Analysis and Comparison of OFDM and SC-FDE characteristics in frequency selective channels

Justin Hu, EE 239 w/ Professor Ian Roberts

December 13, 2024

1 Introduction & Motivation

In wireless communication, we wish to send information from some transmitter to some receiver(s) when they are separated by the environment without a direct link. This is done by transmitting and receiving electromagnetic waves through the air (or medium) via antennas. Ideally the transmitted signal travels to the receiver without any distortion, allowing the receiver to trivially decode the transmitted information, but this is rarely if ever the case. The environment in which the electromagnetic waves propagate introduce various channel effects that need to be accounted for to properly decode the received signal, a central part of wireless communication. The act of processing the received signal to account for channel effects and recover the original data is known as ”equalization” [Mar64].

Orthogonal frequency domain multiplexing, otherwise known as OFDM, has emerged as the preferred wireless communication system used by industry, ranging from cellular data to satellite communications. When learning about OFDM, its success is often justified by its simple and robust equalization method. However, I realized that the equalization techniques used in OFDM can easily be applied to the classic, single-carrier system as well, which has been dubbed Single-carrier w/ frequency domain equalization (SC-FDE). Thus, this project serves as a personal foray into both OFDM and SC-FDE, analyzing the capabilites of both systems through simulation to understand and evaluate their relative strengths.

2 Theoretical Background (Multi-tap, Frequency selective, wideband channels)

Wireless engineers have come up with various abstractions to model the ”wireless channel” and describe its effects. The channel model used in this project is a tap delay channel model (TDL), where the ”channel response” consists of a set of impulses (taps), each of which with a particular delay and gain [HTM20]. Essentially, the received signal consists of many copies of the transmitted signal, corresponding to the multiple paths a signal can travel to reach the receiver within the complex, open-space environments where wireless communications occur. Each of these copies is scaled and delayed by some amount before being added together at the receiver. This resulting received signal looks nothing like the transmitted signal and must be equalized before decoding in order to remove the channel effects and recover (to the best of our abilities) the transmitted signal along with the desired data.

The ”delay spread” of a tap delay channel model describes how spread apart these impulse responses are, and can be understood as a measure of how different the various transmission paths are between the transmitter and the receiver. In this project, it is defined as the range in which non-zero channel impulse responses occur. It should be obvious upon closer inspection that a large delay spread is undesirable, since they cause different parts of the transmitted signal to overlap with each other in the received signal making them impossible to tell apart. This effect is generally known as ”inter-symbol interference” (ISI) and is a major problem in wireless communication [GD61].

Theoretically, any non-zero delay spread will cause ISI. However, if the delay spread is small enough relative to the length of a symbol, then the effect of the channel can be approximated by a single tap constituting their average, forming a ”single-tap” channel. Such a channel is trivial to equalize since

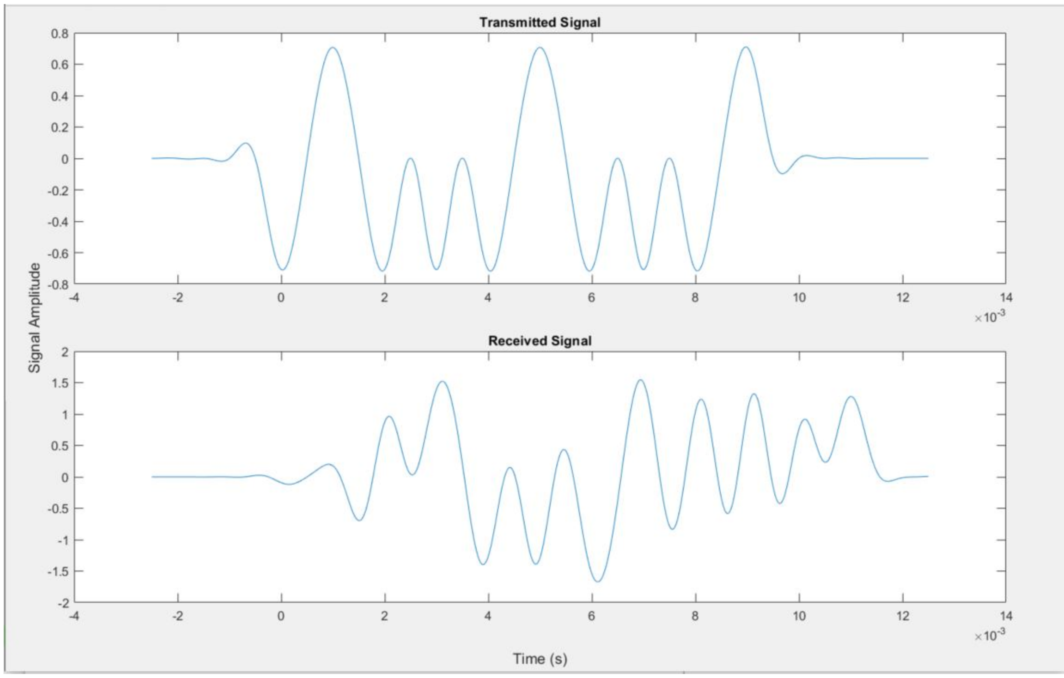


Figure 1: 2PAM in TDL channel model with x3 symbol time delay spread (no noise).

the received signal is just a scaled and delayed version of the transmitted signal with (approximately) no ISI, and estimation of these 2 parameters can be done very easily. On the other hand, if the delay spread is large relative to the length of a symbol, possible extending across multiple symbols, the channel must instead be modeled with multiple taps, forming a "multi-tap" channel. Equalization is no longer trivial and requires specialized techniques, one of which will be explored thoroughly in this project. For a single carrier system, as the bandwidth increases (gets wider), the symbol time decreases, which makes the delay spread larger relative to the symbol time and ISI more severe. Thus, single-tap channels are also called "narrow-band" channels and multi-tap channels are "wide-band" channels.

