Algebraic Subtyping for Algebraic Effects and Handlers

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What are algebraic effects and handlers?

Algebraic effects and handlers

Model impure behaviour such as mutable state or I/O [1]

Exceptions on steroids

[1] Pretnar, M., 2015. An introduction to algebraic effects and handlers. Electr. Notes Theor. Comput. Sci, 319, pp.19-35.

Algebraic effects and handlers

Model impure behaviour such as mutable state or I/O [1]

Exceptions on steroids

```
effect Op : unit -> int

let someFun b =
    handle (
        if (b == 0) then
            let a = #Op ()
            print a
        else
            print b
    ) with
        | #Op () cont -> cont 1
```

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Exception handlers + “Continue”

Eff programming language [1]

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Functional language

[1] http://www.eff-lang.org/

Eff programming language [1]

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Algebraic effects and handlers

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Functional language

Algebraic effects and handlers

ML style language

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Functional language

Algebraic effects and handlers

ML style language

Type inference

[1] http://www.eff-lang.org/

Type inference

```
let a = 5
```

Functional language

Algebraic effects and handlers

ML style language

Type inference

Type inference

```
let a = 5
```

‘a’ has type ‘int’

Functional language

Algebraic effects and handlers

ML style language

Type inference

Eff programming language [1]

Functional language

Algebraic effects and handlers

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- Type inference

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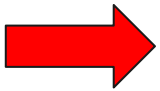
Eff programming language [1]

Functional language

Algebraic effects and handlers

ML style language

Type inference



Problems

[1] http://www.eff-lang.org/

Eff programming language [1]

Functional language

Algebraic effects and handlers

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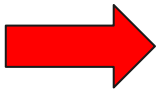

Problems

Slow to compile

Hard to debug

[1] http://www.eff-lang.org/

Example: Twice

```
let twice f x =
    f (f x)
```

What does this function do?

Example: Twice

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What does this function do?

f is a function

Example: Twice

```
let twice f x =
    f (f x)
```

What does this function do?

f is a function
    accepts x and (f x)

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

$x : \alpha$

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

x : α

f : β -> γ

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

$x : \alpha$

$f : \beta \rightarrow \gamma$

$\alpha \leq \beta$

Example: Twice

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let twice f x =
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```

Subtyping

$x : \alpha$

$f : \beta \rightarrow \gamma$

$\alpha \leq \beta, \gamma \leq \beta$

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

$x : \alpha$

$f : \beta \to \gamma$

$twice : (\beta \to \gamma) \to \alpha \to \gamma \mid \alpha \leq \beta, \gamma \leq \beta$

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

x : α

f : β -> γ

twice : (β -> γ) -> α -> γ) α ≤ β, γ ≤ β

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

$x : \alpha$

$f : \beta \to \gamma$

$twice : (\beta \to \gamma) \to \alpha \to \gamma \mid \alpha \leq \beta, \gamma \leq \beta$

Example: Twice

```
let twice f x =
    f (f x)
```

Subtyping

x : α

f : β -> γ

twice : (β -> γ) -> α -> γ | α ≤ β, γ ≤ β

If constraints solved:

Get actual type

Problem

https://cdn-images-1.medium.com/max/888/1*XJ7YrEAwzoFSql6HFT4d4g.jpeg

Algebraic Subtyping for Algebraic Effects and Handlers

Dolan, Stephen, and Alan Mycroft. "Polymorphism, subtyping, and type inference in MLsub." In ACM SIGPLAN Notices, vol. 52, no. 1, pp. 60-72. ACM, 2017.

Algebraic Subtyping for Algebraic Effects and Handlers

Stephen Dolan

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Algebraic Subtyping

Example: Twice

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let twice f x =
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Algebraic Subtyping

Result of f

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

Result of f

    Output of twice

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

Result of f

    Output of twice

    Input of f

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

x : α

return : β

f : α -> ?

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

x : α
return : β
f : α -> α & β

=> accept both α AND β

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

twice : (α -> α & β) -> α -> β

Example: Twice

```
let twice f x =
    f (f x)
```

Algebraic Subtyping

twice : (α -> α & β) -> α -> β

Subtyping:

twice : (β -> γ) -> α -> γ | α ≤ β, γ ≤ β

Effects?

Algebraic effects and handlers

Algebraic subtyping

???

Algebraic subtyping + effects

(pure) type A, B	$::=$	$\texttt{bool}$	bool type
	$\mid$	$A \to \underline{C}$	function type
	$\mid$	$\underline{C} \Rightarrow \underline{D}$	**handler type**
	$\mid$	α	type variable
	$\mid$	$\mu\alpha.A$	recursive type
	$\mid$	$\top$	top
	$\mid$	$\bot$	bottom
	$\mid$	$A \sqcap B$	intersection
	$\mid$	$A \sqcup B$	union
dirty type $\underline{C}, \underline{D}$	$::=$	$A \,!\, \Delta$	

Algebraic subtyping + effects

dirt Δ	$::=$	Op	operation
	$\mid$	δ	dirt variable
	$\mid$	$\emptyset$	empty dirt
	$\mid$	$\Delta_1 \sqcap \Delta_2$	intersection
	$\mid$	$\Delta_1 \sqcup \Delta_2$	union

What are the semantics?

Algebraic subtyping + effects

What are the semantics?

Set operations?

(Op ⊔ Op2) ⊓ (Op ⊔ Op3)

Algebraic subtyping + effects

What are the semantics?

Set operations?

(Op ⊔ Op2) ⊓ (Op ⊔ Op3)

=> Op

Algebraic subtyping + effects

What are the semantics?

Set operations?

(Op ⊔ Op2) ⊓ (Op ⊔ Op3)

=> Op

α & β

accept both α AND β

Algebraic subtyping + effects

What are the semantics?

Set operations?

(Op ⊔ Op2) ⊓ (Op ⊔ Op3)

=> Op

α & β

accept both α AND β

Input vs Output

=> Inputs cause ⊓

=> Outputs cause ⊔

Design of a type-&-effect system

Formulation of typing rules

Define relationship to subtyping

Biunification algorithm (input <-> output)

Type inference algorithm

Keep design close to Dolan’s design

Proofs

Reason to stay close to Dolan

Instantiation
Weakening
Substitution
Soundness
Type preservation

Implementation

Instead of using a ‘toy’ language

Use Eff programming language

Replace the type inference engine

~2700 loc ⇔ ~5700

Implementation

Instead of using a ‘toy’ language

- Use Eff programming language

Replace the type inference engine

- ~2700 loc ⇔ ~5700

Testing against other systems

- Coercion subtyping
- Subtyping
- Row polymorphism

Result

Extension of Algebraic Subtyping
    For algebraic effects and handlers

Type System

Algorithmic

Proofs

Implementation

Simplify constraint generation for types AND EFFECTS by using extended algebraic subtyping

Questions?