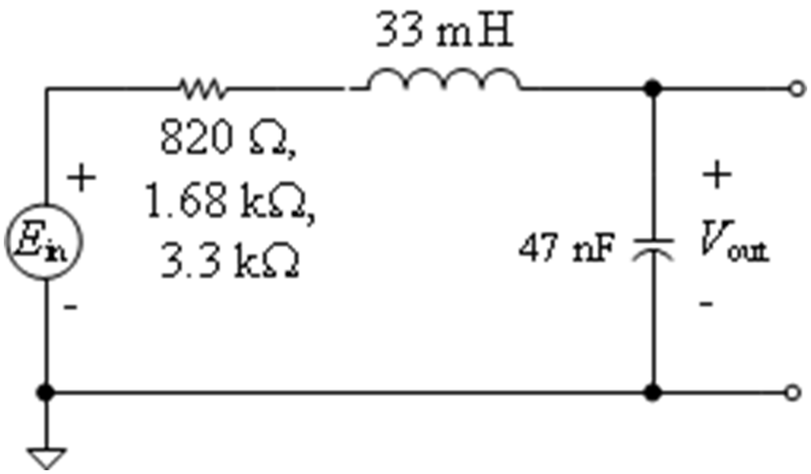
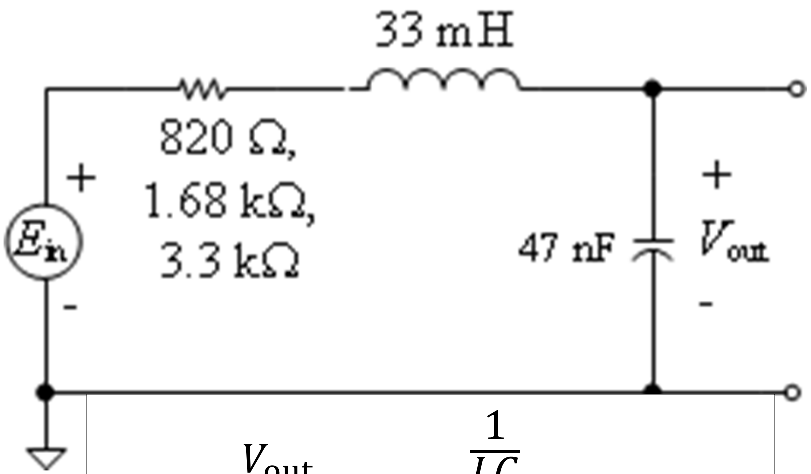
Laplace Second Order – Series RLC



1. Add the Laplace impedances.
2. Find the transfer function.

Laplace Second Order – Series RLC



$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Quadratic equation *roots*

$$s^2 + 2\zeta\omega_{\text{n}}s + {\omega_n}^2 = 0$$

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

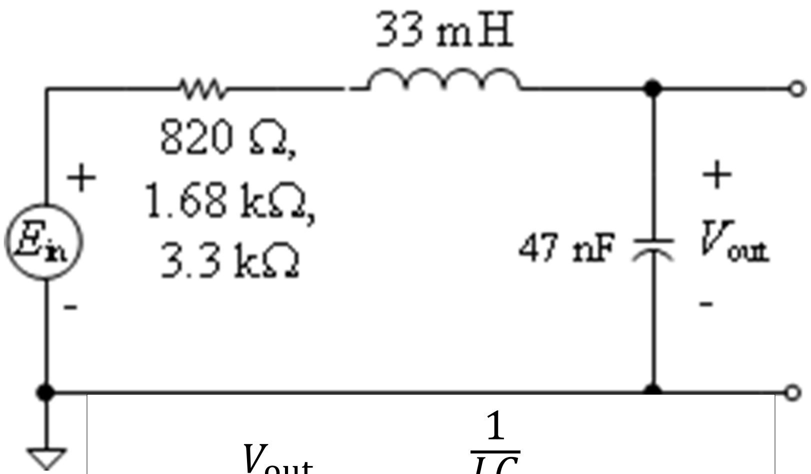
A = 1 $\quad$ $\text{B} = 2\zeta\omega_{\text{n}}$ $\quad$ $C = {\omega_n}^2$

