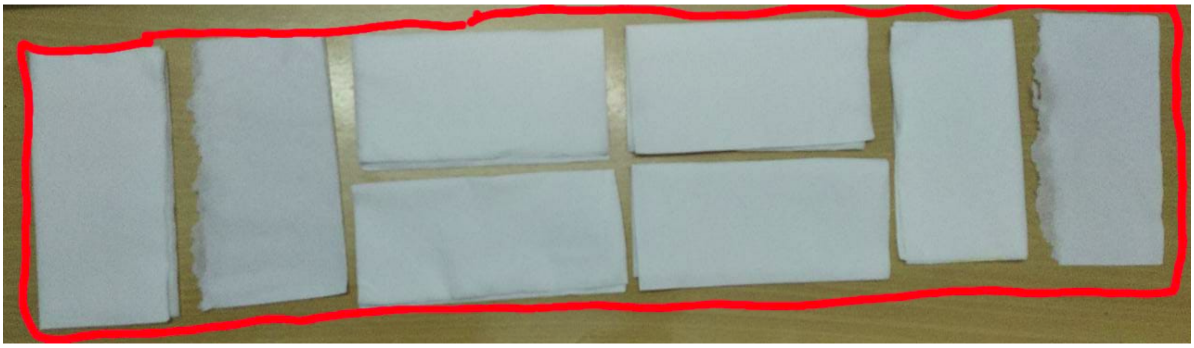
Dynammic Programming

Main Focus: Tiling Problem

DP for Tiling Purposes:

Assume that we use dominoes measuring 2×1 to tile an infinite strip of height 2. How many ways can one tile a 2 $\times$ n strip of square cells with 1 $\times$ 2 dominoes?

This is super relatable with respect to fitting things optimally in a suitcase or fitting bricks optimally on a wall and such.



Notice that we can place tiles either vertically or horizontally. For placing vertical tiles, we need a gap
of at least 2 $\times$ 2. For placing horizontal tiles, we need a gap of 2 $\times$ 1. In this manner, the problem is reduced to finding the number of ways to partition n using the numbers 1 and 2 with order considered relevant.

For example: 11 = 1+2+2+1+2+2+1 (Seen in the above figure)

If we have to find such arrangements for 12, we can either place a 1 at the end or add 2 in the arrangements possible with 10. Similarly, let us say we have F_n, possible arrangements for n. Then for $(n+1)$, we can either place just 1 at the end or we can find possible arrangements for $(n-1)$ and put a 2 at the end. Using the above theory:

> $F_{n+1} = F_n + F_{n-1}$

Now, we verify the above theory and make it more self-explanatory:

1. In how many ways can we fill a 2 $\times$ 1 strip: 1 $\longrightarrow$ Only 1 vertical tile.
2. In how many ways can we fill a 2 $\times$ 2 strip: 2 $\longrightarrow$ Either 2 vertical tiles or 2 horizontal tiles.
3. In how many ways can we fill a 2 $\times$ 3 strip: 3 $\longrightarrow$ Either put 2 vertical tiles along with the 2 $\times$ 2 solution or 2 horizontal tiles along with the 2 $\times$ 3 solution.
4. Similarly, now, how do we fill 2 $\times$ n strip: Either put a vertical tile in the solutions possible for 2 $\times$ $(n-1)$ strip or put 2 horizontal tiles in the solution possible for a 2 $\times$ $(n-2)$ strip. [$F_{n-1} + F_{n-2}$]
5. The base case for the solution is: $F_1 = 1$ and $F_2 = 2$.
