A little book about waves

> "As one scans the Contemporary Panopticon of Present and Past Concepts for crucial turning points in the history of science, no more important experiment stands out – in classical physics, in quantum mechanics, or perhaps in science altogether – than the celebrated 'double-slit' experiment of Thomas Young. It was, any textbook will tell you, carried out by Young in the year 1800. It was announced by him before the full assembly of the Royal Society of London. And it was the experiment that conclusively proved, after a century of debate, that light was not composed of particles, as Newton had believed, but was a wave."
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> — Rothman, Tony, *Everything's relative*, 2003, pp. 12

Lesson 4: Two source interference

Interference is a general term we use to describe the situation of multiple waves existing simultaneously in a medium and superposing to create a net effect. The example of standing waves on a string (or in a pipe) is a one-dimensional interference effect: As two identical waves travel in opposite directions through the same medium, they superpose and – under certain conditions – create stationary nodes and antinodes.

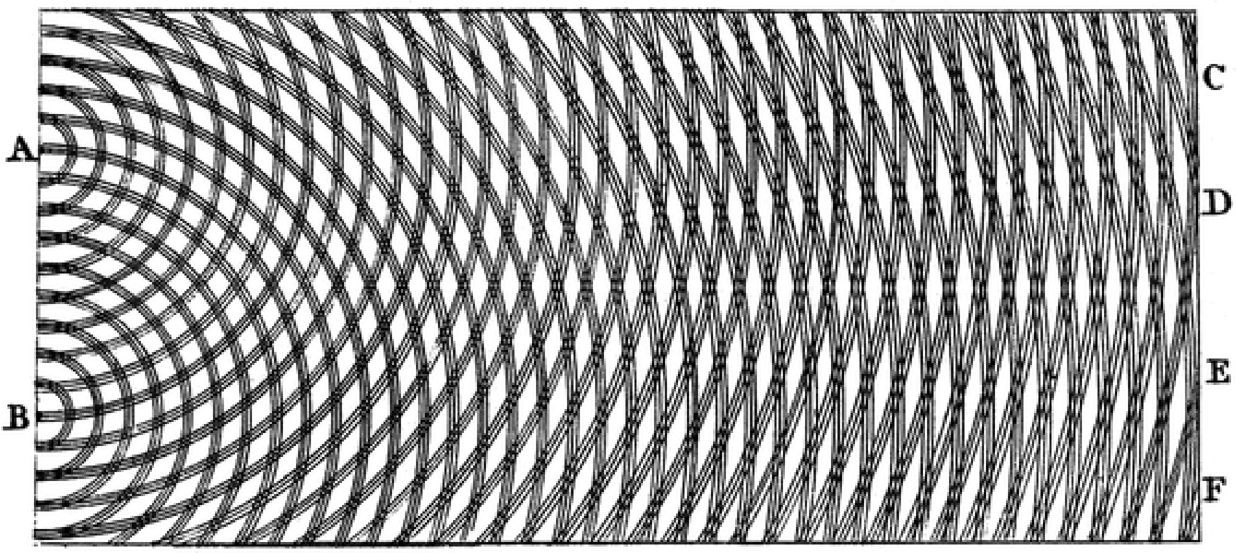


Figure 1: Thomas Young's famous double slit experiment demonstrated interference between two light sources. The experiment played a major role in the general acceptance of the wave theory of light.

Another common situation is when two identical wave sources are close to each other. As the sources emit waves into the surrounding medium, the waves superpose and create a specific **interference pattern**. Figure 1 shows the two-dimensional pattern arising from two identical wave sources. We will see in this lesson that the main result is to decrease the energy flowing in certain directions (for example C, D, E, and F shown in figure 1) while increasing the energy flowing in other directions (between the letters C, D, E, and F).

Interferometry is a technique that uses the interference of waves to extract information. Interferometry has played an important role in the advancement of physics, and has a wide range of applications in science and engineering. It played a key role in experiments related to both of Einstein's theories of relativity: The Michelson-Morley experiment (1887) for special relativity and the detection of gravitational waves (2015) for general relativity.

Concept #1: Path difference

Let us first consider two wave sources oscillating in phase[1]. Assume they also have equal frequency, f, and amplitude, A. As they disturb their medium, waves travel outwards in all directions with a constant wave speed (which only depends on the medium). Both waves have the same wavelength, λ (why?).

