

- $$\omega' + \frac{1}{\tau} \omega = \frac{A}{\tau} \nu$$

-

- $\omega =$ _____

- $\nu =$ _____

-

- $A =$ _____

- τ _____.

Motor Differential Equation

$$\omega' + \frac{1}{\tau} \omega = \frac{m}{\tau} v$$

ω = speed of the motor
 $\frac{\text{revolutions}}{\text{minute}}$ (RPM)

τ = time constant
second

m = motor's gain
 $\frac{\text{RPM}}{\text{V}}$

v = applied voltage

Motor Differential Equation

$$\omega' + \frac{1}{\tau} \omega = \frac{m}{\tau} v$$

ω = speed of the motor

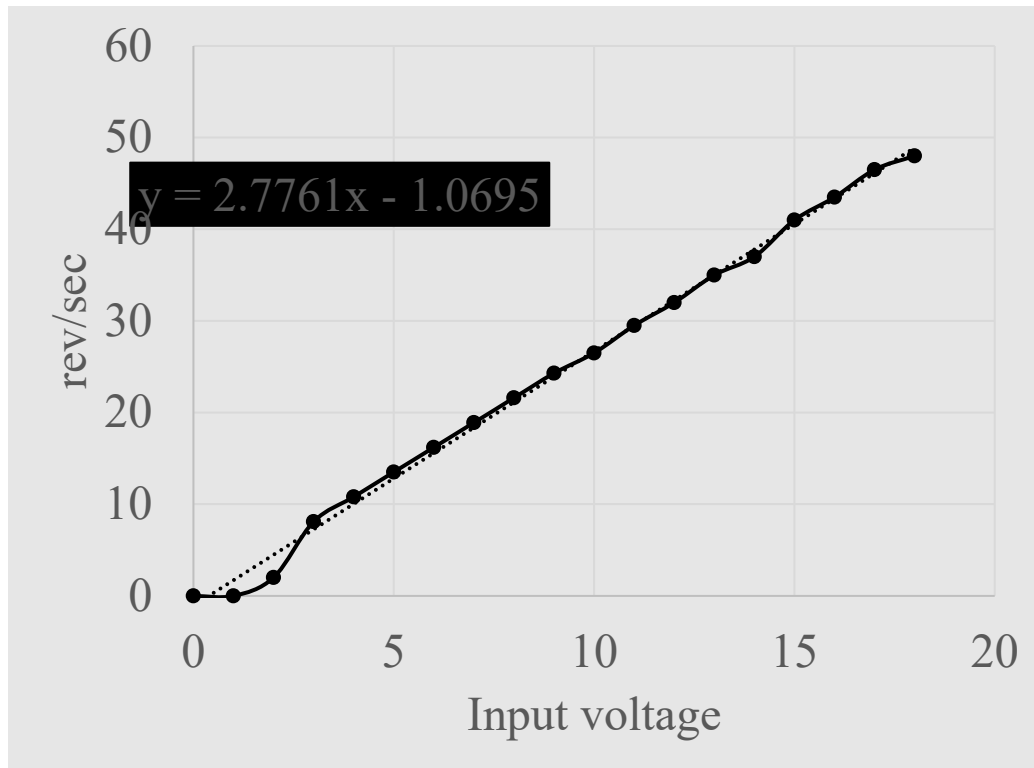
$\frac{\text{revolutions}}{\text{minute}}$ (RPM)

