

A little book about matter

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Lesson 4: Phase changes



Figure 1: A beautiful winter landscape with H_2O present in all three states of matter: Solid (ice), liquid (water), and gas (steam).

Specific heat capacity

We saw in the last lesson that a gas can be thought of as a collection of particles moving around in random directions and not experiencing any forces except when they elastically collide with another particle or the container walls. The energy in such a system is purely kinetic and we used this to define temperature as a measure of the average kinetic energy per particle.

When we add more energy, ΔE , to a fixed amount of gas (e.g. by heating it) the average kinetic energy of the particles increases, hence the temperature increases. If we remove energy from a fixed amount of gas (e.g. by cooling it), the average kinetic energy of the particles decreases, hence the temperature decreases. It is an experimental fact that this temperature change is *directly proportional to the amount of energy transferred and inversely proportional to the mass of the gas*¹:

$$\Delta T \propto \frac{\Delta E}{m}$$

Another way of expressing this is that the energy change is proportional to the mass and the temperature change

$$\Delta E = cm\Delta T \quad (1)$$

Here we have written the constant of proportionality as c . This constant is called the **specific heat capacity** and it represents *the amount of energy required per unit mass to change the temperature 1°C* . When you express c as

$$c = \frac{\Delta E}{m\Delta T}$$

¹ This makes perfect sense: The more energy you add the more the temperature will rise and the more particles you have the less the temperature will rise because there are more particles sharing the energy.

it's easy to see that the units of c could be

$$\frac{\text{J}}{\text{g } ^\circ\text{C}} = \text{J g}^{-1} ^\circ\text{C}^{-1} = \frac{\text{J}}{\text{g K}} = \text{J g}^{-1} \text{K}^{-1}$$

Notice how K and $^\circ\text{C}$ are interchangeable here because we are only dealing with temperature *changes*, not absolute values: A temperature change of $\Delta T = 1 \text{ K}$ is the same as a temperature change of $\Delta T = 1 ^\circ\text{C}$.

For example, the specific heat capacity of "steam" (= water vapour, gaseous H_2O) is

$$c_{\text{H}_2\text{O, gas}} = 2.1 \frac{\text{J}}{\text{g K}}$$

which means it requires 2.1 J of energy to change the temperature of 1 g of steam by 1 K (or $1 ^\circ\text{C}$). Pay good attention to the units, e.g. I could also have expressed this as

$$c_{\text{H}_2\text{O, gas}} = 2.1 \times 10^3 \frac{\text{J}}{\text{kg K}}$$

which says it requires $2.1 \times 10^3 \text{ J} = 2.1 \text{ kJ}$ of energy to change the temperature of 1 kg of steam by 1 K (or $1 ^\circ\text{C}$). A common mistake is to not pay close enough attention to these units.

Equation (1) is often used in the following way: Say we want to raise the temperature of 10 grams of steam² from $100 ^\circ\text{C}$ to $250 ^\circ\text{C}$. This is a temperature change of $\Delta T = 150 \text{ K}$ and it will therefore require the following amount of energy³:

$$\Delta E = \left(2.1 \frac{\text{J}}{\text{g K}} \right) \cdot 10 \text{ g} \cdot 150 \text{ K} \approx 3.2 \text{ kJ}$$

In a temperature vs. energy diagram, such changes are represented by straight lines as shown in figure 2: Using equation (1) we can express *the rate of change in temperature with energy*, $\Delta T / \Delta E$, as

$$\Delta E = cm\Delta T \quad \Rightarrow \quad \frac{\Delta T}{\Delta E} = \frac{1}{cm}$$

and this shows that for a given amount of mass (when m is constant), the graph is a straight line with a slope equal to $1/cm$.

Water (H_2O in its liquid state) has a specific heat capacity that is around twice that of water vapour (this is one of the characteristic properties of water)

$$c_{\text{H}_2\text{O, liquid}} = 4.2 \frac{\text{kJ}}{\text{kg K}}$$

hence the rate of change of temperature with energy

$$\frac{\Delta T}{\Delta E} = \frac{1}{cm} \approx 0.02 ^\circ\text{C/J}$$

is smaller than for vapour, and the slope of the straight line is therefore less steep as shown in figure 3.

