1 a) Factor $2^{15}-1 = 32,767$ into a product of two smaller positive integers

15 factors are 3 and 5

let $a=3$ and $b=5$

$x = 2^5 - 1 \quad y = 1 + 2^5 + 2^{2(5)}$

$x = 31 \quad y = 1057 \quad 31 \cdot 1057 = 32,767$

b) Find an integer x such that $1 < x < 2^{32,767}-1$ and $2^{32,767}-1$ is Divisible by x

let $a = 7$ $b = 4681$

$2^{ab}-1 = 2^{7(4681)} = 2^{4681}(1 + 2^{4681} + 2^{2(4681)}$
$+ 2^{3(4681)} + 2^{4(4681)} + 2^{5(4681)} + 2^{6(4681)}$

2. Make conjecture about n for $3^n - 1$ being or values of n for which $3^n - 2^n$ is prime

n	$3^n - 1$
2	8
3	26
4	80

Suppose n is an integer larger than 1 then $3^n - 1$ is not prime.

Suppose n is an integer larger than 1 then

4	80
5	242
6	278
7	2186
8	6560
9	19682
10	59048

suppose n is an integer larger than 1 then $3^n - 1$ is even.

3. a) Find a prime different from 2, 3, 5, and 7.

let $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, and $p_4 = 7$

let $m = (2 \cdot 3 \cdot 5 \cdot 7) + 1$

$m = 211$ which is prime

b) let $p_1 = 2$, $p_2 = 5$, and $p_3 = 7$

let $m = (2 \cdot 5 \cdot 7) + 1$

$m = 71$ which is prime

4. let $n = 5$

let $x = (5+1)! + 2$

not prime

1 $= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) + 2 = 362$

2 $= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) + 3 = 363$

3 $= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) + 4 = 364$

4 $= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) + 5 = 365$

5 $= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) + 6 = 366$

5. Find 2 perfect numbers $(2^{n-1})(2^n-1)$ is perfect

let $n=2$ $(2^{2-1})(2^2-1)=6$

$2\cdot 3$

let $n=7$ $(2^{7-1})(2^7-1)=8128$

$64\cdot 127$

6. Find a triplet pair other than $(3,5,7)$

There are no other.

All other primes >2 are even

and prime triplets are multiple of 3

let the odd triplets by

$n, n+2, n+4$ either $n \bmod 3 = 0$

or $n+2 \bmod 3 = 0$

or $n+4 \bmod 3 = 0$

$n \bmod 3$ is either 0, 1, or 2

Case 1

$n \bmod 3 = 0$

$n \bmod = 1$

$n+2 = 1+2 = 3 \bmod 3 = 0$

$n \bmod 3 = 2$

$n+4 = 2+4 = 6 \bmod 3 = 0$

7. (m,n) amicable all positive

integers $< n$ that divide n is m,

integers $< n$ that divide n is m,
and sum of positive integers $< m$ that divide m is n.

show that $(220, 284)$ is amicable

$n = 220$, $1+2+4+5+10+11+20+22+44+55+110 = 284 = m$

$m = 284$, $1+2+4+71+142 = 220 = n$