Real
Overdamped

Equal
Critically damped

Complex
Under damped

Laplace Second Order – Series RLC



$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Quadratic equation ***roots***

$$s^2 + 2\zeta\omega_{\text{n}}s + \omega_n{}^2 = 0$$

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Real
Overdamped

Equal
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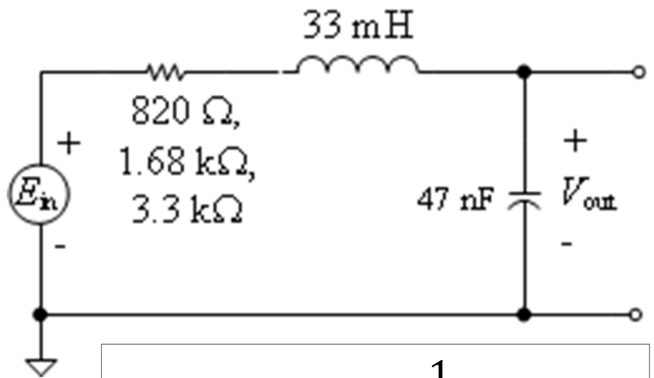
Under damped

Which R => which damping?



$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + 2\zeta\omega_{\text{n}}s + \omega_n{}^2 = 0$$

$$Ax^2 + Bx + C = 0$$

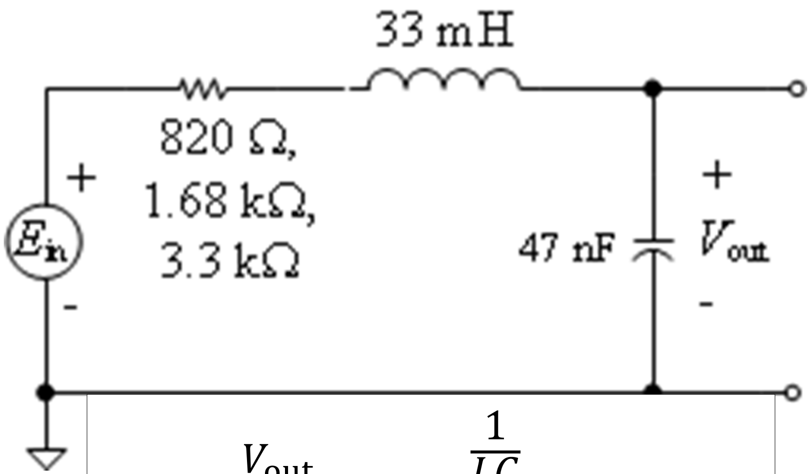
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

820 Ω
______ damped

1.68 kΩ
______ damped

3.3 kΩ
______ damped

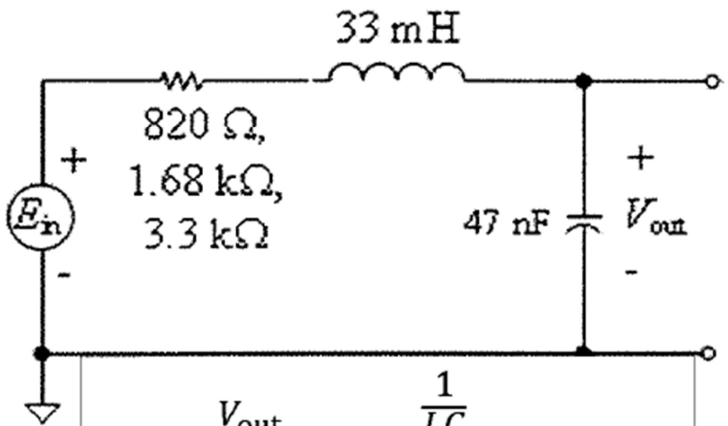
Write the time domain equation for $V_{\text{out}}(t)$ Overdamped



$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{A}{s(s+a)(s+b)} \rightarrow \frac{A}{ab}\left(1 + \frac{ae^{-bt} - be^{-at}}{b-a}\right)$$