[1] Do you remember what that means? If not, go back to lesson 1 and review this important detail. We will come back to this point later in the section.

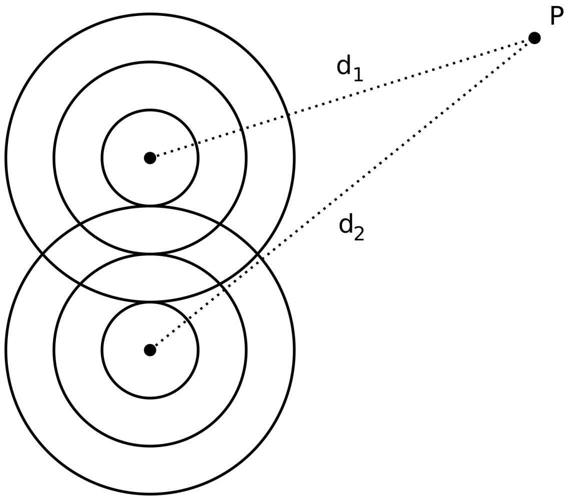


Figure 2: Two identical wave sources emit waves into their surrounding medium. Any point P is a certain distance away from each source and the difference between those distances is called the path difference. In this figure, both wave sources are just about to emit another wavefront.

Figure 2 shows a point P in the medium and a few approaching wavefronts. The point is a certain distance away from each source and *the difference between those distances* is called the **path difference**, hence the path difference can be expressed as

$$\text{path difference } = d_2 - d_1$$

All depending on where point P is, the path difference can be positive, negative or zero.

1. Draw points that have positive, negative, or zero path differences. Can two different points have the same path difference? Do points far away have larger path differences?

When the waves eventually reach point P, the net displacement of the medium at that point will be the sum of the two separate displacements caused by each wave. *What condition must we have in order for the medium to be oscillating in phase at point P?* Well, since the two sources are already oscillating in phase at their origins (wavefronts are emitted at exactly the same time), *the phase difference at a given point is only due to the difference in distance travelled by the two waves*. Hence, if the path difference at P happens to be *a whole number of wavelengths, then the arriving waves will always cause the medium* *to oscillate in phase at P*. At that point we say the waves undergo **constructive interference** and the point will oscillate with an amplitude equal to the sum of the two amplitudes (which would be $2A$ in this case). Hence for $n \in \mathbb{Z}$,

$$\text{path difference } = d_2 - d_1 = n\lambda \quad \text{(constructive)}$$

The situation is illustrated in this desmos animation. It shows the two waves travelling through point P and being in phase there. Note how the point is oscillating with a maximum possible amplitude.

2. In figure 3, the wave sources are just about to emit a new wavefront. What is the wavelength of the waves? Calculate the path differences for points P_1, P_2, and P_3.

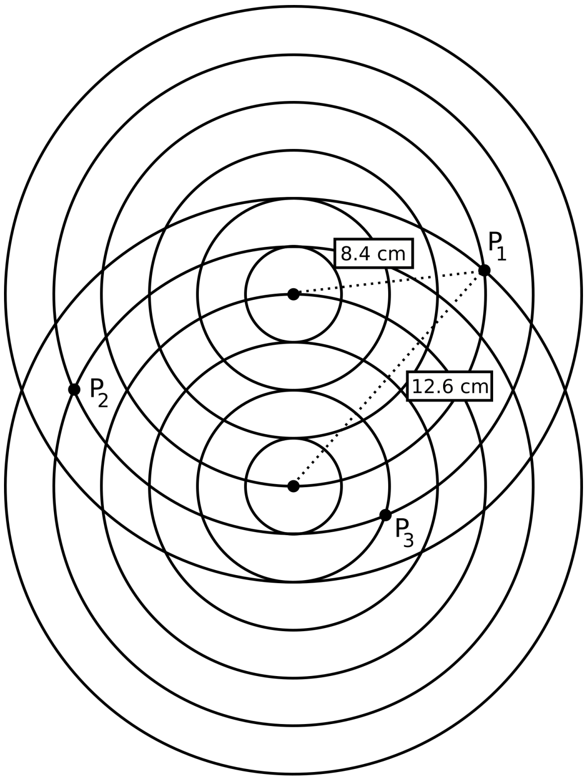


Figure 3: Three points with integer wavelength path differences.

