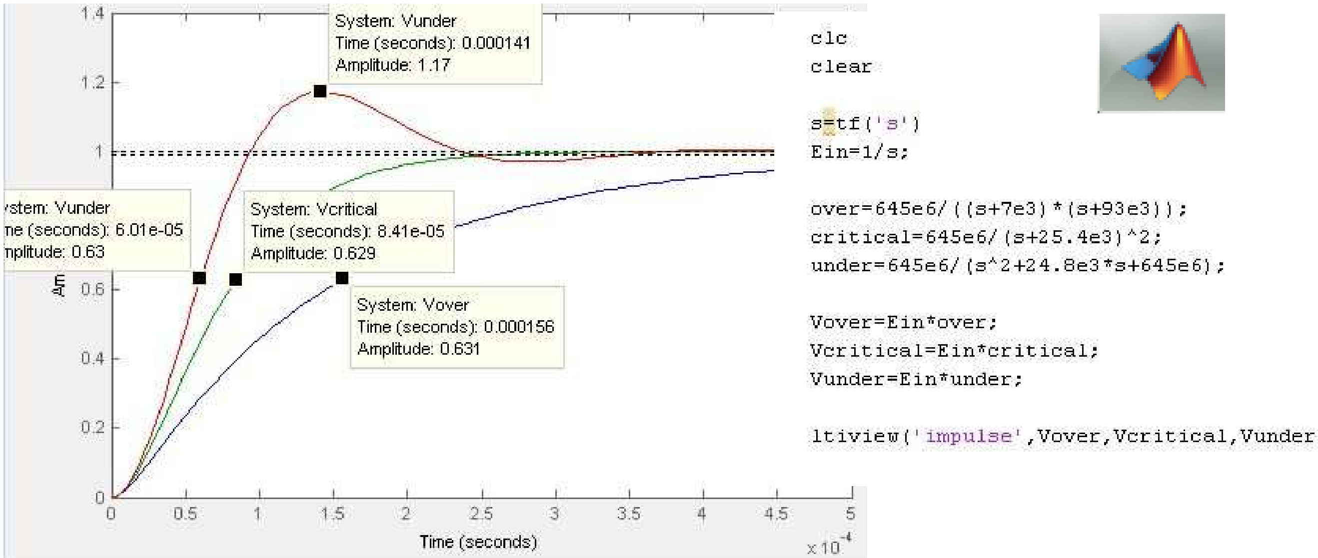


Figure 4-15 Matlab and ltiview step response of all three dampings

4.5 The LC Second Order Low Pass Filter

Two of the major uses of RLC passive circuits are in switching power supplies and in Class D audio amplifiers. In each, transistor switches pass large pulses of current to a resistive load. But, it is the *average* value of the pulse width modulated square wave that is wanted. All of the high frequency edges must be rejected, passing only the relatively slow, low frequency or DC average value.

That is the role of the low pass filter. An entire later chapter is dedicated to the details of building a host of sophisticated, op amp based, active filters. For now, a simple, *passive* low pass filter can be built with an inductor and a capacitor. It is shown in Figure 4-16.

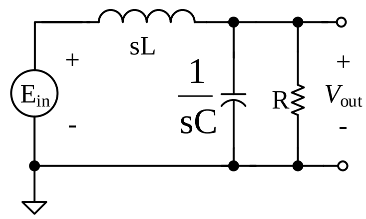


Figure 4-16 LC low pass filter driving a resistive load

This is different from the previous RLC circuit. The resistor is in *parallel* with the capacitor. Handle that first by defining Z_o as that parallel impedance.

$$Z_o = \frac{\frac{1}{sC} \times R}{\frac{1}{sC} + R}$$

Multiply the numerator and the denominator by *sC* to clear the complex fractions.

$$Z_o = \frac{\frac{1}{sC} \times R}{\frac{1}{sC} + R} \frac{sC}{sC} = \frac{R}{RCs + 1}$$

The transfer function is $\frac{V(s)_{out}}{E(s)_{in}}$ and can now be written directly using the voltage divider law.

