A little book about matter

"From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."

— Feynman, *Lectures on Physics*, Vol. II, section 1–6

Lesson 5: Electric fields

Figure 1: One of the classic demonstrations showing the effect of an electric field. As the girl's hair gets positively charged (by removing electrons), the strands of hair repel each other.

Gravitational fields are created by **mass**, one of the fundamental properties of matter. Another fundamental property of matter is **charge** and this is responsible for creating another type of force field, the **electric field**[1]. Charge is associated with elementary particles, such as protons and electrons, and unlike mass, charge comes in two forms, **positive** ($+$) and **negative** ($-$). The proton carries a positive amount of charge called the **elementary charge** and this happens to be the smallest amount of observed charge. The unit of charge is the **coulomb**[2] (C) and the elementary charge has the exact value[3]

$$\mathrm{e} \equiv 1.602\,176\,634 \cdot 10^{-19}\,\mathrm{C}$$

We often use the letter q to describe a quantity of charge (similar to how m is often used for mass), so the proton has the charge

$$q_{\text{proton}} = +\mathrm{e}$$

The electron carries the exact equal but opposite charge:

$$q_{\text{electron}} = -\mathrm{e}$$

These two charges add to zero,

$$q_{\text{total}} = q_{\text{proton}} + q_{\text{electron}} = 0$$

[1] More accurately, this field is called the **electromagnetic field**, because charge is also responsible for creating **magnetic** fields. We'll be discussing that later. For the moment, we will be ignoring magnetic effects.

[2] The coulomb is a unit derived from the fundamental SI units 'ampere' and 'second', more on that later.

[3] It used to be a measured quantity (Robert Millikan measured it for the first time in 1909), but now it is defined by this value.

so a hydrogen atom, which consists of these two particles, has an overall (= total) charge of zero and is said to be **neutral**. Atoms become charged when they gain or lose electrons. For example, if a neutral helium atom loses both its electrons then it becomes a positive helium ion with a charge determined by the two remaining protons

$$q = +2\mathrm{e} = 3.2 \cdot 10^{-19}\,\mathrm{C}$$

Such a positive helium nucleus is also called an **alpha particle**.

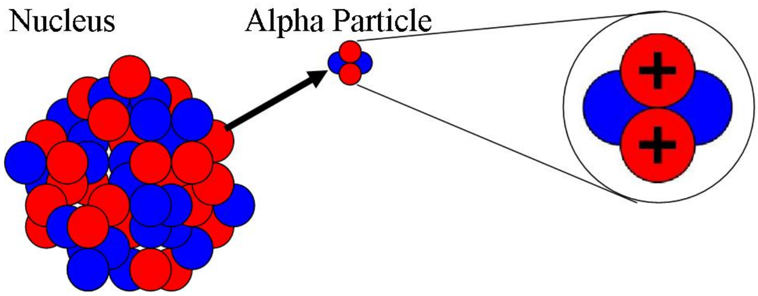


Figure 2: An alpha particle is a positively charged (+2e) collection of two protons (red) and two neutrons (blue). It is the nucleus of a helium-4 atom. An alpha particle is a particularly stable arrangement of nucleons and one form of radioactivity is the emission of such a particle from a much larger unstable nucleus as illustrated here.

It is an experimental fact that *the total charge in an isolated system always remains constant* – charge never just appears or disappears without a trace. Furthermore, *all observed quantities of charge are integer multiples of* e and this suggests that the overall charge of any amount of matter is determined by the number of protons and electrons (or other charged particles that we will learn about later). Consider, for example, the tip of your finger. It consists of roughly $1\,\mathrm{cm}^3$ of water (since the human body is mainly made of up water) and this volume has a mass of 1 g. Water is made up of H_2O molecules and these have a molar mass of 18 g/mol. So in the tip of your finger, there are

$$\frac{1\,\mathrm{g}}{18\,\mathrm{g\,mol}^{-1}} \approx 5 \cdot 10^{-2}\,\mathrm{mol} \approx 5 \cdot 10^{-2} \cdot 6 \cdot 10^{23} = 3 \cdot 10^{22}\ \text{molecules}$$

of water. Each neutral water molecule contains 10 protons and 10 electrons (two hydrogen atoms and one oxygen atom), so the amount of positive charge in your fingertip is

$$3 \cdot 10^{23}\,\mathrm{e} = 3 \cdot 10^{23} \cdot 1.602 \cdot 10^{-19}\,\mathrm{C} \approx 5 \times 10^{4}\,\mathrm{C} \tag{1}$$

with an equal but opposite amount of negative charge due to the electrons.