Substance	$c / \text{J g}^{-1} \text{K}^{-1}$
Lead	0.13
Zinc	0.38
Copper	0.39
Iron	0.41
Aluminium	0.92
Sodium	1.2
Wax	2.5
Animal tissue	3.5
Solid H_2O (ice)	2.1
Liquid H_2O (water)	4.2
Gaseous H_2O (steam)	2.1

Table 1: Some specific heat capacities.

² What is the volume of 10 grams of steam at $100 ^\circ\text{C}$ and 1 atm?

$$V = \frac{10/18 \cdot 8.31 \cdot 373}{10^5} \approx 17 \text{ L}$$

³ Again, notice how K and $^\circ\text{C}$ are interchangeable because we are only working with temperature *changes*, not absolute values.

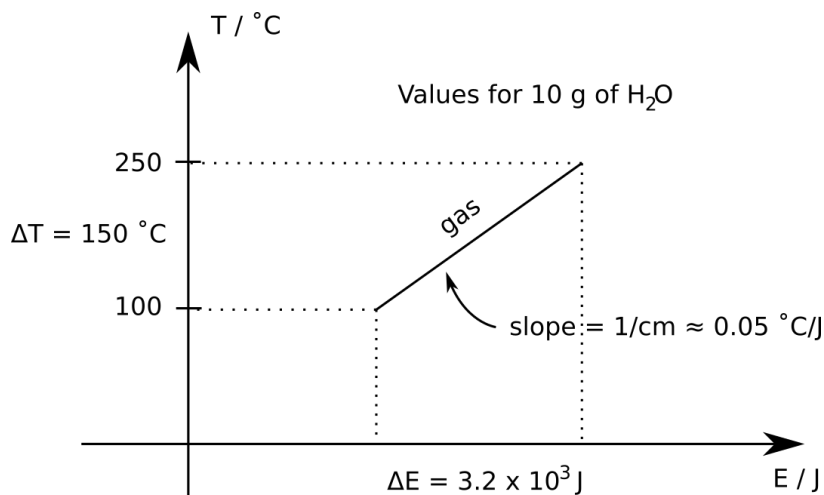


Figure 2: Here a small amount of steam, $m = 10 \text{ g}$, is being heated 150 K . When substances are heated they often undergo a temperature change that is proportional to the energy change. As we will see later, this is only true when there are no phase changes occurring.

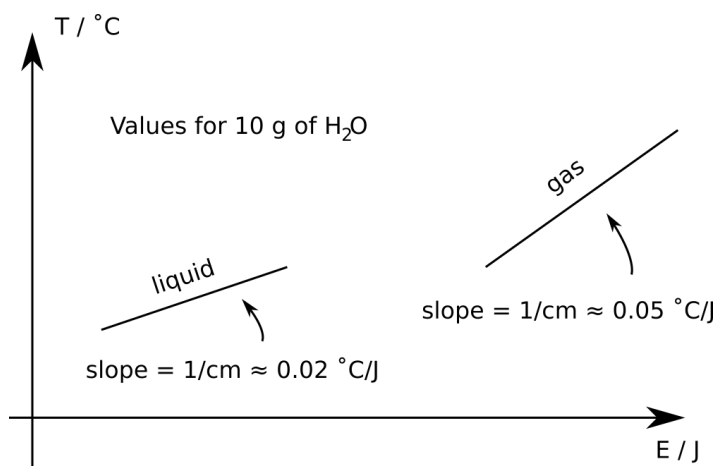


Figure 3: The temperature increase per unit of energy is less for liquid H_2O than for gaseous H_2O , hence the rate of change of temperature with energy (the slope of the line here) is less. This is because the specific heat capacity of water is larger and the slope of the line is $1/\text{cm}$.

Latent heat

If we remove enough energy from a gas (and hence lower the temperature) it will start to **condense** into a liquid. Likewise, if we add enough energy to a liquid it will start **vaporising (boiling)** and turn into a gas. This is called a **phase change**. For example liquid water at normal atmospheric pressure starts boiling at $100 ^\circ\text{C}$. *It is an experimental fact that even though more energy is added during a phase change, the temperature remains constant.* This suggests that the average kinetic energy per particle remains constant during a phase change. But since energy is still being added, where does that energy go? This "**latent**" (= hidden) energy turns out to be in the form of **potential energy** associated with intermolecular forces that are present in liquid and solid states of matter: When a liquid turns into a gas, the added energy increases the potential energy rather than the kinetic energy. And vice versa, when a gas condenses into a liquid, the energy removed is potential energy, not kinetic energy.

