Matlab Project

Adrian Font
Andres Saladrigas

Functions Descriptions

all vectors are return or input in column form unless it specifies otherwise
all angles are in radians unless it specifies otherwise

1. **axisangle2matrix:**

Input:

This function takes a Euler axis and angle

Function Purpose and notes:

The vector first is normalized and reshaped. Using the Rodrigues Formula we obtain the Rotation matrix.

Notes:

- angles must be in radians
- vector is normalized inside the function

Output:

Returns a rotation matrix.

2. EulerAngle_to_Quat:

Input:

This function takes Euler angles

Function Purpose and notes:

Using the given formula we transform the angles to quaternions

$$
\mathbf{q}_{\mathrm{IB}} = \begin{bmatrix} \cos(\psi/2) \\ 0 \\ 0 \\ \sin(\psi/2) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ 0 \\ \sin(\theta/2) \\ 0 \end{bmatrix} \begin{bmatrix} \cos(\phi/2) \\ \sin(\phi/2) \\ 0 \\ 0 \end{bmatrix}
= \begin{bmatrix} \cos(\phi/2)\cos(\theta/2)\cos(\psi/2) + \sin(\phi/2)\sin(\theta/2)\sin(\psi/2) \\ \sin(\phi/2)\cos(\theta/2)\cos(\psi/2) - \cos(\phi/2)\sin(\theta/2)\sin(\psi/2) \\ \cos(\phi/2)\sin(\theta/2)\cos(\psi/2) + \sin(\phi/2)\cos(\theta/2)\sin(\psi/2) \\ \cos(\phi/2)\cos(\theta/2)\sin(\psi/2) - \sin(\phi/2)\sin(\theta/2)\cos(\psi/2) \end{bmatrix}
$$

Notes:

- angles must be in radians

Output:

Returns a quaternions

3. EulerAnglesToRotMat:

Input:

This function takes in Euler angles

Function Purpose and notes:

Using Composition of rotation

$$\mathbf{R} = \begin{pmatrix} \cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \cos\phi\sin\psi & \cos\psi\cos\psi\sin\theta + \sin\psi\sin\phi \\ \cos\theta\sin\psi & \sin\psi\sin\theta\sin\phi + \cos\phi\cos\psi & \sin\psi\sin\psi\cos\theta - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix}$$

Notes:

- angles must be in radians

Output:

Returns a rotation matrix.

4. Obt_RotVec:

Input:

It takes an axis of rotation a and angle

Function Purpose and notes:

The vector first is normalized. Make a Rotation Vector with information of an axis and an angle.

Notes:

- angles must be in radians
- vector is normalized inside the function

Output:

Returns a rotation vector

5. QuatMult:

Input:

Two quaternions

Function Purpose and notes:

Calculate the product of two quaternions

$$\mathring{q}\mathring{p} = \begin{pmatrix} q_0 p_0 - \boldsymbol{q}^\top \boldsymbol{p} \\ q_0 \boldsymbol{p} + p_0 \boldsymbol{q} + \boldsymbol{q} \times \boldsymbol{p} \end{pmatrix}$$

$$\mathring{q}\mathring{p} = \underbrace{\begin{pmatrix} q_0 & -\boldsymbol{q}^\top \\ \boldsymbol{q} & q_0 \mathbf{I}_3 + [\boldsymbol{q}]_\times \end{pmatrix}}_{\mathbf{Q}(\mathring{q})} \mathring{p} = \underbrace{\begin{pmatrix} p_0 & -\boldsymbol{p}^\top \\ \boldsymbol{p} & p_0 \mathbf{I}_3 - [\boldsymbol{p}]_\times \end{pmatrix}}_{\tilde{\mathbf{Q}}(\mathring{p})} \mathring{q}$$

Output:

Returns a new quaternion

6. RotMatToEulerAngles:

Input:

It takes a rotation matrix

Function Purpose and notes:

Using rotation out of the angle composition.

$$\mathbf{R} = \begin{pmatrix} \cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \cos\phi\sin\psi & \cos\psi\cos\psi\sin\theta + \sin\psi\sin\phi \\ \cos\theta\sin\psi & \sin\psi\sin\theta\sin\phi + \cos\phi\cos\psi & \sin\psi\sin\psi\cos\theta - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix}$$

with

$$r_{31} = -\sin\theta \rightarrow \theta = \arcsin(-r_{31})$$

with angle θ (pitch) known:

$$\phi = \text{arctan2}\left(\frac{r_{32}}{\cos\theta}, \frac{r_{33}}{\cos\theta}\right) \rightarrow \text{roll}$$

$$\psi = \text{arctan2}\left(\frac{r_{21}}{\cos\theta}, \frac{r_{11}}{\cos\theta}\right) \rightarrow \text{yaw}$$

Notes:

- checks determinant of rotation matrix == 1

<u>Output:</u>

Returns a set of Rotation Angles

7. RotMatToEulerAxis_Angle:

<u>Input:</u>

It takes a rotation matrix

<u>Function Purpose and notes:</u>

Using inverse mapping from rotation matrix

- $\text{trace}(\mathbf{R}) = 3\cos(\phi) + \left(u_1^2 + u_2^2 + u_3^2\right)(1 - \cos(\phi)) = 1 + 2\cos(\phi) \Rightarrow$

$$\phi = \arccos\left(\frac{\text{trace}(\mathbf{R}) - 1}{2}\right) \rightarrow \text{Euler's angle } \phi$$

- $\mathbf{R} - \mathbf{R}^\intercal = 2\,[\boldsymbol{u}]_\times \sin(\phi)$

$$[\boldsymbol{u}]_\times = \frac{\mathbf{R} - \mathbf{R}^\intercal}{2\sin(\phi)} \rightarrow \text{Euler's axis } \boldsymbol{u}$$

Notes:

- checks determinant of the rotation matrix

Output:

Returns a Euler axis and Euler angle

8. RotVec:

Input:

It takes a radius and two coordinates of the plane (x and y)

Function Purpose and notes:

We project a vector into the rotating sphere from the x and y position to a point of the sphere. using later the vector to calculate the motion of the mouse pointer.

Output:

Returns a rotation vector

9. TwoVec_To_Quat:

Input:

two columns vector

Function Purpose and notes:

quaternion formula, from two vectors to quaternion.

Output:

Returns a quaternion

Quat_To_RotMat:

Input:

Quaternion

Function Purpose and notes:

Calculate the rotation matrix from a quaternion. using a formula

Output:

Returns a rotation matrix