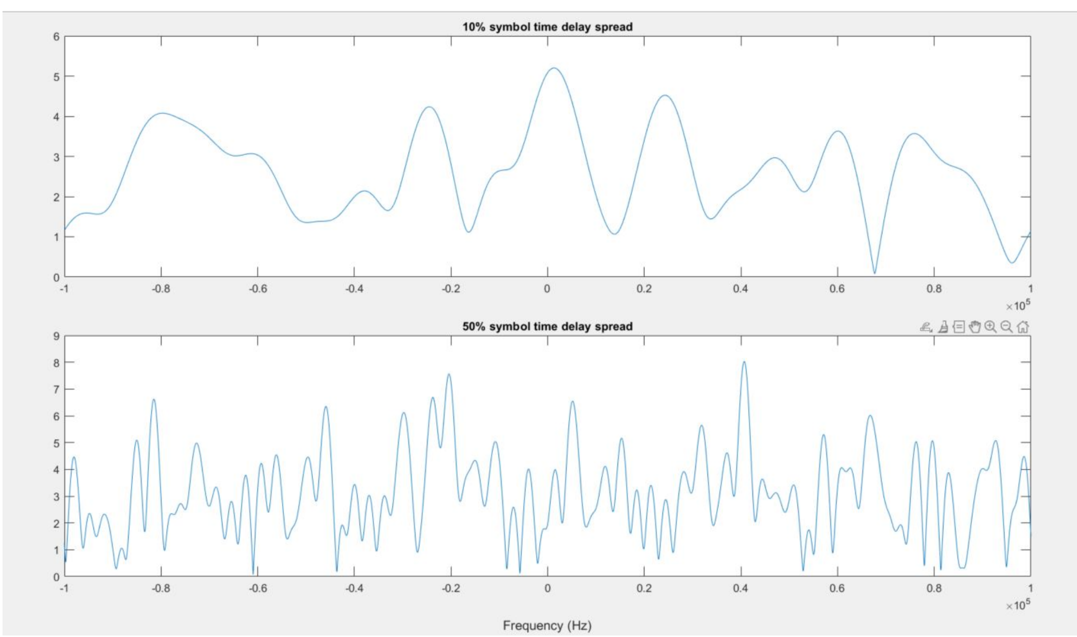


Figure 2: Random 10-tap TDL Channel frequency response comparison.

Another common way to understand this concept is by analyzing the frequency response of the channel. A truly single tap channel is "frequency-flat", meaning it responds the same to all frequency components since it just scales and delays the entire original signal. When multiple taps are present, the linear combination of different time delays creates constructive and destructive interference in parts of the signal depending on the phase difference between the taps which in turn depends on the frequency component. Thus, the channel no longer responds the same to different frequencies and becomes "frequency selective". More taps and wider delay spreads lead to higher frequency selectivity, which can be visualized in a frequency domain plot of the channel response.

Given a certain channel response, a lower bandwidth signal is affected by a smaller portion of the channel response. Thus, even if the channel response is selective over a large frequency range, a sufficiently narrow-band signal can approximate the channel response to be frequency flat, while a wide-band signal cannot do so. Thus, all 3 terms in the section title refer to the same fundamental concept by viewing it from different perspectives. A primary goal of this project is to understand how OFDM and SC-FDE are able to deal with the severe ISI caused by such a channel.

3 Overview of Communication Systems

The classic single carrier communication system transmits data using a series of pulses in time with varying magnitude and phase. Information is encoded by modulating the amplitude and phase of each pulse through a "modulation scheme", and the rate at which pulses are transmitted determines the bandwidth of the system. To avoid interference from other users due to overlapping frequency components in the transmitted signals, the transmitter multiplies the original "base-band" signal by a "carrier frequency" in a process known as "upconversion" which shifts all frequency components in the original signal upwards in the frequency domain. The receiver must do the same process but in reverse to "downconvert" the signal to its original form and recover the information. If upconversion and downconversion are properly implemented, the carrier frequency itself is not particularly important, since the receiver decoder never observers the pass-band signal. It is merely a tool to "carry" the desired signal in a different frequency region to avoid interference with other communications. Since this system uses a single carrier frequency to do so, it is a "single-carrier" system.

3.1 Orthogonal Frequency Division Multiplexing (OFDM)

Prior to OFDM, when given a certain amount of bandwidth, communication engineers would design their system to simply transmit at the rate defined by the bandwidth via a single carrier system, which works fine when the channel is sufficiently flat for the bandwidth of operation. However, once the channel becomes frequency selective, severe ISI occurs. Narrowing the bandwidth lowers the data rate and wastes part of the spectrum, which defeats the purpose of having a wide bandwidth. Although time domain equalization techniques have been developed to counteract ISI, they are generally complex and more cumbersome to use. The key insight of OFDM is realizing that a given bandwidth doesn't have to be covered by a single carrier system with the same bandwidth. Instead, it can be covered by multiple single carrier systems, each of which only takes up a portion of the total bandwidth, making OFDM a "multi-carrier" system [KB17]. Since each "subcarrier" has a much narrower bandwidth than the total bandwidth, they naturally become more resistant to ISI and observe a flatter channel response, meaning they can usually be equalized via narrow-band techniques, even if the overall system spans a wide bandwidth.

Although the original 1966 publication [Cha66] formulated OFDM in terms of multiple independent single carrier frequency systems that are processed in parallel through filtering in the time domain, modern OFDM systems do not use this method. Referencing figure 3, we notice that the transmitted OFDM signal looks like a series of pulses in the frequency domain, each one carrying an independent symbol. Using modern DSP techniques, we can directly synthesize the entire OFDM signal in frequency domain before using the IFFT algorithm to create a corresponding time domain signal. On the receiver end, the data can be decoded within the frequency domain by evaluating the FFT of the entire received signal instead of filtering out the individual carriers and evaluating in time domain. These two methods are mathematically identical and ideally achieve the same result, but the FFT algorithms are faster and simpler to implement than traditional analog filters [Don06].

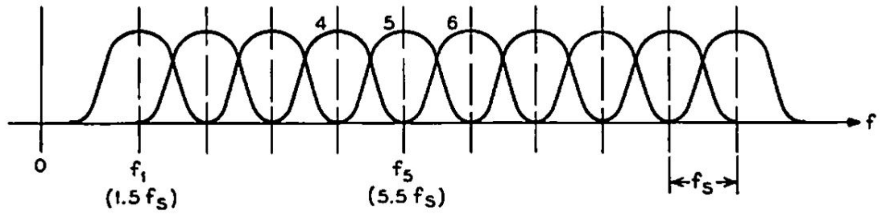


Figure 3: Division into multiple narrower-band single carrier systems (Frequency domain).