3. In exercise 2 above you should have noticed that point P_1 had a path difference of two wavelengths $= 2\lambda$. But it's not the only one! Mark off more points that have a path difference of 2λ. It turns out that there are infinitely many points having a path difference of 2λ and you can show[2] that all these points lie on a hyperbola. We call it the $n = 2$ **anti-nodal line**, since it consists of all the points oscillating with maximum amplitude due to a two wavelength path difference.

4. Mark off points that have a path difference of zero (hence points lying on the $n = 0$ anti-nodal line.)

[2] That exercise might be a good IB Mathematics IA topic. Check out this.

5. On figure 4 anti-nodal lines corresponding to $n = 0, 1$, and 2 have been drawn. Draw anti-nodal lines for path differences corresponding to $n = -1$ and $n = -2$.

6. On figure 4 draw anti-nodal lines for path differences corresponding to $n = \pm 3$ and $n = \pm 4$. Do anti-nodal lines for $|n| > 4$ exist?

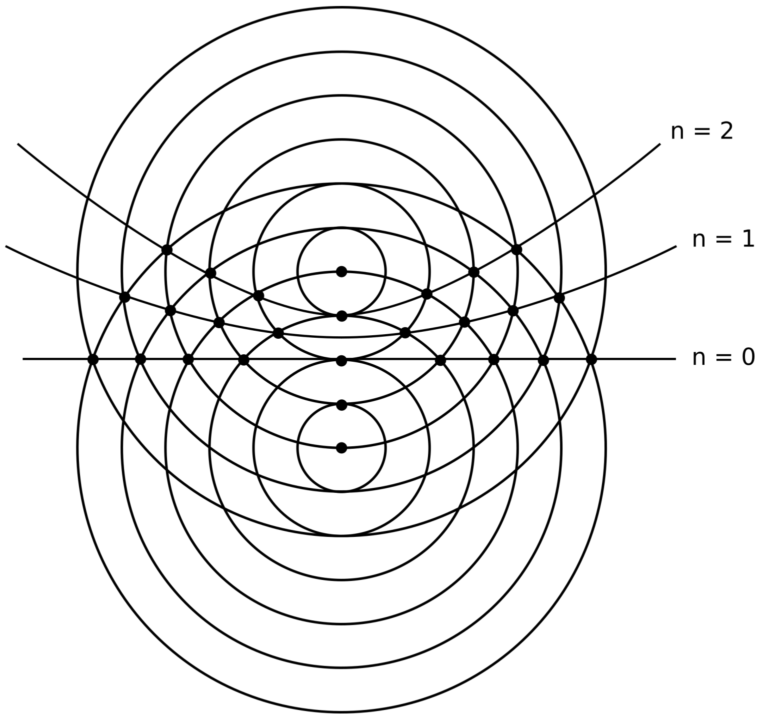


Figure 4: Anti-nodal lines for $n = 0, 1$, and 2 are shown. Draw anti-nodal lines for path differences corresponding to $n = -1$ and $n = -2$.

Let us now return to figure 2 and consider the following question: *What condition must we have in order for the medium to be oscillating completely out of phase at point P?* Again, since the two sources are already oscillating in phase at their origins (wavefronts are emitted at exactly the same time), *any phase difference at a given point is only due to the path difference*. If the path difference at P happens to be *an odd multiple of half-wavelengths, then the arriving waves will always cause the medium to oscillate completely out of phase P*. At that point we say the waves undergo **destructive interference** and the point will not oscillate at all (provided the amplitudes are equal and cancel out completely). Hence[3] for $n \in \mathbb{Z}$,

$$\text{path difference } = d_2 - d_1 = \left(n + \frac{1}{2}\right)\lambda \quad \text{(destructive)}$$

The situation is illustrated in this desmos animation. Note how the point doesn't oscillate despite two waves passing through it.

7. In figure 5, try to find some points that have a path difference of one half wavelength $= \lambda/2$. It turns out that there are infinitely many points having a path difference of $\lambda/2$ and you can show[4] that all these points lie on a hyperbola. We call these hyperbolas

[3] This is sometimes also called "half-integer wavelengths" although that might be a bit misleading. It can also be expressed as either

$$(2n-1)\frac{\lambda}{2} \quad \text{or} \quad (2n+1)\frac{\lambda}{2}$$

Either way, you end up with the same sequence of "half integers" $\ldots, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \ldots$

[4] Very similar exercise to the previous one.