τ = time constant
second

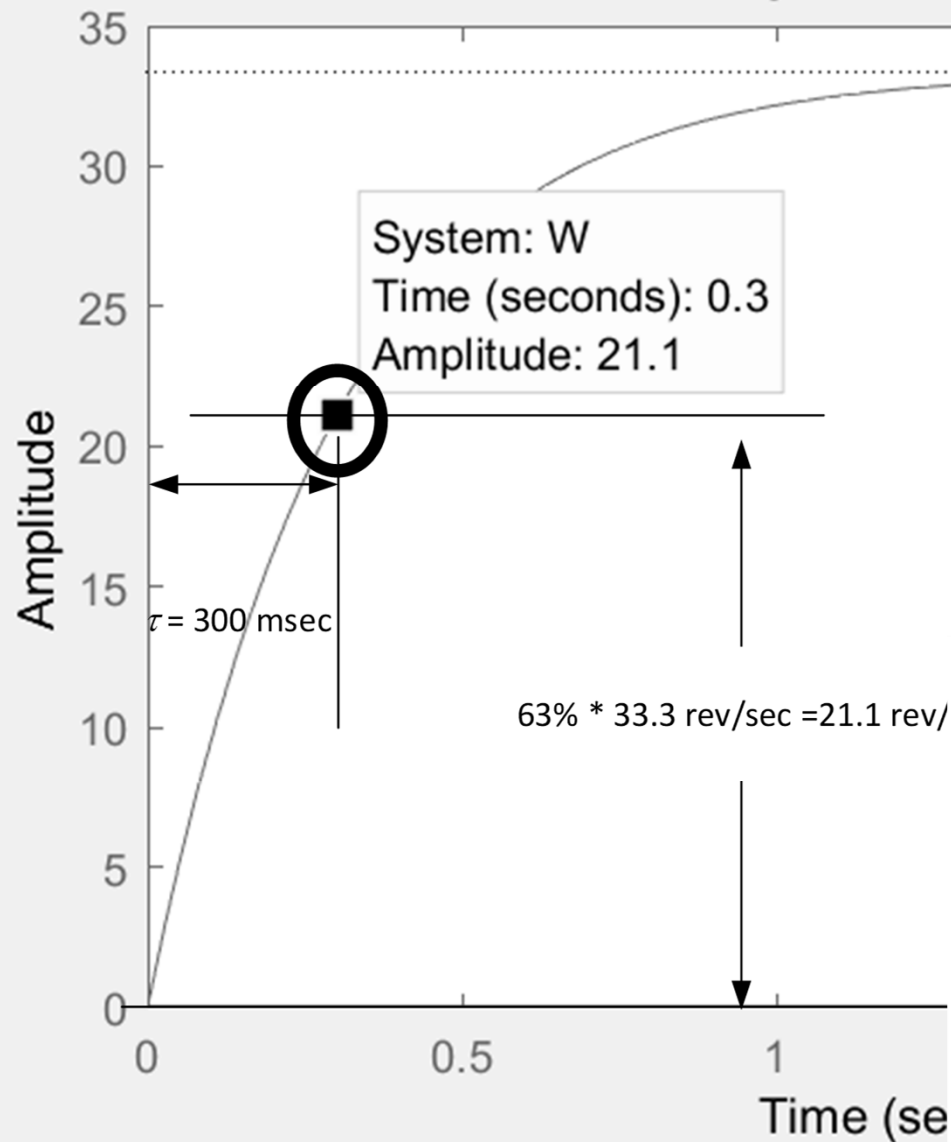
m = motor's gain

$\frac{\text{RPM}}{\text{V}}$

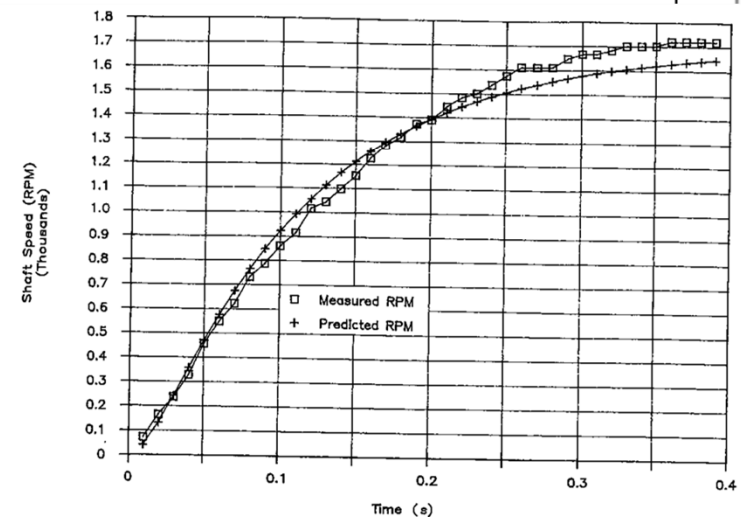
v = applied voltage



12 V step => Time Constant



$$\omega' + \frac{1}{\tau} \omega = \frac{m}{\tau} v$$

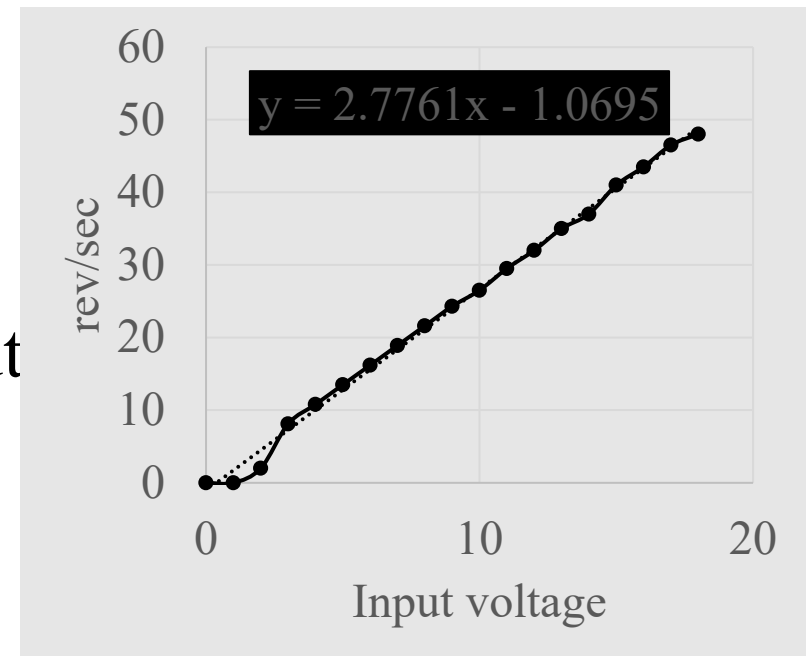


$$\omega' + \frac{1}{\tau} \omega = \frac{m}{\tau} v$$

$$\omega' + \underline{\hspace{2cm}} \omega = \underline{\hspace{2cm}} \times V$$

ω in rev/sec. Input in volts

Output starts at _____, _____ V input