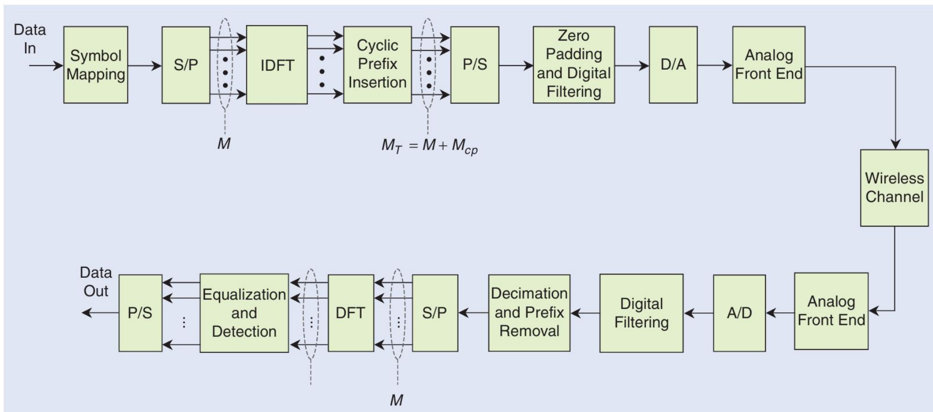


Figure 4: Modern OFDM System block diagram. [Pan+08]

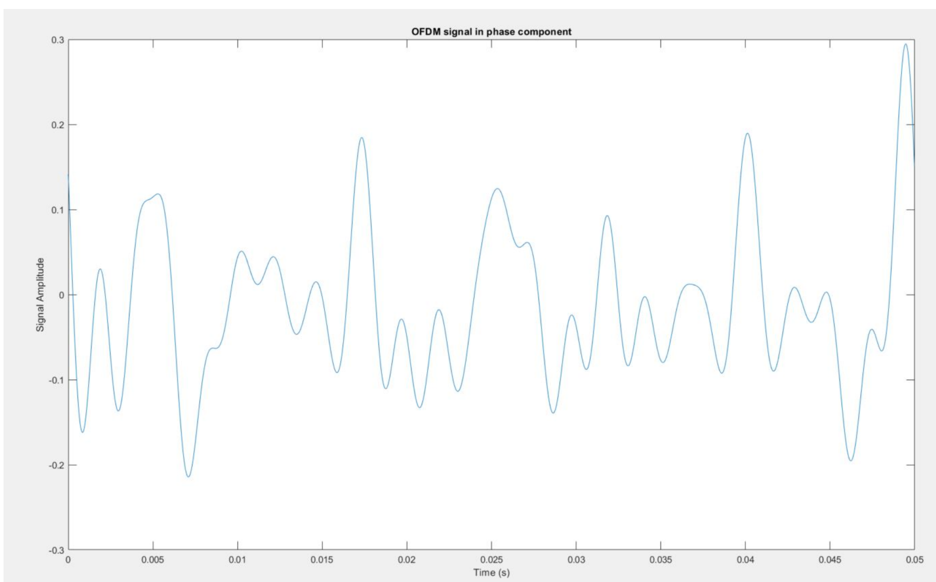


Figure 5: 2kHz Bandwidth, 50 subcarrier OFDM signal in-phase component in time domain.

Analyzing an example OFDM signal in time domain, it looks nothing like the pulse train type signals we are used to seeing in traditional single carrier systems. This is because it is a linear combination of many subcarriers, each one at a different frequency with different amplitude and phase which causes complex constructive and destructive interference patterns. However, taking the FFT of this signal allows one to cleanly separate each of the subcarriers and the symbols they carry. Note that the total length of the time domain signal is 0.05 seconds. If 2kHz bandwidth is divided among 50 subcarriers, each subcarrier gets 40Hz (two-sided) bandwidth, meaning that their symbol time (1 cycle) is $2/40 = 0.05$. Thus, the math checks out for our frequency domain formulation.

Perhaps due to the realization of how OFDM can be implemented more efficiently in the frequency domain, people realized that equalization can also be directly performed in the frequency domain. This is the central topic I wanted to explore in this project, and will be detailed in the following section.

3.2 Cyclic prefixing and Frequency Domain equalization

From basic signal processing theory, we know that a convolution in the time domain is a multiplication in the frequency domain. When the transmitted signal passes through the channel, it is convolved with the channel impulse response before reaching the output. In a tap delay channel model, the channel impulse response is just an impulse train of various magnitude and delay, as detailed earlier above. However, convolutions are complicated and difficult to work with [MA09]. Rather than deal with unwinding a complicated convolution in time domain, we can perform equalization in the time domain instead by dividing the frequency response of the received signal by the channel frequency response to retrive the transmitted signal. This is especially desirable for modern OFDM systems, since the received symbols are decoded in the frequency domain anyways. With a frequency domain equalization scheme, the symbols can be decoded directly after equalization without needing to be transformed again, which may introduce additional noise/non-linearities.

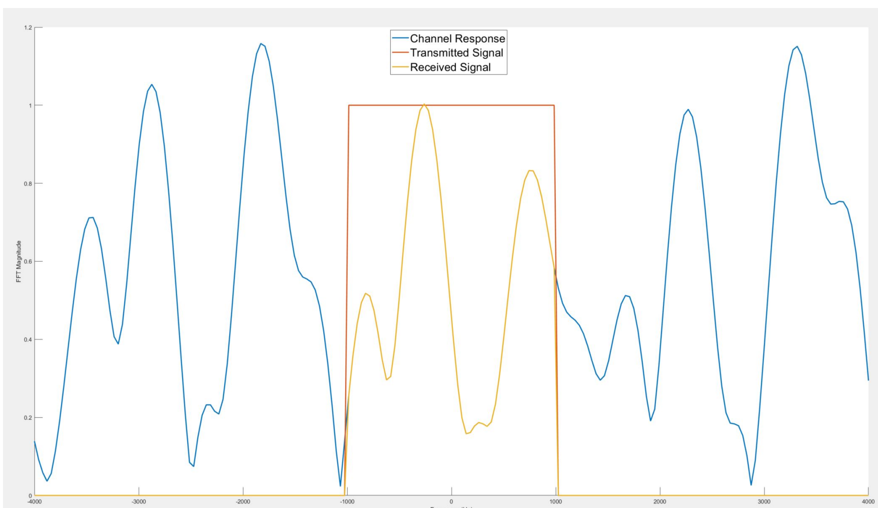


Figure 6: Frequency response illustration.

In figure 6, I convolved an input signal with the channel response in the time domain and plotted their frequency responses separately. In the absence of noise, the received signal is simply multiplied by the channel response in the frequency domain. Then, the transmitted signal can be recovered by simply dividing the received signal's frequency response by the channel's frequency response to "undo" the signal transformation that the channel has applied.

In reality, there is one more step that needs to be taken. The FFT implementation of the fourier transform, crucial in modern DSP systems for its speed and efficiency, simplify the operation by assuming the input signals are periodic. This means that the convolution to multiplication property listed above only holds true for circular convolutions when the FFT is applied, when real systems employ linear convolutions. In a circular convolution, the tail end of the input signal influences the beginning of the output signal since the signals are assumed to be periodic/infinitely repeating in time. Thus, for ideal frequency domain equalization, the linear convolution needs to be transformed into a circular convolution.

This can be done with what's known as a cyclic prefix. As the name suggests, it is a signal segment attached to the beginning of the transmitted signal (prefix) in order to imitate the effect of a circular (cyclic) convolution. As mentioned above, the circular convolution assumes signals are periodic, meaning the tail end of the input influences the beginning of the output. Then, we can simply copy a portion of the tail end of our input signal and attach it to the very front. When the input signal is convolved linearly with the channel response, the resulting output excluding the cyclic prefix looks as if it had undergone a circular convolution and can be equalized through division with channel frequency response via FFT.

