1.2 Thevenin's Theorem

Adapted from DC/AC Circuits and Electronics by Robert Herrick

Any *linear* system can be simplified to a single voltage source in series with a single impedance. This makes figuring out how your circuit will interact with a large, complex, even unknown system *much* simpler. You only have to complete two tests or two calculations and you are done.

Figure 1-13 demonstrates this theorem. Figure 1-13(a) represents a complicated circuit and Figure 8-28(b) represents its simple Thévenin model equivalent.

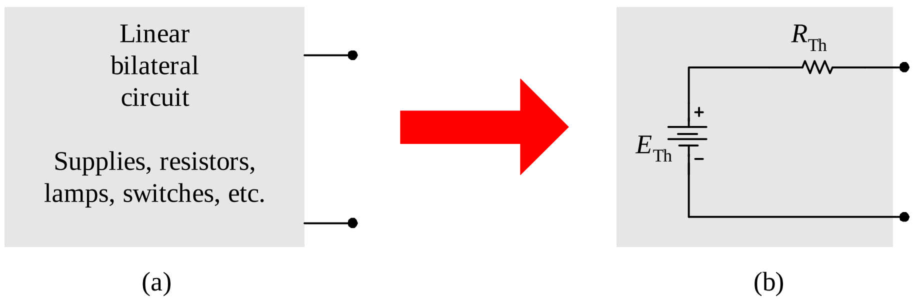

Figure 1-13 Thévenin model equivalent of a circuit

The model represents the actual circuit in terms of all possible load conditions. Once the model is established, it is easier to work with than an original complicated circuit.

E$_{Th}$ by *Measurement*

So there is an electronic gismo with a pair of output terminals where the load is attached (such as a function generator). How can the Thévenin voltage E_{Th} of Figure 1-13(a) be measured? You cannot go inside the circuit and measure this voltage; you can only work at the terminals of the original circuit of Figure 1-13(a). Just *measure* the *open circuit voltage* at the terminals. Since there is no current with the unloaded circuit, the voltmeter measures the value of E_{Th}. Good voltmeters have very high internal resistances (1 MΩ or higher). If R_{Th} is a much lower resistance than this, then the voltmeter provides an accurate measurement of E_{TH}.

$$E_{TH} = V_{no\ load} = V_{open\ circuit}$$

R_{Th} by *Measurement*

To measure R_{Th} attach a load and draw a load current through R_{Th}. Then, measure two of the following: (1) the load voltage, and either (2) the load current, or the load resistance. Ohm's law then can be used to calculate R_{TH}. Look at Figure 1-14.

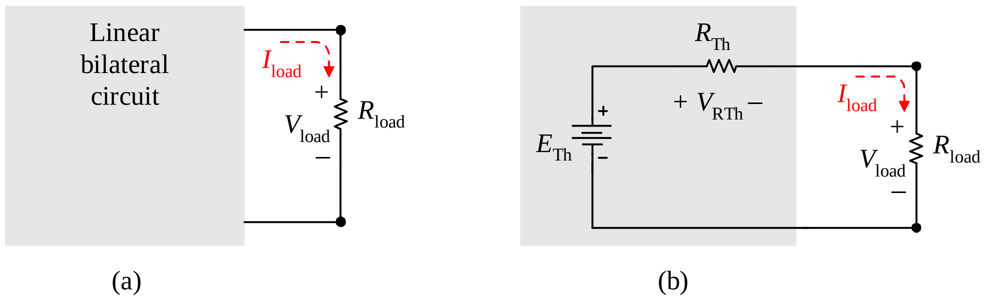

Figure 1-14 Finding R_{Th} with loaded circuit and model

Example 1-4

a. A half-bridge circuit is analyzed in the lab. The load is removed and its open circuit voltage is measured to be 10 V. A load resistance of 4 kΩ is reattached and the load voltage is measured to be 8 V. Find and draw the Thévenin model for this circuit.

b. If a 1 kΩ resistor were attached as a load instead of the 4 kΩ resistor, what would the load voltage and current be?

Solution

a. Based upon the no load measurement,

$$\boldsymbol{E_{TH} = 10\ \mathrm{V}}$$

Figure 1-15(a) represents the Thévenin model for this circuit with known values: open circuit voltage, loaded circuit voltage, and load resistance.

