Fundamentals of Mathematical Logic by Hinman

Frosty

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1 Propositional Logic and Other Fundamentals

1.1 The Propositional Language

Problem 10

Give a precise definition of the set of sentences using infix notation and prove unique readability for your definition.

Solution:

Definition of infix sentences:

1. $S_0 := $ the set of L-atomic sentences.
2. For each $n \in \omega$,

$$\begin{aligned} S_{n+1} := \; & S_n \cup \{\neg\phi : \phi \in S_0\} \cup \{(\neg\phi) : \phi \in S_n \sim S_0\} \\ & \cup \{(\phi) \bullet (\psi) : \phi, \psi \in S_n \sim S_0, \bullet \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}\} \\ & \cup \{(\phi) \bullet \psi : \phi \in S_n \sim S_0, \psi \in S_0, \bullet \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}\} \\ & \cup \{\phi \bullet (\psi) : \phi \in S_0, \psi \in S_n \sim S_0, \bullet \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}\} \\ & \cup \{\phi \bullet \psi : \phi, \psi \in S_0, \bullet \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}\} \end{aligned}$$

3. Let $S_f := \bigcup_{n \in \omega} S_n$.

Definition 1 (Parenthesization of binary connectives)**.** *Let ϕ and ψ be sentences, and let $\bullet$ be any binary connective. Then the four possible parenthesizations of $\phi \bullet \psi$ are:*

1. $(\phi) \bullet (\psi)$

2. $(\phi) \bullet \psi$

3. $\phi \bullet (\psi)$

4. $\phi \bullet \psi$

Proposition 1 (Unique readability for propositional sentences)**.** *For any sentence $\theta \in S_f$, exactly one of the following holds:*

1. θ is atomic;

2. *for some sentence ϕ, $\theta = \neg\phi$;*
3. *θ is a **conjunction** $\phi \wedge \psi$ for some sentences ϕ and ψ, with one of the four parenthesizations from Definition 1;*
4. *θ is a **disjunction** $\phi \vee \psi$ for some sentences ϕ and ψ, with one of the four parenthesizations from Definition 1;*
5. *θ is an **implication** $\phi \to \psi$ for some sentences ϕ and ψ, with one of the four parenthesizations from Definition 1;*
6. *θ is a **bi-implication** $\phi \leftrightarrow \psi$ for some sentences ϕ and ψ, with one of the four parenthesizations from Definition 1.*

Proof of unique readability:

Proof. ∎

Problem 11

For any expression $\phi = s_0 \dots s_k$, a **proper initial segment** of ϕ is any sequence of symbols $s_0 \dots s_l$ with $l < k$. Prove that no proper initial segment of a sentence is a sentence, and show that this can be used as an alternative to (4) as a technical lemma for the proof of Proposition 1.1.5.

Problem 12

Give a careful proof of Proposition 1.1.9, and show how Theorem 1.1.7 is an application of it.