# Algebraic subtyping for algebraic effects and handlers

Axel Faes
student : undergraduate
ACM Student Member: 2461936
Department of Computer Science
KU Leuven
axel.faes@student.kuleuven.be

Amr Hany Saleh
daily advisor
Department of Computer Science
KU Leuven
amrhanyshehata.saleh@kuleuven.be

Tom Schrijvers
promotor
Department of Computer Science
KU Leuven
tom.schrijvers@kuleuven.be

## Abstract

Algebraic effects and handlers are a very active area of research. An important aspect is the development of an optimising compiler. Eff is an ML-style language with support for effects and forms the testbed for the optimising compiler. However, the type-&-effect system of Eff is unsatisfactory. This is due to the lack of some elegant properties. It is also awkward to implement and use in practice.

***Keywords*** algebraic effect handler, algebraic subtyping, effects, optimised compilation

Contents



List of Figures



List of Tables

## 1 Introduction

The specification for a type-&-effect system with algebraic subtyping for algebraic effects and handlers is given in this document. The formal properties of this system are studied in order to find which properties are satisfied compared to other type-&-effect systems. The proposed type-&-effect system builds on two very recent developments in the area of programming language theory.

***Algebraic subtyping*** In his December 2016 PhD thesis, Stephen Dolan (University of Cambridge, UK), has presented a novel type system that combines subtyping and parametric polymorphism in a particulary attractive and elegant fashion. A cornerstone of his design are the algebraic properties that the subtyping relation should respect.

***Algebraic effects and handlers*** These are a new formalism for formally modelling side-effects (e.g. mutable state or non-determinism) in programming languages, developed by Matija Pretnar (University of Ljubjana) and Gordon Plotkin (University of Edinburgh). This approach is gaining a lot of traction, not only as a formalism but also as a practical feature in actual programming languages (e.g. the Koka language developed by Microsoft Research). We are collaborating with Matija Pretnar on the efficient implementation of one such language, called Eff. Axel Faes has contributed

to this collaboration during a project he did for the Honoursprogramme of the Faculty of Engineering Science.

### 1.1 Motivation

Algebraic effects and handlers benefit from a custom type-&-effect system, a type system that also tracks which effects can happen in a program. Several such type-&-effect systems have been proposed in the literature, but all are unsatisfactory. We attribute this to the lack of the elegant properties of Dolan's type system. Indeed the existing type-&-effect systems are not only theoretically unsatisfactory, but they are also awkward to implement and use in practice.

***Research questions***

- How can Dolan's elegant type system be extended with effect information?
- Which properties are preserved and which aren't preserved?
- What advantages are there to an type-&-effect system based on Dolan's elegant type system?

### 1.2 Goals

The goal of this thesis is to derive a type-&-effect system that extends Dolan's elegant type system with effect information. This type-&-effect system should inherit Dolan's harmonious combination of subtyping (in our case induced by a lattice structure on the effect information) with parametric polymorphism and preserve all of its desirable properties (both low-level algebraic properties and high-level metatheoretical properties like type soundness and the existence of principal types). Afterwards this type-&-effect system The following approach is taken:

1. Study of the relevant literature and theoretical background.
2. Design of a type-&-effect system derived from Dolan's, that integrates effects.
3. Proving the desirable properties of the proposed type-&-effect system: type soundness, principal typing, ...
4. Time permitting: Design of a type inference algorithm that derives the principal types of programs without type annotations and proving its correctness.
5. Time permitting: Implementation of the algorithm and comparing it to other algorithms (such as row polymorphism based type-&-effect systems).

## 2 Background

## 3 Related Work (Eff)

The type-&-effect system that is used in Eff is based on subtyping and dirty types [1].