The Coulomb force

Charges exert forces on each other just like masses exert forces on each other. The formula for this **electric force** is very similar to the gravitational force formula: If you have two point charges at rest, q_1 and q_2, a distance r apart, then they will exert a force on each other (along the line joining them) with a magnitude of

$$F = k\frac{|q_1 q_2|}{r^2}$$

where $k \approx 8.99 \cdot 10^9\,\mathrm{N\,m^2\,C^{-2}}$ is a constant called **Coulomb's constant**. This law is also often referred to as **Coulomb's law**, named after the French scientist who first wrote it down in 1785. The above equation gives you the *magnitude* of the force since the product of the charges is inside an absolute value sign. The *direction* of the force is always *attractive when the signs are opposite and repulsive when the signs are equal* - so unlike gravitational forces, electric forces can also be repulsive, an important difference.

An obvious first question is: How does the Coulomb force compare in strength to the gravitational force? Consider the hydrogen atom consisting of one proton with an electron in orbit around it. These two particles exert an electric force and a gravitational force on each other. The distance between these two particles in hydrogen's lowest energy level is called the **Bohr radius** and it is around half an ångström ($r = 0.5 \cdot 10^{-10}$ m). For the proton we have

$$q_p = +1.602 \cdot 10^{-19}\,\mathrm{C}$$
$$m_p = 1.67 \cdot 10^{-27}\,\mathrm{kg}$$

while for the electron,

$$q_e = -1.602 \cdot 10^{-19}\,\mathrm{C}$$
$$m_e = 9.11 \cdot 10^{-31}\,\mathrm{kg}$$

Now lets' calculate the ratio between the electric force and the gravitational force (notice the distance squared cancels out):

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \frac{k\frac{|q_e q_p|}{r^2}}{G\frac{m_e m_p}{r^2}} = \frac{9.0 \cdot 10^9 \cdot |(-1.6 \cdot 10^{-19}) \cdot (+1.6 \cdot 10^{-19})|}{6.67 \cdot 10^{-11} \cdot 9.11 \cdot 10^{-31} \cdot 1.67 \cdot 10^{-27}} \approx 10^{39}$$

The electric force is 10^{39} times larger than the gravitational force - an unbelievable difference in strength! Gravitational effects at the atomic scale are completely insignificant for this reason. If all the electrons suddenly disappeared from the tip of your left and right index finger, the force of repulsion between your fingers would be roughly (using our result (1) from before and assuming the fingers are one meter apart)

$$8.99 \cdot 10^9 \frac{(5 \cdot 10^4)^2}{1^2} \approx 2 \cdot 10^{19}\,\mathrm{N}$$

despite the fact that you only lost half a microgram of mass!

Paradoxically, the enormous strength of the electric force compared to gravity *is exactly the reason you don't notice it that much*. Tiny imbalances in positive and negative charge are very quickly balanced out due to the enormous forces in play[4]. **Static electricity** and **lightning bolts** are the only everyday effects where you directly experience the effect of relatively large charge imbalances. Static electricity and lightning bolts naturally occur due to something called the **triboelectric effect**. The triboelectric effect is when two materials in contact with each other get electrically charged

Figure 3: Charles-Augustin de Coulomb (1736–1806) was a French officer, engineer, and physicist. He is best known as the discoverer of what is now called Coulomb's law, the description of the electrostatic force of attraction and repulsion, though he also did important work on friction. The SI unit of electric charge, the coulomb, was named in his honour in 1880. (Wikipedia)

[4] "With such enormous forces so perfectly balanced in this intimate mixture, it is not hard to understand that matter, trying to keep its positive and negative charges in the finest balance, can have a great stiffness and strength. The Empire State Building, for example, swings less than one inch in the wind because the electrical forces hold every electron and proton more or less in its proper place." (Feynman)

as they separate (if you've ever taken off a wooly sweater in dry weather, you know what I'm talking about). Rubbing two materials together increases the contact between their surfaces, and hence the triboelectric effect. The sign and strength of the charges differ according to the materials, surface roughness, temperature, strain, and other properties, but in all cases, it is important to know that in most cases *only electrons are transferred between the objects*: Electrons are roughly two thousand times lighter than protons (which is the reason they orbit the protons and not the other way around) so they are much easier to move around.

The triboelectric effect is very unpredictable, and only broad generalisations can be made. Well-known examples of the effect are rubbing glass with fur, running a plastic comb through your hair or pulling off a wooly sweater. Amber can also acquire an electric charge by contact and separation with wool and this property was first recorded by the ancient Greek philosopher Thales of Miletus. The word "electricity" is actually derived from the Greek word for amber: 'elektron'.

Figure 4: A beautiful piece of 'elektron' (Greek for amber).

