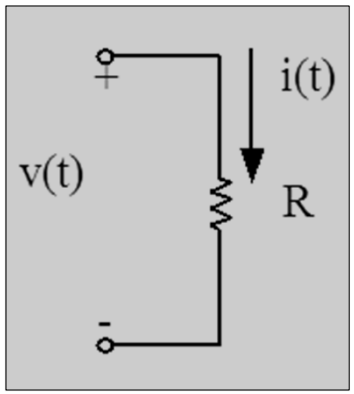
RL Laplace

- Laplace Impedances
- First Order RL
  - Schematic
  - Gain (Transfer Function)
  - Response to step (manual, MATLAB, Multisim)

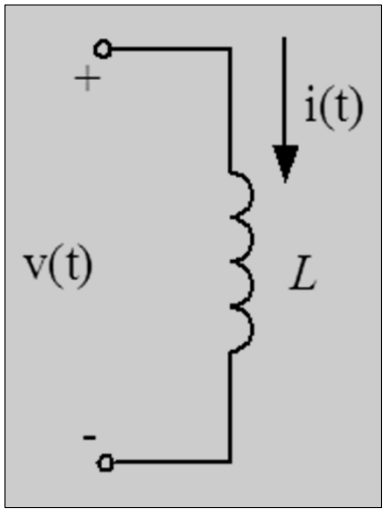
Resistor's Laplace Impedance



$$v_R(t) =$$

$$Z_R(s) =$$

Inductor's Laplace Impedance



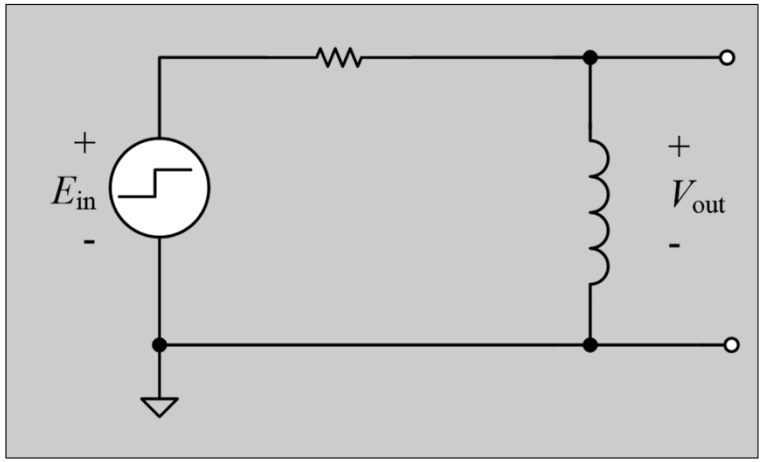
$v_L(t) =$

TABLE A-1 *(continued)*

No.	$F(S)$	
7.	$\frac{As}{s^2+\omega^2}$	$A\cos\omega t$
8.	$aF(s)$	$af(t)$
9.	$\frac{n!}{s^{n+1}}$	t^n
10.	$sF(s)-f(0)$	$\frac{df(t)}{dt}$
11.	$s^2F(s)-sf(0)-\frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$
12.	$\frac{F(s)}{s}$	$\int f(t)\,dt$
13a.	$\frac{A}{\tau s+1}$	$\frac{A}{\tau}e^{-t/\tau}$
13b.	$\frac{A}{s+a}$	Ae^{-at}
14a.	$\frac{A}{}$	$\frac{A}{}$

