CHENNAI MATHEMATICAL INSTITUTE

Probability Theory, B.Sc. I, 2025, Jan-April

Mid-term Examination.

You can score a maximum of 100 marks, combined from both sections A and B. 12本十五十十五十3 Part A

The problems in this section are either already discussed in the class or a slight variation of that. Give the required proof completely. You can score up to a maximum of 45 marks from this section.

 \sim 1. Show that a non-negative integer valued random variable X satisfies

 $\frac{P(\{X=n+k/X>n\})}{P(X>n)} = P(\{X=n+k/X>n\}) = P(\{X=k\}), \ \forall k, n \ge 0$ if and only if it is geometric.

2. Show that a random variable X with hypergeometric distribution with parameter (N, n, k), can be written as sum of k dependent Bernoulli random variables. Using 15 that calculate the mean and variance of X.

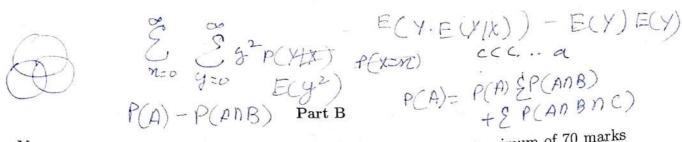
3. Let X be a random variable having a negative binomial distribution with parameters α and p. Derive the generating function of X. Find the mean and variance of X.

4. State and prove the strong law of large numbers.

15

5. A hen lays N eggs, where N has the Poison distribution with parameter λ . Each egg hatches with probability p independently of the other eggs. Let K be the number of chicks. Find the mass function of K. Also find E(K/N), E(K) and E(N/K).

 $\frac{2(N-K)!}{p(N)(K)} = \frac{p(K(N))p(N)!}{p(K)} = \frac{1}{p(K(N))} \frac{1}{p(K)} = \frac{1}{p(K(N))} \frac{1}{p(K)} = \frac{1}{p(K(N))} \frac{1}{p(K)} = \frac{1}{p(K(N))} \frac{1}{p(K)} = \frac{1}{p(K(N))} \frac{1}{p(K(N))} = \frac{1}$



You may use any result proved in the class. You can score up to a maximum of 70 marks from this section p-(n-1) q+(n-1) x from this section.

- a, Ca, cça, 1. Let $\Omega = \{a, b, c\}$ represents the sample space of a random experiment. Suppose we repeat this experiment indefinitely and independently. Calculate, in terms of $p = P(\{a\})$ and $q = P(\{b\})$, the probability that a occurs before b.
 - 2. Let $(\Omega, \mathcal{P}(\Omega), P)$ be a discrete probability space with $P(\{a\}) > 0$ for all $a \in \Omega$. Define $d(A, B) = P(A\Delta B)$ for $A, B \in \mathcal{P}(\Omega)$. Show that d is a metric on $\mathcal{P}(\Omega)$. \times
 - 3. Suppose it is given in a probability space that at least one, but no more than three, of the events A_r , $1 \le r \le n$, occur, where $n \ge 3$; the probability of at least two occurring is 1/2. Further if $P(A_r) = p$, $P(A_r \cap A_s) = q$, $r \neq s$ and $P(A_r \cap A_s \cap A_t) = x$, r < s < t, then show that $p \geq \frac{3}{2n}$ and $q \leq \frac{4}{n}$. np-(2)9+(n)n=1
 - 4. Let X and Y be discrete random variables with mean 0, variance 1 and covariance c. 20 Prove that $E(Max\{X^2, Y^2\}) \le 1 + \sqrt{1 - c^2}$.
 - 5. Define the conditional variance of Y given X by $Var(Y/X) = E((Y E(Y/X))^2/X)$. Show that Var(Y) = E(Var(Y/X)) + Var(E(Y/X)). 2EQ2 (X)

6 Let X and Y be independent random variables each having a geometric density with parameter p. Find E(Y|X+Y).

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