### 3.1 Types and terms

***Terms*** Figure 1 shows the two types of terms in Eff. There are values $v$ and computations $c$. Computations are terms that can contain effects. Effects are denoted as operations $Op$ which can be called.

| | | | |
|---|---|---|---|
| value $v$ | ::= | $x$ | variable |
| | \| | $\texttt{true}$ | true |
| | \| | $\texttt{false}$ | false |
| | \| | $\lambda x.c$ | function |
| | \| | $\{$ | handler |
| | | $\quad\texttt{return}\ x \mapsto c_r,$ | return case |
| | | $\quad[\texttt{Op}\ y\ k \mapsto c_{\texttt{Op}}]_{\texttt{Op}\in O}$ | operation cases |
| | | $\}$ | |
| comp $c$ | ::= | $v_1\ v_2$ | application |
| | \| | $\texttt{let rec}\ f\ x = c_1\ \texttt{in}\ c_2$ | rec definition |
| | \| | $\texttt{return}\ v$ | returned val |
| | \| | $\texttt{Op}\ v$ | operation call |
| | \| | $\texttt{do}\ x \leftarrow c_1\ ;\ c_2$ | sequencing |
| | \| | $\texttt{handle}\ c\ \texttt{with}\ v$ | handling |

**Figure 1.** Terms of Eff

***Types*** Figure 2 shows the types of Eff. There are two main sorts of types. There are (pure) types $A, B$ and dirty types $\underline{C}, \underline{D}$. A dirty type is a pure type $A$ tagged with a finite set of operations $\Delta$, which we call dirt, that can be called. This finite set $\Delta$ is an over-approximation of the operations that are actually called. The type $\underline{C} \Rightarrow \underline{D}$ is used for handlers because a handler takes an input computation $\underline{C}$, handles the effects in this computation and outputs computation $\underline{D}$ as the result.

| | | | |
|---|---|---|---|
| (pure) type $A, B$ | ::= | $\texttt{bool}$ | bool type |
| | \| | $A \to \underline{C}$ | function type |
| | \| | $\underline{C} \Rightarrow \underline{D}$ | handler type |
| dirty type $\underline{C}, \underline{D}$ | ::= | $A\ !\ \Delta$ | |
| dirt $\Delta$ | ::= | $\{\texttt{Op}_1, \ldots, \texttt{Op}_n\}$ | |

**Figure 2.** Types of Eff

### 3.2 Type System

#### 3.2.1 Subtyping

The dirty type $A\ !\ \Delta$ is assigned to a computation returning values of type $A$ and potentially calling operations from the set $\Delta$. This set $\Delta$ is always an over-approximation of the actually called operations, and may safely be increased, inducing a natural subtyping judgement $A\ !\ \Delta \leq A\ !\ \Delta'$ on dirty types. As dirty types can occur inside pure types, we also get a derived subtyping judgement on pure types. Both judgements are defined in Figure 3. Observe that, as usual, subtyping is contravariant in the argument types of functions and handlers, and covariant in their return types.

**Subtyping**

$$\textsc{Sub-bool}\ \frac{}{\texttt{bool} \leqslant \texttt{bool}} \qquad \textsc{Sub-}\longrightarrow\ \frac{A' \leqslant A \qquad \underline{C} \leqslant \underline{C}'}{A \to \underline{C} \leqslant A' \to \underline{C}'}$$

$$\textsc{Sub-}\Rightarrow\ \frac{\underline{C}' \leqslant \underline{C} \qquad \underline{D} \leqslant \underline{D}'}{\underline{C} \Rightarrow \underline{D} \leqslant \underline{C}' \Rightarrow \underline{D}'} \qquad \textsc{Sub-!}\ \frac{A \leqslant A' \qquad \Delta \subseteq \Delta'}{A\ !\ \Delta \leqslant A'\ !\ \Delta'}$$

**Figure 3.** Subtyping for pure and dirty types of Eff

### 3.2.2 Typing rules

Figure 4 defines the typing judgements for values and computations with respect to a standard typing context $\Gamma$.

***Values*** The rules for subtyping, variables, and functions are entirely standard. For constants we assume a signature $\Sigma$ that assigns a type $A$ to each constant k, which we write as $(\mathsf{k} : A) \in \Sigma$.

