

# Divide And Conquer

## Main Focus: Closest Pair of Points

### NOTE: See Video To Understand This Well!

$O(n \log n)$  **solution:** Divide the points into 2 equal halves based on median of  $x$ -coordinates.

#### Algorithm:

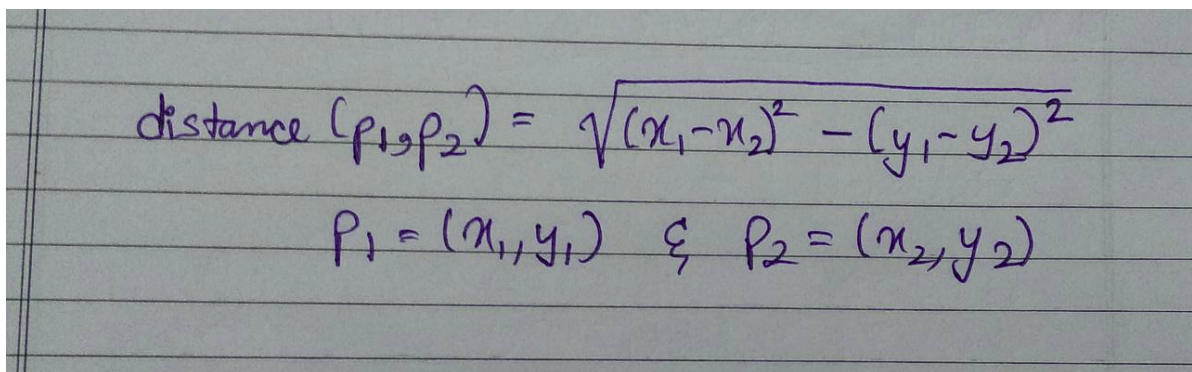
1. Sort the given points in  $S$  (given set of points) based on their  $x$ -coordinates. Partition  $S$  into two subsets  $S_1$  and  $S_2$ , about the line  $l$  through median of  $S$ .  $\leftarrow$  Divide Part of DnC.
2. Find the closest-pairs in  $S_1$  and  $S_2$  and call them  $L$  and  $R$  recursively.
3. Now, steps 4 to 8 form the Combining component of the DnC technique.
4. Let us assume that  $\delta = \min(L, R)$ .
5. Eliminate points that are farther than  $\delta$  apart from  $l$ .
6. Consider the remaining points and sort based on their  $y$ -coordinates.
7. Scan the remaining points in the  $y$  order and compute the distances of each point to all its neighbours that are distanced no more than  $2 \times \delta$  (that's the reason for sorting according to  $y$ ).
8. If any of these distances is less than  $\delta$  then update  $\delta$ .

#### Analysis:

1. Step 1 and 2 take  $O(n \log n)$  for sorting and recursively finding the minimum.
2. Step 4 takes  $O(1)$ .
3. Step 5 takes  $O(n)$  for scanning and elimination.
4. Step 6 takes  $O(n \log n)$  for sorting.
5. Step 7 takes  $O(n)$  for scanning.

**Therefore, Total Complexity:**  $O(n \log n) + O(1) + O(n) + O(n) + O(n) \approx O(n \log n)$

Images Used In The Video:



Handwritten formula for the distance between two points  $P_1$  and  $P_2$ :

$$\text{distance}(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Below the formula, the points are defined:

$$P_1 = (x_1, y_1) \quad \& \quad P_2 = (x_2, y_2)$$

