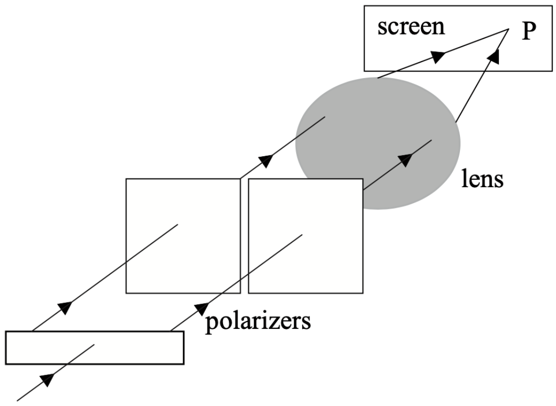


Figure 13: Problem 12.

State and explain whether any light would be observed at P, in the case in which the polarisers have their transmission axes

(a) parallel

(b) at right angles to each other

13. Here's a good YouTube video explaining why light slows down in transparent media.

14. In problem 11 you drew the graph that shows the transmitted intensity vs. polarisation angle. When unpolarised light of intensity I_0 is incident on a polariser, all polarisation angles are present and the average intensity transmitted is therefore

$$I_{\text{average}} = I_0 \frac{1}{\pi} \int_0^{\pi} \cos^2 \theta \, d\theta$$

Show that this equals $I_0/2$.

Answers to all the problems.

Now take a quiz

Check your understanding of this lesson: Here is a quiz.