A little book about matter

"In the discovery of hidden things and the investigation of hidden causes, stronger reasons are obtained from sure experiments and demonstrated arguments than from probable conjectures and the opinions of philosophical speculators of the common sort."

— William Gilbert in *De Magnete* (1600)

Lesson 8: Magnetism

We haven't talked much about the *magnetic* part of the electromagnetic force, but now is the time to do that. It turns out that magnetism, like electricity, is also caused by electric charge. Hence there is no new 'magnetic' charge, only the usual electric charge.[1] The subtle difference is that **magnetic fields are only created when charge is moving**: *When a charged particle moves it creates a magnetic field and this field can exert a magnetic force on other moving charges.*

[1] The deep connection between electricity and magnetism is one of the most beautiful things to learn in physics and it is closely related to Einstein's theory of special relativity.

Concept #1: Basic properties of magnets

Naturally occurring magnets have been known for millennia. These small rocks (called **lodestones**) contain the mineral magnetite (Fe_3O_4) and were probably magnetised by lightning striking the ground[2]. They were found and described in **Magnesia** (a part of ancient Greece) which is where the name 'magnet' comes from. Lodestones, suspended so they could turn, were the first magnetic **compasses**. The earliest known surviving descriptions of magnets and their properties are from Greece, India, and China around 2500 years ago.

[2] The large current in a lightning strike briefly creates a strong magnetic field that has a magnetising effect, see the next section.

Figure 1: A lodestone with small iron nails stuck to it.

Earth is surrounded by a large magnetic field and lodestones work as compasses because they react to being in this field. The first rigorous scientific treatment of magnetism was done by William Gilbert. In 1600 he published an influential book called *De Magnete*, in which he discussed and performed experiments relating to Earth's magnetic field. *De Magnete* was influential because of the rigorous way in which Gilbert described his experiments and his rejection of ancient theories of magnetism[3]. Even today we know surprisingly little about how Earth's magnetic field really works. We do know that it is created by a hot, iron-rich core at the center of the Earth and we also understand very well how the field protects us from the Sun's dangerous radiation and turns it into beautiful auroras at the poles, see figures 2 and 3.

[3] Hence he contributed to the development of the Scientific Method of Inquiry. He was also the person to introduce the terms "electric" and "electricity".

Figure 2: An aurora borealis.

The **north pole of a magnet**, N, is per definition *the end of a magnet that points to the geographic north* and the other pole is called **south**, S.[4] Magnets always have two poles (this is a consequence of Maxwell's equations and the fact that there is no 'magnetic' charge) and like poles repel, while unlike poles attract. If you break a magnet, two new opposite poles appear. Either pole of a magnet can attract unmagnetised materials containing iron and such materials

[4] Another interesting fact – aren't there just too many! – is that the magnetic poles of the Earth sometimes swap positions.