The cyclic prefix also has an additional effect of shielding each OFDM symbol from the previous one, making them fully independent and easy to process individually. Since the cyclic prefix is meant to account for the effect of the circular convolution from the tail part of the signal, it is also long enough such that when OFDM symbols are transmitted in sequence one after the other continuously, the effects from the previous symbol only last long enough to effect the cyclic prefix and not the actual payload, which is fine since the cyclic prefix is discarded prior to equalization and decoding anyways.

There is a caveat to this process. If the channel frequency response at any point is 0, then that part of the input signal cannot be recovered through equalization since information about the part of the input has been completely destroyed. Since OFDM decodes symbols in the frequency domain, these ”frequency nulls” can be an issue for reliable transmission that cannot be solved via equalization.

The cyclic prefix is actually not strictly necessary for this equalization scheme to work. Treating a linear convolution as a circular one will cause distortion effects, but they are tolerable if the delay spread of the channel is small enough relative to the symbol period. By using too long of a cyclic prefix, the transmitter wastes too much time sending out signals that don’t actually carry data, which hurts the overall throughput of the system. A cyclic prefix at least as long as the delay spread of the channel is necessary for perfect equalization, but one that is slightly shorter may provide similar performance with higher overall throughput. It is also theoretically possible to just use an increasing number of sub-carriers to increase the symbol period and make them very narrowband, but this could be undesirable due to effects I will cover in a later section.

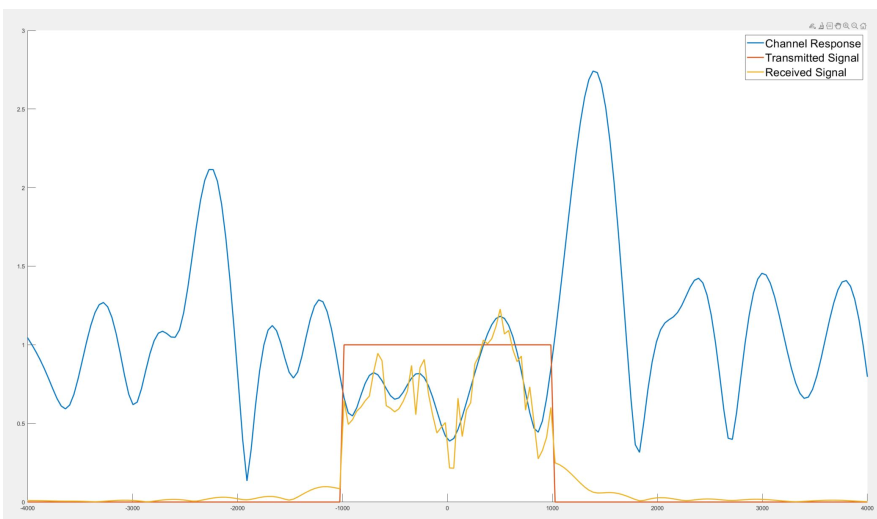


Figure 7: Distortion effects from linear –> circular convolution w/ 10% delay spread.

3.3 Single Carrier - Frequency Domain Equalization (SC-FDE)

After frequency domain equalization was introduced via OFDM systems, it was soon realized that the exact same technique can be applied to traditional single carrier systems. This is labeled as SC-FDE to differentiate them from traditional single carrier system without using FDE.

In OFDM, equalization is applied on each OFDM symbol separately, since they are mutually separated by the cyclic prefix. Additionally, the splitting of bandwidth into multiple carriers forcibly elongates the symbol time by reducing the bandwidth of each individual subcarrier, allowing the system to maintain higher throughput since the cyclic prefix takes up a smaller percentage of the overall symbol. These are two desirable properties from OFDM that we wish to also utilize in SC-FDE. This can be achieved by grouping multiple symbols into chunks and processing each chunk like you would an OFDM symbol at the receiver, taking the FFT of the entire chunk and not individual symbols [Zha+09].

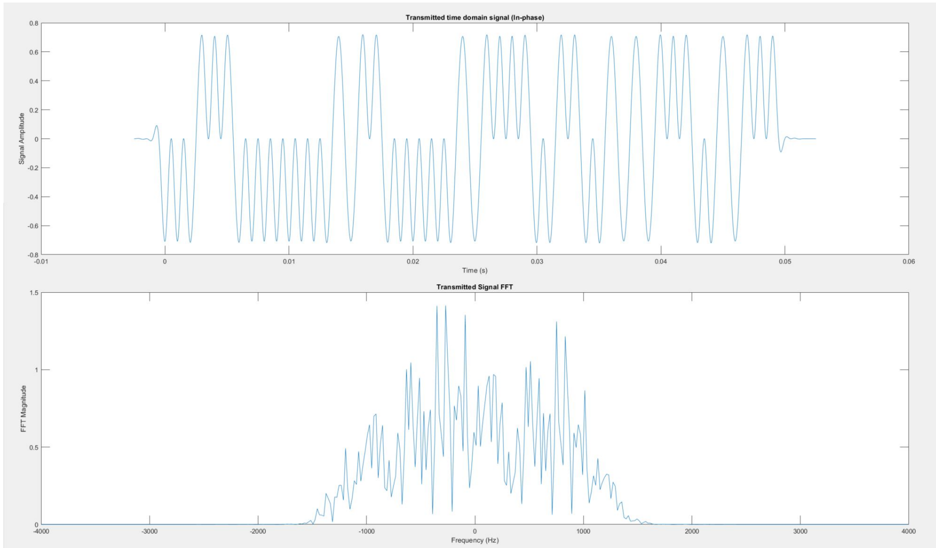
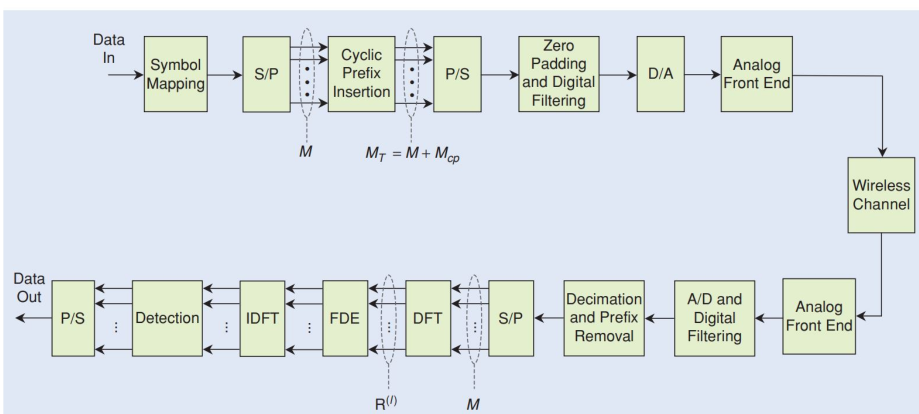


Figure 8: 2kHz bandwidth 4QAM Signal chunk in time and frequency domain.