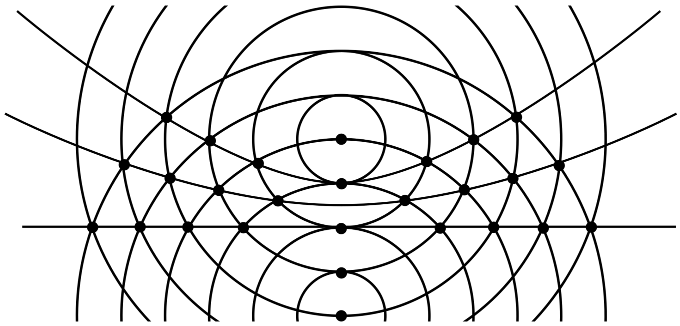


Figure 5: A few anti-nodal lines are shown. Try to find points with a path difference of half a wavelength.

nodal lines, since they consists of all the points oscillating with zero (or minimum) amplitude due to destructive interference taking place.

8. On figure 5 draw nodal lines for path differences equal to $\lambda/2$, $3\lambda/2$, and $5\lambda/2$. How many nodal lines are there in total?

9. Go to this PhET simulation and investigate the two source interference pattern! See figure 6 – set the separation and frequency to maximum and use the "water level meter" to measure the oscillations at given points in the medium.

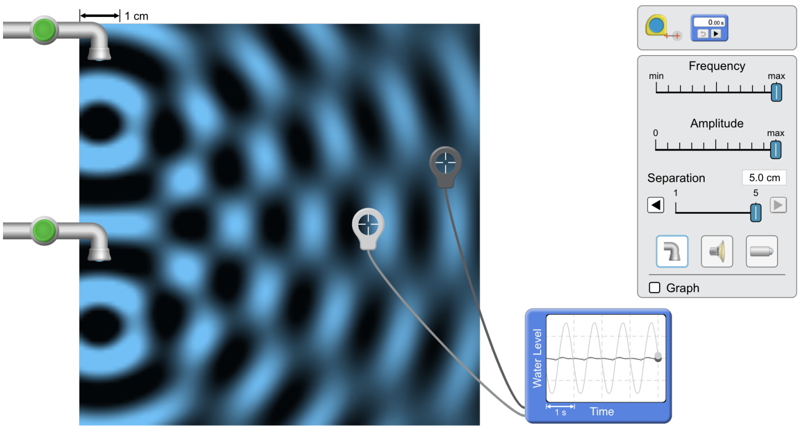


Figure 6: Play around with the excellent PhET simulation of two-source interference.

10. Here is another very nice website with good 2D and 3D animations and explanations. In this simulation, you can make the sources oscillate completely out of phase. Do you still get an interference pattern in this case? How is the pattern different from when they oscillate in phase? What if they were oscillating a little bit out of phase? *As long as the wave sources have the same frequency and are **coherent**[5] you will get an interference pattern.*

[5] Remember this means having a *constant phase difference.*

Concept #2: Hyperbolas "look like" straight lines

As we saw in the previous section, the interference pattern created by two identical sources is characterised by hyperbolic nodal and anti-nodal lines. We are now going to introduce an assumption that will allow us to approximate the hyperbolas to straight lines. This assumption has to do with the fact that hyperbolas when "viewed from far away" look like straight lines set at a particular angle, see figure 7.

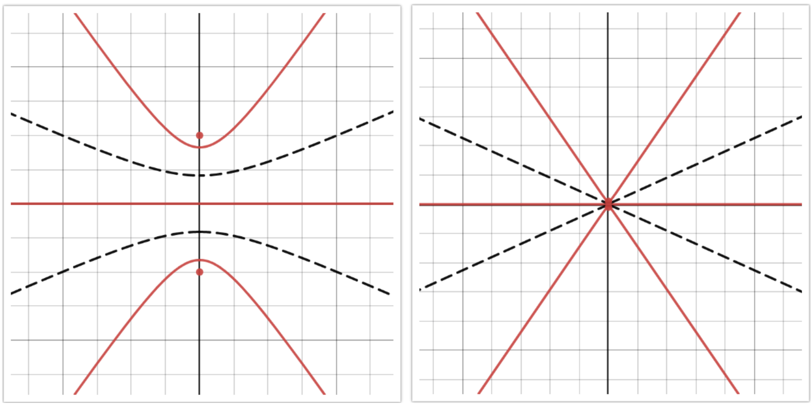


Figure 7: When hyperbolas are "viewed from far away" they look like straight lines. On the left we have zoomed in on the two wave sources and it is clear that the lines are hyperbolas. On the right we have zoomed out from the same sources to distances that are much greater than the separation between them. Notice how the hyperbolas now look like straight lines.

