A little book about waves

Lesson 2: Mainly about mechanical waves

Concept #1: Mechanical waves need a medium

When you throw a stone in water (who hasn't tried that?) you first see the stone disturb the water at a given point and then ripples spread out across the surface of the water. Here's an explanation of why that happens: First the stone pushes some water molecules aside at the point of impact, and when they move they bump into other nearby molecules that are also set in motion. Those nearby molecules then set other nearby molecules in motion, and over time, the disturbance spreads out to all molecules in the water.[1] Overall, *energy is transferred from a single point of impact to other regions of the water through particle interactions*, and as that energy spreads out over a larger region, the ripples get smaller and eventually become too small to notice (and ultimately the energy gets absorbed by the surroundings). If you *repeatedly* disturb water at a given point, you get what is normally referred to as a (travelling/propagating) **wave**.

[1] Obviously this is a very simplified description of what happens when you throw a real stone into real water! Describing the motion of a real fluid is extremely complicated. In an introductory physics course like this one, we are only interested in developing simple models that can explain basic systems. More advanced models typically build on these basic models.

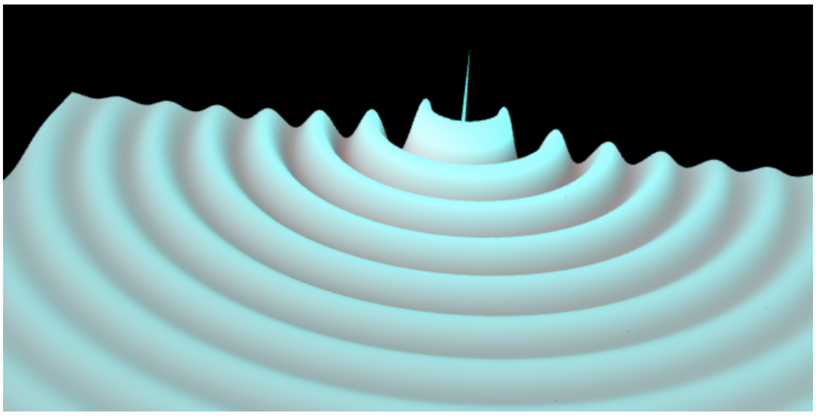


Figure 1: A disturbance gives rise to a travelling wave.

Figure 1 shows a wave created by disturbing a point repeatedly (here it is in motion, check the 3D box). Notice how the heights of the ripples (the peaks/crests) decrease with distance from the impact point. This is due to energy being spread out over a larger area (see concept #6 later in this lesson). Also notice how peaks of the same height all lie on concentric circles. The same applies to valleys/troughs of the same depth or, more generally, to all points having the same displacement. We call these circles **wavefronts** and the distance between successive wavefronts[2] is called a **wavelength**, but more about all that later.

[2] The technical definition of a wavefront is *a collection of points that have exactly the same phase*. 'Phase' being the entire argument of the sinusoidal function as mentioned in lesson 1.

Throwing a stone into water is not only something we can see, it is also something we can hear. Sound is similar to the water waves described above, but instead of being a disturbance in water, it is a disturbance in air. The stone's impact with the water also pushes nearby air molecules and as those air molecules are set in motion, they push other nearby air molecules, that again push other air molecules, etc. see figure 2 (here it is in motion).

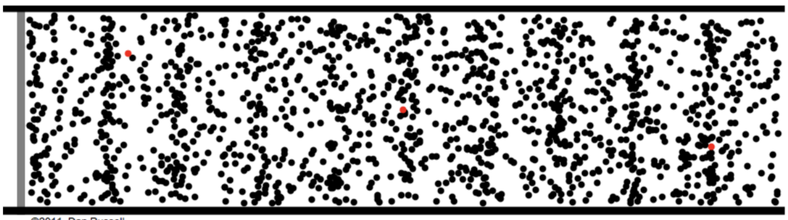


Figure 2: Sound is a disturbance travelling through air. As the air molecules get pushed and pulled sideways due to pressure differences, they collectively form regions of high pressure (and low pressure) that define a wavefront.

When a "wall" of air molecules on the left gets pushed to the right, the gas undergoes a **compression**, causing an increase in pressure that pushes on nearby air molecules, etc. The regions of high pressure (where air molecule are close together) propagate to the right[3], hence sound is simply a travelling pressure wave. Notice that the regions of high pressure form a collection of bunched up particles that we can refer to as wavefronts which are again one wavelength apart. Whenever a sound is produced, e.g. by a tuning fork or a loudspeaker, air is being disturbed in this way. Eventually, that disturbance reaches your eardrum which is also set in motion and your auditory system senses the sound.

[3] Regions of low pressure, **rarefactions** (or expansions), also travel from left to right, but the high pressure regions are more noticeable so we tend to focus on those.