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{Z_o}{sL + Z_o}$$

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{\frac{R}{RCs + 1}}{sL + \frac{R}{RCs + 1}}$$

Convert the denominator to have a common denominator

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{\frac{R}{RCs+1}}{\frac{sL(RCs+1)}{RCs+1} + \frac{R}{RCs+1}}$$

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{R}{sL(RCs+1)+R}$$

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{R}{RCLs^2+sL+R}$$

Finally, divide the numerator and denominator by RCL to set the coefficient of s^2 to 1.

LC low pass filter transfer function

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Remember, the standard second order transfer function is

$$\frac{V_{\mathrm{out}}}{E_{\mathrm{in}}} = \frac{A\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Equating coefficients

$$\omega_n^2 = \frac{1}{LC} \qquad \boldsymbol{\omega_n = \frac{1}{\sqrt{LC}}}$$

$$A\omega_n^2 = \frac{1}{LC} \qquad \boldsymbol{A = 1}$$

These two parameters are the same as with the series RLC circuit seen in the preceding section. But,

$$2\xi\omega_n = \frac{1}{RC}$$

Several steps of algebra gives $\boldsymbol{\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}}$

This has the same components, arranged considerably differently from the series RLC circuit.

Generally, the load resistance, R is specified. So, there are two remaining parameters, ω_n and ξ, and two components to set them, L and C.

The damping is going to determine how quickly the output responds, and whether it overshoots. Generally the fastest possible response *without* overshoot is preferable. Design the system to be critically damped, or maybe a little over damped.

$$\xi = 1$$

Selecting the right natural (also called *critical)* frequency requires looking at the circuit's frequency response plot, shown in Figure 4-17. *Much* more will be discussed in the chapter on active filters.

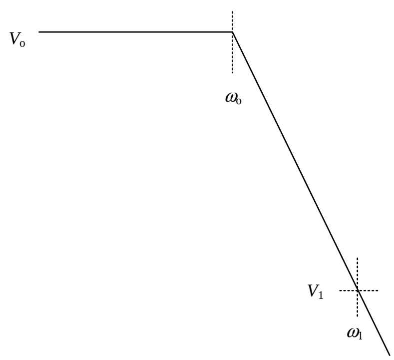


Figure 4-17 Simple frequency response plot

The output voltage is plotted on the vertical axis, starting at a large value, V_o. The horizontal axis is frequency. At frequencies below $\omega_n = \omega_o$, the desired signal is passed as V_o. But above this corner (resonant, natural, critical) frequency the output falls off proportionally as frequency increases. These are the signals that are to be rejected.

An inverse ratio and proportion is usually shown as

$$\frac{V_1}{V_0} = \frac{\omega_0}{\omega_1}$$

But, for filters, this is enhances to

$$\frac{V_1}{V_0} = \left(\frac{\omega_0}{\omega_1}\right)^n$$

The exponent *n* is the *order* of the filter. It is set by the highest exponent of *s* in the transfer function, the highest derivative in the differential equation, the number of energy storage components, the number of L and C. For this circuit $n = 2$. The higher the order, the more steeply the frequency response plot falls, and the smaller the output becomes, the more the unwanted, high frequency signals are rejected. But, that requires a more complex circuit.

Knowing what is being applied to the circuit, V_o, and how much output is acceptable at a higher frequency, V_1 at ω_1 allows the circuit's natural frequency to be calculated.

$$\omega_0 = \omega_1 \sqrt[n]{\frac{V_1}{V_0}}$$

With the transfer function, the load resistance, the damping and the natural frequency, the circuit can be designed.