Charge can move more easily through some materials compared to others. Materials that allow charge to flow easily through them are called **conductors**, while materials that don't do that very easily are called **insulators**. In most solid conductors (e.g. metals) electrons are the charged particles in motion, but in ionic aqueous solutions (e.g. salt water) you typically have positive and negative ions carrying the charge around. Materials are not always easily classified as a conductor or an insulator and each materials **electrical conductivity** depends on a number of factors, e.g. its atomic structure and the strength of any applied electric fields. This knowledge is the foundation of modern electronics and after covering electric field strength and electric potential in the next two sections, we will be ready to talk about electric circuits.

Electric field strength

When we discussed gravity earlier, we introduced the concept of **gravitational field strength**, $\vec{g}$. This was a measure of how much force per unit mass is exerted on a point mass when it is placed in a gravitational field. For example, close to the surface of the Earth, 1 kg of mass experiences a gravitational force of magnitude 9.8 N, so we can write $|\vec{g}| = 9.8\,\mathrm{N/kg}$. The technical definition is[5]

$$\vec{g} = \frac{\vec{F}_{\text{gravity}}}{m} \quad \text{(point mass } m\text{)}$$

and notice that gravitational field strength is a vector quantity which points in the direction of the force.

[5] Recall that $\vec{g}$ is also an acceleration as is apparent from the definition and the unit since

$$\frac{\mathrm{N}}{\mathrm{kg}} = \frac{\cancel{\mathrm{kg}} \cdot \mathrm{m/s^2}}{\cancel{\mathrm{kg}}} = \mathrm{m/s^2}$$

We'll do the same thing now for electric fields. However, since there are two types of charge, we need to be a bit more careful with our definition: The **electric field strength**, $\vec{E}$, *is a measure of how much electric force per unit charge is exerted on a* ***positive*** *point charge* *when it is placed in an electric field.*[6]

$$\vec{E} = \frac{\vec{F}_{\text{Coulomb}}}{q} \quad \text{(\textbf{positive} point charge } q\text{)}$$

[6] Compared to gravitational field strength, this is however not an acceleration. Be aware of that difference!

This is again a vector quantity which points in the direction of the electric force on a **positive** point particle and it has the unit N/C.

Let's look at an example: Consider the electric field created by a positive point charge of $Q = 2.00\,\text{nC}$. If a positive point charge $q = +\text{e}$ is placed 40.0 cm away from it, then the electric force experienced by q (due to Q) is given by Coulomb's law:

$$F = k\frac{|qQ|}{r^2} = (8.99\times10^9)\frac{(1.602\times10^{-19})(2.00\times10^{-9})}{(0.400)^2} \approx 1.80\times10^{-17}\,\text{N}$$

At this point there is therefore an electric field strength of magnitude

$$|\vec{E}| = \frac{F}{q} = \frac{1.80\times10^{-17}}{1.602\times10^{-19}} \approx 112\,\text{N/C}$$

Notice that we could have arrived at this straight away by simply calculating it directly as

$$|\vec{E}| = \frac{F}{q} = \frac{k\frac{|qQ|}{r^2}}{q} = k\frac{|Q|}{r^2} = (8.99\times10^9)\frac{(2.00\times10^{-9})}{(0.400)^2} \approx 112\,\text{N/C}$$

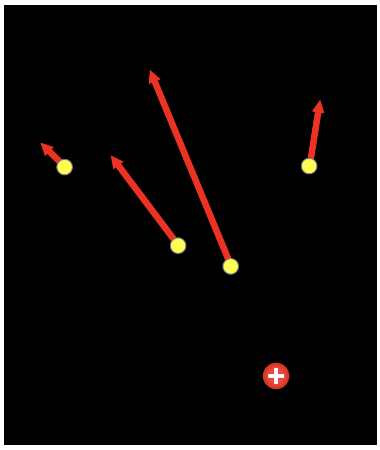


Figure 5: The positive point charge Q creates an electric field and here are shown four electric field strength vectors at different points in the field. The field is stronger at points that are closer to the point charge.

The direction of this electric field strength would be pointing away from Q since positive charges repel (see figure 5).

The above example leads to the result that *the magnitude of the electric field strength at distance r from a single point charge* is given by the formula

$$|\vec{E}| = k\frac{|Q|}{r^2} \quad \text{(for a point charge ONLY!)}$$

and the direction is *pointed away from positive charges and towards negative charges*. As also shown above, electric field strength is typically measured in units of N/C.

Drawing electric fields strength vectors at lots of points in the field is one way of visualising the field (as shown in figure 5), but it quickly gets messy. Instead of doing that, we often draw **electric field lines**. These are continuous lines that show the direction of the electric field strength vector at any given point, see figure 6. By definition, the lines always point from positive charge to negative charge and the density of lines in a certain region indicates the strength of the field in that region. We will explore these lines in greater detail later.