What would happen if there was no air to push? Then we wouldn't hear a thing! Many waves require a **medium** to travel through, e.g. water waves require water, sound waves require air, etc. A wave can travel through a medium because the particles that make up the medium exert forces on each other (on a water surface there is surface tension and in air, pressure differences make the air molecules move). Such waves are mechanical in nature and Newton's laws of motion can therefore be used to work out all the details. We call such waves **mechanical waves**. The mathematical models required to accurately describe waves are quite complex and is something you will learn over many years as your mathematical literacy increases.[4]

[4] One important step on this journey is understanding how to solve the wave equation,

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \quad \text{(in 1D)}$$

which is done in most standard multivariable calculus courses.

Concept #2: Transverse and longitudinal waves

Start this PhET simulation and adjust the settings to 'oscillate, no end, damping = none'. You should see figure 3, which is an example of a wave travelling on a string. The string is the medium of the wave and notice how *each individual string particle is oscillating*. Each peak/crest represents a wavefront and the **wave speed** is the speed at which it moves through space.

1. Determine the wave speed by using the ruler and clock provided in the simulation (measure the time it takes for a crest to travel a certain distance - click on 'slow motion' to make it easier to

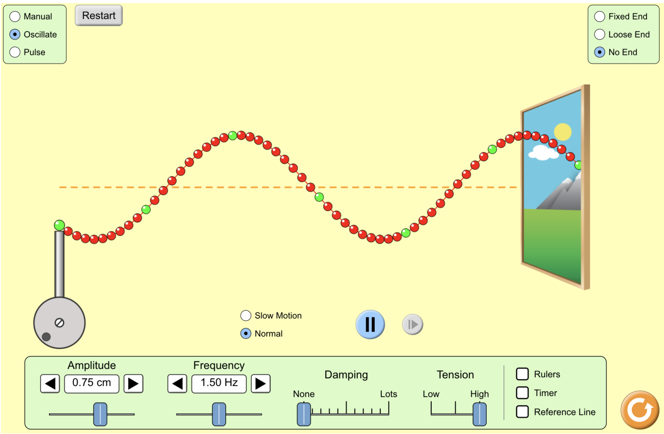


Figure 3: A screenshot from the excellent PhET string simulation.

measure). Instead of measuring a peak/crest could you just as well have measured a valley/trough? Or any other point for that matter?

2. For a string, the wave speed depends on the tension of the string. Try changing the tension of the string and notice how the wave speed changes. What relationship (roughly) does there seem to be between tension and wave speed?

3. The wave speed on a string also depends on the length density of the string.[5] Using Newtonian mechanics one can show that the wave speed on a string is in fact equal to

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension and μ is the length density of the string. If a string of length density $\mu = 0.69\,\text{g/m}$ has a tension of $5.0\,\text{N}$, what is the speed of a wave travelling through this string?

[5] Length density is just the mass per unit length. We also have area density (mass per unit area) and of course the usual volume density (mass per unit volume).

In general, the stronger the forces between particles in a medium (the larger the "tension"), the greater the wave speed, e.g. sound in air travels[6] at around 340 m/s, it travels around 1500 m/s in water, while sound in solid metal travels at around 5000 m/s. *The speed of a wave through a given medium depends only on the properties of that medium* and this is one example of how physical properties of a material depend on the atomic structure. E.g. the speed of sound in air depends on density, temperature, humidity etc.

[6] Newton's *Principia* from 1687 includes a computation of the speed of sound in air and he got around 300 m/s. The discrepancy was due to Newton neglecting the (then unknown) effect of temperature on a sound wave. This error was later rectified by Laplace.

4. In what direction is the string wave traveling? In what direction is the medium oscillating (focus on one string particle)? Since *the wave is moving perpendicular to the direction of oscillation of the medium*, this wave is an example of a **transverse wave**.

5. Click on this link and notice the first animated image showing a **longitudinal** wave. How would you define a longitudinal wave in terms of the direction of the wave motion and the direction of the oscillating particles?

Here is another good website with some nice animations of transverse and longitudinal waves. Notice that for both transverse and longitudinal waves there is *no overall transfer of mass from left to right*, the individual particles are just vibrating/oscillating around an equilibrium point. So what is in fact traveling? The answer is energy! *A wave is a transfer of energy, and the wave speed is the speed at which energy is being transferred through the medium*.

More complex wave motion can be both transverse and longitudinal at the same time. Click again on this link and scroll down to see the water wave example. In this case the medium is moving sideways and up and down at the same time. The particles in the medium are moving along circles, with smaller circles the deeper you go, which is how deep water waves work (shallow-water waves tend to be closer to being pure transverse waves).

6. What type of wave is "the wave" created by football fans in a stadium: (i) transverse (ii) longitudinal or (iii) a combination of transverse and longitudinal?