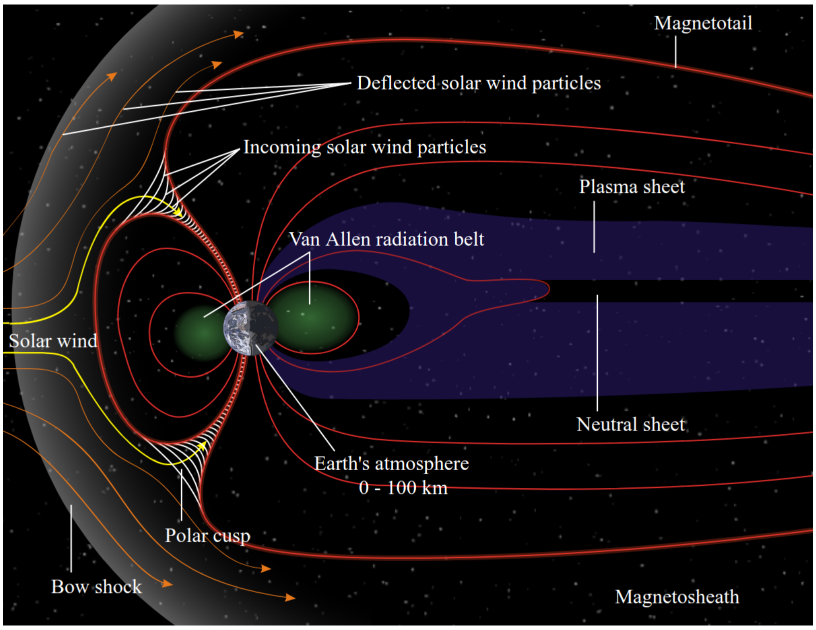

Figure 3: Earth's magnetic field extends into space and protects us from the Sun's dangerous "solar wind". The energetic charged particles in the solar wind are deflected by the field and some of it makes its way into our atmosphere at the poles where it creates beautiful polar lights.

are called **ferromagnetic**[5]. Many pure metals (like aluminium) are not ferromagnetic, hence magnets don't stick to them (try sticking a fridge magnet onto an aluminium pot). Iron rods can be magnetised by 'stroking" them with an already existing magnet[6]. A magnet can lose its magnetism again, for example by hitting it hard multiple times or by heating it beyond its Curie point.

The magnetic field is similar in nature to the electric or gravitational field and it is characterised by a **magnetic field strength vector** $\vec{B}$ at any point in the field. The $\vec{B}$-field vector points in the direction of north on a small compass needle. Similar to electric and gravitational fields, **magnetic field lines** are often drawn instead of individual $\vec{B}$-field vectors – the lines always point in the same direction as the vector at any given point. Hence, magnetic field lines point towards south and away from north. Figure 4 shows the magnetic field lines surrounding a normal bar magnet (note the direction of the lines). In contrast to electric and gravitational field lines, *magnetic fields lines never have a start or an end point – they always form closed loops* as indicated in figure 4. A unit of magnetic field strength will be defined in the next section.

[5] From the latin word for iron: 'ferrum'.

[6] 'Soft' iron is easier to magnetize than 'hard' iron, but hard iron is harder to unmagnetise.

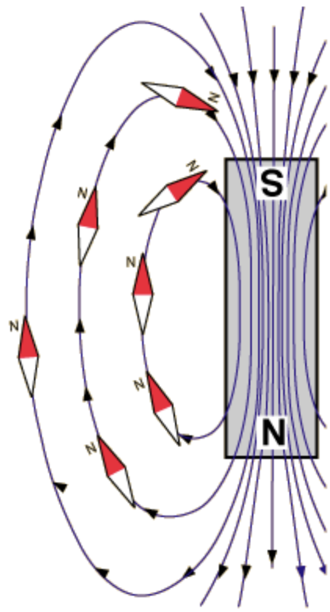

Figure 4: Magnetic fields lines created by a bar magnet. A $\vec{B}$-field vector at any point in the field is tangent to a field line and points in the same direction as north on a small compass needle.

Concept #2: Moving charges create magnetic fields

It was long thought that magnetism and electricity were separate phenomena, but in 1820 Hans Christian Ørsted discovered the deflection of a compass needle in the presence of an electric current. *It was the first demonstration of electricity causing magnetism*. In 1831 Michael Faraday discovered the opposite effect, namely *how to create electric current from magnetic fields*. This is called **electromagnetic induction** and it's the way in which we create electricity in today's modern world (we will cover that topic at a later time). We now fully understand the intimate relationship between electric and magnetic fields, and that knowledge is encoded in Maxwell's Equations (see our previous lesson on that).

When a current flows in a wire a magnetic field is created around the wire as shown in figure 5. The figure also illustrates the **right-hand rule for a wire** that we use to remember the direction of the field: Point your right-hand thumb in the direction of the conventional current direction and your fingers will curl in the direction of the circular $\vec{B}$-field lines.

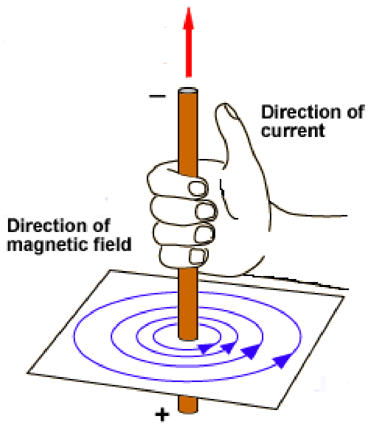

Figure 5: The magnetic field created around a current flowing through a wire.