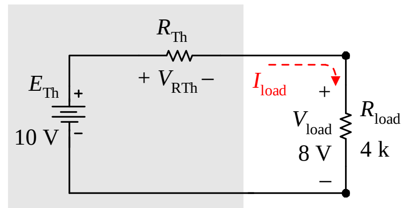

Figure 1-15(a) Model loaded with 4 kΩ load resistor

Find the voltage drop.

$$V_{RTh} = 10\text{ V} - 8\text{ V} = 2\text{ V}$$

Next, calculate the load current I_{load}.

$$I_{RTh} = I_{load} = \frac{8\text{ V}}{4\text{ k}\Omega} = 2\text{ mA}$$

$$\text{R}_{\textbf{Th}} = \frac{2\ \textbf{V}}{2\ \textbf{mA}} = 1\ \textbf{k}\boldsymbol{\Omega}$$

b. Figure 1-15(b) represents the Thévenin model of the half-bridge circuit with a 1 kΩ load attached for analysis. Find the total circuit resistance of the series circuit.

$$R_{total} = 1\text{ k}\Omega + 1\text{ k}\Omega = 2\text{ k}\Omega$$

Find the series circuit current.

$$I_{load} = \frac{10\text{ V}}{2\text{ k}\Omega} = 5\text{ mA}$$

Find the load voltage.

$$V_{load} = 5\text{ mA} \times 1\text{ k}\Omega = 5\text{ V}$$

So, the original bridge circuit with a 1 kΩ load resistor instead of a 4 kΩ resistor would have a load current of 5 mA and a load voltage drop of 5 V. Notice that the details of the original circuit were not even known, but using Thevenin's Theorem its interaction with the rest of the world was calculated.

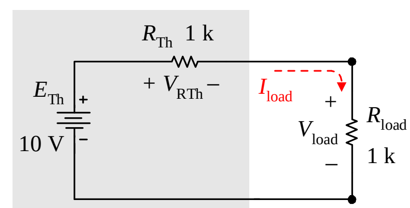

Figure 1-15(b) Model with a 1 kΩ load

Thevenin's Values by *Analysis*

The response of the RC charge circuit in Figure 1-16 is covered elsewhere. In summary, when e_{in} steps up, current begins to flow, limited by the resistor. As time goes on, charge from that current accumulates on the plates of the capacitor. Its voltage rises. Given enough time, the voltage on the capacitor matches the input voltage. At that point current stops and the capacitor continues to hold that charge (voltage) until the input drops to discharge the capacitor. How far the capacitor charges depends on the input voltage, and how quickly it can charge. That charge rate depends on both R and C. The mathematical relationships are developed elsewhere.

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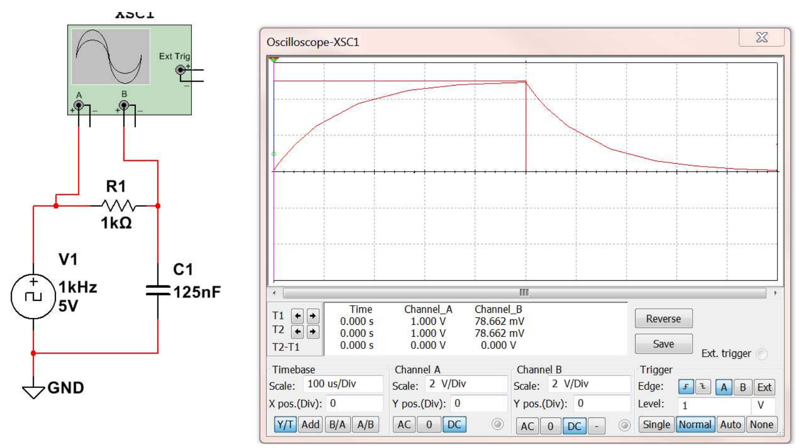
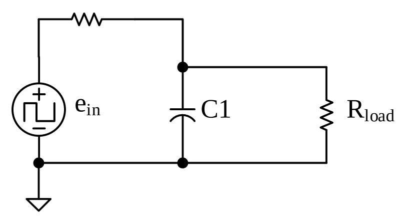


Figure 1-16 RC charge circuit

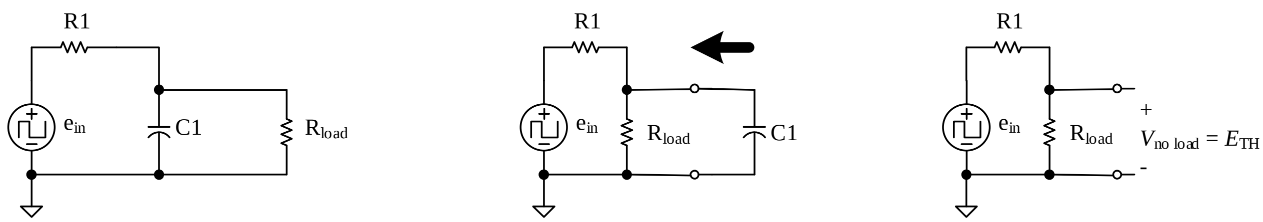
What happens when a load resistance is placed in parallel with C1, to use this ramp in a following circuit? Look at Figure 1-17.



(a) Loaded RC network

Figure 1-17 RC circuit loading

The first step is to rearrange the components, placing the one of interest to the far right. Since the charging of the capacitor is the process being considered, the capacitor has been moved in Figure 1-17b.