$Z_L(s) =$

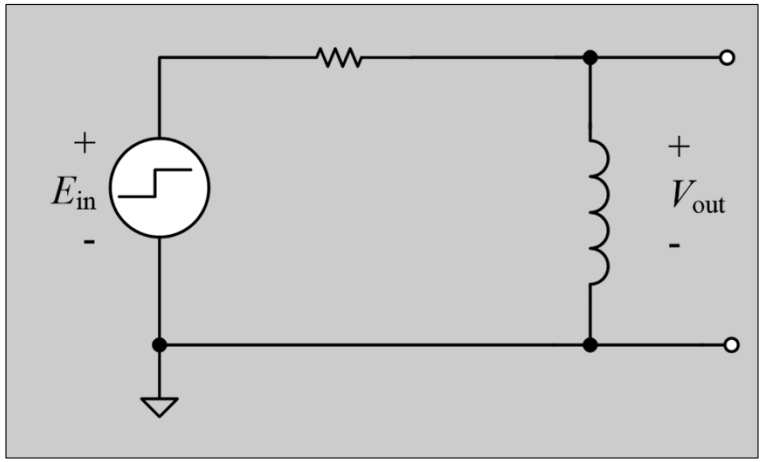
RL Circuit – Transfer Function



$$V_{out}(s) =$$

$$\frac{V_{out}(s)}{E_{in}(s)} =$$

RL Circuit – Response to 5 V step



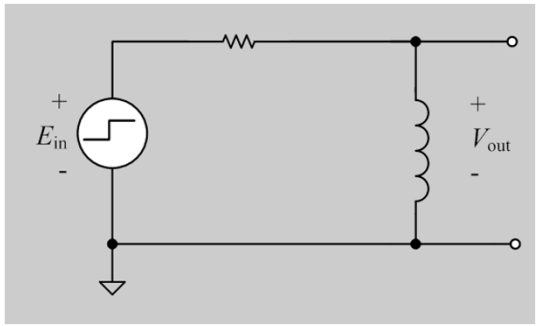
$$\frac{V_{out}(s)}{E_{in}(s)} =$$

$$V_{out} =$$

$$E_{in\ step} =$$

TABLE A-1 LAPLACE TRANSFORMS

No.	$F(S)$	$f(t)$	Comments
1.	1	$\delta(t)$	Unit impulse
2.	$\frac{A}{s}$	$A(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$	Step
3.	$\frac{1}{s}$	$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$	Unit step
4.	$\frac{A}{s^2}$	At	Ramp
5.	$\frac{2A}{s^3}$	At^2	Parabola
6.	$\frac{A\omega}{s^2 + \omega^2}$	$A \sin \omega t$	Sine



RL Circuit – Response to 5 V step

$$V_{out}(s) =$$

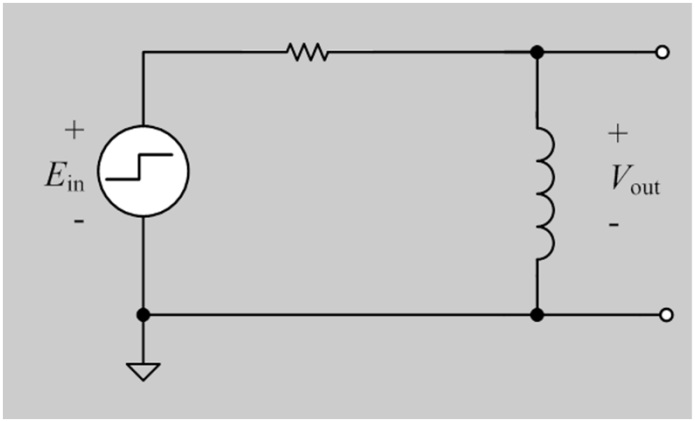
TABLE A-1 *(continued)*

No.	$F(S)$	$f(t)$	Comments
7.	$\frac{As}{s^2+\omega^2}$	$A\cos\omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s)-f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s)-sf(0)-\frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t)\,dt$	
13a.	$\frac{A}{\tau s+1}$	$\frac{A}{\tau}e^{-t/\tau}$	Free response of first-order system
13b.	$\frac{A}{s+a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{A}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$	Free response of second-order system ($\xi>1$)
14b.	$\frac{A}{(s+a)(s+b)}$	$\frac{A}{b-a}(e^{-at}-e^{-bt})$	
15a.	$\frac{A}{(\tau s+1)^2}$	$\frac{At}{\tau^2}e^{-t/\tau}$	Free response of second-order system ($\xi=1$)
15b.	$\frac{A}{(s+a)^2}$	Ate^{-at}	
16.	$\frac{A\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t)$	Second-order system, free response ($\xi<1$)
17a.	$\frac{A}{s(\tau s+1)}$	$A(1-e^{-t/\tau})$	First-order system response to a step input
17b.	$\frac{A}{s(s+a)}$	$\frac{A}{a}(1-e^{-at})$	
18a.	$\frac{A}{s^2(\tau s+1)}$	$A\tau\left(e^{-t/\tau}+\frac{t}{\tau}-1\right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s+a)}$	$\frac{A}{a^2}(e^{-at}+at-1)$	

$$V_{out}(t) =$$

$$V_{out}(t=0+) =$$

$$V_{out}(t\to\infty) =$$



RL Circuit – Response to 5 V step

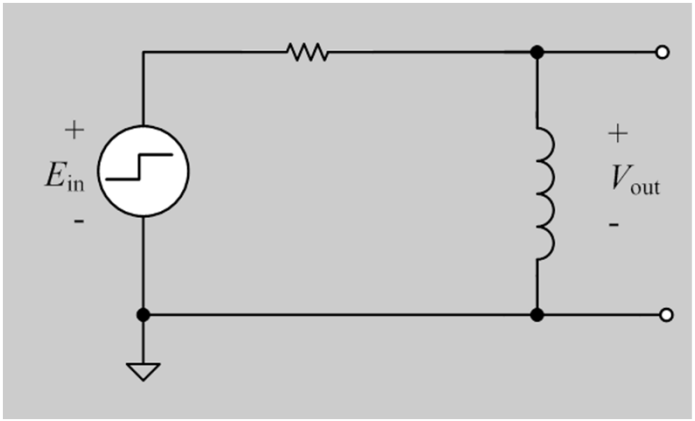
$$V_{out}(s) =$$

$$V_{out}(t) =$$

```
clc
clear

s=tf('s')
R=
L=
tau=
Ein=


G=
Vout=

ltiview(Vout)
```