Example 4-6

A switching power supply has a switching frequency of 100 kHz, an amplitude of 15 V_{PP}, and can tolerate only 50 mV_{PP} of that "noise" at the 100 Ω resistive output.

a. Calculate the desired transfer function.

b. Verify the step response and frequency response using Matlab and ltiview.

c. Calculate the values of L and C.

d. Verify the transient performance and the AC noise with Multisim.

e. Explain the effect of raising and of lowering the load resistance.

Solutions

a.

$$\omega_1 = 2\pi f_1 = 2\pi \times 100\ \text{kHz} = 628\ \text{k}\frac{\text{rad}}{\text{sec}}$$

$$\omega_0 = \omega_1 \sqrt[n]{\frac{V_1}{V_0}} = 628\ \text{k}\frac{\text{rad}}{\text{sec}} \sqrt[2]{\frac{50\ \text{mV}}{15\ \text{V}}}$$

$$\omega_n = \omega_0 = 36.3\ \text{k}\frac{\text{rad}}{\text{sec}}$$

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{A\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1 \times \left(36.3\ \text{k}\frac{\text{rad}}{\text{sec}}\right)^2}{s^2 + 2 \times 1 \times 36.3\ \text{k}\frac{\text{rad}}{\text{sec}} s + \left(36.3\ \text{k}\frac{\text{rad}}{\text{sec}}\right)^2}$$

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{1.32 \times 10^9}{s^2 + 72.6\ \text{k}\frac{\text{rad}}{\text{sec}} s + 1.32 \times 10^9}$$

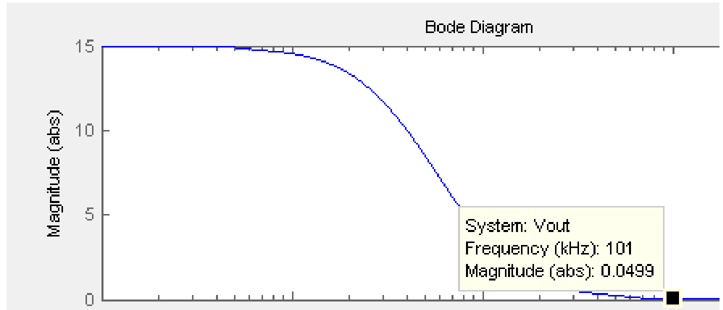
b.

```
clc
clear

s=tf('s')
Ein=15;

G=1.32e9/(s^2+72.6e3*s+1.32e9);
Vout=Ein*G;

ltiview('bode',Vout)
```



(a) Bode (frequency response plot)

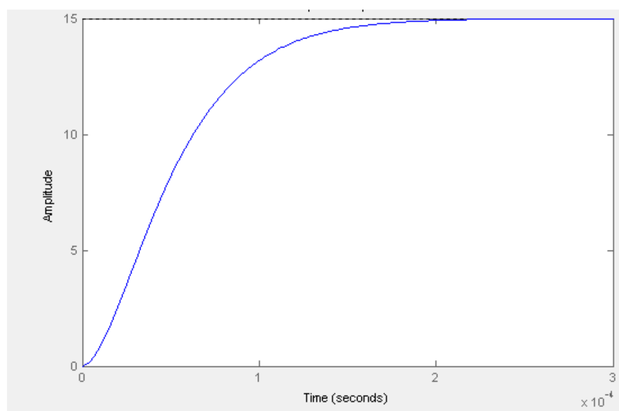
For a frequency response plot, select **'bode'** in the **ltiview** statement. Also set **Ein** at the amplitude, **15**, not as a step (which would be **15/s**). Once the plot is displayed, under the edit menu, select viewer preferences/magnitude/abs and frequency/kHz. As specified, the filter provides an output of $V_o = 15$ V, with $V_{1\ @\ 100\ \mathrm{kHz}} = 50$ mV.

```
clc
clear

s=tf('s')
Ein=15/s;

G=1.32e9/(s^2+72.6e3*s+1.32e9);
Vout=Ein*G;

ltiview('impulse',Vout)
```



(b) Step response, *not* under damped

Figure 4-18 Matlab ltiview frequency and transient response plots

Matlab performs the step response by changing the Ein statement to a step (**15/s**) and using **'impulse'** in the ltiview statement. The result is a quick rise to 15 V without an overshoot, characteristic of a critically damped circuit.