and ends at _____, _____ V input.



$$\omega' + \frac{1}{\tau} \omega = \frac{A}{\tau} v$$

$$10. \quad sF(s) - f(0)$$

$$\frac{df(t)}{dt}$$

Transform into the Laplace domain.

$$\text{Transfer function } \frac{W}{V} =$$

Motor's response to a 12 V step

3. $\frac{1}{s}$

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} .$$

Unit step

17a. $\frac{A}{s(\tau s + 1)}$

$$A(1 - e^{-t/\tau})$$

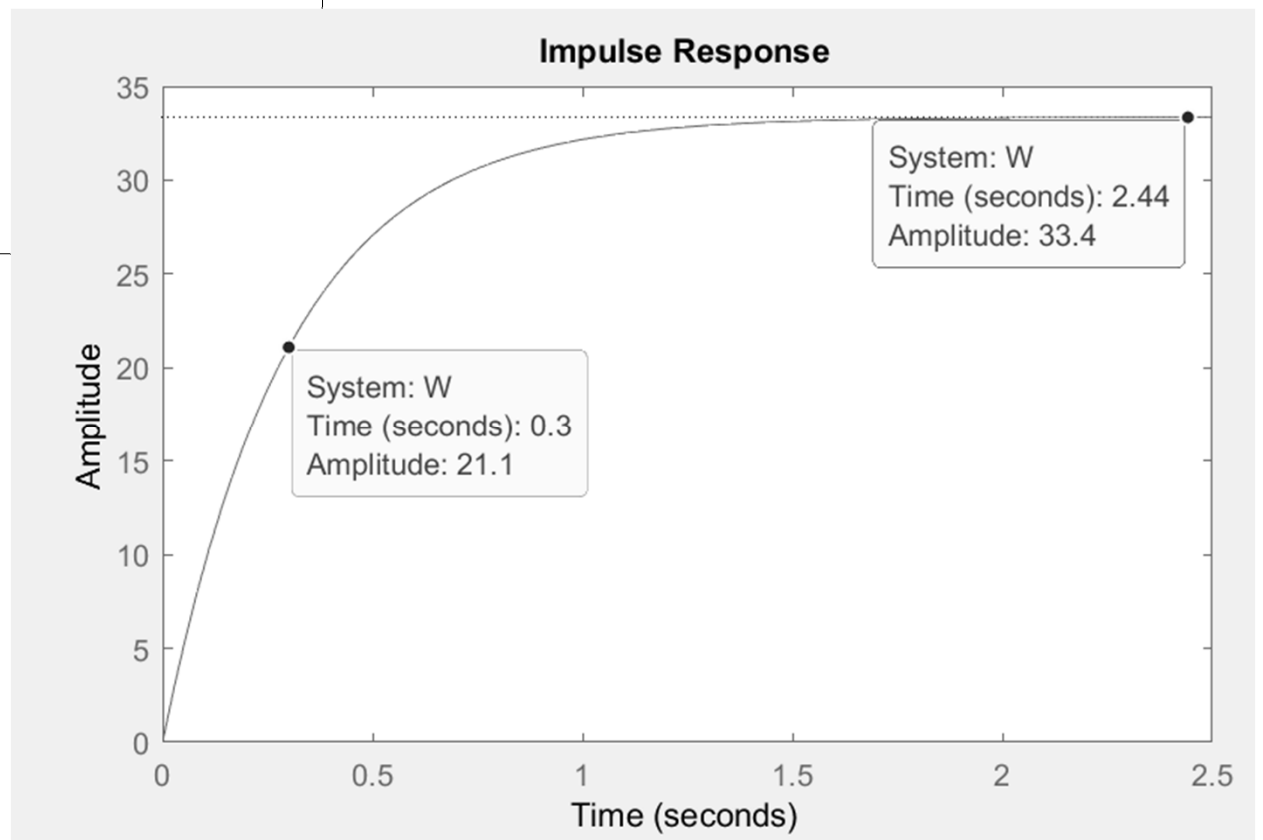
First-order
system re-
sponse to a
step input

```
clc  
clear
```

```
s=tf('s')  
tau=0.3;  
m=2.78;
```

