Problems 113

Wednesday, April 2, 2025 10:23 PM

113, $x^2-9x+20=0$

$(x-5)(x-4)$, $x=5, x=4$

$c=20=5\cdot 4$

$-b=+9=5+4$

$x^2-2x-8=0$

$(x-4)(x+2)$, $x=4, x=-2$

$c=4\cdot -2=-8$

$-b=4+-2=2$

$x^2+4x+2=0$

$(x^2+4x)=-2$

$(x^2+4x+4)=2$

$(x+2)^2=2$

$x=-2\pm\sqrt{2}$

$x=-2+\sqrt{2}$

$x=-2-\sqrt{2}$

$c=2=(-2+\sqrt{2})(-2-\sqrt{2})$

$=4-2=2$

$-b=-4=-2+\sqrt{2}-2-\sqrt{2}=-4$

$x^2+bx+c=0$ roots r_1, r_2

$x^2-(r_1+r_2)x+r_1r_2$ $c=r_1r_2$ $b=-(r_1+r_2)$

$$x=\frac{(r_1+r_2)\pm\sqrt{(-(r_1+r_2))^2-4(r_1r_2)}}{}$$

$$= \frac{r_1 + r_2 \pm \sqrt{r_1^2 + 2r_1r_2 + r_2^2 - 4r_1 4r_2}}{2}$$

$$= \frac{r_1 + r_2 \pm \sqrt{r_1^2 + r_2^2 - 2r_1r_2}}{2}$$

$$= \frac{r_1 + r_2 \pm \sqrt{(r_1 - r_2)^2}}{2}$$

$$= \frac{(r_1 + r_2) \pm (r_1 - r_2)}{2}$$

$$= \frac{r_1 + r_2 + r_1 - r_2}{2} \quad , \quad = \frac{r_1 + r_2 - (r_1 - r_2)}{2}$$

$$= \frac{2r_1}{2} = r_1 \quad , \quad = \frac{r_1 + r_2 - r_1 + r_2}{2} = r_2$$

114. $x - \sqrt{x} - 2 = 0$ | $x - \sqrt{x} - 2 = 0$

$x - 2 = \sqrt{x}$ | $u^2 - u - 2 = 0$

$x-2=\sqrt{x}$

$x^2-4x+4=x$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4,\ x=1$

$\frac{12}{(x-3)^2}+\frac{10}{(x-3)}+1=0$

$u^2-u-2=0$

$(u-2)(u+1)=0$

$u=2,\ u=-1$

$x=4,\ x=1$

$\frac{12}{(x-3)^2}+\frac{10}{(x-3)}+1=0$

$\frac{12}{u^2}+\frac{10}{u}+1=0$

$12u+10u^2+1=0$

$12+10u+u^2=0$

$u^2+10u+12=0$

$u^2+10u=-12$

$u^2+10u+25=13$

$(u+5)^2=13$

$u=-5\pm\sqrt{13}$

$(x-3)=-5\pm\sqrt{13}$

$x=-2\pm\sqrt{13}$

$x = -2 \pm \sqrt{13}$

$$\frac{12}{(x-3)^2} + \frac{10}{(x-3)} + 1 = 0$$

$$12(x-3) + 10(x-3)^2 + (x-3)^2(x-3) = 0$$

$$(x-3)(12 + 10(x-3) + (x-3)^2) = 0$$

$$(x-3)(12 + 10x - 30 + x^2 - 6x + 9) = 0$$

$$x^2 + 4x - 9 = 0$$

$$x^2 + 4x = 9$$

$$x^2 + 4x + 4 = 9 + 4$$

$$(x+2) = 13$$

$$x = -2 \pm \sqrt{13}$$