Concept #3: The wave speed equation

In the previous sections you should have noticed that a wave is really just a *correlated collection of oscillations*. Consider again this PhET simulation and adjust the settings again to 'oscillate, no end, damping = none'. Notice how each individual string particle is oscillating up and down, and the variation of displacement for such an individual particle can be graphed in a *displacement vs. time* diagram. If we assume the particles are oscillating in SHM, then we will get a nice sinusoidal variation as we saw in the previous lesson.

But now consider the whole string/medium. Notice that consecutive particles are slightly out of phase. This is because it takes time for a particle to exert a force on (and transfer energy to) the next particle. If you pause the motion, you can see that the *shape of the wave itself in space* across the entire string/medium is also sinusoidal, but this graph is a *displacement vs. distance* plot of many different oscillating particles in the string/medium.

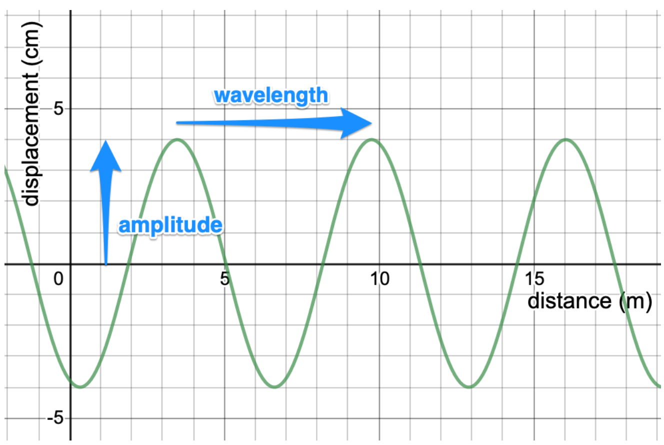


Figure 4: A displacement vs. distance graph shows the displacement of *all the particles in the medium at a given moment in time*.

On such a graph we can easily read off the **wavelength** of the wave which is defined as *the smallest distance between particles that are in phase*. We often use the Greek letter 'lambda', λ, to denote wavelength. The **amplitude** of the wave is equal to the amplitude of the SHM that each particle is undergoing.

1. A wave is travelling through a medium. Figure 5 is a displacement vs. time graph *for a single particle in the medium*. Can this graph be used to determine the wavelength? The period? The amplitude.

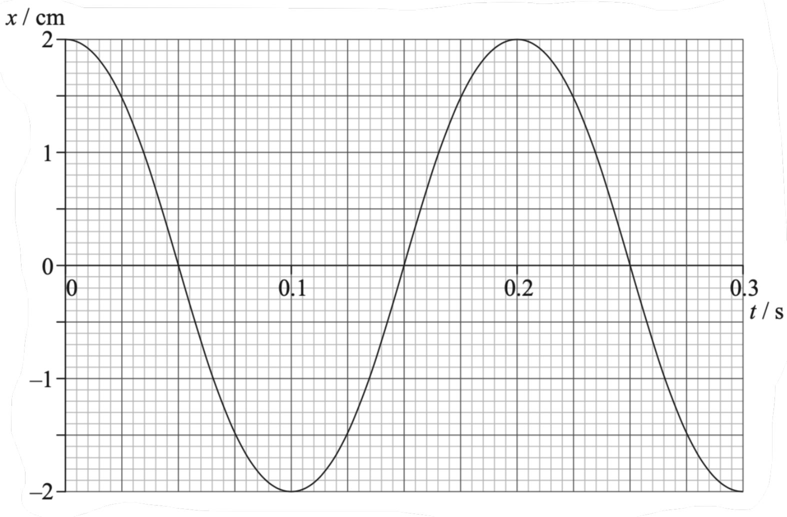


Figure 5: A displacement vs. time graph *for a single particle in the medium*.

2. Figure 6 is a displacment vs. distance graph *for the entire wave* (same wave as in 1). Can this graph be used to determine the wavelength? The period? The amplitude.

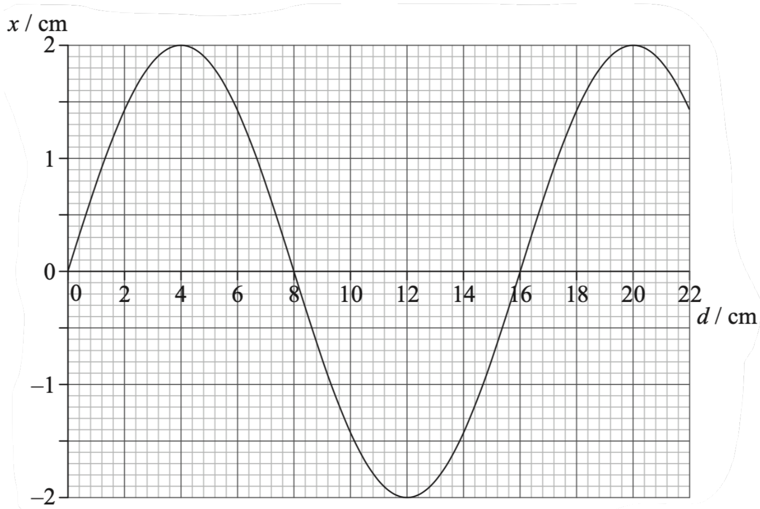


Figure 6: A displacement vs. distance graph *for the entire wave*.