Write the time domain equation for $V_{out}(t)$ Overdamped



$$\frac{V_{out}}{E_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{\frac{1}{33mH * 47nF}}{s^2 + \frac{3.3k\Omega}{33mH} s + \frac{1}{33mH * 47nF}}$$

$$= \frac{645M}{s^2 + 100k\, s + 645M}$$

$A = 1 \qquad B = 100k \qquad C = 645M$

$$\frac{A}{s(s+a)(s+b)} \rightarrow \frac{A}{ab}\left(1 + \frac{ae^{-bt} - be^{-at}}{b-a}\right)$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{-100k \pm \sqrt{(100k)^2 - 4 \times 645M}}{2}$$

$$x = -50k \pm 43k = -7k \; (a), \; -93k \; (b)$$

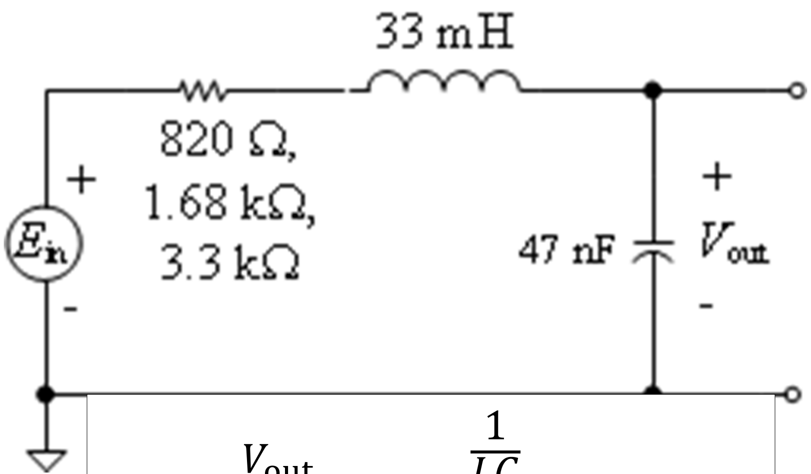
$$(s + 7k)(s + 93k)$$

$$V_{out} = \frac{1V}{s} * \frac{645M}{(s+7k)(s+93k)}$$

$$v_{out} = \frac{645M}{7k * 93k}\left(1 + \frac{7k e^{-93k t} - 93k e^{-7k t}}{93k - 7k}\right)$$

$$\boxed{= 0.991\left(1 + 0.081 e^{-t/11\mu sec} - 1.08 e^{-t/143\mu sec}\right)}$$

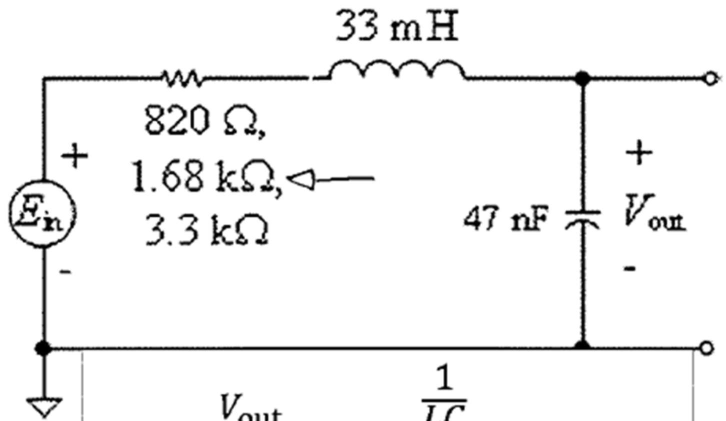
Write the time domain equation for $V_{\text{out}}(t)$
Critically damped



$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{A}{s(s+\alpha)^2} \rightarrow \frac{A}{a^2}\,[1-(1+at)e^{-at}]$$

Write the time domain equation for $V_{out}\,(t)$
Critically damped



$$\frac{V_{out}}{E_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{645M}{s^2 + 50.9k\,s + 645M}$$

$A = 1 \quad B = 50.9k \quad C = 645M$

$$\frac{A}{s(s+\alpha)^2} \rightarrow \frac{A}{a^2}\left[1-(1+at)e^{-at}\right]$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2}$$

$$x = \frac{-50.9k \pm \sqrt{(50.9k)^2 - 4(645M)}}{2}$$

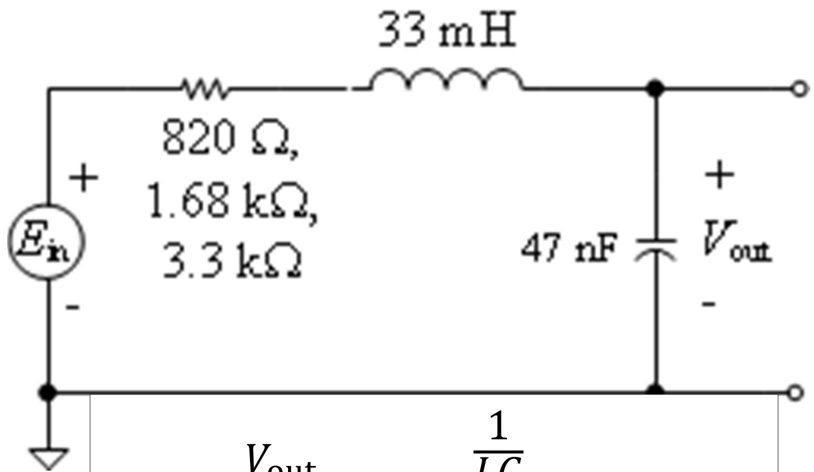
$$x \simeq -25.5k \pm 0$$ ← almost with standard components