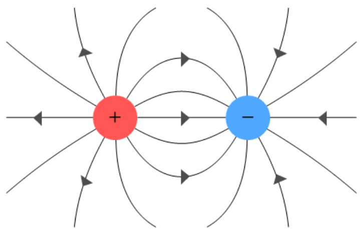


Figure 6: Electric field lines surrounding a positive and negative point charge of equal magnitude. This arrangement is also called a **dipole** and it's a very configuration because many more complicated systems behave approximately like a dipole system.

Electric potential

When we discussed the gravitational force we saw how we could define gravitational potential energy due to this force being a conservative force (= the work down by the force was independent of the path taken). It turns out that the Coulomb force is also a conservative force so we can likewise define **electrical potential energy**. It's not very hard to understand: When you push two protons together you have to do positive work on them (since they repel), and this positive work is stored as electrical potential energy between them. If you let go, this potential energy will be converted into kinetic energy as they fly apart. Likewise, when you bring an electron closer to a proton, you have to do negative work on it to prevent it from flying towards the proton (your force holds it back), hence electrical potential energy is being removed from the system.[7]

We will discuss electrical potential energy in greater detail at a later time, but here we will define the concept of **potential difference** (or **voltage**) so that we can more easily understand electrical circuits and other simple electric effects. As with gravitational potential energy, the actual value of the electric potential energy at a given point is less important than the *change* in potential energy between two points. Let's consider a positive charge, q, that moves between two points in an electric field. The electric force that acts on this charge will do work on it, W, and the **potential difference**, ΔV, (or the **voltage**, V) between these two points[8], is per definition this *work done by the electric force per unit charge*:

$$\Delta V \equiv \frac{W}{q} \quad \text{or} \quad V \equiv \frac{W}{q}$$

This definition is often expressed in terms of the work done as

$$W = q\Delta V \quad \Rightarrow \quad W = qV$$

The unit of potential difference is J/C which we also define as one **volt**[9], V, so

$$\text{volt} \equiv \frac{\text{joule}}{\text{coulomb}} \quad \Rightarrow \quad \mathrm{V} \equiv \frac{\mathrm{J}}{\mathrm{C}} = \mathrm{J\,C^{-1}}$$

For the time being, don't worry too much about the sign of the potential difference (remember that work, W, can be positive or negative, as can the charge q), but when that becomes relevant we will address it. At this moment, you should just be able to understand statements such as: If a charge of $q = 2\,\mathrm{C}$ is moved across a potential difference of $\Delta V = 9\,\mathrm{V}$, then the work done on it is

$$W = q\Delta V = (2\,\mathrm{C}) \cdot (9\,\mathrm{V}) = 18\,\mathrm{J}$$

All depending on the direction in which the charge moves between the two points (and the sign of the charge) this could be energy lost or energy gained, but we won't worry too much about that right now.

If a battery is said to have a voltage of $V = 1.5$ V, it simply means there is a potential difference of 1.5 V between the positive terminal (+) and negative terminal (−). This furthermore means that whenever 1 C of charge travels between the two terminals (which it can only do through an external circuit attached to the battery) then it delivers 1.5 J of energy to the circuit.

[7] In atomic physics we will learn that as electrons move closer to the nucleus they go from a higher energy state to a lower energy state, and the release of this energy is on the form of an electromagnetic wave (which can also be thought of as a particle of electromagnetic energy called a **photon**). What prevents the electron from falling completely into the proton? The laws of quantum mechanics as you will learn another day!

[8] Physicists tend to be a bit sloppy here and the notations ΔV and V are commonly used interchangeably.

[9] Named after Alessandro Volta (1745–1827) who was an Italian physicist, chemist, and pioneer of electricity. He is credited as the inventor of the electric battery (the voltaic pile) in 1799 and the discoverer of methane.

Maxwell's equations

In this physics course, we will only be scratching the surface of the vast field of electromagnetism. At the heart of this fascinating (and extremely useful) area of physics lies Maxwell's four equations which are

> "... a set of coupled partial differential equations that, together with the Lorentz force law, form the foundation of classical electromagnetism, classical optics, and electric circuits. The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc. They describe how electric and magnetic fields are generated by charges, currents, and changes of the fields. The equations are named after the physicist and mathematician James Clerk Maxwell, who, in 1861 and 1862, published an early form of the equations that included the Lorentz force law. Maxwell first used the equations to propose that light is an electromagnetic phenomenon." (Wikipedia)