A handler expression has type $A\ !\ \Delta \cup O \Rightarrow B\ !\ \Delta$ iff all branches (both the operation cases and the return case) have dirty type $B\ !\ \Delta$ and the operation cases cover the set of operations $O$. Note that the intersection $\Delta \cap O$ is not necessarily empty. The handler deals with the operations $O$, but in the process may re-issue some of them (i.e., $\Delta \cap O$).

When typing operation cases, the given signature for the operation $(\mathsf{Op} : A_{\mathsf{Op}} \to B_{\mathsf{Op}}) \in \Sigma$ determines the type $A_{\mathsf{Op}}$ of the parameter $x$ and the domain $B_{\mathsf{Op}}$ of the continuation $k$. As our handlers are deep, the codomain of $k$ should be the same as the type $B\ !\ \Delta$ of the cases.

***Computations*** With the following exceptions, the typing judgement $\Gamma \vdash c : \underline{C}$ has a straightforward definition. The `return` construct renders a value $v$ as a pure computation, i.e., with empty dirt. An operation invocation $\mathsf{Op}\, v$ is typed according to the operation's signature, with the operation itself as its only operation. Finally, rule With shows that a handler with type $\underline{C} \Rightarrow \underline{D}$ transforms a computation with type $\underline{C}$ into a computation with type $\underline{D}$.

typing contexts $\Gamma ::= \epsilon \mid \Gamma, x : A$

**Expressions**

$$\textsc{SubVal}\ \frac{\Gamma \vdash v : A \qquad A \leqslant A'}{\Gamma \vdash v : A'} \qquad \textsc{Var}\ \frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}$$

$$\textsc{True}\ \frac{}{\Gamma \vdash \texttt{true} : \mathit{bool}} \qquad \textsc{False}\ \frac{}{\Gamma \vdash \texttt{false} : \mathit{bool}}$$

$$\textsc{Fun}\ \frac{\Gamma, x : A \vdash c : \underline{C}}{\Gamma \vdash \lambda x.c : A \to \underline{C}}$$

$$\textsc{Hand}\ \frac{\Gamma, x : A \vdash c_r : B\ !\ \Delta \qquad \Big[(\mathsf{Op} : A_{\mathsf{Op}} \to B_{\mathsf{Op}}) \in \Sigma \quad \Gamma, x : A_{\mathsf{Op}}, k : B_{\mathsf{Op}} \to B\ !\ \Delta \vdash c_{\mathsf{Op}} : B\ !\ \Delta\Big]_{\mathsf{Op} \in O}}{\Gamma \vdash \{\texttt{return}\ x \mapsto c_r, [\mathsf{Op}\, y\, k \mapsto c_{\mathsf{Op}}]_{\mathsf{Op} \in O}\} : A\ !\ \Delta \cup O \Rightarrow B\ !\ \Delta}$$

**Computations**

$$\textsc{SubComp}\ \frac{\Gamma \vdash c : \underline{C} \qquad \underline{C} \leqslant \underline{C}'}{\Gamma \vdash c : \underline{C}'} \qquad \textsc{App}\ \frac{\Gamma \vdash v_1 : A \to \underline{C} \qquad \Gamma \vdash v_2 : A}{\Gamma \vdash v_1\, v_2 : \underline{C}}$$

$$\textsc{LetRec}\ \frac{\Gamma, f : A \to \underline{C}, x : A \vdash c_1 : \underline{C} \qquad \Gamma, f : A \to \underline{C} \vdash c_2 : \underline{D}}{\Gamma \vdash \texttt{let rec}\ f\ x = c_1\ \texttt{in}\ c_2 : \underline{D}}$$

$$\textsc{Ret}\ \frac{\Gamma \vdash v : A}{\Gamma \vdash \texttt{return}\ v : A\ !\ \emptyset} \qquad \textsc{Op}\ \frac{(\mathsf{Op} : A \to B) \in \Sigma \qquad \Gamma \vdash v : A}{\Gamma \vdash \mathsf{Op}\, v : B\ !\ \{\mathsf{Op}\}}$$

$$\textsc{Do}\ \frac{\Gamma \vdash c_1 : A\ !\ \Delta \qquad \Gamma, x : A \vdash c_2 : B\ !\ \Delta}{\Gamma \vdash \texttt{do}\ x \leftarrow c_1\ ;\ c_2 : B\ !\ \Delta}$$

$$\textsc{With}\ \frac{\Gamma \vdash v : \underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c : \underline{C}}{\Gamma \vdash \texttt{handle}\ c\ \texttt{with}\ v : \underline{D}}$$

**Figure 4.** Typing of Eff

## 4 Core language (EffCore)

EffCore is a language with row-based effects, intersection and union types and effects and is subtyping based.