Electrons in orbit around atomic nuclei define small flowing currents and these create small magnetic fields. Electrons and protons additionally spin around an axis and this spinning (hence moving) charge also creates a magnetic field. *These tiny atomic magnetic fields are the original source of your everyday fridge magnet*: In a few metals (particularly iron), the unbalanced spins of large numbers of electrons can be aligned (by using another magnetic field) which ends up creating one large magnetic field. Think about that next time you buy a fridge magnet!

Concept #3: Magnetic fields can exert forces on moving charges

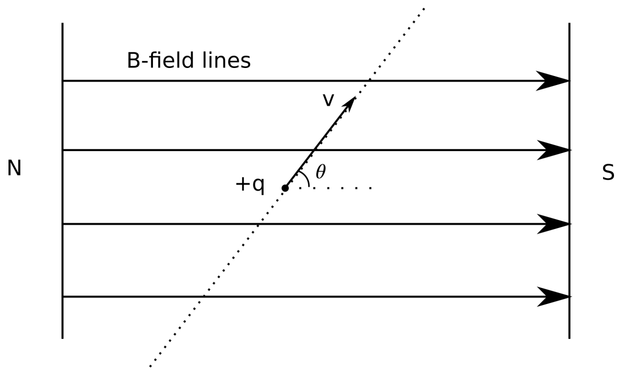

Figure 6: When a charge moves through a magnetic field it experiences a force (except in the special case where it travels parallel to the field lines).

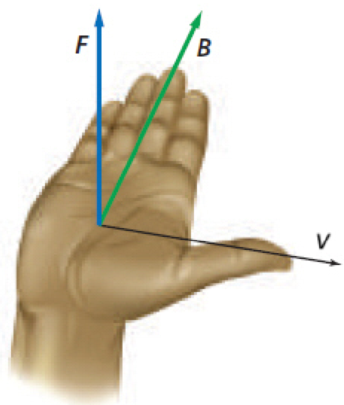

Figure 7: The right hand slap rule. One of many hand rules!

It's an experimental fact that when a charge moves through a magnetic field it experiences a force that is different from the other forces we have examined so far. We call this the **magnetic force** and it has the following properties:

- *The force is always perpendicular to the velocity and the magnetic field strength* ($\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$). For a *positive* charge the direction of the force is given by the **right-hand slap rule**: Point your thumb in the direction of the positive charges velocity, the rest of your fingers in the direction of the magnetic field lines, and the force will come out of the palm of your hand (you 'slap' the charge). *For a negative charge the force is in the opposite direction to the right-hand "slap" rule* (some people like to use their left hand for negative charges!). In figure 6 the force on the positive charge will be pointing into the paper. If the moving charge had been negative the force would have pointed out of the paper.

- *The magnitude of the force is proportional to the amount of charge that is moving* ($|\vec{F}| \propto |q|$.)
- *The magnitude of the force is proportional to the speed of the charge.* ($|\vec{F}| \propto |\vec{v}|$.)
- *The magnitude of the force is proportional to the magnitude of the magnetic field strength.* ($|\vec{F}| \propto |\vec{B}|$.)
- *The magnitude of the force is proportional to the sine of the angle between the velocity and the field.* ($|\vec{F}| \propto \sin\theta$, where θ is the angle between the $\vec{v}$ and $\vec{B}$.) This implies that the force is zero if the velocity is parallel to the field lines, and the force is maximum when the velocity is perpendicular to the field lines.

The above properties can all be summarised in the following (where the vector notation has been omitted for clarity)[7]:

$$F = qvB\sin\theta \quad + \quad \text{the right hand slap rule}$$

We can rearrange the above equation to isolate $|\vec{B}|$,

$$B = \frac{F}{qv\sin\theta}$$

and we then define **one tesla**, 1 T, as being the unit of magnetic field strength that would exert a force of one newton on a one coulomb charge moving at a velocity of one meter per second (alternatively, a 1 T magnetic field strength would exert a force of one newton on a one ampere current of length one meter):

$$\mathrm{T} \equiv \frac{\mathrm{N}}{\mathrm{C\,m\,s^{-1}}} = \frac{\mathrm{N}}{\mathrm{A\,m}}$$

The Earth's magnetic field has a strength that is a bit less than one **gauss**, 1 G, which is defined as $1\,\mathrm{G} \equiv 10^{-4}\,\mathrm{T}$.

