Title by Author

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Contents

1 Ordered Sets

Problem 5

Prove that the ordered set $\sum^{**}$ of all binary strings is a **tree** (that is, an ordered set P with $\bot$ such that $\downarrow x$ is a chain for each $x \in P$). For each $u \in \sum^{**}$ describe the set of elements covering u.

Problem 7

Let P and Q be ordered sets. Prove that $(a_1, b_1) \prec (a_2, b_2)$ in $P \times Q$ if and only if $(a_1 = a_2 \ \& \ b_1 \prec b_2)$ or $(a_1 \prec a_2 \ \& \ b_1 = b_2)$.

Problem 8

Draw the diagrams of the products shown in Figure 1.12.

Problem 13

Draw and label a diagram for $\mathcal{O}(P)$ for each of the ordered sets P of Figure 1.13.
FIXME DRAW

Problem 14

Let P be a finite ordered set.
(i) Show that $Q = \downarrow MaxQ$ for all $Q \in \mathcal{O}(P)$.
(ii) Establish a one-to-one correspondence between the elements of $\mathcal{O}(P)$ and antichains in P.
(iii) Hence show that, for all $x \in P$, $|\mathcal{O}(P)| = |\mathcal{O}(P \setminus \{x\}) + |\mathcal{O}(P \setminus (\downarrow x \cup \uparrow x))|$.

Problem 17

Problem 24

Problem 25

Problem 26

Problem 27