$$V_{OUT} = \frac{1}{s} \times \frac{645M}{(s+25.5k)^2}$$

$$v = \frac{645M}{(25.5k)^2}\left[1-(1+25.5k\,t)e^{-25.5k\,t}\right]$$

$$\boxed{v = 0.992\left[1-(1+25.5k\,t)e^{-t/39\,\mu sec}\right]}$$

Write the time domain equation for $V_{out}(t)$
Underdamped



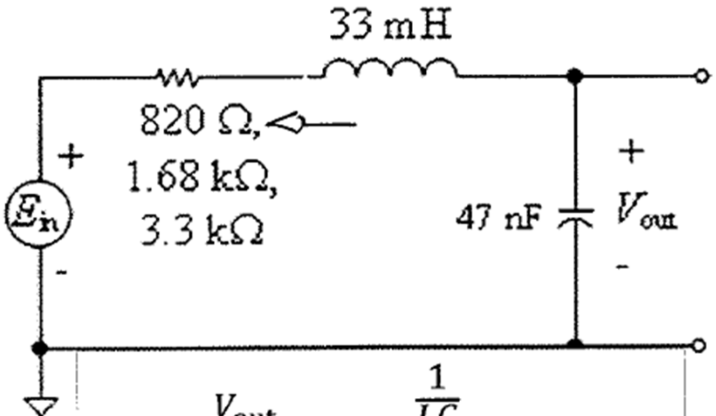
$$\frac{V_{out}}{E_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

22. $$\frac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$A\left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t - \psi\right)\right]$$

where $\psi = \tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{-\zeta} \quad (0 < \psi < \pi)$

Write the time domain equation for $V_{out}(t)$
Underdamped



$$\boxed{\frac{V_{out}}{E_{in}} = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}}$$

$$= \frac{645M}{s^2 + 24.8k\, s + 645M}$$

$\omega_n^2 = 645M$

$\boxed{\omega_n = 25.4K}$

$A\omega_n^2 = 645M$

$\boxed{A = 1}$

$2\zeta\omega_n = 24.8k$

$$\zeta = \frac{24.8k}{2 \times 25.4k}$$

$\boxed{\zeta = 0.49}$

22. $\dfrac{A\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ $\quad A\left[1+\dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t-\psi\right)\right]$

where $\psi = \tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{-\zeta}$ $(0<\psi<\pi)$

$$v = 1\left[1 + \frac{e^{-0.49\times 25.4k\, t}}{\sqrt{1-0.49^2}}\sin\left(25.4k\sqrt{1-0.49^2}\, t - \psi\right)\right]$$

$$= 1 + 1.147\, e^{-12.4k\, t}\sin(22.1k\, t - \psi)$$

$$\boxed{= 1 + 1.147\, e^{-t/81\mu sec}\sin(2\pi * 3.5kHz\, t - \psi)}$$

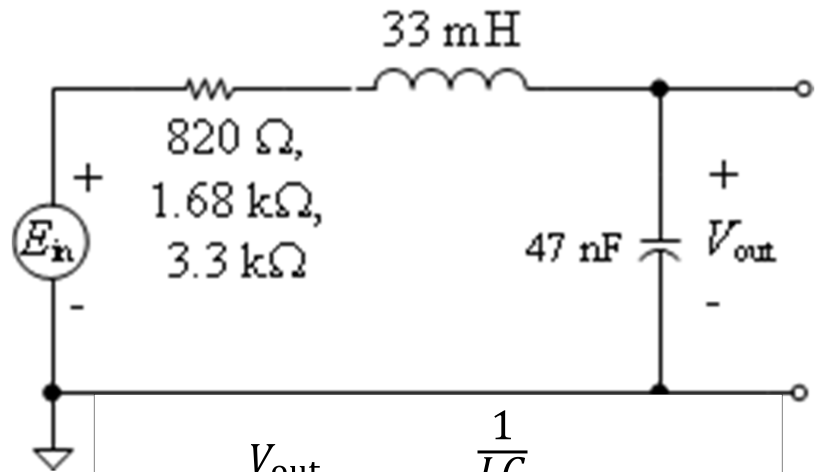
eventually $v_{out} = E_{IN}$

eventually dies out (in ~400μsec)