Here's how we determine the angle: Figure 8 shows two sources separated by a distance d. When considering a point in the medium that is very far away from the sources, d_1 and d_2 are very close to being parallel lines that make roughly the same angle θ to the horizontal[6] (see the enlarged box in figure 8). Using a bit of trigonometry, the path difference in this case can be expressed as

[6] If you need more convincing about this, then play around with this desmos simulation.

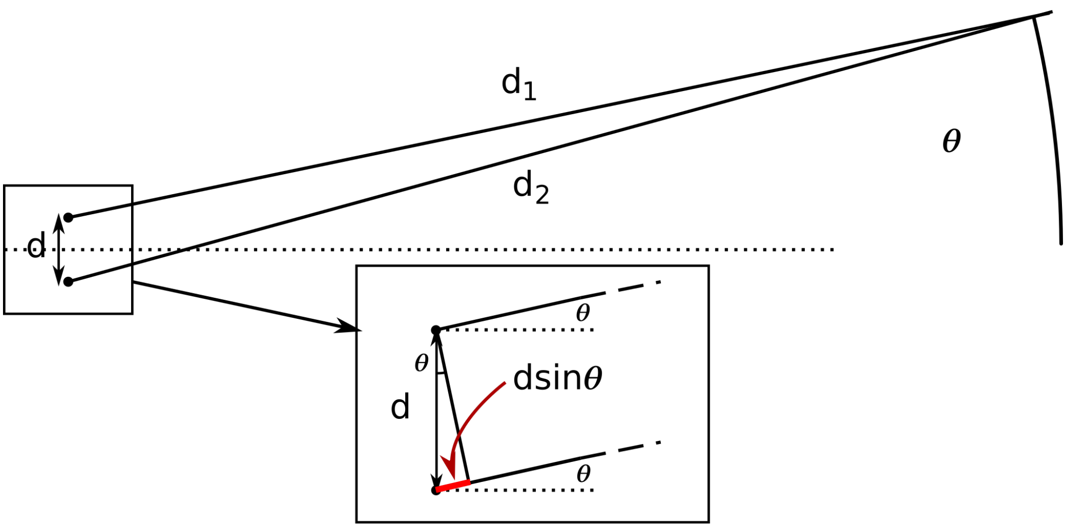


Figure 8: If the point under consideration is very far away from the sources, then the path difference is approximately $d\sin\theta$, where θ determines the direction to the point.

$$d \sin\theta$$

hence the conditions for constructive and destructive interference become (for $n \in \mathbb{Z}$)

$$d\sin\theta_n = n\lambda \quad \text{(constructive)}$$
$$d\sin\theta = \left(n + \frac{1}{2}\right)\lambda \quad \text{(destructive)}$$

We are often only interested in the constructive interference angles *(because that is the direction in which all the energy is "channeled")*, and for this case the angle/direction corresponding to a given n is called the n**th order**. We sometimes label the corresponding angle using a subscript: θ_n.

1. Assume two wave sources, separated by $d = 2.0$ m, emit waves of wavelength $\lambda = 0.10$ m.
   (a) For a point in the medium far away from the sources, find the angle of the 3rd order constructive interference line.
   (b) How many lines of constructive interference are there?
2. It is often desirable to radiate most of the energy from a radio transmitter in particular directions rather than uniformly in all directions. Pairs (or rows) of antennas are often used to produce the desired radiation pattern. As an example, consider two identical antennas 400 m apart, operating at 1500 kHz (near the top end of the AM broadcast band) and oscillating in phase.
   (a) At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?
   (b) In which directions will the radio signal not be able to be picked up?

Concept #3: Small angles simplify things even more

In this section we will introduce another assumption that will allow us to derive a simple formula that we will later use to measure the wavelength of light. The assumption is to only view the two wave sources "at small angles".