(c).

It is tempting to go directly to the equations that relate R, L, and C to ω_n and ξ. But that gives two equations in two unknows (L and C) that are *not* simple to unravel. Instead, notice that

$$\frac{V(s)_{out}}{E(s)_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \qquad \frac{V_{\text{out}}}{E_{\text{in}}} = \frac{A\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The coefficient of *s* contains only one unknown.

$$\frac{1}{RC} = 2\xi\omega_n$$

$$C = \frac{1}{2\xi\omega_n R} = \frac{1}{2 \times 1 \times 36.7\ \text{k}\frac{\text{rad}}{\text{sec}} \times 100\Omega}$$

***C* = 136 nF**

$$\omega_n^2 = \frac{1}{LC}$$

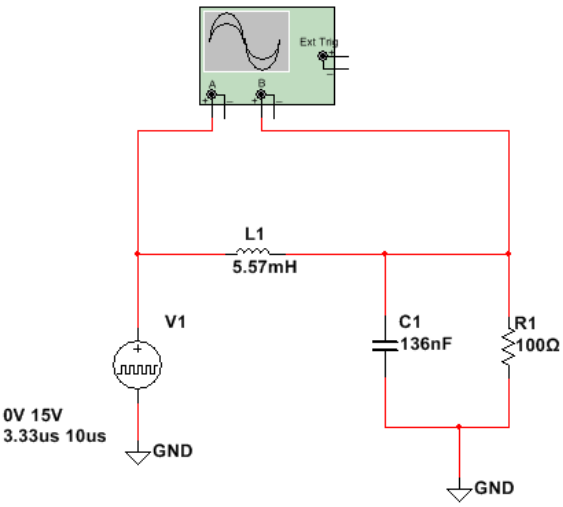
$$L = \frac{1}{C \times \omega_n^2} = \frac{1}{136\ nF \times 1.32 \times 10^9}$$

***L*=5.57 mH**

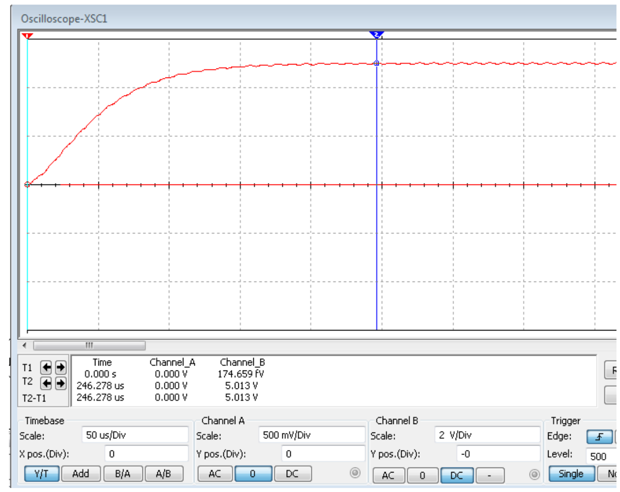
MULTISIM

(d)

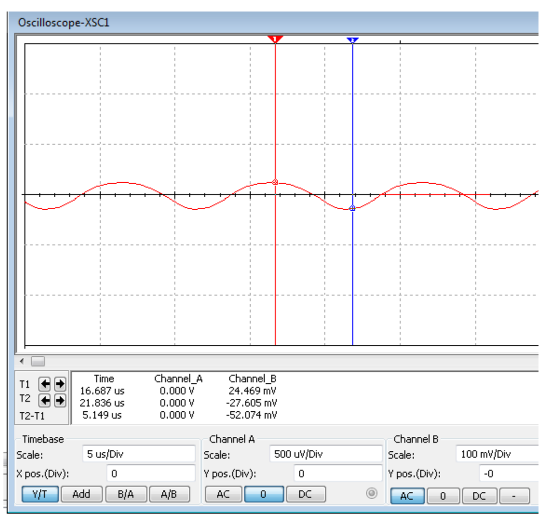
The Multisim schematic is given in Figure 4-19(a). To provide a 5 V output, the input pulse is set with levels from 0 V to 15 V, a 10 µsec period (100 kHz) and a 33% duty cycle.