oscillation with decaying amplitude with 286μsec period

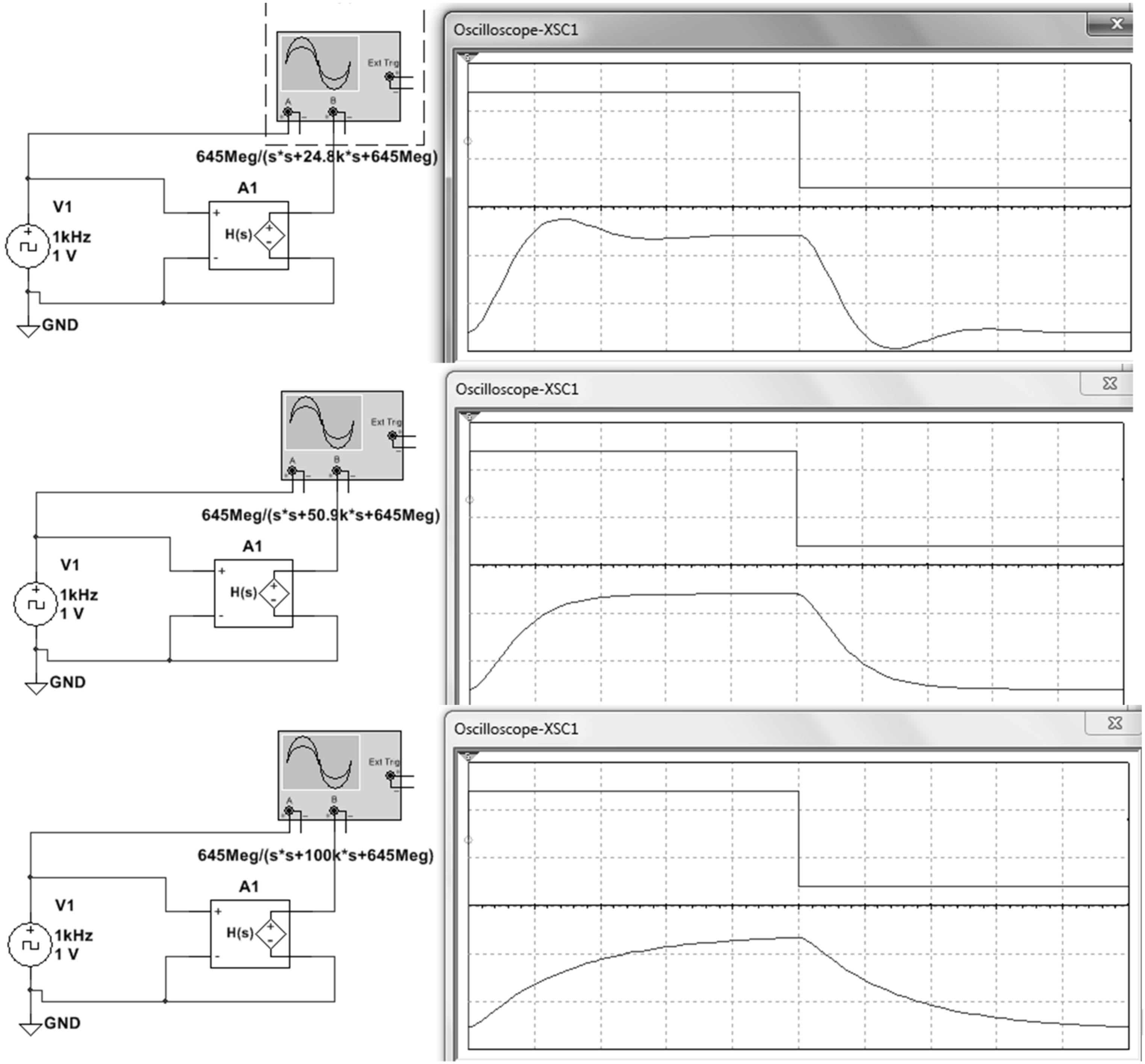
MATLAB calculated responses

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

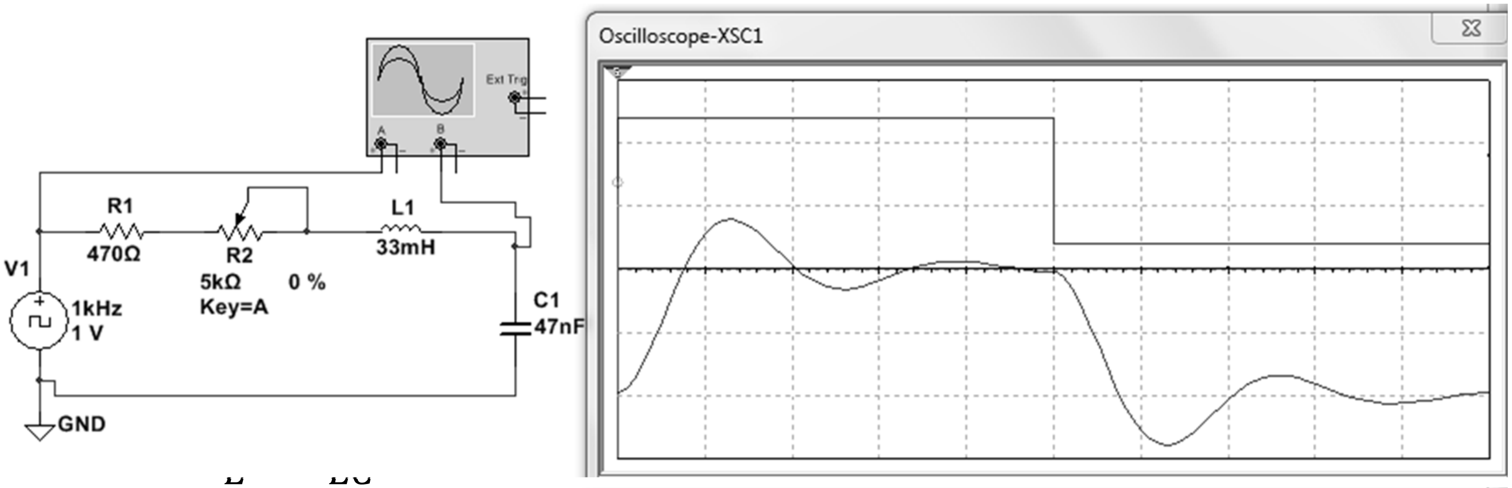


$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