### 4.1 Types and terms

***Terms*** Figure 5 shows the two types of terms in EffCore. There are values $v$ and computations $c$. Computations are

terms that can contain effects. Effects are denoted as operations $Op$ which can be called. The function term is explicitly annotated with a type and type abstraction and type application has been added to the language. These terms only work on pure types.

| | | | |
|---|---|---|---|
| value $v$ | ::= | $x$ | $\lambda$-variable |
| | \| | $\hat{\mathbf{x}}$ | let-variable |
| | \| | `true` | true |
| | \| | `false` | false |
| | \| | $\lambda x.c$ | function |
| | \| | { | **handler** |
| | | `return` $x \mapsto c_r$, | return case |
| | | $[\mathtt{Op}\ y\ k \mapsto c_{\mathtt{Op}}]_{\mathtt{Op} \in O}$ | operation cases |
| | | } | |
| comp $c$ | ::= | $v_1\ v_2$ | application |
| | \| | `let` $\hat{\mathbf{x}} = c_1$ `in` $c_2$ | let |
| | \| | `if` $e$ `then` $c_1$ `else` $c_2$ | conditional |
| | \| | `return` $v$ | **returned val** |
| | \| | Op $v$ | **operation call** |
| | \| | `handle` $c$ `with` $v$ | **handling** |

**Figure 5.** Terms of EffCore

***Types*** Figure 6 shows the types of EffCore. There are two main sorts of types. There are (pure) types $A, B$ and dirty types $\underline{C}, \underline{D}$. A dirty type is a pure type $A$ tagged with a finite set of operations $\Delta$, which we call dirt, that can be called. It can also be an union or intersection of dirty types. In further sections, the relations between dirty intersections or unions and pure intersections or unions are explained. The finite set $\Delta$ is an over-approximation of the operations that are actually called. Row variables are introduced as well as intersection and unions. The .(DOT) is used to close rows that do not end with a row variable. The type $\underline{C} \Rightarrow \underline{D}$ is used for handlers because a handler takes an input computation $\underline{C}$, handles the effects in this computation and outputs computation $\underline{D}$ as the result.

### 4.2 Type system

### 4.3 Subtyping rules

The subtyping rules are given in Figure 7, Figure **??** and Figure 8. Figure 7 contains all subtyping rules related to types. Figure **??** contains the distributative subtyping rules. Finally Figure 8 contains the subtyping rules that govern the dirts.

The dirty type $A \,!\, \Delta$ is assigned to a computation returning values of type $A$ and potentially calling operations from the set $\Delta$. This set $\Delta$ is always an over-approximation of the actually called operations, and may safely be increased, inducing a natural subtyping judgement $A \,!\, \Delta \leq A \,!\, \Delta'$ on dirty types. As dirty types can occur inside pure types,

| | | | |
|---|---|---|---|
| (pure) type $A, B$ | ::= | `bool` | bool type |
| | \| | $A \to \underline{C}$ | function type |
| | \| | $\underline{C} \Rightarrow \underline{D}$ | **handler type** |
| | \| | $\alpha$ | type variable |
| | \| | $\mu\alpha.A$ | rec type |
| | \| | $\top$ | top |
| | \| | $\bot$ | bottom |
| | \| | $A \sqcap B$ | intersection |
| | \| | $A \sqcup B$ | union |
| dirty type $\underline{C}, \underline{D}$ | ::= | $A \,!\, \Delta$ | |
| dirt $\Delta$ | ::= | {R} | |
| R | ::= | Op | operation |
| | \| | $\delta$ | row variable |
| | \| | . | closed row |
| | \| | $R_1 \sqcap R_2$ | intersection |
| | \| | $R_1 \sqcup R_2$ | union |
| All operations $\Omega$ | ::= | $\{\mathtt{Op}_i \mid \mathtt{Op}_i \in \Sigma\}$ | |

**Figure 6.** Types of EffCore

we also get a derived subtyping judgement on pure types. Observe that, as usual, subtyping is contravariant in the argument types of functions and handlers, and covariant in their return types.