3. If figure 6 represents the medium at time $t = 0$ s, then which particle in the medium is figure 5 a representation of?

Consider again this PhET simulation and adjust the settings to 'oscillate, no end, damping = none'. Notice that *in one period (when a single particle performs a full cycle) the wave moves forward one wavelength*. This means that the wave travels a distance of one wavelength in one period, hence we arrive at the much-used **wave speed equation**:

$$v = \frac{\lambda}{T} = \lambda f$$

where the last step was due to $f = 1/T$.

4. Determine the wave speed of the wave in exercises 1-3 above.

5. In concept #2, exercise 1 you measured the wave speed of a wave using a ruler and timer. Go back and check with $v = \lambda f$ that you get roughly the same result. This time pause the motion and measure the wavelength while reading off the frequency.

Concept #4: Waves can reflect

The properties of a wave can change when it encounters an obstacle or moves into a different medium. Consider again this PhET simulation and adjust the settings to 'pulse, fixed end, damping = none'. Figure 7 shows what it looks like when you fire off a **pulse** (a pulse is a non-repeating disturbance).

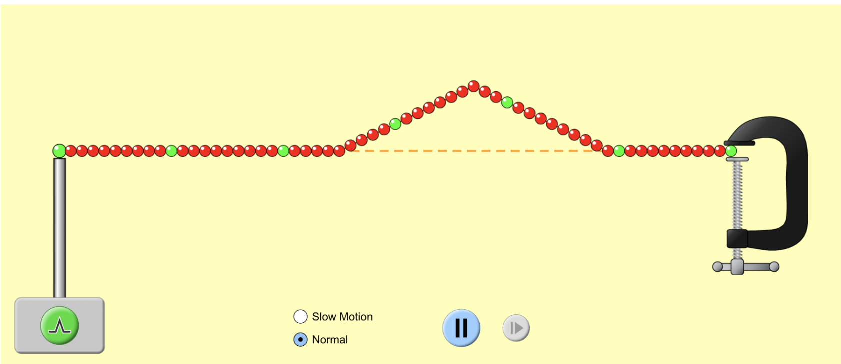


Figure 7: A pulse is fired.

1. What happens to the pulse after it hits the fixed end? Read the explanation for that behaviour at this link.

2. Change the setting to 'loose end'. What happens now? Read the explanation for that behaviour at this link.

3. Scroll to the bottom of this link and see what happens when a string wave goes from a less dense string to a more dense string (and vice versa). Notice how the wave gets partially reflected and partially transmitted. You can explore the transmission aspect in more detail here. Notice how the frequency of the transmitted wave is the same as the original wave, but both the wave speed and wavelength change. Does the change seem to be in agreement with the wave speed equation $v = \lambda f$? The idea of waves travelling between different media will be very important when we learn about light waves.

Concept #5: Superposition and standing waves

For multiple waves travelling through the same medium, the principle of superposition implies that *the net displacement of the medium at any point is the sum of the individual displacements at that point*. Play this animation and see that in action.

1. Figure 8 shows two pulses of equal width and height travelling in opposite directions on the same string. Draw the shape of the string when the pulses completely overlap.

2. The two pulses in figure 9 travel at $1\,\mathrm{cm\,s^{-1}}$ and both have width 2 cm. The heights are indicated on the diagram. In each case, draw the shape of the resulting pulse according to the principle of superposition at times $t = 0.5\,\mathrm{s}$, $t = 1.0\,\mathrm{s}$ and $t = 1.5\,\mathrm{s}$. Take $t = 0\,\mathrm{s}$ to be the time when the pulses just meet.

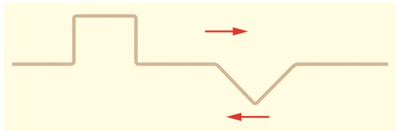


Figure 8: Two pulses meet.

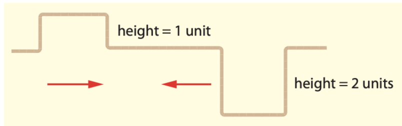


Figure 9: Two pulses meet again!

Consider again this PhET simulation and adjust the settings to 'oscillate, fixed end, damping = none, slow motion, frequency 1.68 Hz'. After a while you should notice that the overall wave seems to be 'standing still' while single particles are just oscillating up and down with large amplitudes. Here is a mathematical model of what is happening (figure 10 is a screenshot). What you see is called a **standing wave**: When the wave first travels along the string, it reflects (and inverts) at the fixed end, resulting in the creation of *an identical wave travelling in the opposite direction*. Those two waves (red and blue in figure 10) now superpose to create a resultant wave (black in figure 10) with some points that are not moving (**nodes**) and some points with maximum amplitude (**antinodes**).