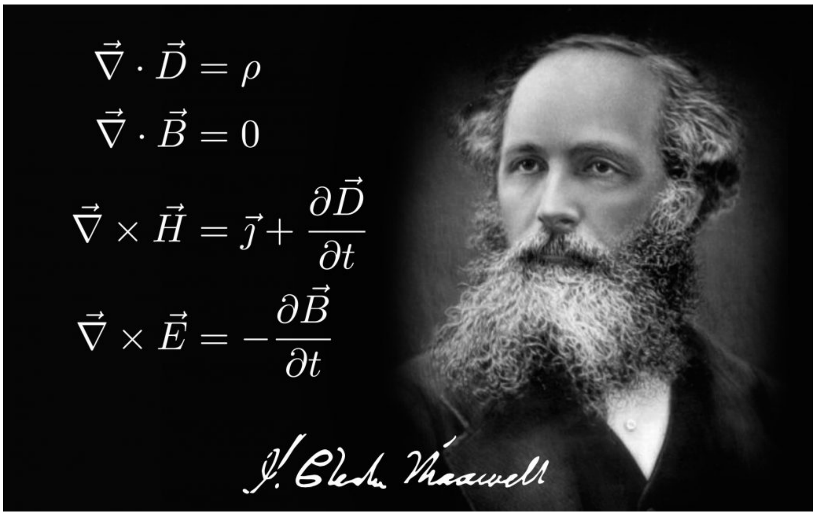
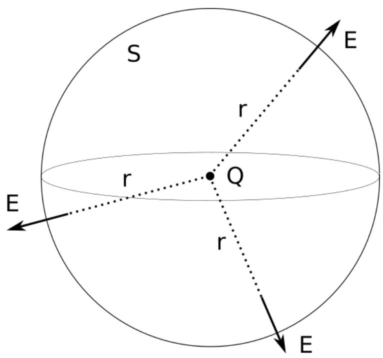


Figure 7: Maxwell's four equations in differential form and applied to ordinary matter. The first equation is referred to as **Gauss' law** and the Coulomb force law is a consequence of it. The second equation is referred to as **Gauss' law for magnetism** and it expresses the fact that there is no such thing as a "magnetic charge". The third equation on this list is often referred to as **Ampere's circuit law** and it expresses the fact that magnetic fields can be generated by currents or varying electric fields. The final equation is often referred to as **Faraday's law of induction** and this law describes how a changing magnetic field produces an electric field (this is how we produce electricity in most power plants).

Everything you can think of that is related to electromagnetism (including optical phenomena because light is an electromagnetic wave) can be derived from these equations. For example, Maxwell's first equation (also called Gauss' law) states that *the flux of the electric field through any given closed surface, S, is proportional to the net charge inside the surface*[10]. In mathematical terms this can be expressed as (the left-hand side is called a surface integral and ϵ_0 is the vacuum permittivity (see exercise 13))

$$\oiint_S \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\epsilon_0}$$

[10] This is the integral form of the first equation and we are assuming the charge is in a vacuum.



Consider the simplest possible example, a point charge Q enclosed in and placed at the center of a spherical surface, S, of radius r. Due to the spherical symmetry of the arrangement, the E-field at any point on the surface will be pointing in the radial direction and have a constant magnitude E. The surface area of the sphere is $4\pi r^2$, so Gauss' law reduces to (the constant E is taken outside of the integration and the integral is simply the surface area of a sphere)

$$\oiint_S \vec{E} \cdot \mathrm{d}\vec{A} = |\vec{E}| \oiint_S \mathrm{d}\vec{A} = |\vec{E}| 4\pi r^2 = \frac{Q}{\epsilon_0}$$

which then becomes the well-known expression for the E-field surrounding a point charge:

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

The above example shows how a special case (the electric field around a point charge) can be derived from Maxwell's general equations (specifically just the first one). The entire field of electromagnetism is really just that: An application of Maxwell's equations to many special cases. In the following I will discuss a number of other results that are good to know. Don't worry if you don't understand all the details, but try to memorize the results as students are often assumed to be familiar with them. The justifications can all be found in The Feynman Lectures on Physics, Volume II (chapters 4 & 5) and that is also where most of the quotes are from (in some of the quotes, I've added a few words in square brackets [...]).

Result #1: The electric field surrounding a uniformly charged sphere or a thin, charged spherical shell is the same as a point charge

The proof of this is very similar to what we saw above: Simply enclose the uniformly charged sphere or the thin spherical shell in a slightly larger spherical surface and Gauss' law will give the result.