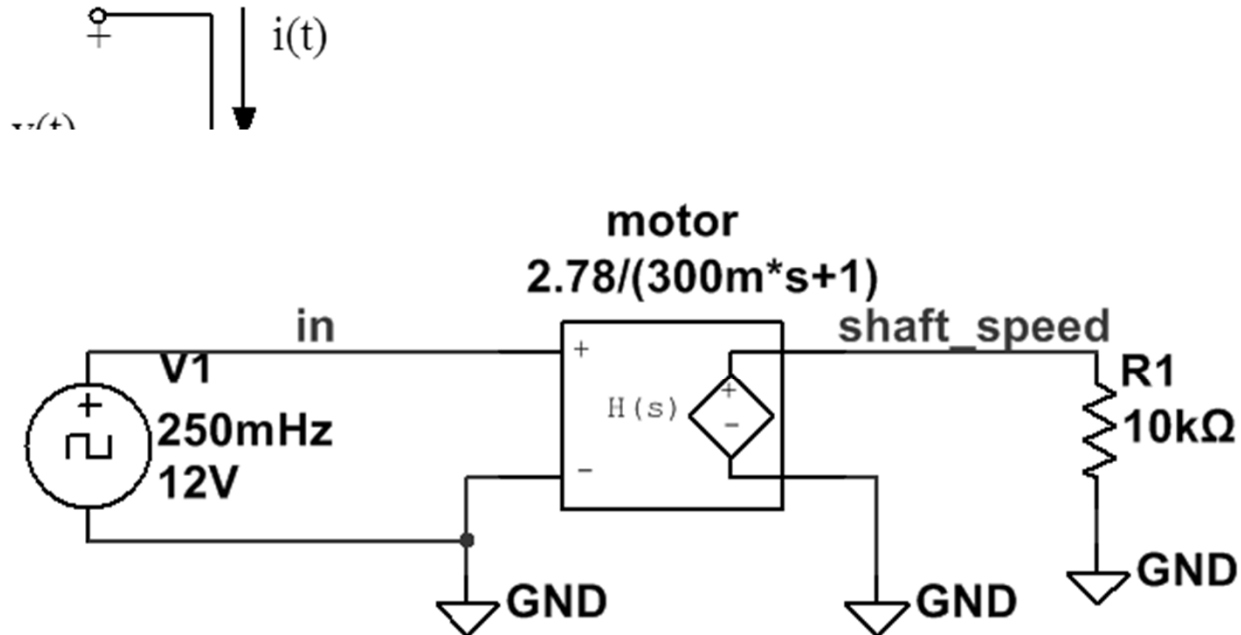
```
W=m*12/(s*(tau*s+1));
```

```
ltiview('impulse',W)
```