***Dirt intersection and union*** There are several possible methods to compute the dirt intersection and union. If row variables were to be disregarded, dirt union and intersection could be defined as set union and intersection. This methods allows unions and intersections to be eliminated. This has an advantage, eliminating unions and intersections simplifies the effect system. However, we cannot disregard row variables.

Thus, set union and intersection cannot simply be used. It would be possible to define $\delta_1 \sqcup \delta_2$ and $\delta_1 \sqcap \delta_2$. Using these, it is possible to use a form of set union and intersection. The following union $\{Op_1, ..., Op_n, \delta_1\} \sqcup \{Op_{n+1}, ..., Op_{n+m}, \delta_2\}$ could be defined as $\{Op_1, ..., Op_n, Op_{n+1}, ..., Op_{n+m}, (\delta_1 \sqcup \delta_2)\}$. A similar construction can be used for intersection. This simplifies the subtyping rules since the more complicated aspects are enclosed within the row variables. The equivalence rules are defined in Figure 9.

### 4.4 Equivalence rules

### 4.5 Typing rules

Figure 10 defines the typing judgements for values and computations with respect to a standard typing context $\Gamma$.

***Values*** The rules for subtyping, variables, type abstraction, type application and functions are entirely standard. For constants we assume a signature $\Sigma$ that assigns a type $A$ to each constant k, which we write as $(\mathsf{k} : A) \in \Sigma$.

**Subtyping of pure and dirty types**

$$\textsc{Sub-Top}\ \frac{}{A \leqslant \top} \qquad \textsc{Sub-Bottom}\ \frac{}{\bot \leqslant A}$$

$$\textsc{Sub-bool}\ \frac{}{\texttt{bool} \leqslant \texttt{bool}}$$

$$\textsc{Sub-Refl}\ \frac{}{A \leqslant A} \qquad \textsc{Sub-Trans}\ \frac{A_1 \leqslant A_2 \qquad A_2 \leqslant A_3}{A_1 \leqslant A_3}$$

$$\textsc{Sub-!}\ \frac{A_1 \leqslant A_2' \qquad \Delta_1 \leqslant \Delta_2'}{A_1\ !\ \Delta_1 \leqslant A_2\ !\ \Delta_2}$$

$$\textsc{Sub-}\rightarrow\ \frac{A_2 \leqslant A_1 \qquad \underline{C}_1 \leqslant \underline{C}_2}{A_1 \rightarrow \underline{C}_1 \leqslant A_2 \rightarrow \underline{C}_2} \qquad \textsc{Sub-}\Rightarrow\ \frac{\underline{C}_2 \leqslant \underline{C}_1 \qquad \underline{D}_1 \leqslant \underline{D}_2}{\underline{C}_1 \Rightarrow \underline{D}_1 \leqslant \underline{C}_2 \Rightarrow \underline{D}_2}$$

**Figure 7.** Subtyping for pure and dirty types of EffCore

A handler expression has type $A\ !\ \Delta \cup \mathcal{O} \Rightarrow B\ !\ \Delta$ iff all branches (both the operation cases and the return case) have dirty type $B\ !\ \Delta$ and the operation cases cover the set of operations $\mathcal{O}$. Note that the intersection $\Delta \cap \mathcal{O}$ is not necessarily empty (with $\cap$ being the intersection of the operations, not to be confused with the $\sqcap$ type). The handler deals with the operations $\mathcal{O}$, but in the process may re-issue some of them (i.e., $\Delta \cap \mathcal{O}$).

