Progressive Ratio Schedules of Reinforcement

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Abstract

Pigeons' pecks produced grain under progressive ratio (PR) schedules, whose response requirements increased systematically within sessions. Experiment 1 compared arithmetic (AP) and geometric (GP) progressions. Response rates increased as a function of the component ratio requirement, then decreased linearly (AP) or asymptotically (GP). Experiment 2 found the linear decrease in AP rates to be relatively independent of step size. Experiment 3 showed pausing to be controlled by the prior component length, which predicted the differences between PR and regressive ratio schedules found in Experiment 4. When the longest component ratios were signaled by different key colors, rates at moderate ratios increased, demonstrating control by forthcoming context. Models for response rate and pause duration described performance on AP schedules; GP schedules required an additional parameter representing the contextual reinforcement.

Keywords

context; fixed ratio; mathematical model; post-reinforcement pause; progressive ratio; reinforcement schedules

One of B. F. Skinner's bequests was his emphasis on the singular importance of the *schedule of reinforcement*, the contingencies that the experimenter or environment arranges for the presentation of events to an organism. The events could be biologically significant or insignificant, positive or negative. Nature, not Skinner, was the first to arrange reinforcement schedules, and many scientists—Pavlov and Thorndike the exemplars—used them to good effect before Skinner. But Skinner, along with his students (e.g., Ferster & Skinner, 1957) made them an end, not means, of inquiry. One of the most fundamental type of schedule arranges reinforcement contingent upon the completion of a fixed number of responses, the Fixed Ratio (FR) schedule. If the ratio is not fixed, but varies widely from one reinforcer to the next, the schedule is called Variable Ratio (VR). Ratio schedules engender high rates of responding, more or less uniform under VR, consisting of a post-reinforcement pause or *break* followed by a *run* of high rate responding under FR.

It is possible to schedule ratio reinforcement intermediate between these extremes of variability. Under Progressive Ratio (PR) schedules the requirements for reinforcement are increased systematically, usually after each reinforcer. Since their introduction by Hodos (1961), PR schedules have been used increasingly to assess reinforcement strength, an assay of special importance to behavioral pharmacologists (e.g., Ping-Teng, Lee, Konz, Richardson,

& Roberts, 1996). The importance of this schedule is attested by the return of over 2000 references to a web search for "progressive ratio schedule/s"; yet no theoretical characterization of PR behavior exists.

The progression by which the number increases can be of any sort, but is typically arithmetic, sometimes geometric. Not much is known about how the progression affects performance on PR schedules, as different progressions have rarely been compared. "To our [Stafford, LeSage and Glowa (1998, p. 177)] knowledge, no investigator has provided a specific rationale for favoring exponential or logarithmic progressions over arithmetic ones, although several authors have outlined positive aspects of the former types of progression", such as a quicker coverage of the range of ratios up to the breaking point, where responding ceases. (The nomenclature for progressions is clarified in the appendix.)

In their review of PR schedules for drug self-administration, Richardson and Roberts (1996) suggested use of the geometric progression $n_j = 5e^{j/5}$-5, but offered a range of other possibilities, advising that the particular progression chosen should depend on the questions asked and the drugs employed. That range of possibilities has been sampled (e.g., Li, He, Parrish, Delich, & Grasing, 2003), but without a unifying theory, relating the results obtained under one progression to those obtained under another is problematic. As we shall see in the results of the first experiment, the choice of a progression is not neutral with respect to the results; geometric progressions yield different breakpoints than arithmetic progressions, as do geometric progressions with different rates of progression. A central purpose of the present research is to reconcile these results, by experimental analysis of the variables controlling PR responding, and theoretical analysis of the resulting patterns of behavior.

The experimental analysis asks to what extent behavior on one component is under the control of the preceding components; to what extent it is under the control of the following components; and to what extent it is under the control of the context as a whole. The theoretical analysis attempts to incorporate the answers to the experimental questions in a more general theory of behavior maintained by PR schedules, for without stable theories to anchor them, empirical results are quickly swept away in the flood of new data.

The Progressive Ratio Situation

The basis for increasing the requirements in a PR schedule may be after each reinforcer, each block of reinforcers, each session (Czachowski, Legg, & Samson, 2003; Czachowski & Samson, 1999), or after blocks of sessions (Powell, 1968). In the limit of a basis of many sessions, the PR schedule becomes a series of FR schedules. The graph of response rate as a function of ratio size for PR schedules has a bitonic shape resembling that found with FR schedules (Foster, Temple, Cameron, & Poling, 1997), and suggests the basic similarity of the behavior under these schedules (Winger & Woods, 1985). But it is undetermined whether this semblance is more than superficial, as different factors are at play in PR and FR schedules (cf. Baron, Mikorski, & Schlund, 1992).

1. In each PR component (ratio), there will be some control of target response rates by the conditions of the preceding component, and possibly by the context as a whole (Baron & Herpolsheimer, 1999). This must be equally true for FR schedules, of course, but those conditions are uniform. Response rates may decrease at large ratio values because the animal's arousal may decumulate during the progress toward extinction, because diminishing returns on invested responses may demotivate the animal, or because of progressive satiation (Bizo, Bogdanov, & Killeen, 1998) or habituation (McSweeney, Hinson, & Cannon, 1996; McSweeney & Roll, 1993). This is especially problematic with drugs as reinforcers, as one or two deliveries of an opiate may satisfy the animal for an extended period. This flux of motivation may

also become conditioned to the experimental context: Animals may become conditioned to higher levels of reinforcement—and thus arousal, and in turn, behavior—toward the start of the session, and lower toward the end of the session. Historical, contemporary, and anticipated levels of reinforcement meld in potential control of PR behavior.

2. Post-reinforcement pause (PRP) length—the break—is an increasing function of requirement on FR schedules (Zeiler, 1979). The requirements are always changing on PR schedules, and the effect of pausing on response rates will interact with the nature of the progression. Because FR schedules do not progress, their study cannot inform us of the possibly differential control that can be exerted by different types of progressions, some of which the animal might anticipate better than others. The first experiment reported here provides basic data on the different progressions, and the subsequent ones are aimed to understand the resulting differences.

The first experiment will establish the fundamental difference in the behavior under geometric progressions (GP) and arithmetic progressions (AP). The second will establish the effect of step-size. The third will schedule ratio requirements in blocks to establish the influence of immediately prior and subsequent requirements, and to introduce a simple model of PR performance. The fourth will reverse the progression to test the model. The fifth will demonstrate control by subsequent requirements, not manifest in the prior experiments. Finally, an attempt is made is reconcile the differences in GP and AP performance by invocation of the reinforcement context as a contributory source of control.

Experiment 1: Geometric and Arithmetic Progressions

Method

Subjects—Four adult male pigeons (*Columba livia*) with previous experimental histories were maintained between 80 and 84% of their *ad libitum* weight by supplementary feedings of fortified mixed grain at the end of the day. They were housed in a vivarium having a 12-hr day/night cycle, with dawn at 7 a.m.

Apparatus—A Lehigh Valley conditioning chamber (31 cm wide, 35 cm deep, 35 cm high) contained a 2.5-cm central response key mounted 20 cm above the chamber floor that could be transilluminated with a white light. The key required a force of 0.15 N to register. A houselight was mounted above the response key, 20 cm above the floor, and an aperture (6×5 cm) permitting access to a food hopper was below it 6 cm from the chamber floor. Activation of the hopper provided 2.5-s access to milo grain. A ventilation fan and white noise contributed to a total masking level of 75 dB. Programs written in Microsoft ® Quick Basic 4.5 controlled and recorded all experimental events from an IBM compatible 386DX computer that was located atop the sound-attenuating chamber.

Procedure—This experiment utilized two sequences of PR schedules. The first sequence comprised an arithmetic progression incrementing by 1 after each reinforcer, yielding the sequence $n_j = j, j = 1, 2, …, 80$. All subjects experienced this schedule first. The second schedule was a geometric progression, with ratios forming the sequence $n_j = 2^{(j+1)/2}, j = 1, 2, …, 19$, rounding up the resulting fractions to the next integer, and incrementing after each reinforcement. Sessions were terminated after the largest ratio or 120 min, whichever came first. Each sequence was in effect for 45 sessions.