[7] For those who know a bit more math, all this can elegantly be expressed as the vector **cross product**

$$\vec{F} = q\vec{v} \times \vec{B}$$

which automatically gives us both the correct magnitude and direction when q contains the appropriate sign.

Concept #4: Drawing vectors in three dimensions

When working with magnetic forces we have to get used to visualising things in three dimensions. This is because we typically have to do with three perpendicular directions: The direction of the charge's velocity $\vec{v}$, the direction of the $\vec{B}$-field, and the direction of the force $\vec{F}$. Since it's much easier to draw two-dimensional pictures, here's a convention we use to indicate the direction of a vector that is going in or out of a page: A vector can be thought of as a small arrow with a tail full of feathers and a pointy tip, see figure 8. When looking at it from behind you see feathers and that is shown as a cross in a circle $\otimes$. When looking at it from the front you see the pointy tip and that is shown as a dot in a circle $\odot$. For example, to show the direction of the force in figure 6, we would draw the force as shown in figure 9.

Figure 8: When a vector points towards us, we see the tip of the arrow and we label it as $\odot$. When a vector points away from us, we see the feathers and we label it as $\otimes$.

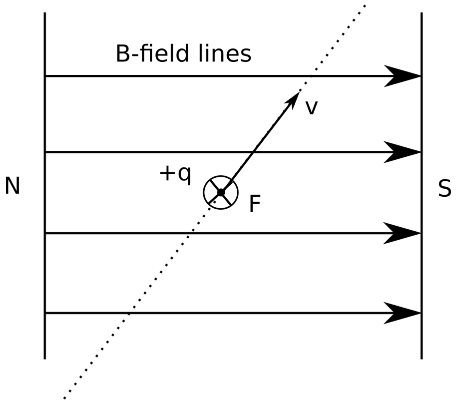

Figure 9: The magnetic force on the positive charge shown here points into the paper (use the right hand slap rule), so the force is shown as a circle with a cross in it.

Concept #5: The magnetic force on a current-carrying wire

When charges move in a magnetic field they experience a magnetic force (unless they are travelling parallel to the field lines as we saw in the previous section). Since current in a wire is the motion of many charges we would expect magnetic fields to exert forces on current-carrying wires. Let's derive a formula for that:

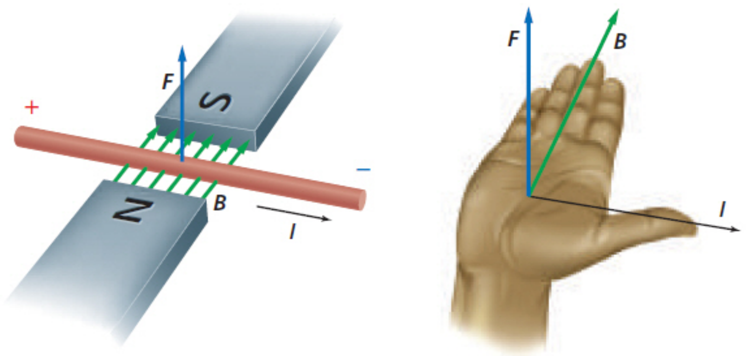

Figure 10: A current-carrying wire in a magnetic field.