(a) Switching power supply filter simulation schematic



(b) Load voltage startup transient, *no* overshoot



(c) Output AC ripple

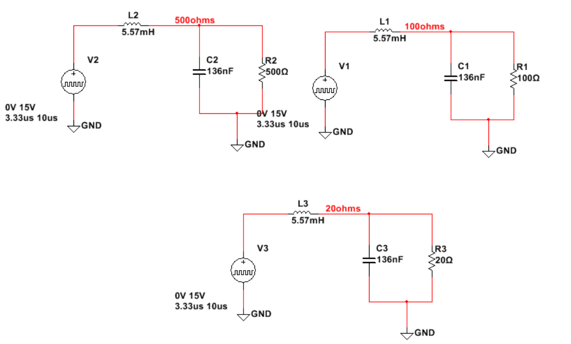
As designed and as predicted by Matlab, the initial startup is smooth, with a quick rise to 5 V and no overshoot. The filter appears to be critically damped.

In Figure 4-19(c), the oscilloscope channel has been AC coupled and then the scales adjusted to show the output voltage ripple. The cursors indicate a 52 mV_{PP} ripple. This aligns well with the 50 mV_{PP} ripple specified.

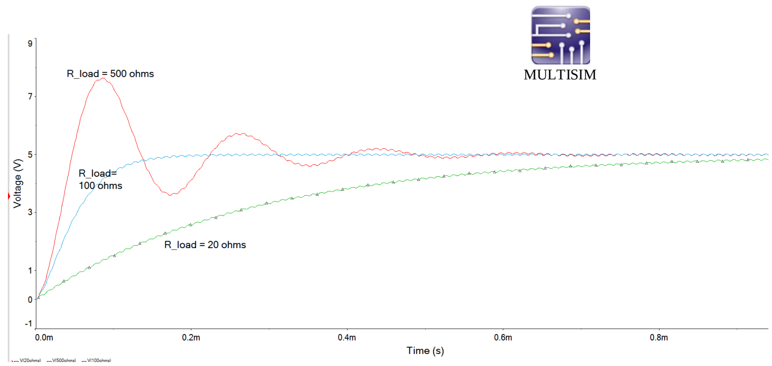
Figure 4-19 Switching power supply filter simulation

(e) The resistance affects the damping. $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$

Increasing R_{load} makes the circuit *under* damped and lowering R_{load} makes the circuit *over* damped. This is the <u>*opposite*</u> in the series RLC circuit of the previous section.



(a) Three schematics to evaluate the effect of load resistance



(b) Transient analysis of three different load resistances produces different damping

Figure 4-20 Multisim transient analysis of the effects of R_{load} on the circuit's transient response

As predicted by the equation above, raising the resistance to 500 Ω produces a very under damped response while asking for more current from the power supply with R_{load} = 20 Ω gives a stable but sluggish response.

It is tempting to say that faster is better. But, connecting a logic circuit which does not require much current, powered from this power supply subjects it to over 7 V on startup. This could seriously damage the logic.

When designing a switching power supply, select R_{load} to draw the *least* current anticipated. Then use that value to create a critically damped filter. When higher loads (lower R_{load}'s) are presented, the power supply will not overshoot and damage the load.

In fact, often, a dedicated load resistor (100 Ω in this example) is hard wired to the output of the power supply. So, even when *no* load is externally connected, the filter is critically damped. It will *never* overshoot.