Data Analysis—In this and all subsequent experiments, data were aggregated using 20% symmetrically trimmed means: Only data between the 20th and 80th percentiles were averaged. This procedure and percentage is recommended by Wilcox (1998), among others. It is more

conservative than the use of a median: It preserves most of the efficiency of a mean (a 0% trim) and much of the robustness of a median (an approximately 49% trim). It is discussed further in the appendix. This procedure was followed for averaging daily response rates within subjects, and across subjects and parameter values. Such precautions are especially prudent in the study of ratio schedules, where distributions of dependent variables are often skewed, and schedule effects may arise from shifts in skewness as much as shifts in means (Baron & Herpolsheimer, 1999). Post-reinforcement pause lengths, which were highly skewed, were summarized by their medians. Other details of data analysis are presented in the appendix.

Results

Figure 1 shows the average response rates from the last five sessions of the two sequences. Under the arithmetic progression, pigeons' response profiles looked as though most would have stopped responding well before the response requirement reached 724. In contrast, under the geometric progression all pigeons continued responding through that ratio—the next to the largest in the right column—and did so even after extended training. Furthermore, rates under the arithmetic progressions showed an approximately linearly decreasing limb, whereas rates under the geometric progressions decreased asymptotically toward zero.

Discussion

This is not the first study to show a linear decrease in arithmetic progressive ratio schedules (Covarrubias, 2006); nor the first to show curvature in the right tail with geometric progressions (Zhang et al., 2004). Those studies used rats as subjects, and did not contrast the two types of progressions in the same animals. The difference in the behavioral effects of the progressions appears real. A common cause could be the preponderance of large step sizes in GPs and small one in APs. That is tested in the next experiment.

Experiment 2: The role of step-size

This experiment attempts to revalidate the form of the response function for arithmetic progressions, and, further, to determine if that form covaries with step size, possibly becoming more curved as step sizes increase.

The experiment utilized four sequences of PR schedules, all arithmetic progressions, distinguished by their step size (SS). The training sequence comprised an arithmetic progression incrementing by 1 (SS 1). All subjects experienced this schedule for the first three sessions, followed by three sessions of SS 2 PR, and then 15 sessions of SS 3 PR, the first experimental condition. In this condition the PR sequence was $n_j = 3j$; 3, 6, …180. A SS 5 progression followed for 15 sessions, and finally a return to the SS 2 sequence for 15 sessions.

Results and Discussion

Figure 2 shows the response rates and PRPs in each of the sequences, averaged over the last six sessions. There was little evidence of convex curvature in the functions, with the exception of the smallest step size around $n = 50$. The arithmetic progression, whatever the step size between successive components, generates response functions consistent with a simple FR model (Bizo & Killeen, 1997a). The functions intercept the x-axis around $a = 100$ responses/reinforcement, showing a slight increase with step size. That increase, if reliable, might be due to slightly greater satiation by the end of the session under the smaller step size. The parameters for Equations 1 and 2 (described below), which drew the curves through these functions, are given in Table 2. The PRPs follow the hyperbolic form predicted by Equation 1, using the same value of the a parameter that was used to fit the FR response function in each case. If plotted on a single graph, these response functions would largely overlap: The negligible effect of step-size on the form of the response function was also noted by Stafford and Branch (1998). The

only threat to the generality of this conclusion comes from the relatively limited range of step sizes studied.

Experiment 3: A Basis-6 PR schedule

Experiment 1 demonstrated performance on geometric PR schedules (GPR) that departed from that seen on arithmetic PR schedules (APR) and FR schedules—a smooth increase turning over into a linearly decreasing limb, descending to zero at the longest ratios. Experiment 3 was conducted to clarify the variables controlling this difference. A candidate cause is the radically increasing requirements in the geometric progression. In particular, if the animal's adaptation to the transition between component ratios lags the operative ratio—if there is hysteresis— then that lag may be magnified by geometric progressions. This is because prior ratios are quite similar on APR, radically different on GPR. Indeed, rats, at least, may be able to anticipate the contemporary ratio in APR (Crystal, Church, & Broadbent, 1997), but not GPR in which case performance on GPR will be more in arrears than that on APR. To get a first appreciation of how slow or fast this adaptation process is, the following experiments holds the ratio at the same value for six reinforcers before incrementing to the next.

Method

Subjects—Six adult homing pigeons with various experimental histories were housed in a room remote from the testing chambers with a 12:12-hr day/night cycle, with dawn at 6 a.m. Free access to water and grit was provided in the home cages. One subject failed to complete most ratio requirements and was dropped from the study. The pigeons' running weights were based on 80% of their free-feeding weights. Each pigeon was weighed immediately prior to an experimental session and was excluded from that session if its weight exceeded 8% of its running weight. Supplementary feedings of ACE-HI pigeon pellets (Star Milling Co.) were given at the end of each day, based 50% on current deviations from target running weight and 50% on a long term-average amount that each pigeon required.

Apparatus—The experiment was conducted in MED Associates modular test chambers measuring 30 cm long, 24 cm wide, and 29 cm high. The floor of the chamber consisted of metal bars spaced 15 mm apart. The ceiling and two of the walls were clear plastic, and the two other walls were aluminum. Three plastic response keys 25 mm in diameter, which could be transilluminated from behind by diodes, were situated on one of the aluminum walls 22 cm from the floor. In the present experiment only the center key was ever lit and operative; it required 0.20 N of force to register. The magazine aperture was 51 mm wide and 57 mm high; its bottom was 22 mm above the floor. During reinforcement, the magazine was illuminated and the hopper was raised, to give the subjects access to milo grain. Each box was enclosed in an individual sound and light attenuating box, which also contained a ventilation fan. A houselight was situated 12 mm from the ceiling on the sidewall opposite the test panel. Experimental events were arranged via a MED-PC® interface connected to a PC. These chambers were used in all subsequent experiments.

Procedure—At the start of each session, the houselight was illuminated and the center key was lit white. Pecks to the center key were recorded, and after the appropriate number of pecks a reinforcer was delivered. After a response no other responses were recorded for 0.1 s. Reinforcement consisted of the hopper raised and illuminated for 2 s, during which time the center key light was dark and inoperative and key-pecks went unrecorded.

Reinforcers were scheduled according to a PR in which the response requirement increased after every sixth reinforcer. The response requirement started at 3 and followed a polynomial progression through 6, 12, 20, 30, 40, 60, 80, 100, 120, 140, and 160. The progression was chosen to provide ample data at short ratios while exploring a large arithmetic step size above

40. The session ended either after all reinforcers had been obtained at all response requirements (72 reinforcers) or after 2 hrs. The experiment ran for 56 sessions; data from the last 20 sessions were analyzed.

Results: Pausing

The first question asked of these data concerned the relation between the duration of an inter-reinforcement interval (IRI) and the subsequent post-reinforcement pause (PRP). Figure 3 confirms archival results on FR schedules, showing a linear relation between pause duration and the duration of the prior IRI (Zeiler, 1979). The data intersect the *y*-axis at 2 s, and increase thereafter with a slope of $m = 0.51$, indicating that these pigeons tended to pause for half the length of the prior IRI. Such linear pausing is consistent with research on progressive interval, fixed ratio, cyclic ratio, fixed interval, cyclic interval and response-initiated delay schedules (Bizo & Killeen, 1997;Capehart, Eckerman, Guilkey, & Shull, 1980;Hanson & Killeen, 1981;Leinenweber, Nietzel, & Baron, 1996;Leslie, Boyle, & Shaw, 2000;Schneider, 1969;Wynne, Staddon, & Delius, 1996). The reason for this invariance is uncertain, but the simple model of Capehart and associates has some face validity. According to those investigators, the probability of an animal initiating a response state at any instant is inversely proportional to the delay to reinforcement at that point in time in the immediately prior run state. Alternatively, if Weber's law holds for estimates of time-to-reinforcement, and animals start to respond when the cumulative probability of reinforcement exceeds, say, *P*, then, for a Weber fraction of *w*, pause duration will be predicted by the inverse of the normal distribution function, $m = \text{NORMINV}(P, 1, w)$. In particular, for $P = 5\%$ and $w = 0.3$, $m = 0.51$.