When typing operation cases, the given signature for the operation $(\mathtt{Op} : A_{\mathtt{Op}} \rightarrow B_{\mathtt{Op}}) \in \Sigma$ determines the type $A_{\mathtt{Op}}$ of the parameter $x$ and the domain $B_{\mathtt{Op}}$ of the continuation $k$. As our handlers are deep, the codomain of $k$ should be the same as the type $B\ !\ \Delta$ of the cases.

***Computations*** With the following exceptions, the typing judgement $\Gamma \vdash c : \underline{C}$ has a straightforward definition. The `return` construct renders a value $v$ as a pure computation, i.e., with empty dirt. In this case, this is defined as a set with the .(DOT) as the only element. An operation invocation $\mathtt{Op}\ v$ is typed according to the operation's signature, with the operation itself as its only operation. Finally, rule WITH shows that a handler with type $\underline{C} \Rightarrow \underline{D}$ transforms a computation with type $\underline{C}$ into a computation with type $\underline{D}$.

**Subtyping of dirts**

$$\textsc{Sub-!-Empty}\ \frac{}{\{.\} \leqslant \Delta} \qquad \textsc{Sub-!-Top}\ \frac{}{\Delta \leqslant \Omega} \qquad \textsc{Sub-!-Trans}\ \frac{\Delta_1 \leqslant \Delta_2 \qquad \Delta_2 \leqslant \Delta_3}{\Delta_1 \leqslant \Delta_3}$$

$$\textsc{Sub-!-Row}\ \frac{}{\{Op_1, ..., Op_n, .\} \leqslant \{Op_1, ..., Op_n, \delta\}} \qquad \textsc{Sub-!-Refl}\ \frac{}{\Delta \leqslant \Delta}$$

$$\textsc{Sub-!-Row-Row}\ \frac{\begin{array}{c} n \geq 0 \qquad m \geq 0 \\ p \geq 0 \qquad \{Op_1, ..., Op_n, Op_{n+m+1}, ..., Op_{n+m+p}, \delta_1\} \leqslant \\ \{Op_1, ..., Op_n, Op_{n+1}, ..., Op_{n+m}, \delta_2\} \end{array}}{\begin{array}{c} \{\delta_1\} \leqslant \{Op_{n+1}, ..., Op_{n+m}, \delta_2\} \\ \{\delta_2\} = \{Op_{n+m}, ..., Op_{n+m+p}, \delta_3\} \end{array}}$$

$$\textsc{Sub-!-Dot-Row}\ \frac{\begin{array}{c} n \geq 0 \qquad m \geq 0 \\ p \geq 0 \qquad \{Op_1, ..., Op_n, Op_{n+m+1}, ..., Op_{n+m+p}, .\} \leqslant \\ \{Op_1, ..., Op_n, Op_{n+1}, ..., Op_{n+m}, \delta_2\} \end{array}}{\begin{array}{c} \{.\} \leqslant \{Op_{n+1}, ..., Op_{n+m}, \delta_2\} \\ \{\delta_2\} = \{Op_{n+m}, ..., Op_{n+m+p}, \delta_3\} \end{array}}$$

$$\textsc{Sub-!-Row-Dot}\ \frac{n \geq 0 \qquad m \geq 0 \qquad \{Op_1, ..., Op_n, \delta_1\} \leqslant \{Op_1, ..., Op_n, Op_{n+1}, Op_{n+m}, .\}}{\{\delta_1\} \leqslant \{Op_{n+1}, Op_{n+m}, .\}}$$

$$\textsc{Sub-!-Dot-Dot}\ \frac{n \geq 0 \qquad m \geq 0 \qquad \{Op_1, ..., Op_n, .\} \leqslant \{Op_1, ..., Op_n, Op_{n+1}, ..., Op_{n+m}, .\}}{\{.\} \leqslant \{Op_{n+1}, Op_{n+m}, .\}}$$

$$\textsc{Sub-Inter-!}\ \frac{\Delta_1 \leqslant \Delta_2 \qquad \Delta_3 \leqslant \Delta_4 \qquad \Delta_1 \neq \Delta_3}{\Delta_1 \sqcap \Delta_3 \leqslant \Delta_2 \sqcap \Delta_4}$$

