3.2 Capacitance in the Time Domain

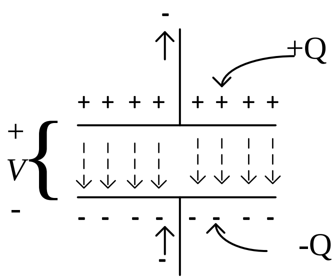


Figure 3-10 Capacitance, charge, electrostatic field, and voltage

Capacitance exists anytime there are two parallel conductors separated by an insulator. Often, this is an intentional component, two plates separated by a film insulator and packaged with an external lead connecting to each plate. This is shown in Figure 3-10. But, any two associated conductors form a capacitance, such as the two traces on the top and bottom of a printed circuit board.

Each electron that arrives at the lower plate forces an electron to leave the upper plate, leaving an equal positive charge there. This charge, *Q*, creates an electrostatic field, shown by the arrows between the plates, storing energy and producing a resulting potential difference, *V*. How much charge and energy that can be stored depends on the size of the plates, how close together they are, and the properties of the insulation. These define the *capacitance*.

$$C = \frac{Q}{V}$$

The more charge (and energy) a capacitor can hold (per volt), the larger its capacitance, (the more water a bucket can hold, the larger it must be).

$$V = \frac{Q}{C}$$

Taking the time derivative of both sides gives

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt}$$

But, current is defined as

$$i = \frac{dQ}{dt}$$

$$\frac{dV}{dt} = \frac{1}{C} i$$

“Ohm’s Law” for capacitors

$$i = C\frac{dV}{dt}$$

Current is the flow of charge onto and off of the capacitor as it charges and discharges. So, even though a charged capacitor looks like an open (to DC), *as* the capacitor charges or discharges, current flows onto and off of it. There is an insulator between the two plates, but *transient* current still flows (onto and off of => through). The more rapidly the voltage across the capacitor *changes* the more quickly charge flows on and off of the capacitor and the higher the resulting current.

Example 3-1

The voltage waveform shown in Figure 3-11 is impressed across a 5 μF capacitor. *Accurately* draw the current flowing into (and out of) the capacitor.

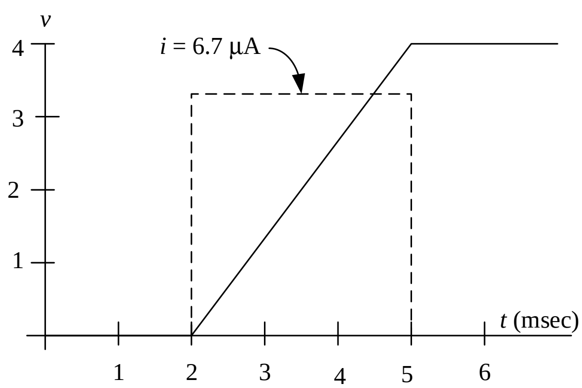


Figure 3-11 Ramp voltage resulting in a pulse of current

Solution

$$i = C\frac{dV}{dt}$$

From t = 0 to 2 msec, the voltage does not *change*.

$$i_{0\text{ to }2\text{ msec}} = 0\text{ A}$$

From t = *2* msec to 5msec, the voltage is a ramp, changing 4 V.

$$i = C\frac{dV}{dt} = 5\ \mu\text{F}\ \frac{4\text{ V}}{5\text{ sec} - 2\text{ sec}} = 6.7\ \mu\text{ A}$$

Since the voltage changes at a steady rate, the current is a constant 5 μA.

For $t > 6$ msec, the voltage does not *change*.

$$i_{t > 6\ \text{msec}} = 0\ \text{A}$$

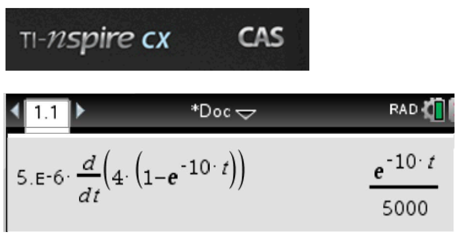
Example 3-2

The voltage across the 5 μF capacitor is changed to an exponential rise

$$v = 4\ \text{V}(1 - e^{-10t})$$

Calculate and plot the equation for the current.

Solution



(a) TI-Nspire derivative

$$i = C\frac{dV}{dt} = 5\ \mu\text{F}\frac{d}{dt}\left(4\ \text{V}(1 - e^{-10t})\right)$$

$$\frac{d}{dt}\left[A\left(1 - e^{\frac{-t}{\tau}}\right)\right] = \frac{A}{\tau}e^{\frac{-t}{\tau}}$$

$$A = 4\ \text{V}$$
$$\tau = 0.1\ \text{sec}$$

$$i = 5\ \mu\text{F}\frac{4\text{V}}{0.1\text{sec}}\text{e}^{-10\text{t}}$$

$$i = 200\ \mu\text{A}\ \text{e}^{-10\text{t}}$$

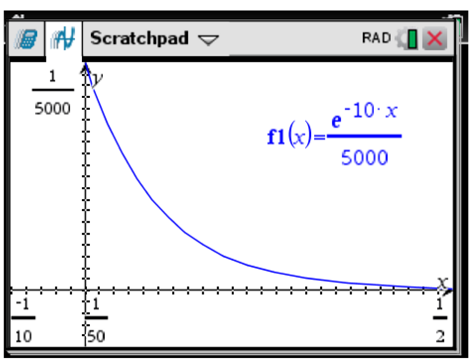


Figure 3-12 (b) Plot of current for Example 3-2

In Figure 3-12(a) the TI-Nspire completed the derivative. The answer was then copied, the calculator mode changed to Graph, and then the function pasted. The graph expects the horizontal axis to be *x*, not *t*, so that variable was changed. Finally the scaling was adjusted to show the full plot, *and* a little of the 0,0 origin.