Discussion: Pausing

The inter-reinforcement interval (IRI) depends on the rate of responding (which depends on the prior IRI, etc). But a linear relation with the prior IRI, attested by the "linear waiting" literature cited above, does not entail a linear relation with the ratio value; indeed, it rules out such a relation. It is not difficult, however, to calculate what the relation between ratio value and pause length should be. This is desirable, as it would permit use of the ratio as a behavior-independent predictor of pause length, rather than use the value of the prior IRI, which is entangled with the prior pause length, itself dependent on … etc. Given, then, that:

a.   Pause length is approximately proportional to IRI, the reciprocal of the rate of reinforcement, *r*: $P \propto IRI = 1/r$, where $\propto$ signifies a proportional relationship.

b.   In turn, under ratio schedules the rate of reinforcement is proportional to overall response rate (*B*): $r = B/n$, where *n* is the ratio requirement, so that:

c. $P \propto 1/r = 1/(B/n)$; therefore:

d. $P \propto n/B$: pause length is proportional to the response requirement over the response rate. Under moderate to long FR schedules, response rate during the run state approximates a linear-decreasing function of ratio value:

e. $b \propto 1 - n/a$ (Bizo & Killeen, 1997). This long chain of inference entails that:

f. $P \propto n/(1 - n/a)$; or, more simply:

$$P \propto n/(a - n), \quad n < a;$$ (1)

pause length is proportional to the response requirement, divided by the distance of that from the extinction ratio *a*. This analysis may be validated against data reported by Powell (1968), who increased the ratio requirement in FR schedules every three days, up to a ratio of 160, and then decreased it in like manner. Figure 4 shows the average post-reinforcement pause resulting

from this very long basis PR. The continuous curve drawn by Equation 1 provides an acceptable description, always within the range of the 3 subjects. There is, however, systematic bias in its predictions. We attribute this to Powell's averaging the data from all three sessions in each block, which would, in the first session of each block at least, be under partial control of the prior ratio. The dashed line shows the fit of Equation 1 if the operative ratio is assumed to be a mixture of 2/3 the nominal ratio and 1/3 that of the prior condition.

The analysis is now applied to the data from Experiment 3. The right panel of Figure 3 shows the PRP from the last ratio in a block of 6 and the first ratio in the next block as a function of their prior ratio value. The curve is given by Equation 1, with $a = 218$ responses/reinforcer. The general proximity of the pairs of points in that panel, coming as they do from the different components, indicates that behavior was under control of the conditions just experienced. Pauses on other ratios in the components fell below these approximately asymptotic values, as IRIs slowly equilibrated after an experienced decrease in rates of reinforcement in the successive ratios. In general, a linear relation between pause length and IRI accounted for more of the variance in all of the data than did any relation between pause length and ratio value. This suggests that it was the IRI in the prior component that controlled subsequent pause length, not the ratio value per se.

Results: Response rates

Response rates on this progression are shown in Figure 5. The top panel shows the across-subject average rate for each of the six reinforcers at each response requirement. The bottom panel shows the average response rates for the first and last reinforcers of each block, along with a model for FR performance derived from Killeen and Sitomer (2003), based on Bizo and Killeen (1997):

$$B = k\left(C_{FRn} - \frac{n}{a}\right) \quad 0 < n \leq a$$

(2)

The foundational model assumes that arousal is proportional to rate of reinforcement (with constant of proportionality a), and that it is directed onto the lever or key by the contingencies of reinforcement (represented by the function C. that ranges from 0 to 1), under temporal constraints (epitomized by δ, the minimum inter-response time). The proportion of the interval spent in the run state is $1 - m$, where m is the slope in the left panel of Figure 3; only that contributes to measured response rate. The parameter k combines the factors of runtime and $1/\delta$, the maximum response rate: $k = (1 - m)/\delta$. In the present paper, δ is fixed at ¼ s, a stereotypic value for pigeons' pecks, and k (or m) is treated as a free parameter. C_{FRn} is the coupling coefficient for FR schedules. It captures the increase in the ability of the reinforcer to strengthen the target response as a function of how many of the target responses are in working memory. Its form depends on the contingencies of the particular reinforcement schedule. For FR schedules it is $C_{FRn} = 1 - (1 - \beta)^n = 1 - e^{-\lambda n\delta}$. What this equation says is that the class of target responses will be strengthened as a diminishing function of the number of such responses occurring before each reinforcer. The rate of increase to the asymptote of 1 depends on the duration of the response, and the average capacity of memory, $1/\lambda$ (Killeen, 2001). Pigeons' memory is saturated once the response requirement reaches a dozen pecks (Killeen, 1969): $C_{FRn \geq 12} \approx 1$. Rats' memory saturates more slowly (Bizo, Kettle, & Killeen, 2001). Equation 2, evaluated at long ratios where coupling approaches unity, gave the prediction for response rates that was used in deriving Equation 1 ($B \propto 1 - n/a$). It slightly overestimates rates at short ratios (where $C_{FR} < 1$), and thus underestimates pausing there. The parameter a depends on incentive motivation, and gives a theoretical measure of the animal's "wanting" the reinforcer (Berridge & Robinson, 1995;Robinson & Berridge, 1993;Salamone

& Correa, 2002). It also gives the point at which the descending limb crosses the *x*-axis, also known as the "breaking point", and "extinction ratio".

Discussion: Response rates

Along with this model, the linear-waiting relationships shown in Figure 3 permit us to make sense of the data in Figure 5. Responding in the components of a PR schedule, like responding on FR schedules, consists of a pause followed by a run of responses emitted at an approximately constant rate. Under FR schedules pause length is proportional to the IRI, just as it is under fixed interval (FI) schedules (Schneider, 1969). In cases where IRI varies, Staddon, Wynne, and Higa (1991) found that a predominant influence on pause length is the duration of the just-prior IRI, a phenomenon that they labeled *linear waiting* or *one-back waiting*, exemplified in Figure 3. As the ratio requirement progressively increases, the animals will continually lag behind the realities of the current situation. A disproportionate amount of the time between reinforcers will be spent in the run state under each subsequent requirement, as shown in Figure 6.

Figures 3 and 5 illustrate an apparently paradoxical effect of such one-back waiting. For example, even though the FR 40 schedule provides an inferior rate of reinforcement compared to the FR 20, the duration of the pause, and thus of the run state, is appropriate to the FR 20. This elevates response rates in the FR 40 over those in the FR 20, because a much smaller proportion of the time in the FR 40 schedule is spent in the pre-run pause. The longer IRI that is obtained on the FR 40 will increase the pause on the subsequent trial, with pause and rate adjustments lagging asymptotic performance. Only after multiple iterations at the same requirement can pre-run pause durations stabilize. Comparing the data for the first and last ratios in a component (bottom panel of Figure 5) shows this to be the case. The data in the top panel suggest that the process of equilibration is complete by six iterations for the shortest (≤ 20) ratios, but not for the longer ones. This suggests that the accommodation to long ratios will take longer than to short ones, which may explain some of the differences between AP and GP progressions: Under AP, the need to adjust grows more slowly and steadily with the progression of reinforcers, giving the animals time to adapt.

The logic represented in Figure 6 and the data in Figures 3 – 5 imply that, under PR schedules, pause times will be shorter than those under an FR schedule of the same magnitude. Run time will be longer, and overall response rate, and thus reinforcement rate, will be higher. Each successive component will produce lower response and reinforcement rates, and thus longer IRIs. Overall, response rates under PR schedules will be higher than under FR schedules with the same requirements.

These considerations entail a simple model for the complex performance shown in Figure 5. It uses pause length and response rates based on the prior ratio, and response requirement for the current ratio, as detailed in the appendix. It is called the *path-dependent PR* model, because its predictions depend on both the current and preceding conditions of reinforcement, and on the predictions of behavior on the preceding component. It drew the jagged line through the rates in Figure 5. The value of *m* was taken from Figure 3 (0.51), and the minimum response time was fixed at $\delta = 0.25$ s. The free parameters took values of $\lambda = 0.35$/s ($\beta = .08$) and $a = 200$. The prediction of response rate at each ratio was based on the prior inter-reinforcement interval, itself based on pausing and the response rate predicted for the prior ratio, B_{i-1}. The envelopes in the bottom panel are drawn by the FR model (Equation 2) for the first ratios in each block and for the last ratios in each block. This permits comparison of the path-dependent PR model in the top panel, with the simple PR/FR model applied to data representative of a standard PR (top curve lower panel, with the prior ratio always being shorter) and to data more representative of an FR (bottom curve lower panel, with 5 prior ratios at the same value). In fitting the model, δ was held at 0.25 s, and in both cases λ settled to 0.32/s, close to the value

for the path-dependent model (0.35/s). For the last (FR-like) ratios of a block, $m = 0.49$, close to the measured value for m, and $a = 188$, close to the value found for the path-dependent model. For the upper function, based on the first ratio of the block, $m = 0.42$ and $a = 217$. These data were under the control of the prior IRI, and thus biased away from the true (i.e., asymptotic) values. This analysis shows how use of the FR model with PR data can lead to biased estimates of the parameters—although in the present case, the bias was relatively small. In addition, the approximate linearity of the top curve shows that large step-size per se is not a cause of the concavity seen on GPR—at least not in the current design.