First imagine a wire perpendicular to the field lines, as shown in figure 10. Consider a positive charge that is moving in the conventional current direction (from $+$ to $-$). Each positive charge drifting through the wire in that direction will experience a magnetic force of magnitude

$$F_{\text{one charge}} = qvB$$

when in the field. In order to get the magnitude of the total force on the wire, we need to find out how many charges experience this force at the same time. If the length of wire exposed to the field is L and the wire has a cross-sectional area A, then the total volume exposed to the field is

$$\text{total volume of wire exposed to the field} = LA$$

and therefore the total number of charges exposed to the field must be

$$\text{total number of charges in the field} = nLA$$

where n is the charge density. The magnitude of the total magnetic force on the wire must be the sum of all those individual forces, hence we get

$$F_{\text{total}} = (nLA)(qvB) = B(nqAv)L = BIL$$

where we simplified the expression using the drift speed formula $I = nqAv$. The above formula only applies if the wire is perpendicular to the direction of the field, but it's not hard to see that if the wire makes an angle θ to the field, then the expression is modified by a factor $\sin\theta$:

$$F = BIL\sin\theta \qquad \text{(force on a wire in a } \vec{B}\text{-field)}$$

Don't forget, the direction of this force is again given by the right hand slap rule as shown in figure 10.

Concept #6: Building a DC motor

Follow these instructions on how to make a simple DC motor!

Lesson 8: Workout

If you really want to learn physics, you have to work on all the following problems!

Lesson 8 Quiz

Check your understanding of this lesson: Here is a quiz.

Lesson 8 Exercises

1. A 10 cm piece of wire carrying a 1.0 A current is placed in a magnetic field with a strength of 100 mT. The wire makes an angle of 60° to the field lines. What is the force experienced by the wire? How can the force be increased?

2. Look at figure 4 again. What does the density of the field lines signify? Knowing this, what does figure 5 imply about how the magnetic field strength depends on radial distance from a wire?

3. A proton moves past a bar magnet as shown in figure 11. Find the direction of the force it experiences in each case.

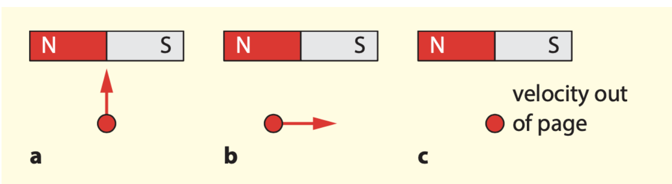

Figure 11: Exercise 3.

4. A long straight wire carries current as shown in figure 12. Two electrons move with velocities that are parallel and perpendicular to the current. Determine the direction of the magnetic force experienced by each electron.

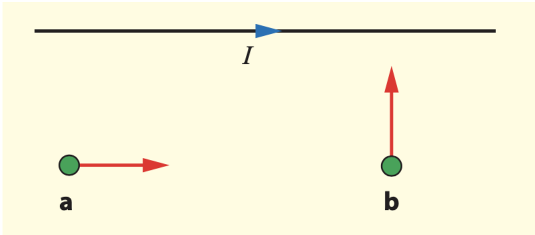

Figure 12: Exercise 4.

5. A current is flowing counterclockwise through a circular loop lying flat on a table. What is the direction of the magnetic field inside the loop? Outside the loop? What happens if the current changes direction?

6. A **solenoid** is a metal coil wound into a tightly packed helix. When current flows through the coil, a magnetic field is produced. Solenoids are useful for generating magnetic fields that are easily controlled and they are used in many different places (e.g. in the Large Hadron Collider at CERN). Figure 13 shows the magnetic field produced by a solenoid.

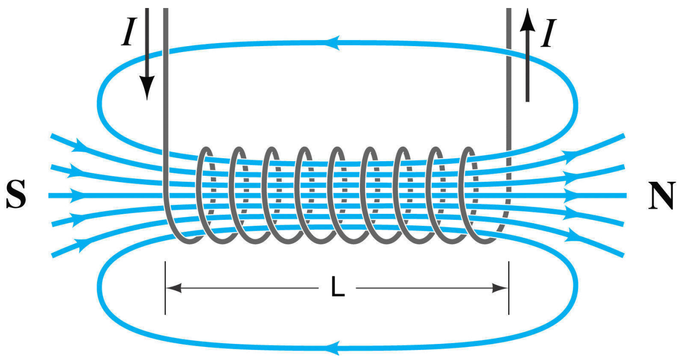

Figure 13: A solenoid and its magnetic field.

(a) Does the shape of the field remind you of the magnetic field of another object?

(b) Convince yourself, using the right hand rule, why the north pole is on the right.