In figure 8, the in-phase (real) component signal of a series of 50 4-QAM symbols in time domain is plotted as well as the corresponding frequency domain equivalent magnitude. Because the signal is generated in time domain, the frequency domain representation is messy and does not correspond to anything in particular. Note that the spectrum of the signal spills out a little outside of the defined 2kHz bandwidth due to pulse shaping with a Beta = 0.5 raised cosine. With the frequency domain representation of the signal chunk obtained, we can simply use the exact same frequency domain technique to perform equalization. Note that in a real system it would be preferable to include a cyclic prefix which I did not include in this simple example.



[FIG1] **Block diagram of an SC digital communication system employing an FDE.**

Figure 9: Block diagram of SC-FDE Implementation. [Pan+08]

The block diagram of an SC-FDE system is very similar to that of an OFDM system with essentially the same blocks. However, there is one crucial difference in how they are arranged. In OFDM, since symbol creation and detection both occur in the frequency domain, detection can occur immediately after equalization without any additional processing. However, since the symbols in SC-FDE are still ultimately time-domain symbols, they need to be reconverted back to the time domain with the IFFT after equalization in order for detection to be done. This means all of the frequency domain processing load is shifted to the receiver in SC-FDE, and the need to reconvert the signal back to time domain after equalization just to detect the symbols could make it more susceptible to noise and distortion.

4 Qualitative Performance Comparison

For this project, I implemented a simple version of both OFDM and SC-FDE communication systems in order to better understand their working principles. With these implementations in place, there are some interesting experiments I can do.

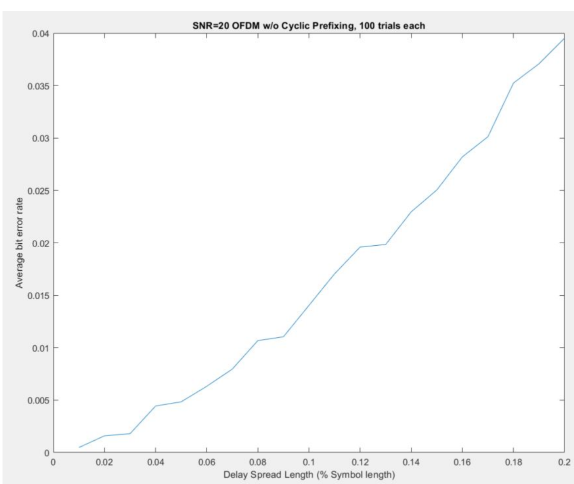
To start with, I will list out the assumptions made in my implementations. I assume the same 2kHz bandwidth for both systems but use raised-cosine pulse shaping for the single carrier system, meaning that the frequency domain sidebands for SC-FDE extend a little past 2kHz. OFDM uses 50 subcarriers and SC-FDE has 50 symbols per chunk so that both systems have the exact same symbol time and system sampling rate. I use a center carrier frequency of 1Mhz, though this is not particularly important. I assume perfect synchronization between transmitter and receiver for the SC-FDE system so that detection is ideal, which theoretically shouldn't be a problem with a high enough system sampling rate. AWGN noise is added to the received time domain signal, with noise power normalized to symbol power such that SNR for both systems is the same. The receiver has perfect channel state information, meaning access to the precise channel impulse response for use in equalization. In real systems, this is often not the case.

Just as described above, a tap delay channel model is used, with delay spread and the number of taps as variables. For the sake of simplicity, I assume the path gain of every tap to be a complex gaussian variable with no assumption about dominant line of sight path. This allows me to generate multiple channel responses very easily, but the resulting channel response and therefore system performance will have a large variation over multiple trials. Thus, these results do not correspond very well to real systems and should be thought of merely as a qualitative evaluation and confirmation with theory.

Another important note is that since I am essentially simulating the DSP part of the communication system and ignoring the analog parts (as is often done in wireless communications), all signals I work with in my simulations are discrete. The granularity/precision I can achieve for the channel impulse response depends on the system sampling rate, and I upsample both systems beyond the minimum required rate by the same amount to have greater control over simulation parameters.

4.1 Cyclic Prefix length and Delay Spread

In the first experiment, I use frequency domain equalization without the cyclic prefix and observe the effect on bit error rate as the delay spread of the channel increases from 1% transmitted signal length to 20% transmitted signal length. Delay spread is increased at 1% transmitted signal length at a time with 100 trials averaged for every data point. SNR of both systems is fixed at 20 and 4-QAM is used.



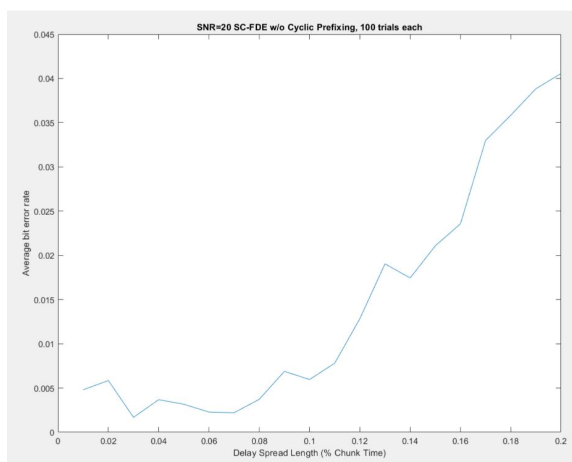


Figure 10: Channel Delay Spread v.s. Bit Error Rate, OFDM (left) and SC-FDE (right).

In the second experiment, I fix the channel delay spread at 60% total signal time and increase the cyclic prefix length from 5% to 100% of channel impulse response delay spread, observing the effect on bit error rate. SNR is still fixed at 20 with 100 trials each and using 4-QAM.