Experiment 4: Testing the model: Regressive ratio schedules

A simple test of the above model is to reverse the progression: If the lagging of the pause is one of the key factors driving the PR performance, then a regressive ratio (RR) schedule, in which each successive component ratio is smaller than the one before, should deviate systematically from a kindred PR schedule. Following the logic of Figure 6, each pause, based on the prior component, should now be longer than its equilibrium value. Therefore response rates should be lower than on the PR schedule. In particular, given the step size of 4 to be used, they should differ as a function of the ratio $(n_j - 4)/(n_j + 4)$, showing the largest difference at small ratios, n_j.

Method

Subjects—The subjects were 6 pigeons with experience on PR and RR schedules, but no appearance in the cast elsewhere in this paper.

Apparatus—The same equipment was used as in Experiment 1.

Procedure—The same basic procedures for housing and maintaining weight described above were employed in this and subsequent experiments. The house light was illuminated at the start of each session, along with a white center key light. Pecks to that key no closer than 0.1 s with prior pecks were recorded. Reinforcement consisted of the hopper being raised and illuminated for 2.5 s. The APR ran from 4 to 120 in steps of 4; the RR ran from 120 to 4 in steps of 4. Ten sessions of the RR condition followed 10 sessions of the PR condition. Average rates from the last session of each condition are reported.

Results

Figure 7 shows the response rates as a function of response requirement on the PR (circles) and equivalent RR schedule (dots). The data follow the familiar decreasing form. The RR data lie below the PR data at small ratios, as predicted, and intermix at largest ratios.

Discussion

The path-dependent PR model was fit to the data, using the same parameters for both functions ($m = 0.25$, $\lambda = 1.9$, $a = 227$). The RR data fell below the PR data, as predicted, but not as much below them as predicted by the model, which made the predictions by predicating pause length on one-back components that were shorter (PR) or longer (RR) than the contemporary one.

One could conger many reasons for the divergence of these patterns. One possibility is progressive satiation, which would take its toll on the right part of the PR function and on the left part of the RR function. That potential confound will be eliminated in the next experiment. One could also wonder why the loci of the data were not as divergent as predicted by the one-back hypothesis. Crystal, Church and Broadbent (1997) noted that the data that Higa, Wynne and Staddon (1991) adduced for one-back waiting could be equally well interpreted as anticipation of the contemporary interval, an alternative which their data from rats supported.

Perfect anticipation in the current experiment would cause the data to superimpose. The obtained data, lying as they do between the two predictions, might simply be telling us that both forces are at work, a possibility that will be addressed in Experiment 5.

Some of the control of response rate observed in Experiments 1 and 2 may have been exerted not by just the one-back contingencies, but by earlier or later contingencies. To further test our interpretation, similar experiments not reported here were conducted using blocks of ratio requirements. Diminishing control was observed by 2, 3, and 4 ratio components back. Behavior is path dependent, and the degree of control falls off gently, like a scarf dragging along the path. The assumption of one-back control, however, accounts for the lion's share of the variance in the data, without having to spend an additional parameter to define the curvature of that scarf.

Experiment 5: Isolating Contextual Control

Not only does the linear one-back pause component of the model give short shrift to more distant past components, it ignores potential control by forthcoming components: Baron and Derenne (2000) have shown that schedules such as FR, extinction, and regressive ratio, postfixed to the end of a 20-component PR schedule, affect pause length and response rates on the preceding PR components.

The presence of contextual control is further attested by phenomena such as spontaneous recovery, conditioned place preference, and stimulus generalization, and inferred by comparator theories such as those used to model classical conditioning phenomena. The present experiment was conducted to evaluate the locus and extent of contextual control in more typical PR schedules. In this experiment, the higher rates of reinforcement in the earlier components were differentially signaled, to weaken the motivational control of the lower rates in the latter components. The hypothesis was that this would preserve higher response rates on components with the early signal, protecting them from generalization from the latter components. By the same token, we expected rates on the longer components, with reduced contextual support from the early components, to show a more precipitous decline.

METHOD

Subjects and Apparatus

Five new pigeons, with some history in PR schedules, but not in the above experiments, served. The same equipment was used as in Experiments 2 and 3.

Procedure

At the start of each session, the house-light was illuminated and the center key light was white. Pecks to the center key were recorded and after the number of pecks appropriate to the response requirement currently operating, a reinforcer was delivered. The programmed ratios were from a power series that grew as: 1, 1, 1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 19, 23, 26, 30, 34, 39, 44, 49, 54, 60, 66, 73, 79, 86, 94, 102, 110, 118, 127, 136, 146, 156, 166, 176, 187. After a response, no other responses were recorded or incremented the ratio counter for 0.1 s. Reinforcement consisted of the hopper being raised and illuminated for two seconds, during which time the key was dark and inoperative. The experiment ran for 20 sessions, with data from the last 12 sessions reported. Sessions lasted until the last (60th) ratio was completed, or 2 hours elapsed, whichever came first. After 20 sessions, the key light was changed from white to red after completion of the 94 ratio.

Results

Figure 8 shows the results averaged over subjects. The control progression (circles) evinces the standard, relatively linear decrease in rates, as expected. The experimental conditions, shown by the squares, at first lie above the control rates, as predicted. The difference between average experimental and control rates on FR 94 is significant (paired $t = 4.78$, df $= 3$, $p < .05$, $p_{rep} = .98$). However, the rates after the key-light change did not plummet, as expected, but continued high, decreasing gracefully to the same low asymptote as the control rates. Behavior was evidently under stronger control by the prior behavior than by the red key color; and since response rates were higher, so also were reinforcement rates. This experiment supports the hypothesis that response are under contextual control in PR schedules, and that some of that control is anticipatory; but the momentum of ongoing response rates, and the higher rates of reinforcement those deliver, are part of that context. The higher response rates going into the key-light change generated a context of higher reinforcement rates, which carried through the light change, at least for a few ratios. The closed-loop properties of responding on ratio schedules thus have a kind of self-fulfilling prophecy about them; as is generally the case in positive feedback systems.

In a follow-up experiment, we interleaved days of VI 8s and PR, to see if contextual conditioning by the VI schedule might raise response rates in the right tail of the PR function. Two of five pigeons were able to complete more ratios in this condition (the others completed the same number), but there was no overall elevation in the right tail of the function. We concluded that whatever contextual conditioning was conferred by the VI schedule had largely, if not completely, washed out by the end of the PR sessions. This contrasts with other experiments in which the largest (moderate) ratio was followed by a step down to much smaller ratios, which elevated the right tail of the PR response function in most pigeons (Posadas-Sanchez, 2001). It is clear from Experiment 3 that performance at each component ratio is under the control of prior ratios; and from Experiment 5 that it is also under some control of forthcoming ratios, particularly in the right tail.

General Discussion

The qualitative difference between responding under PR and FR schedules motivated the experiments and model development reported here. The persistence through large ratios and the asymptotic convexity of the rate functions shown in most geometric progressions diverges from the linear decrease typical of most FR schedules and approximated by arithmetic-progression PR schedules (Figure 1). Several models of PR performance were developed. The first model calculated two times: The pause lengths as a linear function of the interreinforcement interval predicted for the previous component (Figure 3), and the run times from the ratio requirement and the response rate predicted by the FR model (Equation 2). It then predicted overall response rate by dividing the number of responses required in the current component by these two times. This *path-dependent PR model* is effective at predicting transients in responding, as shown in Figure 5, as well as standard response profiles and pauses after reinforcement. It may be simplified to yield a model of pausing and responding on ratio schedules (Equations 1 and 2; Figures 2 and 7) applicable when transients are not a salient aspect of the performance.

This model—neither the path-dependent not the simpler form—does not, however, predict the convexity in the right limb of the response profile, a convexity most apparent in geometric progressive ratios. Other simple models of GPR, not reported here, were likewise ineffective in capturing the form of the response function. It was therefore decided to attack this curvature experimentally. In experiments not reported here, dwelling for 3 iterations of each component did not eliminate the sag in the function, nor did an attempt to force behavior to asymptote by

overshooting the first ratio of each block eliminate it. Would the popular geometric progression introduced by Roberts and Richardson (1992), $n_j = 5e^{j/5} - 5$ eliminate it? This progression has been used by Richardson and Roberts' (1996) and Bradshaw and colleagues (e.g., Zhang et al., 2004) among others. It yields the sequence $n_j = 1, 2, 4, 6, 9, 12, \ldots$. But other experiments, also not reported here also showed a distinct curvature in the right limb; less concave than seen in Figure 1, but clearly nonlinear.

Bit in teeth, we turned finally to theory.