In the liquid and solid state particles are much closer together and they don't move around as fast as in the gas state. Hence the particles "tug" in each other and these intermolecular forces have

potential energy associated with them. You can think of these forces as being tiny springs that connect the particles: In solids the springs are very tight and they hold the particles in fixed positions around which they vibrate (see figure 4). These vibrations still contain kinetic energy, hence solids also have a temperature defined by the average kinetic energy per particle. In liquids the springs are stretched more and they occasionally break and disappear to only reappear again later, hence particles can move around more freely in a liquid but they are still attracted to each other⁴ (and again, there is kinetic energy associated with this motion). In the gas state the springs have completely broken and we define the potential energy in this state to be zero (hence in the liquid and solid states the potential energy is negative, but you don't need to worry about that in this course). [Here a reasonably nice YouTube video](#) going over the three different states of matter in a good amount of detail.

In order to include the intermolecular potential energy into our description, we define the **internal energy**, U , of a system of particles as *the sum of the total (random) kinetic energy and total potential energy of the particles*:

$$U = \text{total KE} + \text{total PE (of all particles)}$$

The amount of energy per unit mass required to change a liquid into a gas is called the **latent heat of vaporisation**, L_v , and it is defined as

$$L_v = \frac{\Delta E}{m} \quad (\text{liquid} \rightarrow \text{gas})$$

For example, the latent heat of vaporisation of H_2O is

$$L_{v,\text{H}_2\text{O}} = 2.3 \cdot 10^3 \frac{\text{kJ}}{\text{kg}}$$

which means it takes $2.3 \cdot 10^3 \text{ kJ}$ of energy to convert 1 kg of water (liquid H_2O) into 1 kg of steam (gaseous H_2O). During this phase change, all the energy is converted into an increase in potential energy of the system, hence the temperature (the average kinetic energy of the particles) remains the same.

A similar phase change occurs when ice (solid H_2O) **melts** into water or when water **freezes** into ice. The latent heat during this phase change is referred to as the **latent heat of fusion**, L_f , and it is the amount of energy per unit mass required to change a solid into a liquid:

$$L_f = \frac{\Delta E}{m} \quad (\text{solid} \rightarrow \text{liquid})$$

For example, the latent heat of fusion of H_2O is

$$L_{f,\text{H}_2\text{O}} = 333 \frac{\text{kJ}}{\text{kg}}$$

Figure 5 shows how the temperature changes with energy for 10 g of H_2O that starts off being solid at -18°C and ends up being a gas at 110°C .⁵

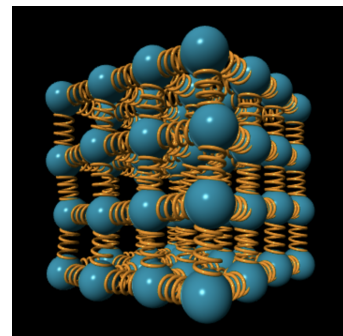


Figure 4: A simple model of what solids look like at the atomic level.

⁴ This explains e.g. surface tension in liquids.

⁵ [Here is the corresponding desmos page.](#)