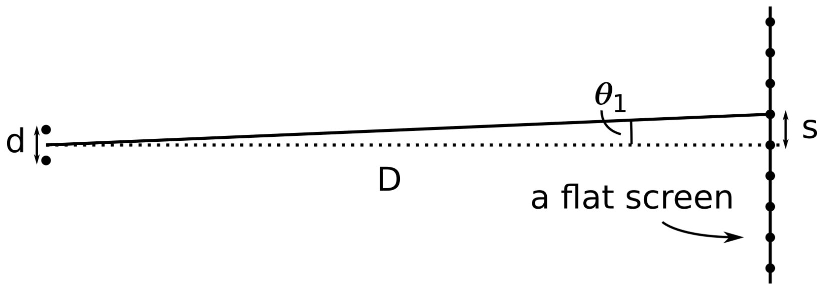


Figure 9: Here we have set up a flat screen at a distance $D \gg d$ and we are only considering small angles away from the center line. The anti-nodal lines intersect the screen at the points shown.

Consider figure 9. Here we are viewing the two sources from a large distance D which is much larger than the source separation d. Hence $d \ll D$ and the approximation from the previous section is valid. On top of that, let us now make the assumption that we are only interested in anti-nodal lines that make a small angle to the middle 0th order line. For small angles θ we remember that

$$\tan\theta \approx \sin\theta$$

so the constructive interferece condition can be expressed as

$$d\sin\theta_n = n\lambda \quad \Rightarrow \quad d\tan\theta_n = n\lambda \quad \Rightarrow \tan\theta_n = \frac{n\lambda}{d} \qquad (1)$$

If we imagine a flat screen placed a distance D away from the sources, then the anti-nodal lines (constructive interference) will intersect the screen at the points shown in figure 9. Let the 1st order angle correspond to a distance s on the screen. Using basic trigonometry,

$$\tan\theta_1 = \frac{s}{D}$$

which, by equating with equation (1) (after inserting $n = 1$), leads to

$$\frac{s}{D} = \frac{\lambda}{d} \quad \Rightarrow \quad s = \frac{\lambda D}{d}$$

The above formula is a very useful formula for measuring the wavelength of a wave, since all you need to do is know the distance between the two sources, d, the distance to a screen, D, and the distance on the screen between the 0th order and the 1st order, s. This is indeed how the first measurements of the wavelength of light were made, and I'll show you this demonstration in class.

1. Find an expression for the distance on the screen between the 0th order and the 2nd order. What is it in terms of of s? Generalise to the nth order.

2. What is the distance on the screen between successive orders, e.g. the 5th and 6th order? Generalise.

3. What is the distance on the screen between the two nodal lines corresponding to path differences of $\pm\lambda/2$? Generalise.

Returning to the screen placed at distance D, we know that we will get constructive interference and therefore maximum amplitude and maximum wave intensity at the points corresponding to anti-nodal lines (the dots shown in figure 9). Likewise, we will get

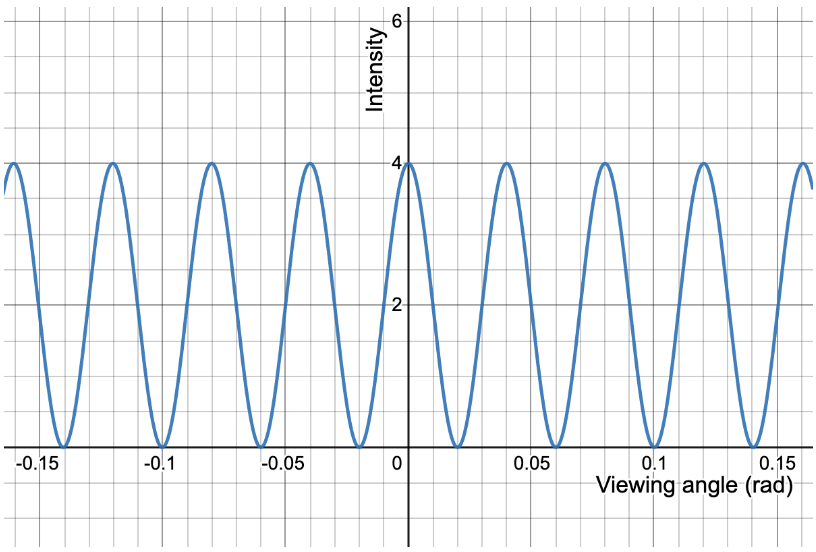


Figure 10: The intensity interference pattern for two wave sources seen from far away and at small angles.

destructive interference and therefore minimum amplitude and minimum wave intensity at points corresponding to nodal lines which lie in the middle between the maximum points. It can be shown[7] that the intensity, I, varies as cosine squared of the distance y (or viewing angle, since they are proportional for small angles) along the screen from the middle line, hence $I \propto \cos^2(y)$. Such a graph, see figure 10, is the typical intensity interference pattern that we often get in interferometry applications.