$$\textsc{Sub-Union-!}\ \frac{\Delta_1 \leqslant \Delta_2 \qquad \Delta_3 \leqslant \Delta_4 \qquad \Delta_1 \neq \Delta_3}{\Delta_1 \sqcup \Delta_3 \leqslant \Delta_2 \sqcup \Delta_4}$$

$$\textsc{Sub-Inter-Greatest-LB-!}\ \frac{\Delta_1 \leqslant \Delta_2 \qquad \Delta_1 \leqslant \Delta_3}{\Delta_1 \leqslant (\Delta_2 \sqcap \Delta_3)} \qquad \textsc{Sub-Inter-LB-!}\ \frac{i \in \{1; 2\}}{(\Delta_1 \sqcap \Delta_2) \leqslant \Delta_i}$$

$$\textsc{Sub-Union-Least-UB-!}\ \frac{\Delta_2 \leqslant \Delta_1 \qquad \Delta_3 \leqslant \Delta_1}{(\Delta_2 \sqcup \Delta_3) \leqslant \Delta_1} \qquad \textsc{Sub-Union-UB-!}\ \frac{i \in \{1; 2\}}{\Delta_i \leqslant (\Delta_1 \sqcup \Delta_2)}$$

**Figure 8.** Subtyping for dirts of EffCore

**Equivalance rules**

Union

$$\overline{(A \sqcup A) = A}$$

Intersection

$$\overline{(A \sqcap A) = A}$$

Other

$$\overline{\ldots}$$

Dist-Func-Inter

$$\overline{(A \to \underline{C}) \sqcap (B \to \underline{D}) = (A \sqcup B) \to (\underline{C} \sqcap \underline{D})}$$

Dist-Func-Union

$$\overline{(A \to \underline{C}) \sqcup (B \to \underline{D}) = (A \sqcap B) \to (\underline{C} \sqcup \underline{D})}$$

Dist-Hand-Inter

$$\overline{(A \Rightarrow \underline{C}) \sqcap (B \Rightarrow \underline{D}) = (A \sqcup B) \Rightarrow (\underline{C} \sqcap \underline{D})}$$

Dist-Hand-Union

$$\overline{(A \Rightarrow \underline{C}) \sqcup (B \Rightarrow \underline{D}) = (A \sqcap B) \Rightarrow (\underline{C} \sqcup \underline{D})}$$

Union-!-Row-Row

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \vee Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, (\delta_1 \sqcup \delta_2)\} = \{Op_1, \ldots, Op_n, \delta_1\} \sqcup \{Op_{n+1}, \ldots, Op_{n+m}, \delta_2\}$$

Union-!-Row-Dot

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \vee Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, \delta_1\} = \{Op_1, \ldots, Op_n, \delta_1\} \sqcup \{Op_{n+1}, \ldots, Op_{n+m}, .\}$$

Union-!-Dot-Row

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \vee Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, \delta_2\} = \{Op_1, \ldots, Op_n, .\} \sqcup \{Op_{n+1}, \ldots, Op_{n+m}, \delta_2\}$$

Union-!-Dot-Dot

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \vee Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, .\} = \{Op_1, \ldots, Op_n, .\} \sqcup \{Op_{n+1}, \ldots, Op_{n+m}, .\}$$

Intersection-!-Row-Row

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \wedge Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, (\delta_1 \sqcap \delta_2)\} = \{Op_1, \ldots, Op_n, \delta_1\} \sqcap \{Op_{n+1}, \ldots, Op_{n+m}, \delta_2\}$$

Intersection-!-Row-Dot

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \wedge Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, \delta_1\} = \{Op_1, \ldots, Op_n, \delta_1\} \sqcap \{Op_{n+1}, \ldots, Op_{n+m}, .\}$$

Intersection-!-Dot-Row

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \wedge Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, \delta_2\} = \{Op_1, \ldots, Op_n, .\} \sqcap \{Op_{n+1}, \ldots, Op_{n+m}, \delta_2\}$$