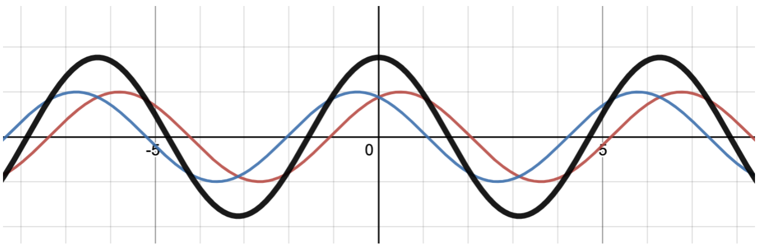


Figure 10: A standing wave is being formed, click here to see it in motion.

There is no overall energy transfer in a standing wave (hence the name 'standing' rather than 'traveling') because as one wave transports energy in one direction, the other wave transports the same amount in the opposite direction (the energy ends up trapped between the nodes).

At the nodes, you can see how the two waves always have displacements that are opposite to each other and the net displacement is always zero (the oscillations at those points are always *completely out of phase*). We call this **destructive interference**.[7] At the antinodes the oscillations are *in phase* and the displacements add to give a maximum net displacement that is equal to the sum of each individual amplitude. This is called **constructive interference**.

[7] The term **interference** is a general term used when waves mix and interact according to the principle of superposition - we will be considering more interference effects later.

3. Click here to see a standing wave. What is the distance between nodes in terms of the wavelength of the waves creating the standing wave? What is the distance between antinodes? You can change the wavelength of the waves by adjusting the parameter k.[8]

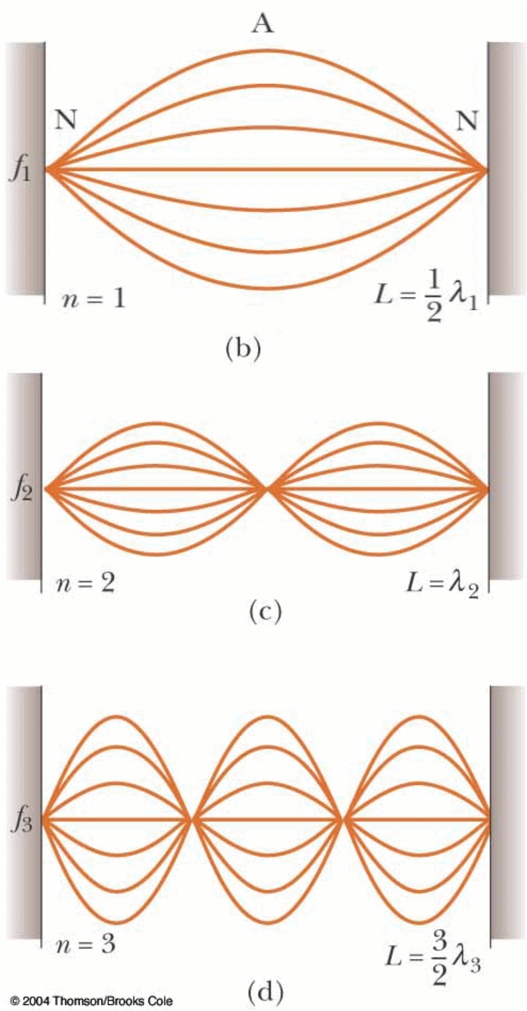
[8] This parameter is called the *wave number*, and it represents the number of radians per unit wavelength. It's not part of the DP syllabus.

From the above exercise, it should be clear that when a string of length L is fixed at both ends, the only way to have a standing wave on it is *if the length of the string is a whole multiple of half-wavelengths*. The first standing wave in the diagram shown below is called the **first harmonic** (or the **fundamental frequency**) and it corresponds to a wavelength of

$$L = \frac{\lambda_1}{2} \quad \Rightarrow \quad \lambda_1 = 2L$$

From the wave speed equation $v = \lambda f$ we can express the frequency of the first harmonic as

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$



Harmonic	#antinodes	#nodes	λ	f
fundamental	1	2	$\frac{2L}{1}$	f_1
second	2	3	$\frac{2L}{2}$	$2f_1$
third	3	4	$\frac{2L}{3}$	$3f_1$

For the second harmonic,

$$L = 2\frac{\lambda_2}{2} \quad \Rightarrow \quad \lambda_2 = L$$
$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$$

and for the third harmonic,

$$L = 3\frac{\lambda_3}{2} \quad \Rightarrow \quad \lambda_3 = \frac{2L}{3}$$
$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$$