[7] Another interesting DP Mathematics IA waiting to happen...

Lesson 4: Time to go to the gym

If you want to learn physics, you have to work on solving problems. It's absolutely fine to make mistakes – it's actually the best way to learn! – but if you are completely stuck, then go back and read the relevant section again. If that doesn't help, then ask for help :)

People don't question the fact that in order to get a stronger body, you need to lift weights. Your muscles need to be pushed to (and beyond) their limit in order to grow back stronger. The same applies to physics, but instead of going to the gym, you need to sit down quietly and solve problems on a piece of paper! Making mistakes is the equivalent of breaking down your muscles. If you never exercise, your body suffers. If you never solve physics problems, your mind suffers.

1. The IB Data Booklet contains the equations shown below. Find all these equations in the text and notice if there are any differences.

Sub-topic 4.4 – Wave behaviour

$$s = \frac{\lambda D}{d}$$

Constructive interference:
path difference $= n\lambda$

Destructive interference:
path difference $= \left(n + \frac{1}{2}\right)\lambda$

Sub-topic 9.3 – Interference

$$n\lambda = d\sin\theta$$

Lesson 4 Quiz

Check your understanding of this lesson: Here is a quiz.

Lesson 4 Problems

1. Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 860 Hz. Point P is 12.0 m from A and 13.4 m from B. Is the interference at P constructive or destructive? Give the reasoning behind your answer.

2. Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves

emitted by each speaker is 172 Hz. You are 8.00 m from A. What is the closest you can be to B and be at a point of destructive interference?

3. Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 12.0 m to the right of speaker A. The frequency of the waves emitted by each speaker is 688 Hz. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk toward speaker B to move to a point of destructive interference?

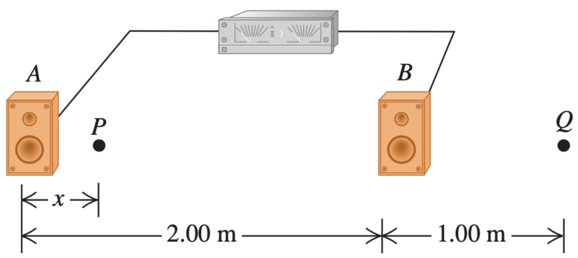
4. Two small speakers A and B 5.0 m apart are emitting a high frequency tone of 10 kHz in phase.

   (a) Explain why they will produce a sound interference pattern.

   (b) If you only consider small angles relative to the centerline, what is approximately the shortest distance between points of constructive interference at a distance of 100 m from the sources.

   (c) The speakers are now brought closer together so they are only 10 cm apart. For points far from the speakers, find all the angles relative to the centerline at which the sound from these speakers cancels. Include angles on both sides of the centerline.

5. Two loudspeakers, A and B, see figure below, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B.



   (a) i. What is the lowest frequency for which constructive interference occurs at Q?

      ii. List all frequencies for which constructive interference occurs at this point.

   (b) i. What is the lowest frequency for which destructive interference occurs at Q?

      ii. List all frequencies for which destructive interference occurs at this point.

(c) Assume the frequency of the waves is 206 Hz. Consider point P between the speakers a distance x to the right of A.

i. For what values of x will destructive interference occur at point P?

ii. For what values of x will constructive interference occur at point P?

(d) All the interference effects described in questions a) - c) above are never an issue when you are listening to music on your stereo at home. Why is that?

6. Interference patterns are not standing waves, though they have some similarities to the standing wave patterns described in lesson 2. In a standing wave, the interference is between two waves propagating in opposite directions; there is no net energy flow in either direction (the energy in the wave is left "standing"). In the situations shown in this lesson, there is likewise a stationary pattern of anti-nodal and nodal curves, but there is a net flow of energy outward from the two sources. All that interference does is to "channel" the energy flow so that it is greatest along the anti-nodal curves and least along the nodal curves. Discuss how you can get what looks like a standing wave pattern on the line joining the two wave sources.

Answers to all the problems.