Intersection-!-Dot-Dot

$$\{Op_i | Op_i \in \{Op_1, \ldots, Op_n\} \wedge Op_i \in \{Op_{n+1}, \ldots, Op_{n+m}\}, .\} = \{Op_1, \ldots, Op_n, .\} \sqcap \{Op_{n+1}, \ldots, Op_{n+m}, .\}$$

**Figure 9.** Equivalance rules

typing contexts $\Gamma ::= \epsilon \mid \Gamma, x : A \mid \Gamma, \hat{\mathbf{x}} : \forall \alpha . B$

**Expressions**

Val

$$\frac{\Gamma \vdash v : A \qquad A \leqslant B}{\Gamma \vdash v : B}$$

Var

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}$$

True

$$\frac{}{\Gamma \vdash \texttt{true} : bool}$$

False

$$\frac{}{\Gamma \vdash \texttt{false} : bool}$$

Fun

$$\frac{\Gamma, x : A \vdash c : \underline{C}}{\Gamma \vdash \lambda x.c : A \to \underline{C}}$$

Hand

$$\frac{\Gamma, x : A \vdash c_r : B \,!\, \Delta \qquad \left[(\mathsf{Op} : A_{\mathsf{Op}} \to B_{\mathsf{Op}}) \in \Sigma \quad \Gamma, x : A_{\mathsf{Op}}, k : B_{\mathsf{Op}} \to B \,!\, \Delta \vdash c_{\mathsf{Op}} : B \,!\, \Delta\right]_{\mathsf{Op} \in O}}{\Gamma \vdash \{\texttt{return}\ x \mapsto c_r, [\mathsf{Op}\ y\ k \mapsto c_{\mathsf{Op}}]_{\mathsf{Op} \in O}\} : A \,!\, \Delta \cup O \Rightarrow B \,!\, \Delta}$$

**Computations**

Comp

$$\frac{\Gamma \vdash c : \underline{C} \qquad \underline{C} \leqslant \underline{D}}{\Gamma \vdash c : \underline{D}}$$

App

$$\frac{\Gamma \vdash v_1 : A \to \underline{C} \qquad \Gamma \vdash v_2 : A}{\Gamma \vdash v_1\, v_2 : \underline{C}}$$

Cond

$$\frac{\Gamma \vdash v : A \qquad \Gamma \vdash c_1 : \underline{C} \qquad \Gamma \vdash c_2 : \underline{D}}{\Gamma \vdash \texttt{if}\ v\ \texttt{then}\ c_1\ \texttt{else}\ c_2 : (\underline{C} \sqcup \underline{D})}$$

Ret

$$\frac{\Gamma \vdash v : A}{\Gamma \vdash \texttt{return}\ v : A \,!\, \{.\}}$$

Op

$$\frac{(\mathsf{Op} : A \to B) \in \Sigma \qquad \Gamma \vdash v : A \qquad \underline{C} : B \,!\, \{\mathsf{Op}\ ; R\}}{\Gamma \vdash \mathsf{Op}\, v : \underline{C}}$$

Let

$$\frac{\Gamma \vdash c_1 : A \,!\, \Delta \qquad \Gamma, x : \forall \alpha . A \vdash c_2 : B \,!\, \Delta \qquad \alpha \notin FTV(\Gamma)}{\Gamma \vdash \texttt{let}\ \hat{\mathbf{x}} = c_1\ \texttt{in}\ c_2 : B \,!\, \Delta}$$

With

$$\frac{\Gamma \vdash v : \underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c : \underline{C}}{\Gamma \vdash \texttt{handle}\ c\ \texttt{with}\ v : \underline{D}}$$

**Figure 10.** Typing of EffCore

## 5 Semantics

## 6 Elaboration

## 7 Constraint Generation

## 8 Proofs

## 9 Implementation

## 10 Evaluation

## 11 Conclusion

## A Dolan's type system

Subtyping is a partial order which is a reflexive transitive binary relation satisfying antisymmetry (subtyping rules). The subtyping order also forms a distributive lattice (equivalence rules).

f = Îżg.f(g true) (ÎijÎś.bool âĘŠ Îś) âĘŠ âŁě
recursive type p.72


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