(a) Loaded RC network **(b)** Network seen by the capacitor **(c)** $V_{\text{no load}} = E_{\text{TH}}$

Figure 1-17 RC circuit loading

First, remember that

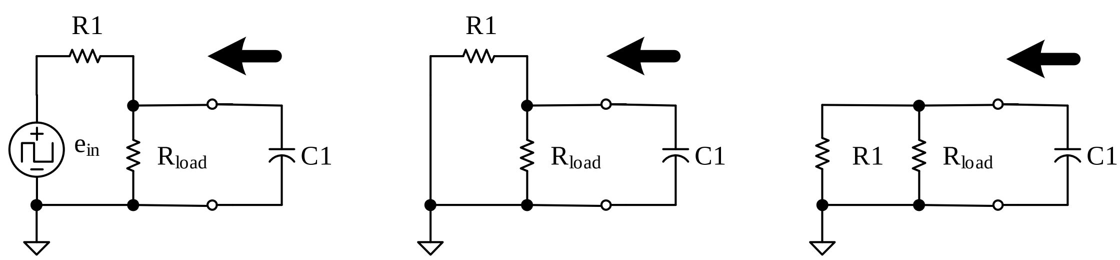
$$E_{\text{TH}} = V_{\text{no load}}$$

So, redraw the circuit with the load (the capacitor) removed, then calculate the resulting output voltage. The schematic is shown in Figure 1-17 (c). For this particular circuit, using the voltage divider law,

$$E_{\text{TH}} = e_{\text{in}} \frac{R_{\text{load}}}{R1 + R_{\text{load}}}$$

A different network provides a different relationship between e_{in} and E_{TH}.

To calculate R_{TH}, sources are replaced by their characteristic impedance. For a voltage source, e_{in}, is replaced by a short. Then, looking back into the circuit from the capacitor, determine the resistance seen. This is shown in Figure 17 (d) and (e).



(b) Network seen by the capacitor **(d)** Source replace with short **(e)** R1 & R_{load} in parallel

Figure 1-17 RC circuit loading

By inspection, for this particular network, R1 and R_{load} are in parallel, *not* series.

$$R_{\text{TH}} = \frac{1}{\frac{1}{\text{R1}} + \frac{1}{R_{\text{load}}}}$$

A different network provides a different relationship between its components and R_{TH}.

Example 1-5

a. For the circuit in Figure 1-16, determine the Thevenin's equivalent circuit when a 2 kΩ resistor is connected as a load.

b. Verify that the model is correct by simulating both the loaded circuit and the model.

Solution

a.

$$\boldsymbol{E_{\text{TH}}} = e_{\text{in}} \frac{R_{\text{load}}}{R1 + R_{\text{load}}} = 5\,V \frac{2\ \text{k}\Omega}{1\ \text{k}\Omega + 2\ \text{k}\Omega} = \mathbf{3.33\ V}$$

$$\boldsymbol{R_{\text{TH}}} = \frac{1}{\frac{1}{\text{R1}} + \frac{1}{R_{\text{load}}}} = \frac{1}{\frac{1}{1\ \text{k}\Omega} + \frac{1}{2\text{k}\Omega}} = \mathbf{667\ \Omega}$$

b.

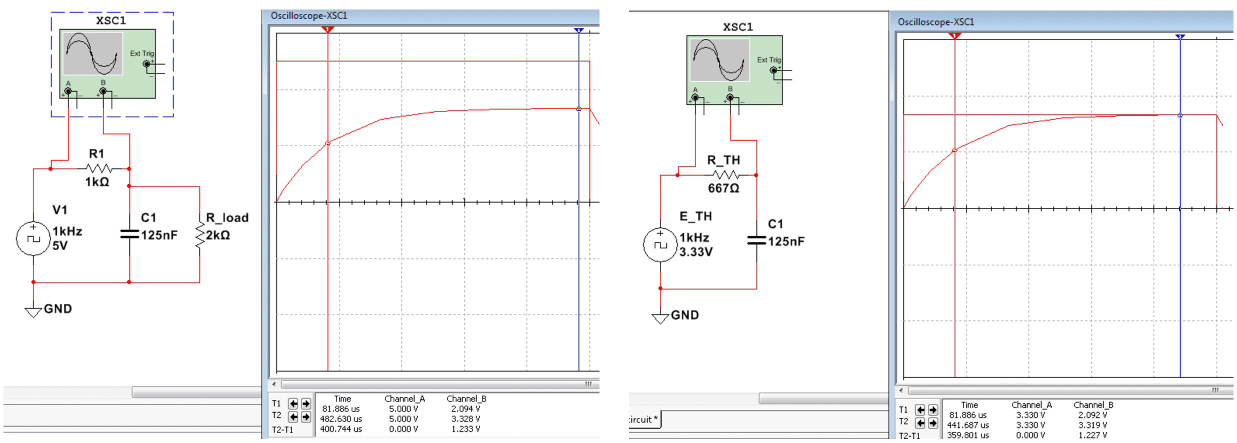


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Figure 1-18 Loaded circuit and Thevenin's model simulations