A Radical Approach to Real Analysis by David M. Bressoud

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2.1 The Archimedean Understanding

Definition 1 (Archimedean Understanding of an Infinite Series)**.** *The* ***Archimedean Understanding*** *of an infinite series is that it is shorthand for the sequence of finite summations. The* ***value*** *of an infinite series, if it exists, is that number T such that given any $L < T$ and any $M > T$, all of the finite sums from some point on will be strictly contained in the interval between L and M. More precisely, given $L < T < M$, there is an integer n, whose value depends on the choice of L and M, such that every partial sum with at least n terms lies inside the interval (L, M).*

2.2 Geometric Series

Definition 2 (Convergence of an Infinite Series)**.** *An infinite series* ***converges*** *if there is a target value T such that for any $L < T$ and any $M > T$, all of the partial sums from some point on are strictly between L and M.*

2.3 Calculating π

Theorem 1 (Newton's Binomial Series)**.** *For any real number a and any x such that $|x| < 1$, we have that*

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!} + \frac{a(a-1)(a-2)}{3!}x^3 + \cdots.$$

2.4 Logarithms and Harmonic Series

Definition 3 (Divergence to Infinity)**.** *When we write that an infinite series equals ∞, we mean that no matter what number we pick, we can find an n so that the partial sums with at least n terms will exceed that number.*

Definition 4 (Euler's constant, γ)**.** *Euler's constant is defined as the limit between the partial sum of the harmonic series and the natural logarithm,*

$$\gamma = \lim_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} - \ln n\right)$$

Definition 5 (Nested Interval Principle). *Given an increasing sequence, $x_1 \leq x_2 \leq x_3 \leq \cdots$, and a decreasing sequence, $y_1 \geq y_2 \geq y_3 \geq \cdots$, such that y_n is always larger than x_n but the difference between y_n and x_n can be made arbitrarily small by taking n sufficiently large, there is exactly one real number that is greater than or equal to every x_n and less than or equal to every y_n.*