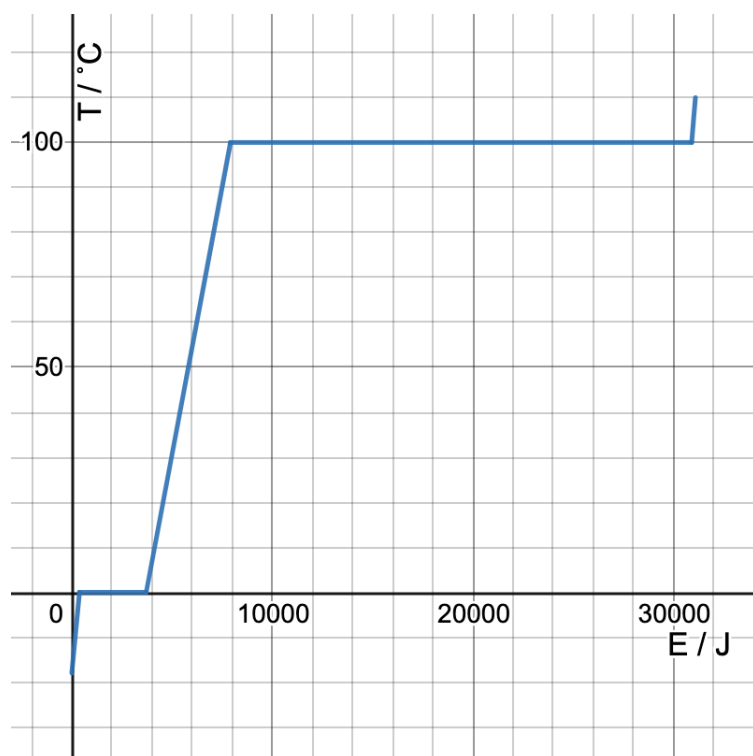


Figure 5: A graph showing how the temperature changes with energy for 10 g of H_2O that starts off being solid at -18°C and ends up being a gas at 110°C

Calorimetry

The word "calorimetry" is derived from the Latin word "calor", meaning heat and the Greek word "metron", meaning measure. It's the word we normally use when describing experiments that measure specific heat capacities or latent heats. Let's look at an example: Say we want to measure the specific heat capacity of copper, c . Heat up a piece of copper of known mass m_1 to a known temperature T_1 . Now place the copper in a body of isolated water of mass m_2 with a lower temperature T_2 . Water has the specific heat capacity c_w . As the hot copper comes into contact with the colder water, energy will flow from the hot copper to the colder water⁶. The energy flow will stop when they reach thermal equilibrium and have a common temperature T . The energy lost by the metal is the amount given by the expression

$$cm_1(T_1 - T)$$

while the energy gained by the water is given by the expression

$$c_w m_2 (T - T_2)$$

If we assume all energy leaving the copper enters the water (hence no energy has been transferred to the surroundings), then we can equate the above two expressions to get

$$cm_1(T_1 - T) = c_w m_2 (T - T_2) \Rightarrow c = \frac{c_w m_2 (T - T_2)}{m_1 (T_1 - T)}$$

Here are some values from an experiment I did:

⁶ What we really mean, of course, is that a transfer of microscopic kinetic energy occurs between the rapidly vibrating copper atoms and the slower moving water molecules.

Quantity	Measurement
Mass of copper	$m_1 = 146.41 \pm 0.01 \text{ g}$
Mass of water	$m_2 = 70.26 \pm 0.01 \text{ g}$
Temperature of copper	$T_1 = 95.3 \pm 0.1 \text{ }^\circ\text{C}$
Temperature of water	$T_2 = 22.2 \pm 0.1 \text{ }^\circ\text{C}$
Final temperature	$T = 32.8 \pm 0.1 \text{ }^\circ\text{C}$

which gives the following measured value of c :

$$c = \frac{c_w m_2 (T - T_2)}{m_1 (T_1 - T)} = \frac{(4.2)(70.26)(32.8 - 22.2)}{(146.41)(95.3 - 32.8)} \approx 0.34 \frac{\text{J}}{\text{g } ^\circ\text{C}}$$

Using error propagation, the uncertainty of this measurement comes out as (neglecting the uncertainty of the mass measurements)

$$\frac{\delta c}{c} = \frac{0.1}{4.2} + \frac{0.2}{10.6} + \frac{0.2}{62.5} \approx 0.02 + 0.09 + 0.02 = 5\%$$

which results in an absolute uncertainty of $\delta c \approx 0.02 \text{ J g}^{-1} \text{ K}^{-1}$ and the overall measurement can be stated as

$$c = 0.34 \pm 0.02 \frac{\text{J}}{\text{g } ^\circ\text{C}}$$

Comparing with the table value, $c_{\text{copper}} = 0.39 \text{ J g}^{-1} \text{ K}^{-1}$, we can see that we are not far off, but still an inaccurate measurement. It's a relatively precise measurement (percentage uncertainty around 5%) hence there must be other uncertainties in play. Indeed, there are plenty of other uncertainties to discuss in this experiment, e.g. one would expect the final temperature, T , to be slightly lower than expected due to minor heat losses to the surroundings. This would imply that the numerator $(T - T_2)$ is smaller than expected and the denominator $(T_1 - T)$ is bigger than expected, which in the end leads to a slightly smaller value of c than expected, which indeed is in agreement with the above measurements.

Lesson 4: Exercises

- Below is an excerpt from the IB DP Physics Data Booklet:

Sub-topic 3.1 – Thermal concepts
$Q = mc\Delta T$
$Q = mL$

What do you think Q stands for?