Modeling Context

The gnawing issue remains how to use the contextual control of behavior, manifest in many of these experiments, as an entree to a theoretical account of the AP-GP difference. Empirical tinkering did not reconcile the two, nor did simple theoretical modifications of the basic models, but two other modifications did, both instantiating contextual control. One was to consider the animal an imperfect filter of the historical nature of the progression, and of its current position in that progression. This was effected with a Gaussian filter on the progressive ratios, convolved with a similar filter on the animal's immediate position on the trial. It bears similarities to some models of way-finding (A. K. Reid & Staddon, 1998) and interval timing (Higa, 1996), giving pause times essentially like human leaving times when planning to meet a friend who typically arrives sometime around noon, but not being really sure what time it presently is. It was successful, but neither transparent nor easily applied.

A gross simplification is to assume that contextual control is not centered around the animal's position through the progression, but uniformly pervades its time in the chamber. This simplification proved surprisingly effective. It can be realized in three ways, all of which make comparable predictions. Context can be added as an alternate source of reinforcement (r_c) to the fundamental equations of motion (Killeen & Sitomer, 2003); to the path-dependent PR model; or to the simple PR model (Equation 2). The easiest way to add it to the PR models is to adjust the effective ratio by adding a virtual ratio concurrent with the actual one; this leads to an adjusted effective ratio, $\bar{n}_i$, of:

$$\bar{n}_i = n_i/(1+cn_i), \tag{3}$$

as shown in the appendix. The free parameter c is proportional to the (hypostatized) contextual rate of reinforcement; realistically, the rate recently experienced and anticipated, but in this simplification uniform through the session.

Figure 9 shows the success of this contextualization, using the same parameters for both the APR and GPR, for the average data from Experiment 1. The continuous curves give the locus of the path-dependent PR model, and the dashed those of the simple PR model (Equation 2), both contextualized by Equation 3. The two models overlap completely behind the data for the APR, and separate negligibly for the GPR. Contextualized rate of reinforcement was $c = 0$ for the AP in both cases, and 0.007 and 0.008 in the path-dependent and simpler models, respectively. The parameters for both curves are reported in Table 1, where they will be seen to be very similar for both versions of the PR model.

Because of the relative simplicity of the contextualized PR model, it is applied to the data from other experiments, replacing n in Equation 2 with $\bar{n}_i$ from Equation 3. The results are seen in the curves sewing together the data. In Figure 1, all parameters except c were the same for the same subject across progressions. Although there are deviations from the theoretical descriptions, and pigeon B1 shows some convexity even under the AP, the fit of model to data

must be considered good, given the parsimony of account. Parameters for these curves are provided in Table 1.

The ability to account for the behavior of the same animal across different progressions is the necessary accomplishment to call this a general model of PR behavior. Results need not be qualified by which of an infinity of possible progressions have been used, if the same parameters can be recovered from any of them. Alas, it is not clear that the same parameters can always be recovered: The rectilinear right limbs of the AP data in Figures 1, 2, and 9 strongly constrain the inferred values of a; in fitting the GP data the parameter c can take up a lot of slack in the data for that value of a. Whereas Equations 1 – 3 provide a parsimonious and general account of PR performance, theirs is not always a well-behaved account, as the same data can often be almost as well-fit by substantial tradeoffs between c and a.

Our recommendation therefore is to not be ad hoc about the progression used, but to seriously consider the use of an AP, increasing step-size uniformly as necessary to achieve empirical goals, while minimizing the need for an additional parameter, whose employment may overpower inferences about a. Investigators interested only in breakpoints may then fit a linear regression to the right limb, to obtain a more efficient estimate of breakpoint that should prove independent of stopping criteria and, possibly, step size. Changes in animals' timing adaptation may be evaluated by regressing pause lengths against the prior inter-reinforcement intervals, yielding measures of impulsivity ($1 - m$, with m the slope of the regression), adaptability (proximity of intercept to 0) and precision (error variance around the regression). Given the other investments in such research, these additional steps of analysis are easily justified, as they permit inferences about a, the animals' motivation, activation, or wanting of the reinforcer, that are independent of δ and m. Even without estimating δ directly from the data, with m independently measured, Equation 2 will provide accurate estimates of the remaining parameters of psychological interest, λ, δ, and a.

Limitations—There is still much to be known about the nuances of PR responding that has not been addressed in this study. "The size of the increases in PR schedules, the possible inhibitory role of the reinforcer, and the behavioral history of the subjects, could cause differential resistance to extinction (or even a PREE), and could explain some of the results, but are not addressed." (Anonymous, 2007); these are taken in turn.

The *size of the increases* of the progressions has not been reduced to a simple predictor of behavior, in that given a GP exponent, the response form can be predicted *a priori*. However, Experiment 2 shows that step size *per se* is not an important factor in APR, and different GPR may be encompassed by the contextualized model. The typical rate of reinforcement (whether calculated as the median rate of reinforcement in the component ratios, or by estimating it from the harmonic mean of the component ratios) is substantially greater on GPR than APR. We suspect this is the salient difference, as the longest ratios are closer to high rates of reinforcement in the GPR than APR. But until systematic experiments have been conducted, this is speculation.

The *inhibitory role of the reinforcer* has been well-documented (Baron, Mikorski, & Schlund, 1992; P Harzem, Lowe, & Priddle-Higson, 1978; Perone & Courtney, 1992; Staddon, 1974) and theoretically developed (Catania, 1973; P. Harzem & Harzem, 1981; Hovland, 1936; R. L. Reid, 1958). The delivery of food initiates timing of the forthcoming component; at long ratios, this leads to long pauses, making the reinforcer a de facto inhibitory stimulus. This will be exacerbated to the extent that there is within-session *habituation* to the reinforcer (Aoyama, 1998; Aoyama & McSweeney, 2001). Progressive *satiation* will clearly play some role in terminal performance under these schedules (Hodos & Kalman, 1963), as has especially been noted when drugs are the reinforcers (Richardson & Roberts, 1996), and for rats, which do not

have a crop to buffer progressive satiation, as do pigeons (DeMarse, Killeen, & Baker, 1999). The current presentation does not address such satiation, although it has a place in the basic model (Killeen, 1995) that can be unpacked where wanted.

Ratio schedules are intrinsically unstable, in that vicissitudes in response rates have a direct effect on reinforcement rate, which feed back on response rate. Animals whose motivation or attention fluctuates will produce lower rates of reinforcement, which will drive motivation lower; and conversely. Lattal, Reilly and Kohn (1998) yoked rats to other rats on a PR schedule, so that the former received food for a single lever press after the PR animal earned it. The yoked animals persisted longer than did the PR animals. They found the same effects with pigeons. For yoked animals, the passage of time without responding was a predictor of reinforcer availability, whereas the opposite was the case for the PR animals. These kinds of feedback relations are incorporated in the foundational theory leading to Equation 2 (Killeen, 1994), which is an equilibrium solution to the fundamental theoretical model (Killeen, 1992). But those asymptotic models do not capture the path dependence of PR schedules, apprehensible in Figures 5 and 7, or the conditioned inhibitory and excitatory values that long pauses might acquire, as nicely demonstrated by Lattal and associates.

Clearly *behavioral history* can matter (L. Morgan, Killeen, & Fetterman, 1993), and the immediate history that is dependent on the particular progression matters. But evidence indicates that PR training will obliterate the effects of experience with prior schedules of food reinforcement (Cohen, Pedersen, Kinney, & Myers, 1994); prior exposure to stimulants may, however, have more long-lasting effects on incentive motivation for them (Lorrain, Arnold, & Vezina, 2000; D. Morgan, Brebner, Lynch, & Roberts, 2002). The expanding steps between reinforcers in the GPR could indeed be generating a kind of partial reinforcement extinction effect (PREE) that would cause the persistence observed at long ratios; although to invoke that as a mechanism would be to explain one unresolved puzzle with another.

The most important theoretical limitations must include our failure to engage Nevin's theory of behavioral momentum (Lattal, Reilly, & Kohn, 1998; Nevin, 2002; Nevin & Grace, 2001; Nevin, Mandell, & Atak, 1983), as the persistence at the long geometric ratios is nothing if not a manifestation of behavioral momentum. Also lacking was a comparison of the present approach with the more modern and general behavioral economic approach (e.g., Bickel, Green, & Vuchinich, 1995; Johnson & Bickel, 2006; Killeen, 1995; Madden, Smethells, Ewan, & Hursh, 2007). We hope that future work will be able to better address these limitations, without in the process creating too many new ones.