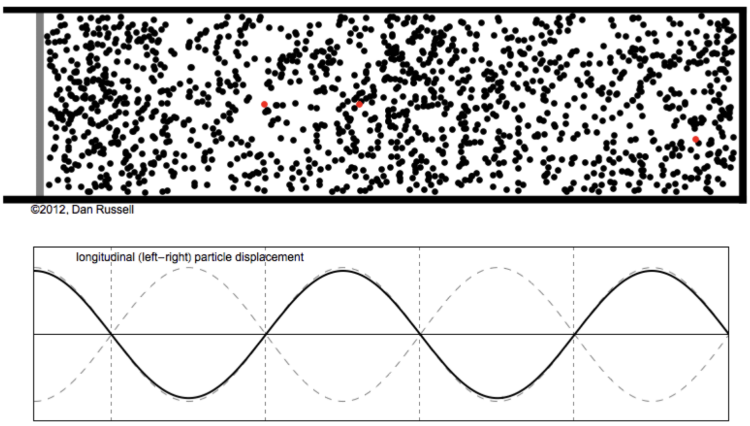
It's not hard to see that this pattern continues so the n'th harmonic of a standing wave that is fixed at both ends has wavelength and frequency given by

$$\lambda_n = \frac{2L}{n} \quad \text{and} \quad f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1$$

Notice how the frequency is simply the n'th multiple of the first harmonic.

It's not hard to imagine standing *transverse* waves as the ones shown above. Longitudinal waves can also create standing waves, but they are a bit harder to visualise. For example, when a sound wave is produced in a pipe that is closed at one end and open at the other (= half-open or half-closed), you can produce standing waves[9] at certain frequencies. These waves have an antinode at the open end and a node at the closed end, click here to see this in motion:

[9] The reason why an open end can cause a reflection is rather technical so you'll just need to take that as an experimental fact. Never forget: The logic of science is the experiment!



4. In the following, you will show that the condition for a standing sound wave to be created in a half-open pipe is for *the length of the pipe to be an odd multiple of quarter-wavelengths*:

   (a) Show that for the first harmonic $\lambda_1 = 4L$ and $f_1 = \frac{v}{4L}$.

   (b) Show that for the second harmonic $f_2 = 3f_1$. How is this different to the standing wave on two fixed ends?

   (c) Show that for the third harmonic $f_3 = 5f_1$. How is this different to the standing wave on two fixed ends?

(d) Derive a general formula for the n'th harmonic of a standing sound wave in a half-open pipe.

5. In class, I'll show you how we can **measure the speed of sound** using a half-open pipe filled with water!

6. Watch this nice video about standing waves in 2 and 3 dimensions!

7. Derive expressions for the wavelength and frequency of the n'th harmonic in a pipe that is open in both ends. How do they compare to a standing wave with nodes at both ends?

Concept #6: The intensity of a wave

Every wave has energy associated with it. To produce a mechanical wave a force is applied to a point in the medium and as that point moves, work is done on the system. As the wave propagates (i.e. travels) through the medium, each point of the medium exerts a force (and does work) on another point and in this way a wave transports energy from one region to another. Recall that the total energy of an oscillating system is

$$\frac{1}{2}m\omega^2 A^2$$

where A is the amplitude. Notice that the energy is proportional to A^2 and since power is energy transfer per unit time, then the power associated with a wave is also proportional to A^2:

$$P \equiv \frac{\Delta E}{\Delta t} \quad \Rightarrow \quad P \propto A^2$$

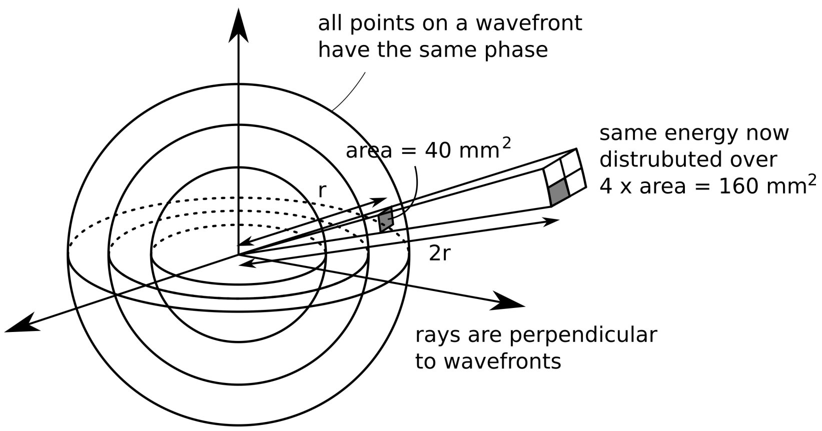


Figure 11: Spherical waves travelling outwards from a single point source. The area 40 mm^2 was chosen because that is the approximate surface area of your eardrum.