Result #2: The electric field inside a conductor is zero

> "An electrical conductor is a solid that contains many 'free' electrons. The electrons can move around freely in the material, but cannot leave the surface. In a metal there are so many free electrons that any [external] electric field [applied to it] will set large numbers of them into motion. Either the current of electrons so set up must be continually kept moving by external sources of energy [e.g. a battery], or the motion of the electrons will cease as they discharge the sources producing the initial field [or create new electric fields that balance out the external field]. In "electrostatic" situations, we do not consider continuous sources of current (they will be considered later when we study magnetostatics), so the electrons move only until they have arranged themselves to produce zero electric field everywhere inside the conductor. (This usually happens in a small fraction of a second.) If there were any field left, this field would urge still more electrons to move; the only electrostatic solution is that the field is everywhere zero inside."

Result #3: The charge on a charged conductor resides on the outer surface.

> "What do we mean when we say a conductor is "charged"? Where are the charges? The answer is that they reside at the surface of the conductor, where there are strong forces to keep them from leaving—they are not completely 'free'. When we study solid-state physics, we shall find that the excess charge of any conductor is on the average within one or two atomic layers of the surface. For our present purposes, it is accurate enough to say that if any charge is put on, or in, a conductor it all accumulates on the surface; there is no [net] charge in the interior of a conductor."

Result #4: The electric field just outside a charged conductor is perpendicular to the surface of a conductor.

> "We note also that the electric field just outside the surface of a conductor must be normal to the surface. There can be no tangential component. If there were a tangential component, the electrons would move along the surface; there are no forces preventing that. Saying it another way: we know that the electric field lines must always go at right angles to an equipotential surface."

The field just outside the surface is given by the formula

$$E = \frac{\sigma}{\epsilon_0}$$

where σ is the local surface charge density (number of coulombs per square meter) and ϵ_0 is the vacuum permittivity (see exercise 13).

Result #5: The surface of a conductor is at the same potential everywhere (it is an equipotential surface)

This follows from result #4 since the motion of a charge along the surface will be perpendicular to the E-field, hence the electric force will do zero work on the charge and the potential is therefore unchanged.

Result #6: The electric field inside a cavity of a conductor is zero and no charges can reside on the inside surface of the cavity.

> "We return now to the problem of the hollow container—a conductor with a cavity. There is no field in the metal, but what about in the cavity? We shall show that if the cavity is empty then there are no fields in it, no matter what the shape of the conductor or the cavity. [. . .] This explains the principle of "shielding" electrical equipment by placing it in a metal can."

The "shielding" mentioned above is the principle behind a "Faraday cage". A Faraday cage operates because an external electrical field causes the electric charges within the cage's conducting material to be distributed so that they cancel the field's effect in the cage's interior. Here's a nice animation that illustrates this phenomenon.

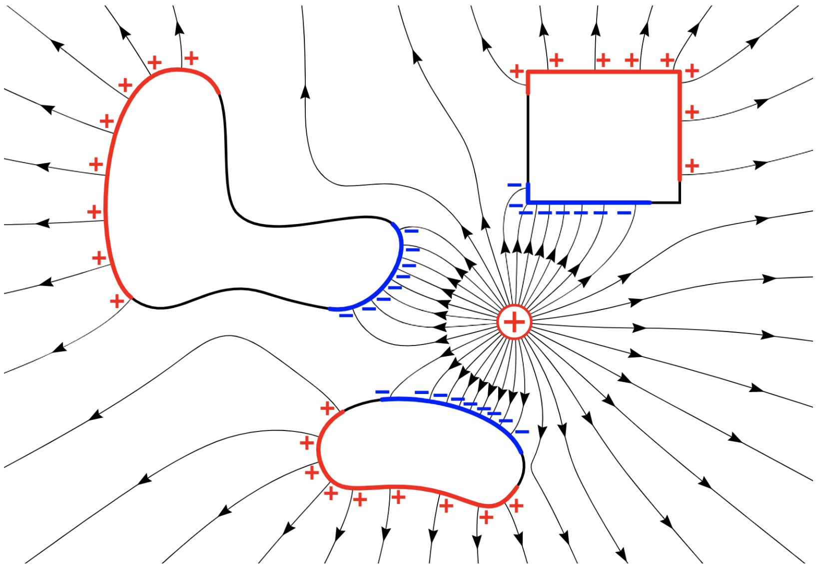


Figure 8: Here's a nice picture demonstrating many of the results mentioned here. You can imagine the three white shapes being solid (or hollow) pieces of metal (conductors). Notice how there is no electric field in their interiors and the field lines are perpendicular their surfaces.

Result #7: The electric field between two charged sheets of metal.

> "The problem of two parallel sheets with equal and opposite charge densities, $+\sigma$ and $-\sigma$, is equally simple if we assume again that the outside world is quite symmetric. Either by superposing two solutions for a single sheet or by constructing a Gaussian box that includes both sheets, it is easily seen that the field is zero outside of the two sheets. By considering a box that includes only one surface or the other, as in (b) or (c) of the figure, it can be seen that the field between the sheets must be twice what it is for a single sheet."