- An object of mass 0.150 kg has its temperature increased by $5.00 \text{ }^\circ\text{C}$ when 385 J of energy is added to it. Calculate the object's specific heat capacity.

3. (a) 50 g of water vapour at 100°C condenses into liquid water at 100°C , How much energy was released in that process?
 (b) 50 g of water vapour at 100°C condenses into liquid water that then cools down to 20°C , How much energy was released in that process?
4. Verify the graph in figure 5 by working out all the amounts of energy associated with each straight line segment. It shows the heating curve of 10 grams of H_2O starting at -18°C and ending up at 110°C .
5. A hot object is brought into contact with a colder object. Assume they make up an isolated system.
 - (a) Which object will lose energy and which one will gain energy?
 - (b) Discuss whether the energy lost by one body is equal to the energy gained by the other
 - (c) When will the energy flows stop? (They are then said to be in **thermal equilibrium**.)
 - (d) Discuss whether the temperature drop of one body will be equal to the temperature rise of the other.
6. A **calorimeter** is typically a metal cup that contains the water used in calorimetry experiments. Since the calorimeter is in thermal contact with the water, it heats up as the water heats up, so we also need to take that energy contribution into account.
 - (a) A calorimeter of mass 90 g and specific heat capacity $420 \frac{\text{J}}{\text{kg K}}$ contains 310 g of a liquid at 15.0°C . The calorimeter is in thermal equilibrium with the water. What is the temperature of the calorimeter?
 - (b) An electric heater warms the liquid at a rate of 20 W for 3.0 min. How much energy does the heater add to the liquid?
 - (c) The liquid heats up to 19.0°C . What temperature does the calorimeter heat up to?
 - (d) Assuming the energy from the heater only flows into the liquid and the calorimeter, calculate the specific heat capacity of the liquid.
7. The **heat capacity** of a system, C (upper case C), is defined as the amount of energy required to change the temperature of the system by 1 K. The unit of C is therefore typically given in J K^{-1} . Note that this is *not* a specific heat capacity as it is not a per mass quantity. Heat capacities are often used to describe systems consisting of different substances.
 - (a) A calorimeter cup is made of 45.0 g of copper. What is the heat capacity of the cup?

- (b) If a pure substance has a mass of m , what is the heat capacity of that mass expressed in terms of the specific heat capacity?
- (c) A calorimeter cup with heat capacity 25 J K^{-1} contains 140 g of a liquid. An immersion heater is used to provide energy at a rate of 40 W for a total time of 4.0 minutes. The temperature of the liquid increases by 15.8°C . Calculate the specific heat capacity of the liquid. State an assumption made in reaching this result.
8. A car of mass 1360 kg descends from a hill of height 86 m at a constant speed. Assuming that all of the gravitational potential energy lost by the car goes into heating the brakes, estimate the rise in the temperature of the brakes when the heat capacity of the brakes is listed as 16 kJ/K.
9. How much ice at -10°C must be dropped into a cup containing 300 g of water at 20°C in order for the temperature of the water to be reduced to 10°C ? The cup itself has a mass of 150 g and is made out of aluminium. Assume that no energy is lost to the surroundings.
10. A quantity of 100 g of ice at 0°C and 50 g steam at 100°C are added to a container that has 150 g water at 30°C . Determine the final temperature in the container. Ignore the container itself in your calculations.
11. Radiation from the Sun falls on the frozen surface of a pond at a rate of 600 W m^{-2} . The temperature of the ice is 0°C .
- (a) Calculate how long it will take to melt a 1.0 cm thick layer of ice. (Take the density of ice to be 900 kg m^{-3} .)
- (b) Suggest why the actual mass of ice that melts is less than your answer to (a).
12. *Demonstration: How hot is a bunsen burner flame?* In class I heated up a small iron cube ($m_1 = 7.66 \pm 0.01 \text{ g}$) in a bunsen burner flame for so long that we can assume it has the same temperature, T_1 , as the flame. I then put the iron cube in some water ($m_2 = 63.95 \pm 0.01 \text{ g}$, $T_2 = 22.6 \pm 0.1^\circ\text{C}$) and after a while I measured the final temperature of the mixture, $T = 34.0 \pm 0.1^\circ\text{C}$. Use this data to determine the temperature of the bunsen burner flame with uncertainty.

Answers to all the exercises.

Lesson 4 Quiz

Check your understanding of this lesson: [Here is a quiz.](#)