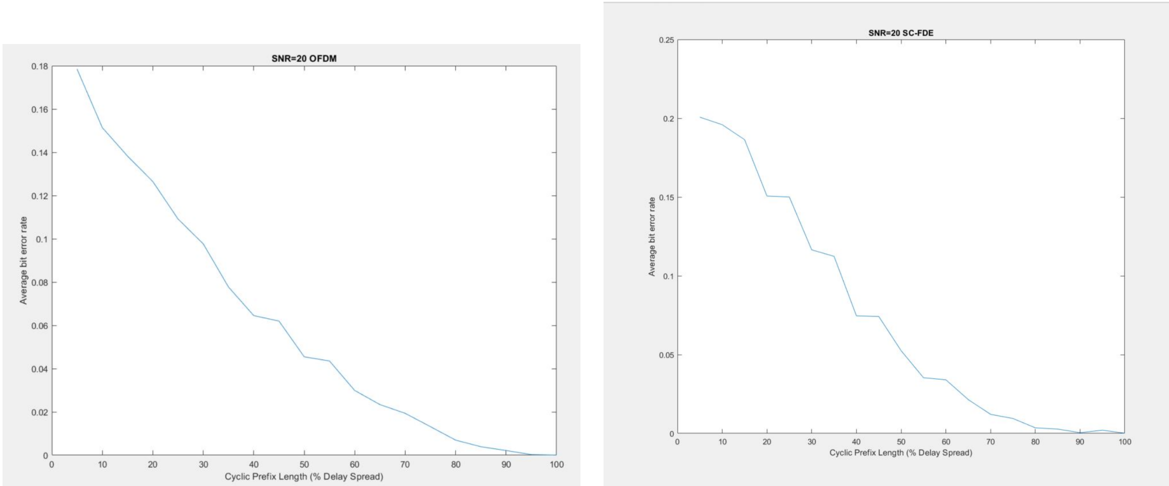


Figure 11: Cyclic Prefix Length v.s. Bit Error Rate, OFDM (left) and SC-FDE (right).

Looking at the results, we see that both systems actually perform very similarly when controlling for the same conditions such as total bandwidth, SNR, channel response, and cyclic prefix length. Although the results of SC-FDE are a little more unstable, both systems show a clear trend of increasing error rate as delay spread of the channel increases when not using a cyclic prefix, and a similar decrease of error rate as the cyclic prefix approaches the length of the channel delay spread. The effects of delay spread seem to be greater than linear, as can be seen both through the plot shape of figure 10 as well as comparison with the results in figure 11. Notably, the error rate scales by the proportion of delay spread length to signal length rather than the actual length of channel delay spread, which matches our theoretical understanding that whether a channel is narrow or wideband depends on the system utilizing the channel.

The results in figure 11 support the theoretical conclusion that a cyclic prefix as long as the channel delay spread is sufficient for reliable communication [KK15]. Out of curiosity, we can try calculating the total efficiency to see whether it is always worth using the maximum cyclic prefix length.

With no cyclic prefix, 1000 symbols per second = 2000 bps, 20% error rate decreases this to around 1600bps.

With a cyclic prefix of 30% signal length, error rate is around 10%, signal time is 0.05*1.3 = 0.065, 1/0.065 = 15.38 signals/second = 769 symbols per second aka 1540bps, decreased to 1390 bps due to errors.

With a cyclic prefix of 60% signal length no errors occur, 0.05*1.6 = 0.08, 1/0.08 = 12.5 signals/second = 625 symbols per second = 1250bps.

Thus, for a simpler modulation scheme like 4-QAM, a small amount of tolerable error may be preferable than an overly lengthy cyclic prefix [NM17]. Of course, realistic communications in such a high SNR environment will not use such a simple modulation scheme in order to maximize transmission rates, with the error rate likely increasing more sharply for more complex modulation schemes as cyclic prefix shortens due to reduced distance between symbols.

In an environment where the delay spread takes up a significant portion of the signal time, it is theoretically better to decrease the proportion of delay spread to signal time by increasing the signal time rather than adding an overly lengthy cyclic prefix. However, the signal time will be upper bounded by other restrictions such as the coherence time in a quasi-static channel and latency requirements for certain applications, in which case a less complex modulation scheme can be used which may decrease the cyclic prefix length required to achieve optimal transmission rate. These restrictions in signal time prevent real systems from achieving shannon rate in highly frequency selective channels [WQQ20].

Overall, these results show that there is no particular advantage of one over the other in terms of equalization capability. This means that the appeal of OFDM over SC-FDE in real communication systems must come from other reasons, which will be explored in the remainder of the report.

4.2 Timing & Complexity

When running experiments of the previous section, I quickly noticed that SC-FDE runs a lot slower in general than OFDM. As mentioned in a previous section, the main difference in equalization between OFDM and SC-FDE is the extra step of processing required to perform time domain detection on a frequency domain equalized signal. Thus, I hypothesize that the slower runtime of SC-FDE is due to the receiver needing to perform more complex operations. To test my hypothesis, I decdied to run a timing experiment on the receiving side using the matlab functions tic and toc.

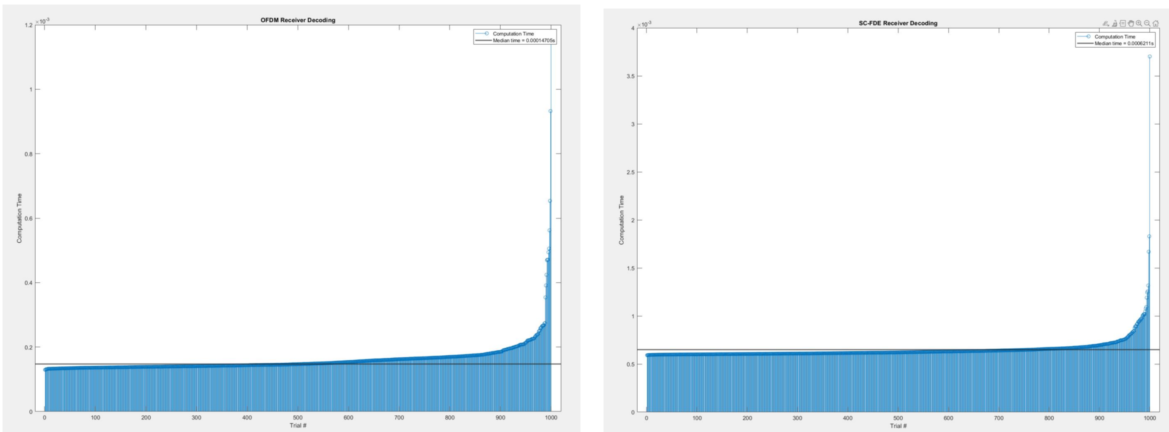


Figure 12: Receiver Decode Computation Time, OFDM (left), SC-FDE (right).

Figure 12 shows total computation time on the receiver side from reception of transmitted time domain signal to making a decision on the symbols, sorted from low to high over 1000 trials. Both systems show a small number of high delay runs likely due to matlab needing to allocate memory for arrays and variables as well as filling up the cache in the initial runs. Disregarding the warm-up outliers, we see that OFDM receiver has an average computation time of 1.47e-4 seconds, while SC-FDE receiver has an average computation time of 6.2e-4 seconds, over x4 the time taken by the OFDM receiver. Of course, these are based on matlab implementations which don't correspond exactly to real systems, but I have done my best to lower computation time by using built-in matlab functions for fft and ifft and performing time-domain detection by direct index sampling assuming perfect synchronization. Addtionally, even if the ratio isn't accurate, there are simply more processing steps that need to occur at the SC-FDE receiver side, in particular the ifft and time domain detection, which will increase computation time. The increase in computation time is significant because it impacts other factors such as device power consumption and application latency as well. These results suggest that the extra processing work required to re-convert back to time domain on the receiver side for SC-FDE systems is a major downside compared to OFDM.