TABLE A-1 LAPLACE TRANSFORMS

No.	$F(S)$	$f(t)$	Comments
1.	1	$\delta(t)$	Unit impulse
2.	$\frac{A}{s}$	$A(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$	Step
3.	$\frac{1}{s}$	$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$	Unit step

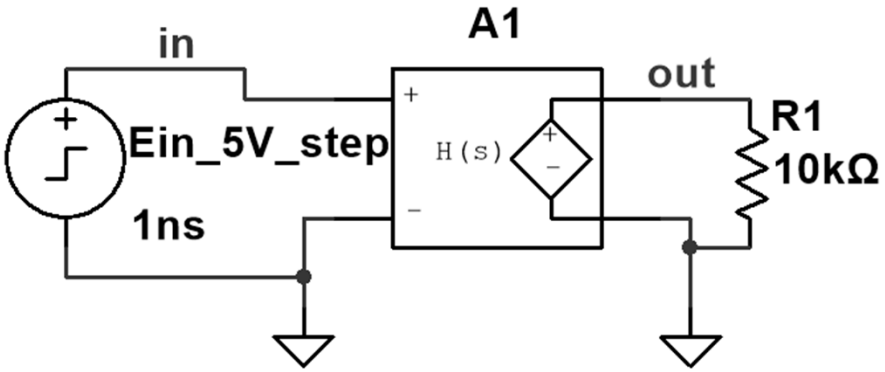


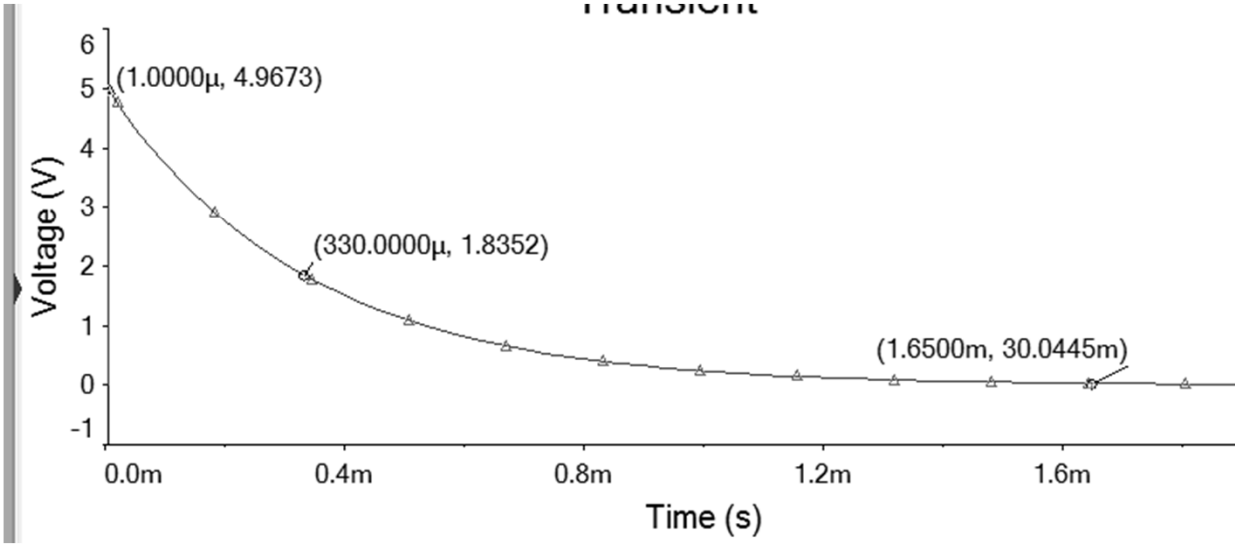
RL Circuit – Response to 5 V step