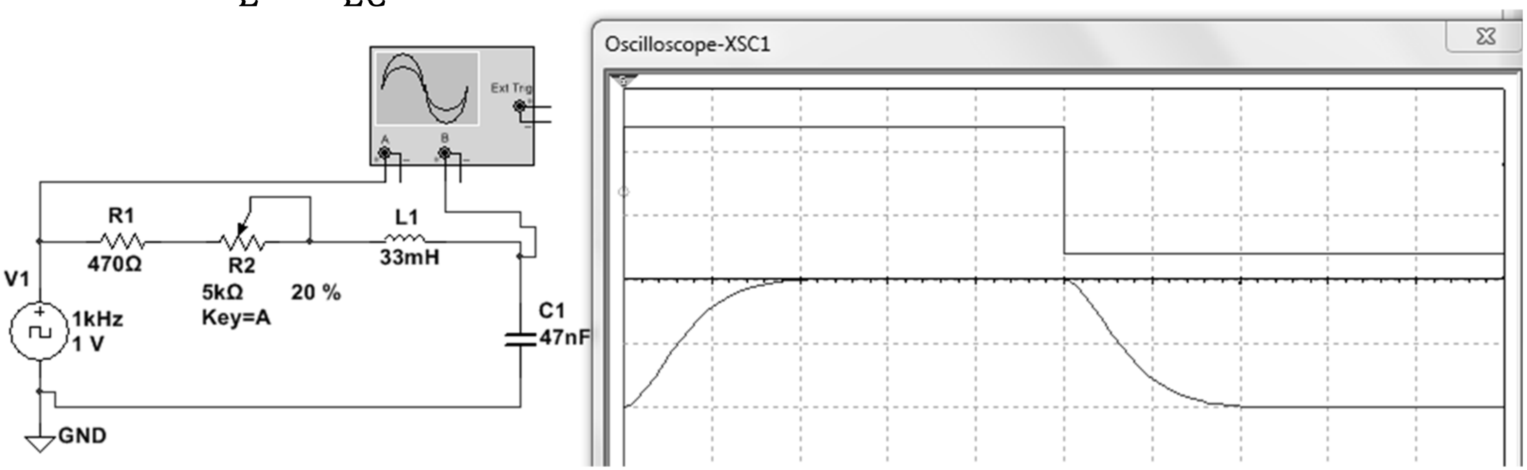
```
clc
clear
s=tf('s')
R1=3.3e3;
R2=1.68e3;
R3=820;
L=33e-3;
C=47e-9;
G_over=(1/L*C)/(s^2+(R1/L)*s+(1/(L*C)))
G_critical=(1/L*C)/(s^2+(R2/L)*s+(1/(L*C)))
G_under=(1/L*C)/(s^2+(R3/L)*s+(1/(L*C)))
ltiview(G_over,G_critical,G_under)
```