5 Additional Practical Considerations

5.1 Ease of multiplexing & Channel Estimation

In the original 1966 formulation of OFDM, their main focus was not on a system that was more resistant to ISI, but rather a system that allowed for orthogonal multiplexing of the subcarriers, such that they can be implemented with a combination of simpler physical filters [Cha66]. The fact that each subcarrier sees a flatter channel that is better approximated by a single-tap channel model seems like an added bonus rather than the main goal. This also highlights the fact that there are many other considerations when designing realistic communication systems in addition to the raw achievable data rate.

The ability to process each subcarrier independently is not to be understated. It's even in the name: *orthogonal* multiplexing. In the previous two experiments, I have implicitly assumed communication from a single transmitter to a single receiver. However, this is often not the case in modern systems. Communication by users is generally sporadic, meaning short bursts of high activity followed by long periods of inactivity. Thus, it is extremely inefficient to switch from serving one particular user to the next, which introduces large amounts of switching overhead and possible delays if the traffic burst of multiple users overlap. OFDM can deal with this problem elegantly by dynamically allocating capacity in both the frequency and time domain.

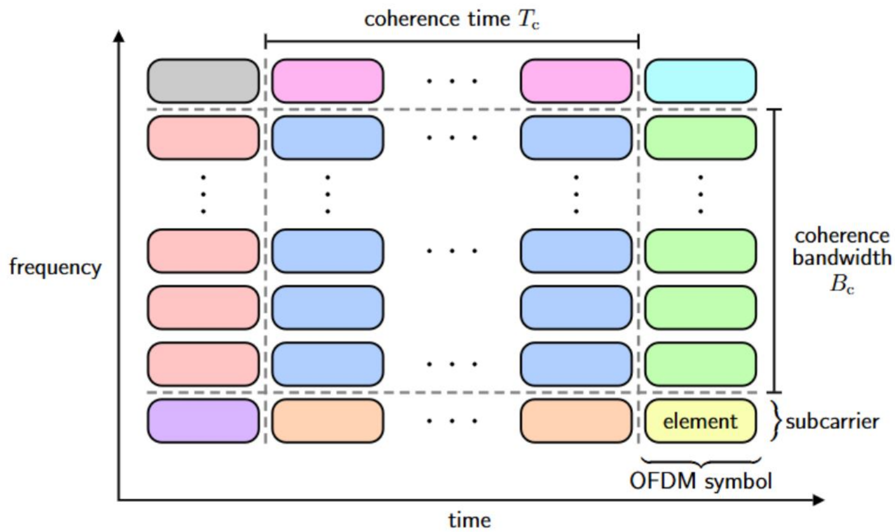


Figure 13: OFDM Transmission on combined frequency and time axis.

Figure 13 shows a visualization of OFDM transmission by splitting up both the subcarriers in frequency domain and individual symbols in time domain. The result is a time-frequency grid consisting of various individual symbols. In OFDM, it is possible to arbitrarily allocate any combination of these symbols to any user very efficiently. This is because each subcarrier is independent, so that both equalization and channel estimation for separate subcarriers can be performed independently. A receiver does not need channel state information (CSI) of the entire bandwidth to perform equalization, only the subcarriers that it has been assigned to. If the system is setup such that each subcarrier or group of subcarriers within some 'coherence bandwidth' see an approximately narrowband channel, channel estimation can be done by transmitting a 'pilot symbol' per coherence block with a known pattern from which the channel gain of a specific coherence bandwidth can be estimated [Zha23]. Then, the receiver can equalize the subcarriers it has been assigned which carry the desired information using the estimated channel gains for those specific subcarriers and directly discard all other received signals without needing to equalize or process them.

This cannot be done for SC-FDE because every time domain symbol contains frequency components which span across the entire bandwidth. If an SC-FDE system were to perform multiple access for the symbols in each chunk, every receiver would need a full estimate of the channel response across the entire bandwidth to perform equalization on the entire received signal and discard the other symbols not allocated to it after going through all the hardwork of equalization. This process is extremely inefficient and makes SC-FDE unsuitable for multiple access schemes meant to serve multiple users. I believe this is likely the main reason OFDM has become the most popular communication scheme currently used. In the modern age of extreme connectivity, communication that cannot easily support multiple user simultaneously is simply unsuited for our primary use cases.

5.2 Peak to Average Power Ratio (PAPR)

Another consideration for communication systems is how it interacts with the physical implementation. A metric often used to describe communication systems is the Peak Average Power Ratio (PAPR), which essentially measures how far apart the average transmitted power is from the peak transmitted power. This is important because a high PAPR hurts the overall energy efficiency of the system [Yan+13].

The power amplifier is the final stage of amplification that takes the analog signal generated from the DAC and amplifies it to a high enough power to be transmitted through an antenna. In the modern era of extremely frequent communication, power consumption is very important, making the efficiency of these power amplifiers a crucial performance metric, which is defined as the ratio of delivered RF power (antenna) to DC power consumption. Like most electronic devices, power amplifiers have an idle (DC) power consumption that cannot be prevented as long as the device is turned on and ready for operation. Thus, it is easy to see that a power amplifier only operates at maximum efficiency when it is driven by its maximum tolerable input signal. When being driven by a less than maximum input, the DC power consumption does not change a lot, but the RF output power drops.

The problem caused by a high PAPR communication scheme is that the power amplifier must be designed such that it is able to handle the peak power output, but most of the time (on average) it is actually being driven by input signals that are much weaker. This means that the power amplifier is operating very far from its maximum output and therefore at a very low efficiency.