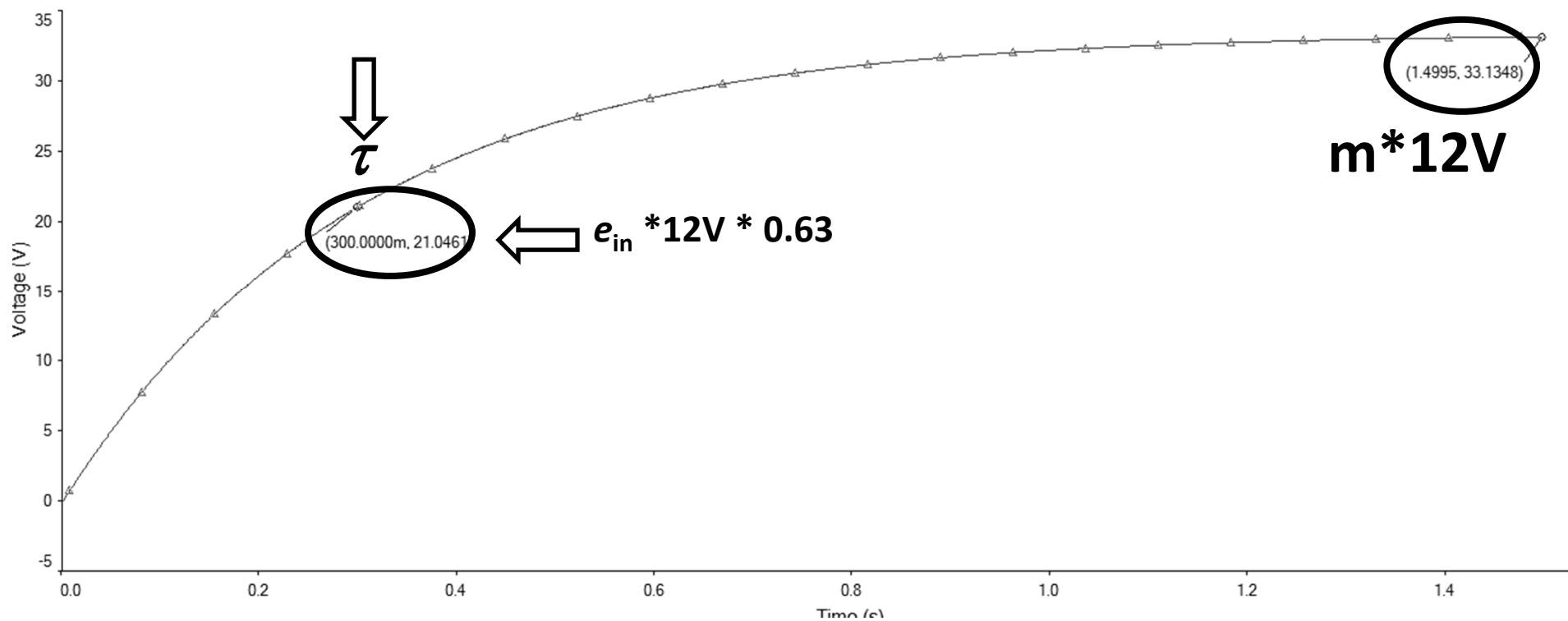


First Order Lag Element

$$A = 2.78 \text{ rev/sec / V} \quad \tau = 0.3 \text{ sec}$$



motor only - Laplace
Transient



$$\omega' + \frac{1}{\tau} \omega = \frac{A}{\tau} v$$

with initial speed

as

$$\omega' + \frac{1}{\tau} \omega = \frac{m}{\tau} v$$

ω = speed of the motor (RPM)

ω_0 = initial speed (RPM)

τ = time constant (sec)

m = motor's gain $\left(\frac{\text{RPM}}{\text{V}}\right)$

v = applied voltage (V)

This equation transforms into the Laplace domain as

$$sW - \omega_0 + \frac{1}{\tau} W = \frac{m}{\tau} V$$

Adding ω_0 to both sides

$$W + \frac{1}{\tau} W = \frac{m}{\tau} V + \omega_0$$

|

Multiplying by τ

$$s\tau W + W = mV + \tau\omega_0$$

Factoring out W

$$W(\tau s + 1) = mV + \tau\omega_0$$

Dividing by the term in ()