For waves travelling in 1 dimension (e.g. a wave on a string), energy travels in 1 direction. For waves in 2 dimensions (e.g. surface waves), energy can travel in 2 dimensions. For waves in 3 dimensions, e.g. sound waves through air (or light waves but we'll cover that later), energy is transferred in 3 dimensions. Figure 11 shows a spherical wave travelling outwards from a point source in 3 dimensions. As mentioned earlier, **wavefronts** consist of points in the medium that all have the exact same phase, and we typically draw them one wavelength apart. In figure 11 the wavefronts are concentric spheres. **Rays** are arrows that indicate the direction of motion of the wave and they are always perpendicular to the wavefronts.

For a wave in 3 dimensions, we define the **intensity**, I, of a wave as the *power received per unit area perpendicular to the direction of propagation*:

$$I \equiv \frac{P}{\text{area}}$$

This implies intensity has units of $\mathrm{W\,m^{-2}}$. Since power is proportional to amplitude squared, we can also conclude that about the intensity of a wave:

$$I \propto A^2$$

If a wave in 3 dimensions spreads out equally in all directions from a source, the intensity at a distance r from the source is inversely proportional to r^2. This follows directly from the conservation of energy[10]: If the power output of the source is P then at distance r all the energy is spread out on a spherical surface with area $4\pi r^2$, so the intensity becomes

$$I = \frac{P}{4\pi r^2} \quad \Rightarrow \quad I \propto r^{-2}$$

For a sound wave, *intensity corresponds to loudness*. If you double your distance to a loudspeaker, then the energy your ear drum receives per unit time decreases by a factor 4, see figure 11.

[10] The 2 dimensional analogy of this was the decreasing ripple height as the water ripples spread out over a larger area in in figure 1

Lesson 2: Brain workout

If you want to learn physics, you must work on all the following problems! It's absolutely fine to make mistakes – it's actually preferable, because that's a great way to learn! If you are completely stuck, go back and read the relevant chapter again. If that didn't help at all, then ask me a question :)

1. The IB Data Booklet contains the equations shown in figure 12. Find all these equations in the text and notice the small differences.

Sub-topic 4.2 – Travelling waves
$c = f\lambda$
Sub-topic 4.3 – Wave characteristics
$I \propto A^2$
$I \propto x^{-2}$

Figure 12: Excerpt from the data booklet.

Lesson 2 Problems

1. Figure 13 shows three points on a string on which a transverse wave propagates to the right.
   (a) Indicate the direction of the velocities of these three points.
   (b) How would your answers to (a) change if the wave were moving to the left?

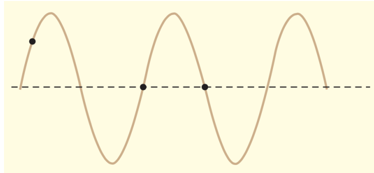


Figure 13: Problem 1

2. Figure 14 shows shows a piece of cork floating on the surface of water when a wave travels through the water to the right. Copy the diagram, and add to it the position of the cork half a wave period later.

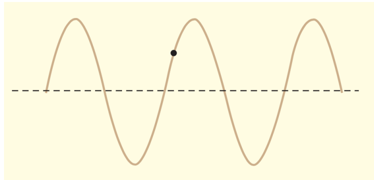


Figure 14: Problem 2

3. Calculate the wavelength that corresponds to a sound frequency of (assume the speed of sound is $340\,\mathrm{m\,s^{-1}}$):
   (a) 256 Hz
   (b) 25 kHz
   (c) The human ear can detect sounds spanning frequencies from around 20 Hz to $20,000$ Hz. Which of the above frequencies could a human hear? (Interesting fact: Dogs can detect high frequencies twice as large as humans.)

4. Figure 15 shows the displacement d of the particles in a medium against position x when a longitudinal wave pulse travels through the medium from left to right with speed $1.0\,\mathrm{cm/s}$. This is the displacement at $t = 0\,\mathrm{s}$.

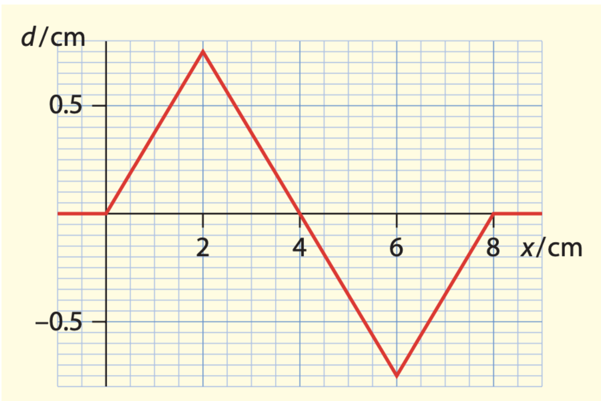


Figure 15: Problem 4

   (a) State what is meant by a longitudinal wave pulse.

   Figure 16 shows a line of nine molecules separated by 1.0 cm. The positions shown are the equilibrium positions of the molecules when no wave travels in the medium

   (b) i. Copy the diagram. Immediately below the copied line draw another line to show the position of these molecules when the pulse travels through the medium at $t = 0\,\mathrm{s}$.
   ii. Indicate on the diagram the position of a compression.