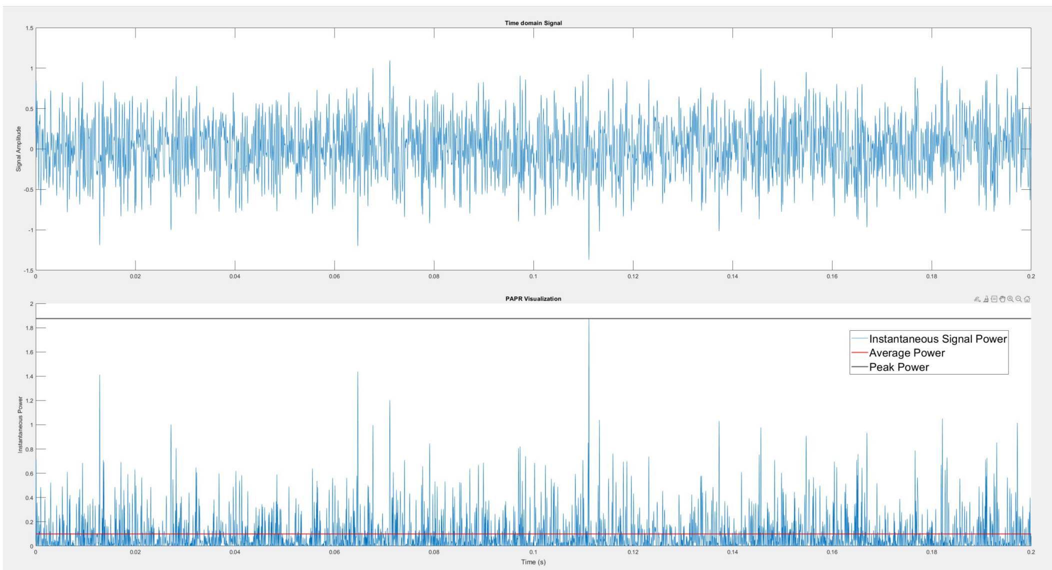


Figure 14: Passband time domain signal & PAPR for OFDM.

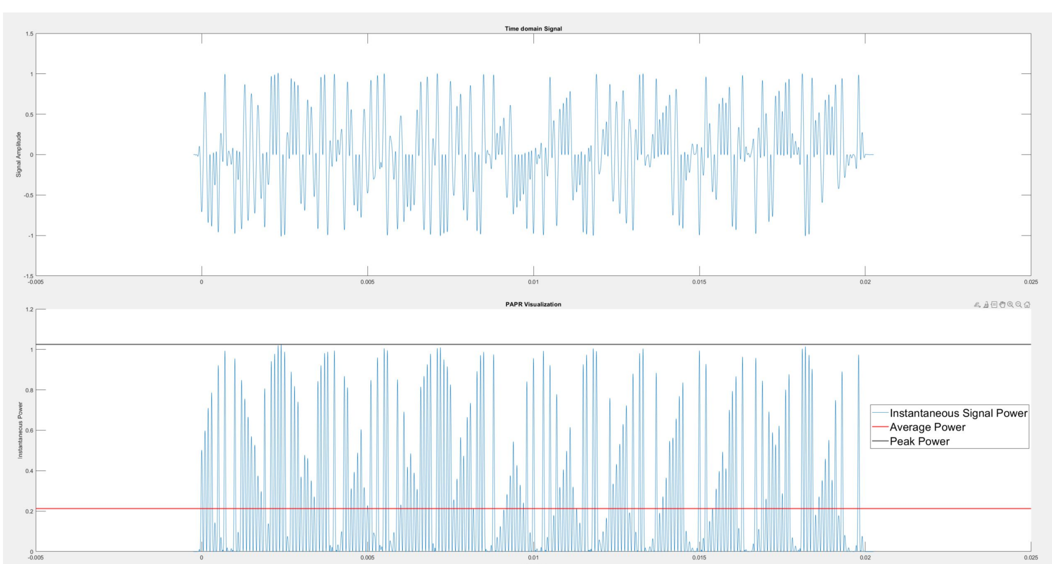


Figure 15: Passband time domain signal & PAPR for SC-FDE.

In this experiment, I compute the PAPR of my implementations of OFDM and SC-FDE. To do this, I combined the in-phase and quadrature components into a single realistic time domain signal that would be sent to the power amplifier. Using this signal, I compute the instantaneous power over time and obtain both peak and average power. Note that for PAPR, the average power defined is RMS power, so it is the square root of the average power as labeled in the figures. Both systems in this experiment have 20kHz bandwidth, a carrier frequency of 1khz, and using 4-QAM modulation. A low carrier frequency was chosen to illustrate the envelope modulation effect while still allowing a higher number of symbols to be visible for consistency in results. Using a realistic carrier frequency did not change the results when testing. The signals contain 200 symbols for SC-FDE and 2000 subcarriers for OFDM. SC-FDE envelope does not change with the number of symbols, which is why I chose to only show 200 for better visibility.

From the plots, we immediately see that OFDM has a higher PAPR than SC-FDE. Because SC-FDE transmits a series of pulses, the overall shape of the signal is fairly constant and the average power is higher. On the otherhand, OFDM has a lot of erratic peaks due to complex constructive interference from the multiple subcarriers, resulting in a low average but occasional spikes. Only one big peak every so often is needed to cause a really high PAPR and be problematic for the amplifier. It is also possible to simply ignore these high peaks and let the amplifier saturate when they occur, but this distorts the transmitted signal and is up to the communications engineer to evaluate cost and benefit.

The average PAPR I computed based on these simulations by shuffling the input symbol string was around 3.4dB for SC-FDE and 6.8dB for OFDM. In my RF design class, 4QAM is cited as having 3.3dB PAPR, which is very close to what I measured in this experiment. OFDM is often cited as having 12dB PAPR, but this does depend on the exact modulation scheme used and density of subcarriers. Higher order modulation schemes will have more variability in amplitude and phases which will increase the PAPR. Thus, my simulations agree with real world tests showing that OFDM generally has a higher PAPR, resulting in worse power amplifier efficiency.

It should be noted that as the modulation order increases, the PAPR of single carrier systems also increases rapidly, which shrinks the relative gap between the two schemes. In the modern era of high throughput communication with something like 128-QAM, the difference may not be significant. Additionally, the design of physical circuits is often informed by the higher level systems that use them and not the other way around. Designing power amplifiers that can handle high PAPR is an active research topic, an example of which is the Doherty amplifier which essentially adds a second efficiency peak, usually around the average power level to maintain better efficiency. Thus, this is not as strong of a consideration as ease of multiplexing, for example [Yu+15].

6 Conclusions

In this project,I analyzed and compared different aspects of OFDM v.s. SC-FDE systems to understand their relative strengths and their usefulness in real systems. I found that in terms of equalizing frequency selective channels, both systems are equally capable when placed under similar operating conditions. Thus, the reason for OFDM's success must not be from its equalization abilities alone.

Looking at other factors such as speed, receiver load, and easy of multiplexing, we find that OFDM is simply a much more flexible system that fits better to the primary use cases of our modern communication infrastructure, which mainly involve a power transmitter sending data to multiple, weaker receivers at the same time. The only advantage SC-FDE can really claim is in regards to PAPR, but even then the advantage is likely insignificant and will only further shrink as power amplifier technology improves.

Through the process of actually implementing not just one but two different communication systems by hand in MATLAB, even if they are rudimentary, really challenged my fundamental understanding of how communication systems worked. I encountered many bugs due to my own misunderstanding of certain topics and had to reshape the mental models I had built thus far to explain the behaviors I was seeing in my simulations. Additionally, writing the theoretical background and thoroughly explaining my experiments required a lot of thought and really helped solidify my understanding of communication systems.

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