Driving an exponentially increasing voltage across a capacitor produces a *spike* of current that falls to 0 when the voltage becomes constant at its maximum value.

Given the voltage, the current can be calculated.

$$i = C\frac{dV}{dt}$$

But, if current is driven into the capacitor, how is the voltage calculated?

$$i\,dt = C\,dv$$

$$dv = \frac{1}{C}i\,dt$$

$$v = \frac{1}{C}\int idt$$

To evaluate this integral requires limits.

$$v = \frac{1}{C}\int_{-\infty}^{t} idt = \frac{1}{C}\left(\int_{-\infty}^{0} idt + \int_{0}^{t} idt\right)$$

Integrating from the beginning of time (t = -∞) could be a problem. So, that part of the integral is separated. Whatever has happened before the beginning of the calculation (t = 0) is just the initial charge on the capacitor, V_o however it got there.

$$v = \frac{1}{C}\int_0^t i\,dt + V_o$$

Given i_{cap}, find v_{cap}.

Example 3-3

Given an exponential *spike* of current into a 5 μF capacitor with:

i_{max} = 1.5 A
τ = 100 μsec
V_o = -10 V

determine the equation for the voltage across the capacitor and plot it.

Solution

$$i = Ae^{-\frac{t}{\tau}} = 1.5\text{ A e}^{-\frac{t}{100\,\mu\text{sec}}}$$

$$v = \frac{1}{C}\int_0^t i\,dt + V_o = \frac{1}{5\,\mu F}\int_0^t 1.5\text{ A e}^{-\frac{t}{100\,\mu\text{sec}}}\text{dt} - 10\text{ V}$$

From the preceding section:

$$\int_0^t \left(Ae^{\frac{-t}{\tau}}\right)dt = A\tau\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$v = \frac{1}{5\,\mu F}\int_0^t 1.5\text{ A e}^{-\frac{t}{100\,\mu\text{sec}}} - 10\,V = \frac{1.5\,A}{5\,\mu F} \times 100\,\mu sec\left(1 - e^{-\frac{t}{100\,\mu sec}}\right) - 10\,V$$

$$v = 30\,V\left(1 - e^{-\frac{t}{100\,\mu sec}}\right) - 10V$$

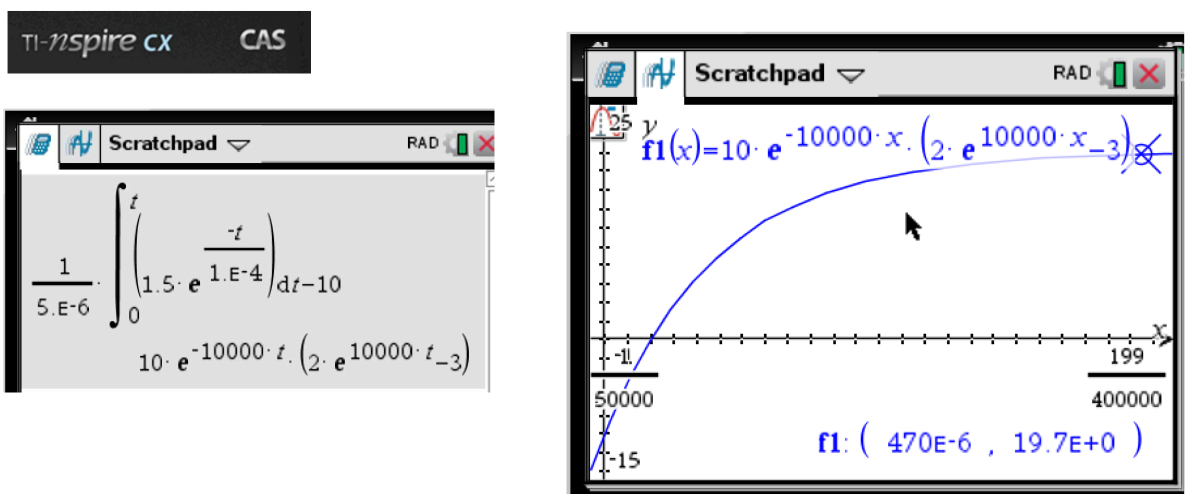


Figure 3-13 Calculator integration and plot

Although the calculator's answer to the integration seems different than the manual solution, a little algebra will show that the two results agree. The plot starts at V_o = -10 V, and in 5 time constants (500 µsec) rises exponentially to 20 V, a total rise of 30 V.