$$W = \frac{mV}{(\tau s + 1)} + \frac{\tau\omega_0}{(\tau s + 1)}$$

$$W = \frac{mV}{(\tau s + 1)} + \frac{\tau \omega_0}{(\tau s + 1)}$$

3. $\frac{1}{s}$

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Unit step

17a. $\frac{A}{s(\tau s + 1)}$

$$A(1 - e^{-t/\tau})$$

First-order
system re-
sponse to a
step input

Position

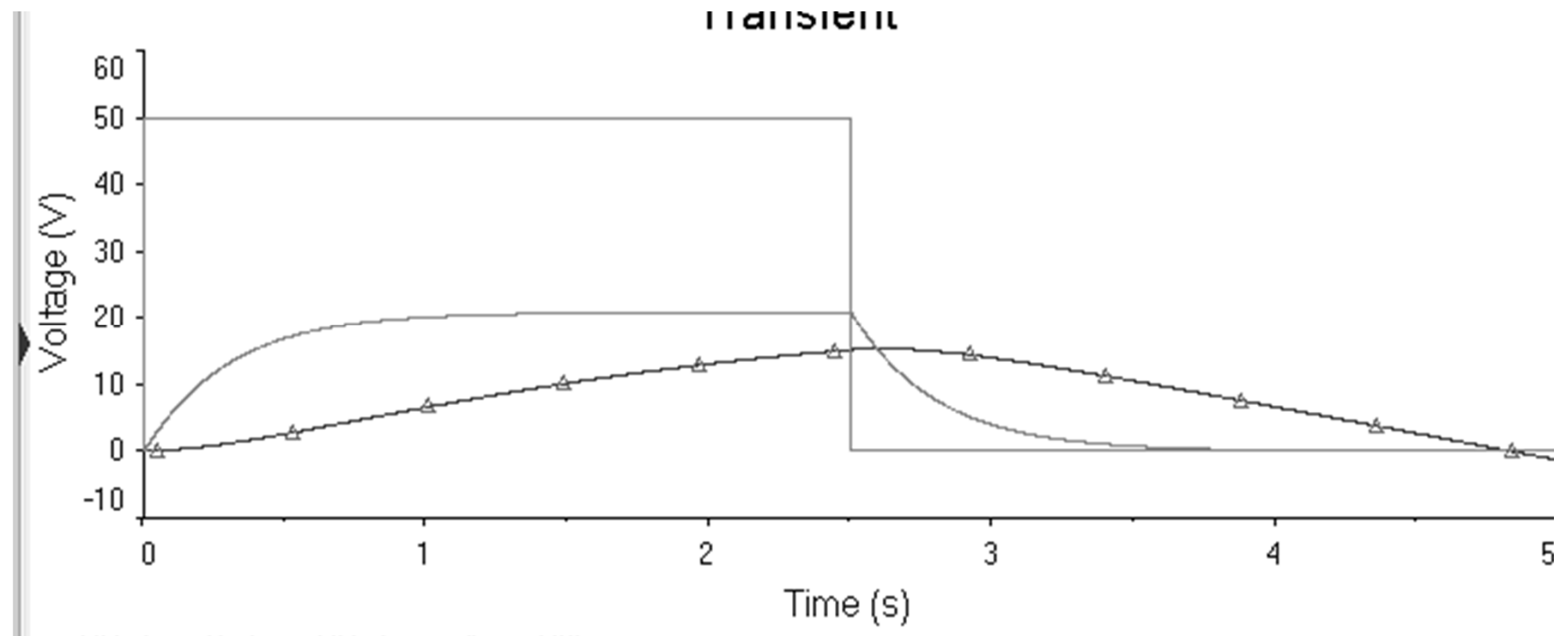
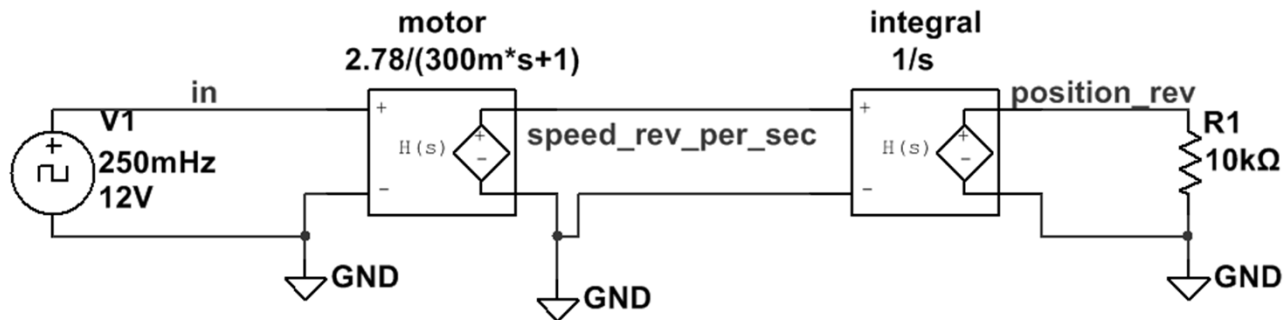
Shaft speed = ω = _____ = rate of change of angular position

18a. $\frac{A}{s^2(\tau s + 1)}$

$$A\tau \left(e^{-t/\tau} + \frac{t}{\tau} - 1 \right)$$

First-order
system re-
sponse to a
ramp input

Position ?????



Position correct