Application—The PR schedule has become a workhorse in behavioral pharmacology (e.g., Arnold & Roberts, 1997; Richardson & Roberts, 1996; Stafford, LeSage, & Glowa, 1998), and yet only one parameter, the ratio at which animals cease to respond (the break point), is typically taken as a dependent variable. "Possibly the most problematic [limitation of PR schedules] is that only a single data point is provided from an entire session." (Arnold & Roberts, 1997, p. 444). This is not only an inefficient use of a datum with fairly high intrinsic variability (Depoortere, Li, Lane, & Emmett-Oglesby, 1993); it is a use that is affected by the criterion for the break point (Lattal & Abreu-Rodrigues, 1994), namely, how long a pause is long enough to say "stopped"? Different criteria yield different results (Stafford & Branch, 1998). The present theory provides a more efficient measure of motivational effects, *a*, that depends on all the data, not just the single ratio at which animals cease to respond. The PR model may be of further use to behavioral pharmacologists in that it offers a way to independently measure response facility $(1/\delta)$ that is more efficient and theoretically based than merely monitoring the obtained distribution of inter-reinforcer intervals (Depoortere, Li, Lane, & Emmett-Oglesby, 1993).

Unfortunately, it is rare for investigators to report the necessary data—response rates at the component ratios. A singular exception is Bradshaw and his collaborators who have reported those rates in numerous publications (e.g., Body et al., 2002; Ho, Body, Kheramin, Bradshaw, & Szabadi, 2003; Kheramin et al., 2005; Zhang, Rickard, Asgari et al., 2005; Zhang, Rickard, Body et al., 2005). Indeed, the reader may turn to Mobini, Chiang, Ho, Bradshaw, and Szabadi (2000), Reilly (2003), Zhang and associates (2005), Kleen, Sitomer, Killeen, and Conrad, (2006) and Sanabria, Acosta, Killeen, Neisewander, and Bizo (2008) for systematic use of the basic FR model to evaluate the mode of action of psychoactive drugs and stress.

Although we were unable to experimentally curtail the convexity of geometric PR functions, the experiments reasserted the orderliness of data issuing from PR schedules, the importance of control by linear waiting, and the utility of simple models of FR performance in describing behavior on PR schedules based on arithmetic progressions. A contextualized theory provided a general account of PR performance. Validating the assumptions of that theory will be the next step in this scientific progression.

Acknowledgments

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Appendix

Nomenclature

Geometric progressions increase each ratio by some fraction of the prior ratio. In Experiment 1 the progression increased by the fraction $2^{j/2}/2^{(j-1)/2} = 2^{1/2} \approx 1.41$; i.e., by about 41%. Richardson and Roberts' (1996) progression $5e^{j/5} - 5$ increases by about 25%; more for the early small ratios, and asymptotically $e^{1/5} \approx 22\%$ for large ratios. Although theirs uses an exponential base, it is only evaluated at integer trial numbers, and thus is strictly a geometric progression (approximately so, because of the subtrahend). "Logarithmic progressions" is probably a misnomer, as those would stipulate a decreasing step-size as trials progress. They could be interesting to use, however, to counter exponentially increasing satiation. Sometimes investigators stipulate an "increase in step size by x log units"; this is actually a geometric progression with ratios increasing by 10^x (or e^x, depending on the base of the logarithm).

A general characterization of progressions, treating each ratio as a function of the prior ratio, is given by: $n_j = \lceil n_{j-1} + g n_{j-1} + a \rceil$, where g is the fractional increment of the geometric progression, a the additive increment, and $|\;|$ indicates that the result is to be rounded up to the next integer. For GP, $a = 0$, and for AP, $g = 0$. The GP of the first experiment is thus $n_j = \lceil n_{j-1} + 0.414\, n_{j-1} \rceil$. Richardson and Roberts' progression is de facto equivalent to $n_j = \lceil n_{j-1} + 0.221 n_{j-1} + 1.107 \rceil$, although that is not obvious. In a few cases a minimal step size is enforced, such as: $n_j = n_{j-1} + \max\left(\lceil g n_{j-1} \rceil, 1\right)$.

Finally, a power series such as $n_j = \lceil b n_{j-1}^g + a \rceil$ has the following features: For $g = 0$ it is an FR $(a+b)$; for $g < 1$ and $b \leq 1$, step sizes decrease with n; for $g = b = 1$ it is an arithmetic progression; for $g = 1$ and $b > 1$, it is a geometric progression; for $b = 1$ and $g > 1$ the absolute value of the step size increases with n, but the increase in step size is less than proportional. Although these can be positive features for some purposes, absent those purposes there is little reason to choose the generality of a power series over a geometric or arithmetic progression. The same may be said for the choice of a geometric progression over an arithmetic one. Given the infinity of possible scheduling arrangements, progressions that focus data collection on the conditions that are most relevant to the psychological questions, and theories that can take data from any such progressions and convert it into method-independent quantification of psychological variables, provide the ideal package. Meanwhile, starting conditioning with an APR step size 1, and increasing the step size during training until half the subjects cease responding during the allotted session time, is a good way to generate orderly and interpretable data using the simple PR model of Equation 2.

Combining data and parameters

Data

The central tendency of data across sessions and subjects are computed using trimmed means. These are more robust to eccentric data than the mean, yet more efficient than the median. Wilcox (1998) recommends a symmetric 20% trim (top 20% and bottom 20%; in some applications such as Excel ® this is called a 40% trim). Such trimming is not throwing away data: it is more conservative and efficient than a median, which is a symmetric $100(n − 1)/2n$ % trim, where n is the number of data summarized.

Another reason for the use of trimmed means is that at long ratios animals cease responding. Up to this point rate may have been decreasing as a linear function of ratio value. The average of a number of such linear functions with different break-points and zero responding thereafter will follow a curvilinear locus. This is the same kind of averaging artifact that can morph the all-or-none break-and-run response patterns on FI and FR schedules to make them appear scalloped. It is avoided by using trimmed means, which are not affected when the smallest 20% of rates fall to zero—and thereafter abridging the average when the next additional score falls to zero. This abridgement may consist of curtailing data analysis at that point or switching to medians. Our algorithm to maximize usable data while avoiding bias was to continue aggregating until one zero datum entered the non-trimmed points, and then to stop reporting data for ratios higher than that. We followed this procedure both within and between animals.

Parameters

The parameters of the models presented here often have skewed distributions. Both logic and the analysis of large data sets recommend the following transformations to reduce the impact of heteroscedasticy: For response duration (δ), and β, use the simple arithmetic mean; for λ, use the geometric or harmonic mean; for specific activation (a), use the harmonic mean, which averages the slopes of the decreasing limbs. As ever, direct measurement of these parameters, where possible (e.g., δ as the first percentile of inter-response times; m as the slope of the PRP function), tightens confidence intervals around both them, and the remaining parameters.

Note that the inferred extinction ratios, a, are much smaller than those that can be supported in closed economies, where subjects earn all their reinforcers within the chamber. This is because: (a) In closed economies, the magnitude of reinforcement is often substantially greater, and that directly affects a (Killeen, 1995); and (b) subjects in such designs have extended opportunities to complete the ratios—hours rather than the minutes allotted in breaking-point criteria—and hunger motivation may increase substantially during that time, or vicissitudes in hunger may carry the animal over a threshold for responding (Posadas-Sanchez & Killeen, 2005).

Developing the PR model from the FR model

Theoretical variables for the ratio models.

Empirical Constructs

$B_i \equiv$ Overall response rate (# resp/s)

$n_i \equiv$ Ratio requirement (# resp/Rf)

$t_i^{Rf} \equiv$ Interreinforcement interval (s)

$t_i^P \equiv$ Pre- run pause (s); time to first response

$t_i^R \equiv$ Run time (s); $t_i^{\text{Rf}} - t_i^P$

$b_i \equiv$ Running rate (# resp/s); $(n_i - 1)/t_i^R$

$m \equiv$ proportion of interreinforcement interval spent pausing (s/s)

Theoretical Constructs

$C_{FR} \equiv$ Coupling Coefficient for FR schedules (dimensionless, $0 \leq C_{FR} \leq 1$)

$\delta \equiv$ Delta: The minimum interresponse time (s)

$a \equiv$ Specific activation (resp/Rf)

$k' \equiv$ Proportion of IRI spent in run state (s/s), when inferred rather than measured

The Progressive Ratio model

1. $t_i^{\text{Rf}} = \dfrac{n_i}{B_i}$, $B_i > 0$; by definition.

2. $t_i^P = m t_{i-1}^{\text{Rf}}$; proportional one-back waiting.