(c) Making certain assumption, it can be shown that the magnetic field strength at the center of a solenoid is equal to

$$B = \mu_0 \frac{NI}{L}$$

where N is the number of turns, I is the current flowing through the coil, L is the length of the coil, and $\mu_0 = 1.3 \times 10^{-6}\,\mathrm{N/A^2}$ is the magnetic constant.

Calculate this magnetic field strength for a solenoid of length 5 cm, current 1 A and $N = 50$ turns.

7. Electrons and protons are charged particles and since they are 'spinning'[8], they have small magnetic fields surrounding them (this field is sometimes called a **magnetic moment**). Neutrons are neutral particles meaning they have no charge and yet they also have small magnetic moments! Try to come up with a simple explanation for this observation. *(Hint: This is partly the reason quarks were discovered.)*

[8] The picture of a spinning charged sphere is an incorrect image to have of a real particle at the atomic level, but nevertheless we think of it like that.

8. Figure 14 shows two charged parallel plates. The electric field has magnitude $2.4 \times 10^3\,\mathrm{NC^{-1}}$. The shaded region is a region of magnetic field normal to the page.

   (a) Deduce the magnitude and direction of the magnetic field so that an electron experiences zero net force when shot through the plates with a speed of $v = 2.0 \times 10^5\,\mathrm{ms^{-1}}$.

   (b) Suggest whether a proton shot with the same speed through the plates experiences zero net force.

   (c) The electron's speed is doubled. Suggest whether the electron would still be undeflected for the same magnetic field found in (a).

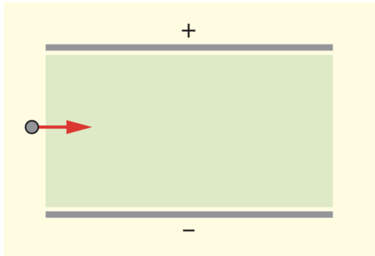

Figure 14: Exercise 8.

9. A proton of mass m and electric charge q enters a region of magnetic field at point X and exits at point Y. The speed of the proton at X is v. The path followed by the proton is a quarter of a circle.

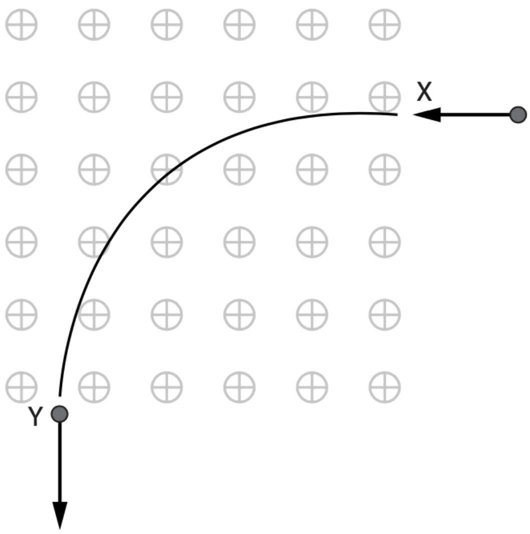

Figure 15: Exercise 9.

   (a) State and explain whether the speed of the proton at Y is the same as the speed at X.

   (b) Suggest why the path is circular.

   (c) i. Show that the radius of the circular path is $R = \frac{mv}{qB}$, where B is the magnetic field strength.

   ii. The speed of the proton is $3.6 \times 10^6\,\mathrm{ms^{-1}}$ at X and the magnetic field strength is $0.25\,\mathrm{T}$. Show that the radius of the path is 15 cm.

iii. Calculate the time the proton is in the region of the magnetic field.

(d) i. The proton is replaced by a beam of singly ionised atoms of neon. The ions have the same speed when they enter at X. The beam splits into two beams: B_1 of radius 38.0 cm and B_2 of radius 41.8 cm. The ions in beam B_1 have mass 3.32×10^{-26} kg. Predict the mass of the ions in beam B_2.

ii. Explain why the ion masses are different.

10. An electron of speed v enters a region of magnetic field B directed normally to its velocity and is deflected into a circular path.

(a) Deduce an expression for the period of the circular motion.

(b) The electron is replaced by a proton. Suggest whether the answer to (a) will change.

Answers to all the exercises.