645Meg/(s*s+24.8k*s+645Meg)
A1
V1
1kHz
1 V
H(s)
GND
Ext Trg
A
B
Oscilloscope-XSC1
645Meg/(s*s+50.9k*s+645Meg)
A1
V1
1kHz
1 V
H(s)
GND
Ext Trg
A
B
Oscilloscope-XSC1
645Meg/(s*s+100k*s+645Meg)
A1
V1
1kHz
1 V
H(s)
GND
Ext Trg
A
B
Oscilloscope-XSC1

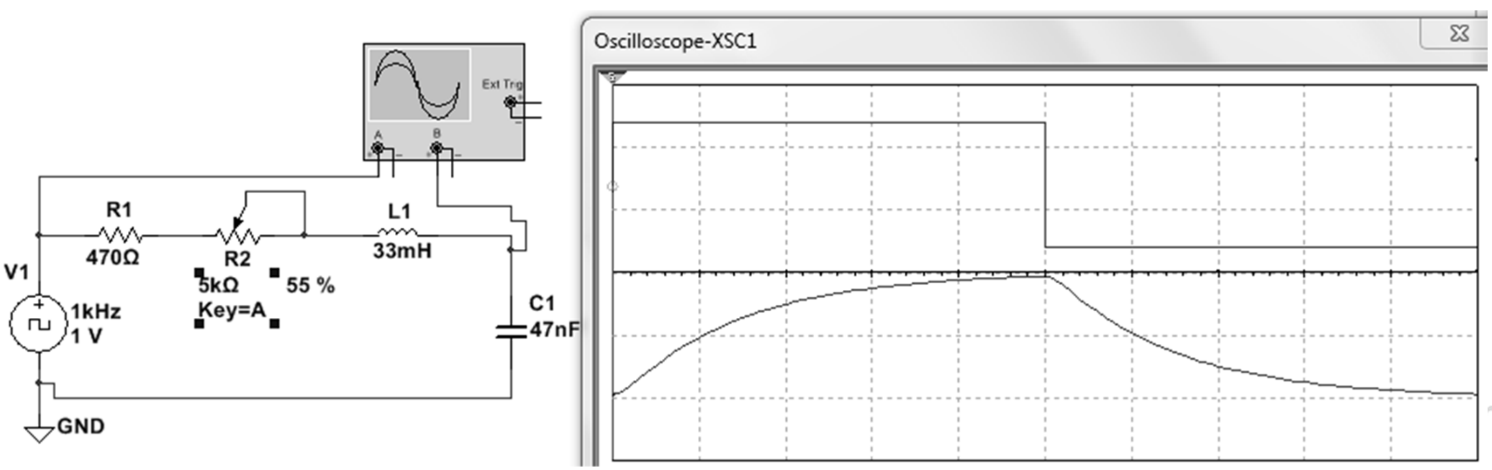
Processing



Oscilloscope-XSC1
R1
470Ω
R2
5kΩ
0 %
Key=A
L1
33mH
V1
1kHz
1 V
C1
47nF
GND



Oscilloscope-XSC1
R1
470Ω
R2
5kΩ
20 %
Key=A
L1
33mH
V1
1kHz
1 V
C1
47nF
GND



Oscilloscope-XSC1
R1
470Ω
R2
5kΩ
55 %
Key=A
L1
33mH
V1
1kHz
1 V
C1
47nF
GND