3. $\begin{aligned} b_i &= \tfrac{1}{\delta}\left(C_{FR,i-1} - \tfrac{n_{i-1}}{a}\right), & \delta, a > 0, n_{i-1} \leq a, \\ b_i &= 0 & \text{otherwise} \end{aligned}$ (based on Killeen & Sitomer, 2003).

4. $C_{FR} = 1 - e^{-\lambda\delta n}$, or
$C_{FR} = 1 - (1 - \beta)^n$, where $\beta = 1 - e^{-\lambda\delta}$;

5. $t_i^R = \dfrac{n_i}{b_i}$; by definition.

6. $B_i = \dfrac{n_i}{t_i^P + t_i^R}$; by definition: total responses divided by total time.

7. $B_i = \dfrac{n_i}{m n_{i-1}/B_{i-1} + n_i/b_i}$, $b_i, B_{i-1} > 0$; from Line 6 by substitution.

This is the path-dependent PR model. Predictions depend on those for the prior ratio: B_i depends on n_{i-1}/B_{i-1}. It is necessary for situations such as those explored in Experiment 3 – 4.

In cases such as FR or closely spaced PR where $B_i \approx B_{i-1}$, the former may be substituted for the latter to yield:

8. $B_i \approx k' b_i$; with b_i given by Line 3 and $k' = 1 - m\, n_{i-1}/n_i$.

This new FR model also suffices for well-behaved PR schedules. It is the path-*in*dependent form of Equation 7, and appears in the text as Equation 2, where it is further simplified by the substitution $k = k'/\delta$.

Commentary on the model

Line 2—A linear, rather than proportional (intercept = 0), waiting time model is more veridical and will increase accuracy for the shortest ratios. Furthermore, fitting an ARIMA model gives evidence for small (20%) effects of IRIs two and more back, as also seen in the results from Experiment 3. But these nuances complicate the model with little return on the investment for the schedules typically analyzed.

Line 3—This constitutes a new interpretation of the ratio model. MPR, like any operating system, gets updated (e.g., as in Killeen and Sitomer, 2003). Here, the basic FR prediction is made for responding during the run-state, not the overall rate, as it had originally been. This permits δ to assume its intended role, the minimum interresponse time, rather than be confounded with pause time. It makes the implicit assumption that running rate is under the control of the rate of reinforcement in the running state, not of overall rate of reinforcement. This was also Schneider's (1969) conclusion for running rates in FI schedules. It is invoked here as a working hypothesis. This revision was forced by the data of Widholm (1997) and others that manifested how significant a role pausing played in the bitonicity of FR response functions. The small positive intercept plays a large role in the lower response rates on short FR schedules.

Because the distribution of *a* is highly skewed, the preferred measures of central tendency are harmonic means and trimmed harmonic means.

The n_{i-1} in Line 3 should be MAX(n_{i-1}, n''), where n'' is the number of responses as they are being emitted in the contemporary component. This would track real-time performance, not snapshot the control from the previous trial. This refinement is not needed for the current analyses; but is the reason the model does not track the rightmost points in Fig. 1.

Line 4—The coupling coefficient was expanded by Killeen and Sitomer (2003) to include δ in the exponent, in recognition of the otherwise very high empirical correlations between lambda (λ) and δ. This modification entails that long-duration responses displace more memory than short ones.

For integer *n*, the two forms given in Line 4 are equivalent; the first is closer to that used in the original theory (Killeen, 1994), the second closer to that used for discrete responses of uniform duration (Bizo & Killeen, 1997b). The parameter beta (β) is more uniformly distributed than λ, and in that sense a better dependent variable; however it is a conflation of response duration and decay rate. Because the distribution of λ is skewed, its average across subjects or conditions should be a geometric mean or trimmed geometric mean.

Line 6—The derivation is complete, but is recapitulated in Line 7. It is called the *Path-dependent PR Model* because the predicted rate for each component in the PR schedule depends on the experimental conditions and behavior of the prior component. Whereas predictions would be more accurate using the measured response rate to calculate the prior IRI, that would use additional information from the data and thus reduce the model's degrees of freedom relative to the data. B_{i-1} is therefore always the rate *predicted* for the prior component. B_0 may be set to an arbitrary value, such as 1; or set to the observed rate in the first component, as long as those are excluded from the calculations of indices of merit. If the progression is geometric, the ratio n_{i-1}/n_i is constant and could be absorbed into k', when that is treated as a free parameter. In the case of FR schedules, $n_{i-1} = n_i$, and $B_i = B_{i-1}$, reducing Line 7 to Line 8 precisely. Because Line 8 is adequate for some purposes of the analysis of PR schedules (see, e.g., Figure 2, and the bottom panel of Figure 5), this new FR model is also referred to as the *simple PR model*.

Line 8—Overall response rate equals the running rate multiplied by the proportion of the interval in the run state. When $k' = 1$, this reduces to the old FR model (Line 3). Because the fundamental model in Line 3 has a coefficient of $δ^{-1}$, it is clear that k and δ are doing similar work; they are collinear. Line 8 is the preferred model for FR schedules, and provides a useful approximation for some PR schedules. It has been our tactic in this paper to hold δ constant at 0.25 s a—representative estimate for pigeons, and for rats on some levers—whenever it or pause durations were not measured. A better tactic is to measure *m* and then the recovered value of δ will provide useful information. The ideal tactic is to measure both k and δ. Delta

may be estimated as the duration of the response at the 1st percentile of the distribution of IRTs. Use of the 1st percentile avoids exceptionally short responses due to key-or lever-bounce.

The Contextualized Ratio Model

The easiest way to add contextual reinforcement to the PR models is to adjust the effective ratio by adding a virtual ratio (n_c) concurrent with the actual one: $r_c \propto 1/n_c$, where $\propto$ means "proportional to". Since it is rates of reinforcement that must be blended, and these are inversely proportional to the ratio requirements, this entails a harmonic mean. It is exactly the came case as when two VI schedules are run concurrently; the resulting average time between reinforcers is given by:

$$VI_{effective} = 1/(1/VI_1 + 1/VI_2).$$

In the case of two VI 60 schedules, for example, the effective VI is given by the equation as 30.

The logic is the same for adjusting the ratio component for contextual reinforcement:

$$\bar{n}_i = 1/(1/n_i + 1/n_c),$$

where $\bar{n}_i$ is the resulting effective ratio. Since n_c is an unknown parameter to be estimated from the data, this may be simplified by multiplying the rhs by n_i, leading to:

