

A Radical Approach to Real Analysis by David M. Bressoud

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January 9, 2026

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1 Crisis in Mathematics: Fourier's Series

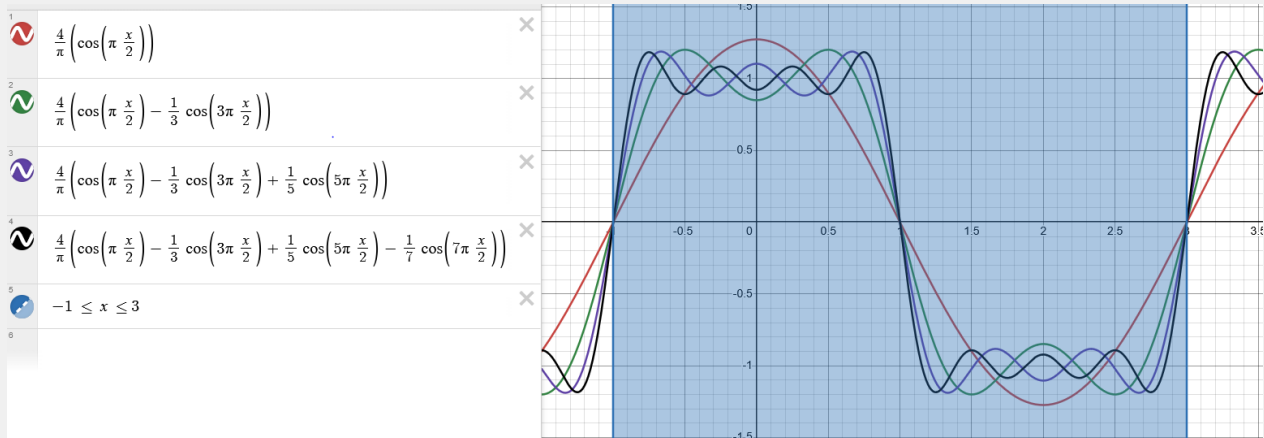
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1 Crisis in Mathematics: Fourier's Series

Problem 1

Graph each of the following partial sums of Fourier's expansion over the interval $-1 \leq x \leq 3$.

1. $\frac{4}{\pi} \cos(\pi x/2)$
2. $\frac{4}{\pi} (\cos(\pi x/2) - \frac{1}{3} \cos(3\pi x/2))$
3. $\frac{4}{\pi} (\cos(\pi x/2) - \frac{1}{3} \cos(3\pi x/2) + \frac{1}{5} \cos(5\pi x/2))$
4. $\frac{4}{\pi} (\cos(\pi x/2) - \frac{1}{3} \cos(3\pi x/2) + \frac{1}{5} \cos(5\pi x/2) - \frac{1}{7} \cos(7\pi x/2))$



Problem 2

Let $F_n(x)$ denote the sum of the first n terms of the Fourier's series evaluated at x :

$$F_n(x) = \frac{4}{\pi} \left(\cos \frac{\pi x}{2} - \frac{1}{3} \cos \frac{3\pi x}{2} + \dots + \frac{(-1)^{n-1}}{2n-1} \cos \frac{(2n-1)\pi x}{2} \right)$$

1. Evaluate $F_{100}(x)$ at $x = 0, 0.5, 0.9, 0.99, 1.1$, and 2 . Is this close to the expected value?
2. Evaluate $F_n(0.99)$ at $n = 100, 200, 300, \dots, 2000$ and plot these successive approximations.
3. Evaluate $F_n(0.999)$ at $n = 100, 200, 300, \dots, 2000$ and plot these successive approximations.
4. What is the value of this infinite series at $x = 1$?