The result is

$$\begin{aligned} E \text{ (between the sheets)} &= \frac{\sigma}{\epsilon_0} \\ E \text{ (outside the sheets)} &= 0 \end{aligned}$$

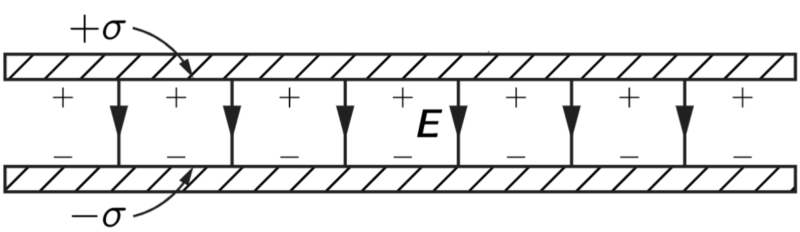


Figure 9: The electric field between two parallel sheets of opposite charge. We will return to this important system later.

If you want to dive more into the complex and fascinating world of electromagnetism, then I can highly recommend The Feynman Lecture os on Physics, Volume II combined with the book by Purcell and Morin (these books are written by Nobel-prize winning physicists, so worth reading!).

Lesson 5: Exercises

1. Two small equally sized charged metal spheres hanging from insulating threads are given charges $-1.0\,\mu$C and $3.0\,\mu$C respectively. Together they make up an isolated system.

   (a) What is the total amount of charge?

   (b) At first they are brought close together without touching. What happens? Estimate the amount of force they exert on each other when they are 10 cm apart.

   They now touch each other and the total charge of the system distributes itself evenly on both spheres (the electrons move around and settle in equilibrium positions).

   (c) How much charge will there be on each sphere after they touch?

   (d) What will happen after this charge redistribution? Estimate the amount of force they now exert on each other when they are 10 cm apart.

2. I have two mystery charges. When placed 10^{-7} m apart, they experience a repulsive force of 5.0×10^{-4} N. What will the force between them be when they are 4.0×10^{-7} m apart?

3. (a) Calculate the force between two charges $q_1 = 2.0\,\mu$C and $q_2 = 4.0\,\mu$C separated by $r = 5.0$ cm.

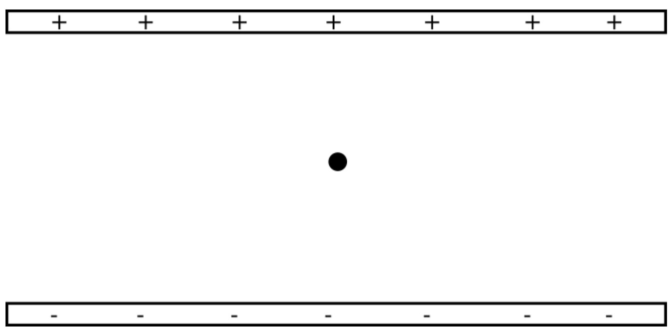
   (b) Let the force in (a) be F. In terms of F and without further calculations, state the force between these charges when:

      i. the separation r of the charges is doubled

      ii. q_1 and r are both doubled

      iii. q_1, q_2 and r are all doubled

4. Two charged metal plates 5 cm apart produce a constant electric field between them. The strength of the field is $E = 600$ N/C. A proton is placed at rest in the middle between the two plates.



   (a) Draw electric field lines in the above diagram.

   (b) Calculate the electric force on the proton and indicate in which direction the force is pointing on the above diagram.

(c) The mass of the proton is 1.67×10^{-27} kg.

i. Calculate the magnitude of the gravitational force on the proton assuming it is close to the surface of the Earth.

ii. How does this gravitational force compare in size with the electric force?

iii. What can you conclude from (ii)?

(d) Calculate the acceleration of the proton.

(e) How long does it take the proton to reach the negative plate?

(f) With what speed does it hit the negative plate? Compare this speed with the speed of light, $c \approx 3 \times 10^8$ m/s.

5. If a proton moves in the direction of the electric force across a potential difference of 1 V, then how much work was done by the electric force? This amount of energy is also called one **electronvolt** (1 eV) and it is a commonly used unit of energy in atomic and nuclear physics. What is the speed of an electron with an energy of 1 eV?

6. Another commonly used unit of electric field strength is V/m (volts per meter). Using the definition of a volt (V) and a joule (J), show that this unit is equivalent to N/C.

