CMI Integration Bee Syllabus

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Please note that this syllabus is more of a guideline of content that will allow you to be able to solve each problem rather than a strict requirement for every problem – a lot of the time advanced techniques/special functions can be avoided with clever substitutions and tricks! We have not mentioned every trick in the book(or on the problem set).

1 Integration Techniques

You should be familiar with the integration techniques listed below. The items at the end will not be required in any form. They have been included to explain what is not expected of the participants.

- ✓ Everything which is on the JEE Advanced and ISI/CMI Mathematics entrance exam syllabus for Integration, including integration by substitution and integration by parts.
- ✓ Differentiation under the integral sign(Feynman Technique)

$$\frac{d}{dt}\left(\int_a^b f(x,t)dx\right) = \int_a^b \frac{\partial}{\partial t}(f(x,t))dx$$

- ✓ Differentiation under the integral sign general version

$$\frac{d}{dt}\left(\int_{a(t)}^{b(t)} f(x,t)dx\right) = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t}(f(x,t))dx + f(b(t),t)b'(t) - f(a(t),t)a'(t)$$

- ✓ The Weierstrass substitution, $t = \tan\left(\frac{x}{2}\right)$ (also known as t substitution) and other popular/standard substitutions.
- ✓ Infinite series and their use in evaluating integrals, swapping an integral and an infinite sum - issues of convergence won't be considered.
- ✓ The reflection property of integrals aka Kings Rule:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

- ✓ Odd and even functions and their use in evaluating integrals.
- ✓ Periodic functions and their use in evaluating integrals.
- ✓ Wallis' Integral

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx \quad m,n \in \mathbb{N}$$

$$= \frac{(m-1)!!(n-1)!!}{(m+n)!!}k \text{ where } k = \begin{cases} \frac{\pi}{2}, & \text{if } m \text{ and } n \text{ are even} \\ 1, & \text{otherwise} \end{cases}$$

- × Green's Theorem, Stokes' Theorem, the Divergence Theorem and other results from vector calculus.

2 Functions & Specific results

Some knowledge of the following special functions and more specific results may be required.

- ✓ The Gamma function,

$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$$

- ✓ The Beta function

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

- ✓ The Riemann zeta function,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \text{ for } s > 1$$

- ✓ The floor function $\lfloor x \rfloor$ which rounds down to the integer less than or equal to x.
- ✓ Useful infinite series, such as

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

,

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} = G \text{ (Catalan's Constant)}$$

- ✓ Euler Mascheroni constant γ

$$\gamma = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k} - \ln n$$

- ✓ Wallis' Product

$$\prod_{n=1}^{\infty} \frac{4n^2}{4n^2-1} = \frac{\pi}{2}$$