Title by Author

Noah Lewis

September 9, 2025

Contents

1 Mathematical Showcase

Theorem 1 (Pythagoras)**.** *For any right triangle with legs a, b and hypotenuse c,*

$$a^2 + b^2 = c^2.$$

Proof. Consider a right triangle with sides a, b, and c. By construction of squares on each side, the areas satisfy

$$\text{Area of square on } a + \text{Area of square on } b = \text{Area of square on } c,$$

hence $a^2 + b^2 = c^2$. ∎

Theorem 2 (Sum of the First n Natural Numbers)**.**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Proof. Consider the sum $S = 1 + 2 + \cdots + n$. Pair terms from opposite ends: $(1 + n), (2 + n - 1), \ldots$. Each pair sums to $n + 1$, and there are $n/2$ pairs. Thus

$$S = \frac{n(n+1)}{2}.$$

∎

Theorem 3 (Integral Example)**.**

$$\int_0^1 x^2 \, dx = \frac{1}{3}.$$

Proof.

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}.$$

∎

Theorem 4 (Geometric Series)**.** *For* $|r| < 1$,

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}.$$

Proof. Let $S = 1 + r + r^2 + \cdots$. Then $rS = r + r^2 + r^3 + \cdots$, so

$$S - rS = 1 \implies S(1-r) = 1 \implies S = \frac{1}{1-r}.$$

∎

Theorem 5 (Quadratic Formula)**.** *For* $ax^2 + bx + c = 0$, *the solutions are*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Proof. Complete the square:

$$ax^2 + bx + c = 0 \implies x^2 + \frac{b}{a}x + \frac{c}{a} = 0,$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2},$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

∎

2 Matrices and Determinants

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 5 & 6 & 0 \end{bmatrix}, \quad \det(A) = 1(-1 \cdot 0 - 4 \cdot 6) - 2(0 \cdot 0 - 4 \cdot 5) + 3(0 \cdot 6 - (-1) \cdot 5) = 9$$

3 Multivariable Calculus

$$f(x, y) = x^2 y + e^y \sin(x), \quad \frac{\partial f}{\partial x} = 2xy + e^y \cos(x), \quad \frac{\partial f}{\partial y} = x^2 + e^y \sin(x)$$

$$\iint_{[0,1]^2} (x + y)\,dx\,dy = \int_0^1 \int_0^1 (x + y) dx\,dy = \int_0^1 \left[\frac{x^2}{2} + xy\right]_0^1 dy = \int_0^1 \left(\frac{1}{2} + y\right) dy = 1$$

4 Limits and Series

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\prod_{k=1}^{\infty}\left(1+\frac{1}{k^2}\right) = \frac{\sinh(\pi)}{\pi}$$