$$\bar{n}_i = n_i/(1 + cn_i),$$

where the free parameter c is proportional to the contextual rate of reinforcement, much like Herrnstein's (1979) r_0. Because it represents the ratio of two count variables (reinforcers per response) it is dimensionless.
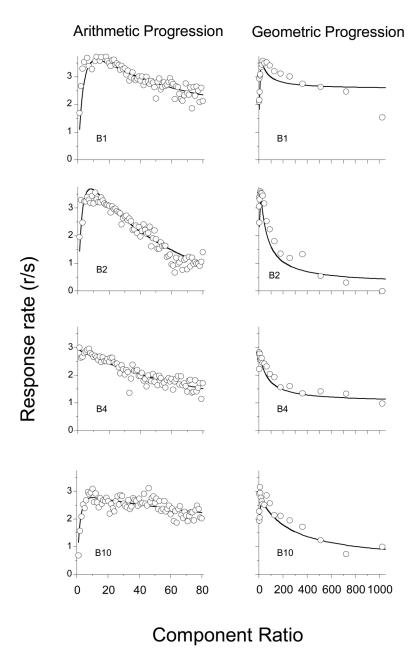
Figure 1.
The average response rates from the last 5 sessions of Experiment 1; note the different abcissae for the two columns. The curves are from the contextualized model, Equations 2 and 3, with parameters listed in Table 1.
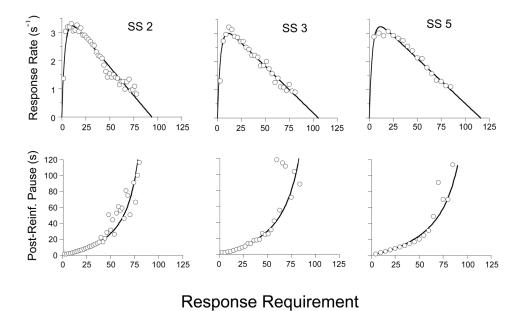
Figure 2.
Response rates and PRPs on the APR schedules of Experiment 2, averaged over subjects over the last 6 sessions. Parameters for the simple PR/FR model, Equation 2, are listed in Table 2; Equation 1 fit the pause data using the same parameter *a* that was employed in the curve above.
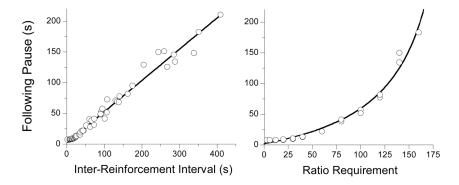
Figure 3.
Post-reinforcement pause durations, averaged across subjects from Experiment 3, are plotted as a function of the inter-reinforcement intervals in the prior component (left panel) and as a function of the ratio requirements in the prior component (right panel). The line in the left panel is given by the one-back linear waiting model; that on the right by Equation 1.
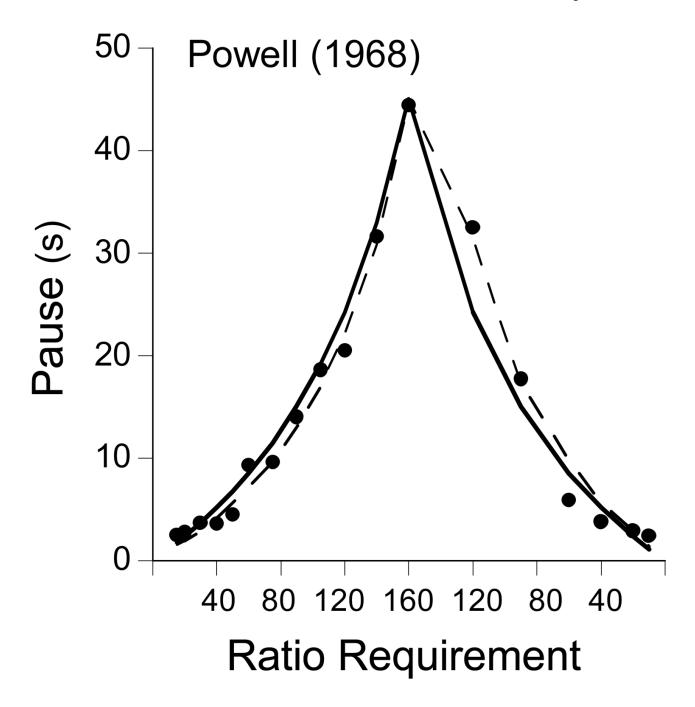
Figure 4.
The median pause lengths of 3 pigeons reported by Powell, as a function of FR requirement. The curves are drawn by Equation 1; the continuous using stipulated ratio sizes, the dashed assuming some hysteresis in control.
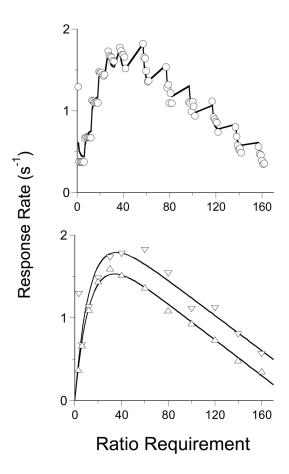
Figure 5.
Response rates at successive ratio requirements from Experiment 3, averaged across subjects over the last 20 sessions. *Top*: The circles representing each set of six reinforcers are separated by a minimum of 1 response for clarity of presentation. For example, the circles for FR 30 take positions at 27 responses (representing the first component of the FR 30 set) through 32. The curve was produced by the path-dependent PR model. *Bottom*: Response rates for the first (down-triangle) and last reinforcer in each component of each ratio. The lines were produced by the simple PR/FR model, Equation 2.
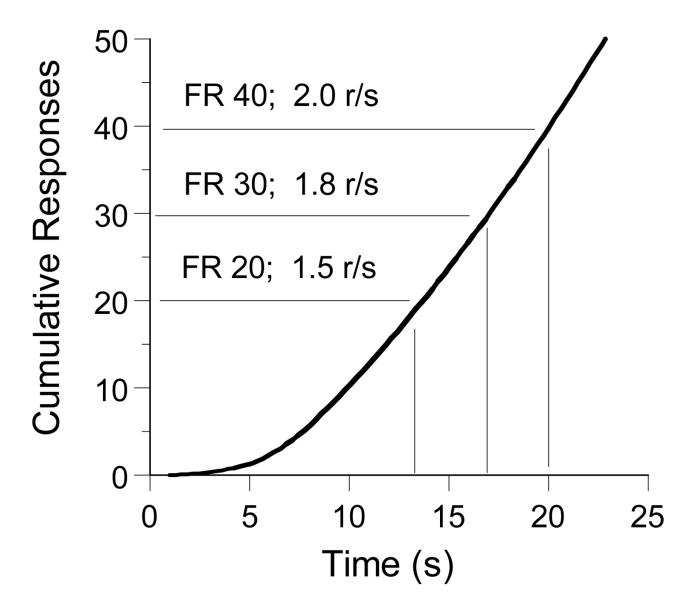
Figure 6.
A cumulative record of the number of responses in a representative FR 20 component. On the first occasion that the requirement is lengthened to 30 or to 40, it will add run time and increase the overall response rate as shown in the figure. Such changes in response rate do not reflect changes in behavior, but rather changes in the proportions of pausing and running phases of behavior that are sampled.
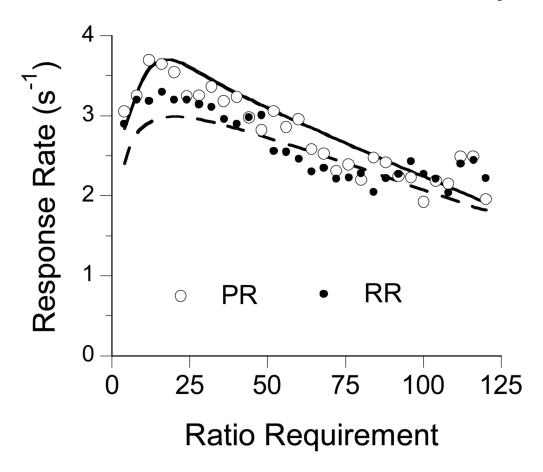
Figure 7.
Response rates of six pigeons on the last session of PR and Regressive Ratio (RR)
reinforcement schedules. The curves are from the path-dependent PR model, using the same
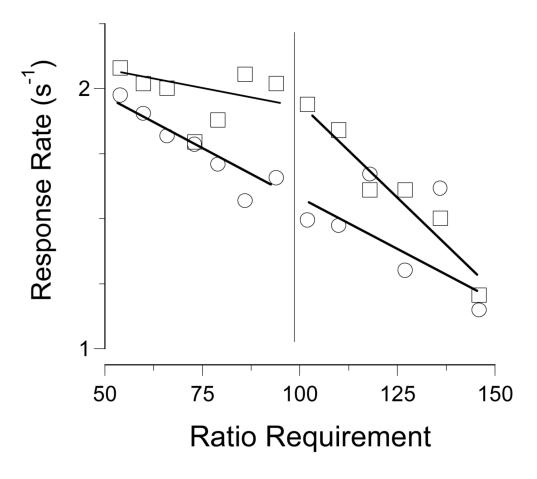parameters for both functions.

Figure 8.
Response rates on the PR schedule of Experiment 6, averaged across subjects over the last 12 sessions. Squares depict overall response rates from six components before and after a ratio of 100, the former to a white key, and the latter to a red key. Circles depict response rates on the same progression without stimulus change.
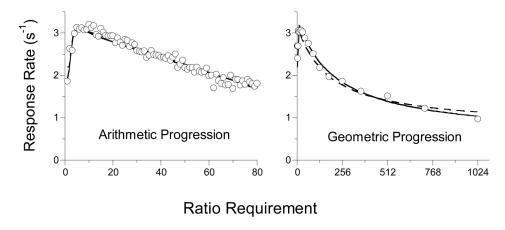
Figure 9.
Average response rates pooled over the four pigeons of Experiment 1. The contextualized PR models—path-dependent (continuous curves) and simple (dashed, Equations 2 and 3)—drew the curves, and delivered the parameters displayed in Table 1.

Table 1

Parameters of the contextualized model (Equations 2 and 3) for Experiment 1.

	Experiment 1				
	k	*λ*	*a*	*c* (**AP**)	*c* (**GP**)
B1	0.06	2.0	182	0.000	0.011
B2	0.03	2.7	86	0.002	0.011
B4	0.28	5.0	112	0.005	0.013
B10	0.28	2.5	320	0.000	0.003
Pooled; Path-Dependent Model	0.22	3.7	178	0.000	0.007
Pooled; Simple Model	0.19	3.4	171	0.000	0.008

Table 2

Parameters of the simple PR model (Equation 2) for the results of Experiment 2.

Step Size	k	λ	a
2	0.05	1.25	94
3	0.13	1.25	105
5	0.07	1.25	116