7. Although air is normally an excellent insulator, when exposed to strong electric fields it can undergo an 'electrical breakdown' and become partially conductive. The strong electric field ionizes the air molecules by ripping electrons off them and the charges then accelerate up to high speeds. As the particles bump into each other they create heat and light which often culminates in a spark. In order to ionize air molecules and create a spark, you need an electric field strength of around 3.0×10^6 N/C.

(a) How much electric force is experienced by an electron in this field?

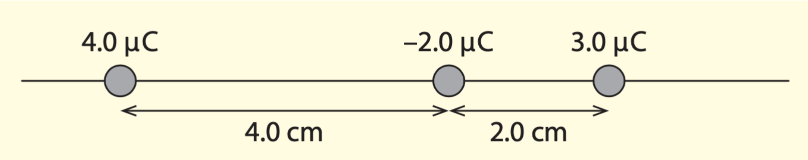
(b) What is the acceleration of the electron in this case?

(c) How long does it take to travel 10 cm if this acceleration is assumed constant?

(d) A field strength of 3×10^6 N/C can also be expressed as 3×10^6 V/m which means that there is a potential difference of three million volts per meter. If we assume this field strength is constant, what is the potential difference across half a meter? Or 10 cm? Or 1 cm?

(e) A Van de Graaff generator is a machine that can create very high electric potential differences. It was invented by the American physicist Robert J. Van de Graaff in 1929. I'll demo one in class and we'll use the length of the sparks to estimate how large a potential difference it is creating!

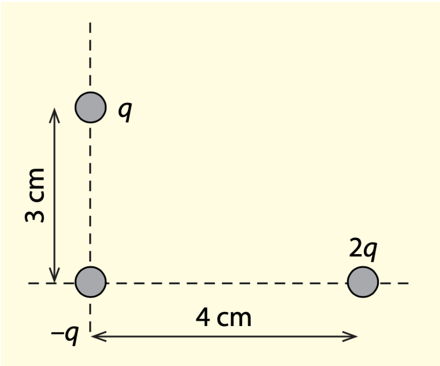
8. *Activity.* Follow the instructions on this activity sheet.



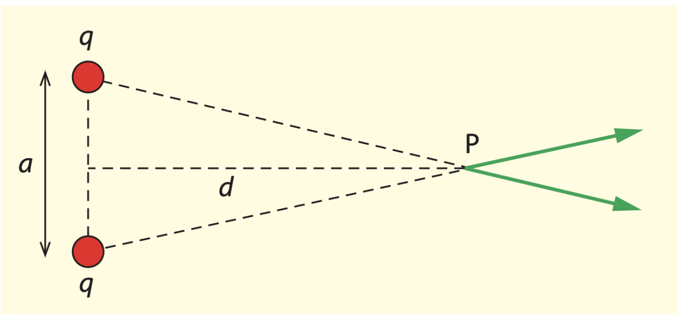
9. Three charges are placed on a straight line as shown in the above diagram. Calculate the net force on the middle charge.

10. In the previous question, determine the position of the middle charge so that it is in equilibrium.

11. Calculate the force (magnitude and direction) on the charge q in the below diagram where $q = 3.0\,\mu$C.



12. The electric field strength is a vector quantity and so two electric field strength vectors at the same point in space must be added according to the laws of vector addition. Consider two equal positive charges, $q = 2.00\,\mu$C, separated by $a = 10.0\,$cm and a point P a distance $d = 30.0\,$cm away as shown in the below diagram. The diagram shows the two electric field strength vectors produced at P by each charge. Determine the magnitude and direction of the net electric field at P.



13. Coulomb's constant, k, is in fact given in terms of another constant, ϵ_0, called the **vacuum permittivity**. The relationship is

$$k = \frac{1}{4\pi\epsilon_0}$$

and the value of ϵ_0 is approximately (don't worry about the unit, you will learn later what 'F' stands for)

$$\epsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$$

Coulomb's law can thus be expressed as

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \quad \text{(in empty space)}$$

The above equation gives the electric force between two charges at rest in empty space (*in a vacuum*), but when the charges are embedded in matter (which they very often are), then the electric force decreases and we can express it by writing

$$F = \frac{1}{4\pi\epsilon_r}\frac{|q_1 q_2|}{r^2}$$

where ϵ_r is the **relative permittivity** for the particular substance that the charges are embedded in.

For water at 50°C, the relative permittivity is around $\epsilon_r = 70$. How much smaller is the electric force between two charges in water?

14. *Above and beyond.* Estimate how many electrons there are in a human body. First do a rough estimate using only values to the nearest power of ten. After that you can do a more accurate calculation using the facts given on this page.

Answers to all the exercises.

Lesson 5 Quiz

Check your understanding of this lesson: Here is a quiz.