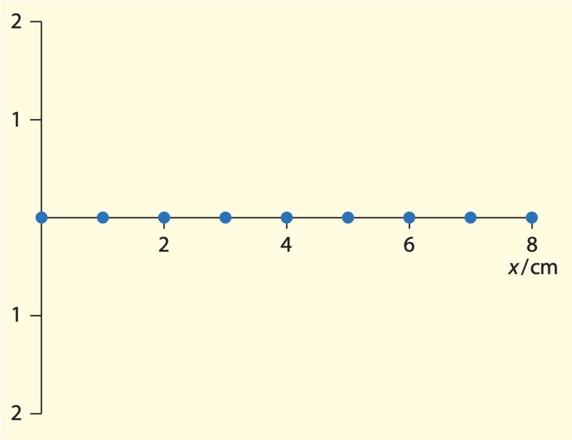


Figure 16: Problem 4

(c) i. Repeat (b) i. to show the position of these molecules at $t = 1$ s.

ii. Indicate on the diagram the position of a compression.

5. Figure 17 shows the variation with distance x of the displacement y of air molecules as a sound wave travels to the right through air. Positive displacement means motion to the right. Assume the speed of sound in air is $340\,\mathrm{m\,s^{-1}}$.

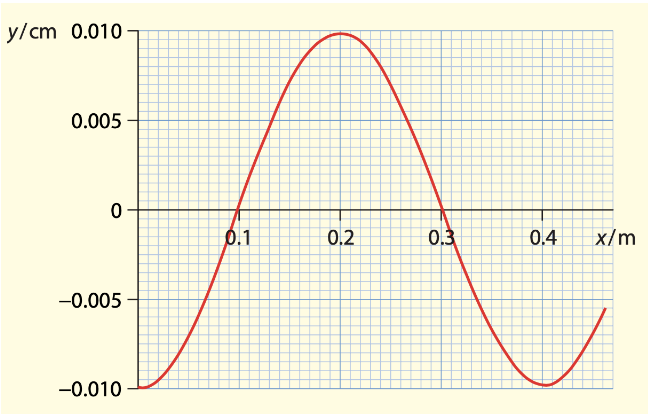


Figure 17: Problem 5

(a) Determine the frequency of the sound wave.

(b) State a distance x at which (i) a compression and (ii) a rarefaction occurs.

6. In the context of standing waves describe what is meant by a node and an antinode.

7. A string is held under tension, with both ends fixed, and has a first harmonic frequency of 250 Hz. The tension in the string is changed so that the speed increases by $\sqrt{2}$. Predict the new frequency of the first harmonic.

8. The velocity of a transverse wave on a string of length 0.500 m is $225\,\mathrm{ms^{-1}}$.

(a) Determine the frequency of the first harmonic of a standing wave on this string when both ends are kept fixed.

(b) Calculate the wavelength of the sound produced in air by the oscillating string in (a). (Take the speed of sound in air to be 340 m/s.)

9. Draw the standing wave representing the third harmonic standing wave in a tube with one closed and one open end.

10. A glass tube is closed at one end. The air column it contains has a length that can be varied between 0.50 m and 1.50 m. A tuning fork of frequency 306 Hz is sounded at the top of the tube. Predict the lengths of the air column at which loud sounds would be heard from the tube. (Take the speed of sound in air to be 340 m/s.)

11. A glass tube with one end open and the other closed is used in an experiment to determine the speed of sound. A tuning fork of frequency 427 Hz is used and a loud sound is first heard when the air column is increased in length to 20.0 cm.

(a) Calculate the speed of sound.

(b) Predict the next length of air column when a loud sound will again be heard.

12. A pipe with both ends open has two consecutive sound wave harmonics of frequency 300 Hz and 360 Hz.

(a) Which number harmonics are these?

(b) Determine the length of the pipe. (Take the speed of sound in air to be 340 m/s.)

13. Consider a string with both ends fixed. A standing wave in the second harmonic mode is established on the string, as shown in figure 18. The speed of the wave is 180 m/s.

(a) Explain the meaning of wave speed in the context of standing waves.

(b) Consider the vibrations of two points on the string, P and Q. The displacement of point P is given by the equation $y = 5.0\cos(45\pi t)$, where y is in mm and t is in seconds. Calculate the length of the string

(c) State the phase difference between the oscillation of point P and that of point Q. Hence write down the equation giving the displacement of point Q

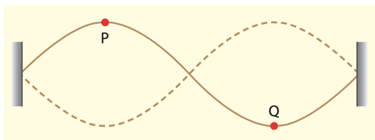


Figure 18: Problem 13

Answers to all the problems.

Lesson 2 Quiz

Check your understanding of this lesson: Here is a quiz.