Key

(11) $\cos x(2\sin x + 1) = 0$

$\cos x = 0 \quad \sin x = -1/2$

$\frac{\pi}{2}, \frac{3\pi}{2}, \quad \frac{7\pi}{6}, \frac{11\pi}{6}$

(12) $\sec x(2\cos x - \sqrt{2}) = 0$

$\sec x = 0 \quad \cos x = \sqrt{2}/2$

$\frac{\pi}{2}, \frac{3\pi}{2} \quad \pi/4, \frac{7\pi}{4}$

or $\varnothing$

(13) $(\tan x + \sqrt{3})(\cos x + 2) = 0$

$60^\circ \quad \tan x = -\sqrt{3} \quad \cos x = -2$

$\frac{2\pi}{3}, \frac{4\pi}{3} \quad \varnothing$

(14) $(2\cos x + \sqrt{3})(2\sin x - 1) = 0$

$30^\circ \quad \cos x = -\frac{\sqrt{3}}{2} \quad \sin x = \frac{1}{2}$

$\frac{5\pi}{6}, \frac{7\pi}{6} \quad \pi/3, \frac{4\pi}{3}$

(15) $\cos x \sin x - 2\cos x = 0$

$\cos x(\sin x - 2) = 0$

$\cos x = 0 \quad \sin x = 2$

$\pi/2, \frac{3\pi}{2} \quad \varnothing$

(16) $\tan x \sin x + \sin x = 0$

$\sin x(\tan x + 1) = 0$

$\sin x = 0 \quad \tan x = -1$

$0, \pi \quad \frac{3\pi}{4}, \frac{7\pi}{4}$

(17) $4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2} \quad 60^\circ$

$\pi/3, \frac{5\pi}{3}$

(18) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2 \quad \sin x = 1$

$\frac{7\pi}{6}, \frac{11\pi}{6} \quad \pi/2$

(20) $3\tan^3 x = \tan x$

$3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0 \quad \tan^2 x = 1/3$

$\pi/2, \frac{3\pi}{2} \quad \tan = \pm\sqrt{3}/3$

30°

$\pi/6, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(22) $2\cos^2 x + \sin x = 1$

$2(1 - \sin^2 x) + \sin x - 1 = 0$

$2 - 2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2 \quad \sin x = 1$

$\frac{7\pi}{6}, \frac{11\pi}{6} \quad \frac{\pi}{2}$

(24) $2\cos 2x + 1 = 0$

$\cos 2x = -1/2$

over interval $0 \le x < 4\pi$

$2x = \cos^{-1}(1/2)$

$60^\circ \to \pi/3$

$2x = \frac{2\pi}{3} \quad 2x = \frac{4\pi}{3}$

$x = \pi/3 \quad x = 2\pi/3$

(26) $\sqrt{3}\tan 3x + 1 = 0$

$\tan 3x = -\sqrt{3}/3$ (30°, $\pi/6$)

$3x = \frac{11\pi}{6}, \frac{5\pi}{6}$

$x = \frac{11\pi}{18}, \frac{5\pi}{18}$

(25) $\cos 3x = \sin 3x$

$\cos^2 3x = \sin^2 3x$

$1 - \sin^2 3x = \sin^2 3x$

$2\sin^2 3x = 1$

$\sin^2 3x = 1/2$

$\sin 3x = \pm\sqrt{2}/2$ (45°)

$3x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$x = \pi/12, \pi/4, 5\pi/12, \frac{7\pi}{12}$

(30) $2\sin\frac{x}{3} + \sqrt{3} = 0$

$\sin x/3 = -\frac{\sqrt{3}}{2}$ (60°, $\pi/3$)

$\frac{x}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = 4\pi, 5\pi$

(39) $2\cos 3x = 1$

$\cos 3x = 1/2$ (60°, $\pi/3$)

$3x = \pi/3, \frac{5\pi}{3}$

$x = \pi/9, 5\pi/9$

(41) $2\sin x\tan x - \tan x = 1 - 2\sin x$

$\tan x(2\sin x - 1) = 1 - 2\sin x$

$\tan x = -1$

$x = 3\pi/4, 7\pi/4$

(42) $\sec x\tan x - \cos x\cot x = \sin x$

$\frac{\sin x}{\cos^2 x} - \frac{\cos^2 x}{\sin x} = \sin x$

$\frac{\sin^2 x}{\sin x\cos^2 x} - \frac{\cos^4 x}{\sin\cos^2 x} = \frac{\sin^2 x\cos^2 x}{\sin\cos^2 x}$

$\sin^2 x - \cos^4 x - \sin^2 x\cos^2 x = 0$

$\sin^2 x - \sin^2 x\cos^2 x - \cos^4 x = 0$

$1 - \cos^2 x - ((1-\cos^2 x)\cos^2 x) - \cos^4 x = 0$

$1 - \cos^2 x - 1 - \cos^2 x + \cos^4 x - \cos^4 x = 0$

$\cos^2 x = 0$ $\quad x = \pi/2, 3\pi/2$

(43) $\tan x - 3\cot x = 0$

$\frac{\sin x}{\cos x} - \frac{3\cos x}{\sin x} = 0$

$\frac{\sin^2 x - 3\cos^2 x}{\sin x\cos x} = 0$

$\sin^2 x - 3\cos^2 x = 0$

$\frac{\sin^2 x}{\cos^2 x} = 3$

$\tan^2 x = \pm\sqrt{3}$

(30°) $\pi/6, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(45) $\tan 3x + 1 = \sec 3x$

$\tan^2 + 2\tan + 1 = \sec^2 x$

$\tan^2 + 2\tan + 1 - (1 + \tan^2 x) = 0$

$2\tan 3x = 0$

$\tan 3x = 0$

$3x = 0, \pi$

$x = 0, \pi/3$

(49) $2\sin^2 x - \cos x = 1$

$2(1 - \cos^2 x) - \cos x = 1$

$2\cos^2 x + \cos x - 2 - 1 = 0$

$2\cos^2 x + \cos x - 3 = 0$

$(2\cos x + 3)(\cos x - 1) = 0$

$\cos x = -3/2$ $\quad \cos x = 1$

$\varnothing$ $\quad 0, 2\pi$

(46) $3\sec^2 x + 4\cos^2 x = 7$

$\frac{3}{\cos^2} + \frac{4\cos^4 x}{\cos^2} = \frac{7\cos^2 x}{\cos^2 x}$

$4\cos^4 x - 7\cos^2 x + 3 = 0$

$(4\cos^2 x - 3)(\cos^2 x - 1) = 0$

(30°) $\cos x = \pm\sqrt{3}/4$ $\quad \cos x = \pm 1$

$\pi/6, \frac{5\pi}{6}$ $\quad \pi, 2\pi$

$7\pi/6, 11\pi/6$

(40) $3\csc^2 x = 4$

$\sin^2 x = 3/4$

$\sin x = \pm\sqrt{3}/2$

$\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$