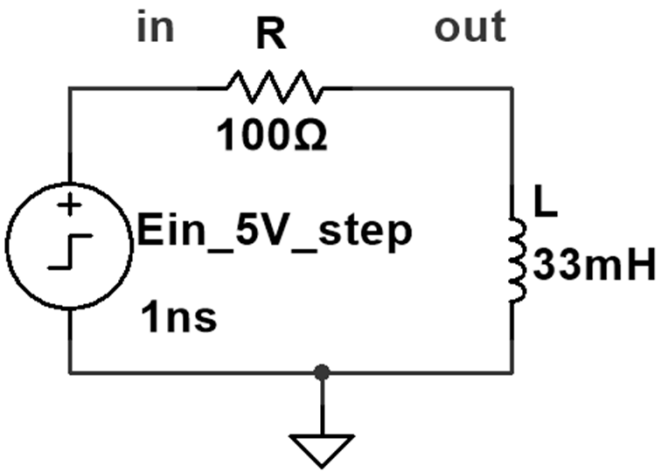
R = _____ L = _____

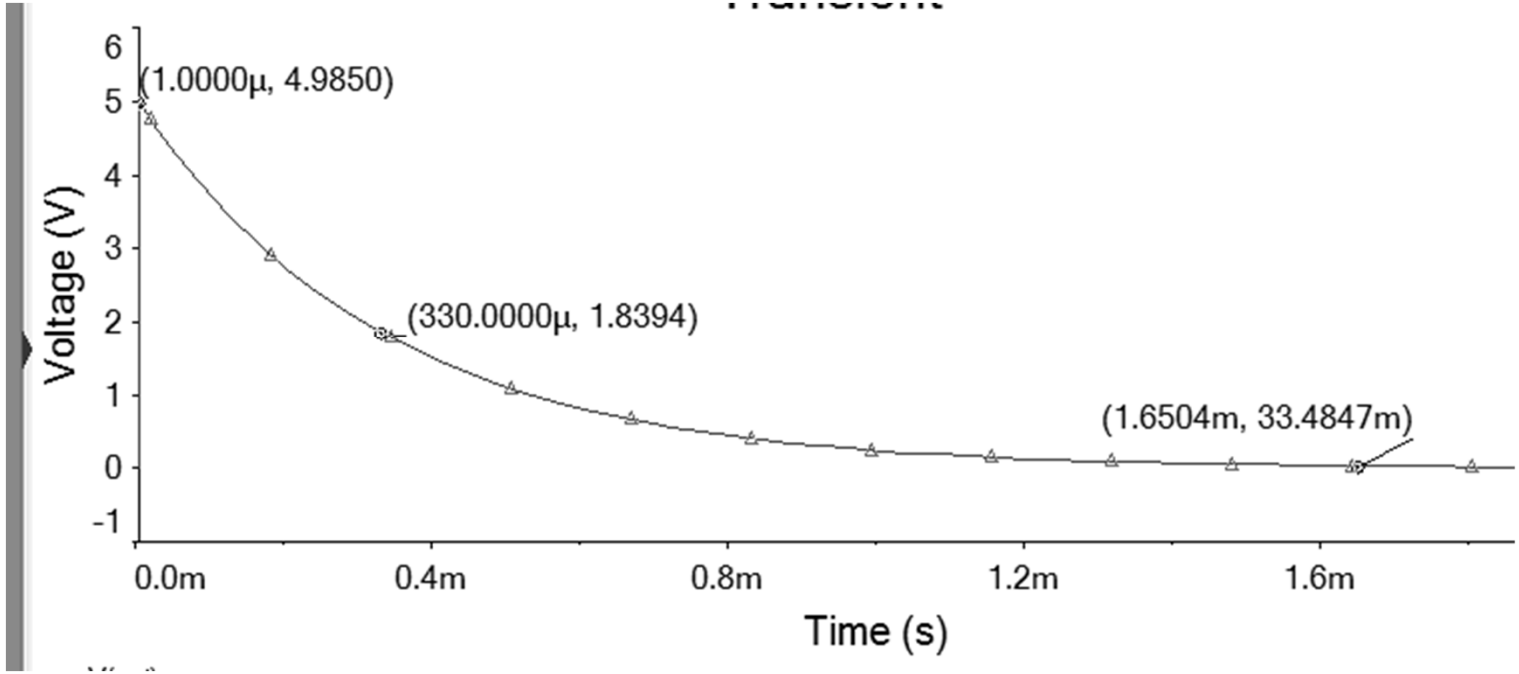
τ = _____ $\frac{V_{out}(s)}{E_{in}(s)}$ = _____

330e-6*s/(330e-6*s+1) $\frac{V_{out}(s)}{E_{in}(s)} = \frac{1}{\tau s+1}$






in
R
out
100Ω
Ein_5V_step
1ns
L
33mH


Transient
(1.0000µ, 4.9850)
(330.0000µ, 1.8394)
(1.6504m, 33.4847m)
Voltage (V)
6
5
4
3
2
1
0
-1
0.0m
0.4m
0.8m
1.2m
1.6m
Time (s)

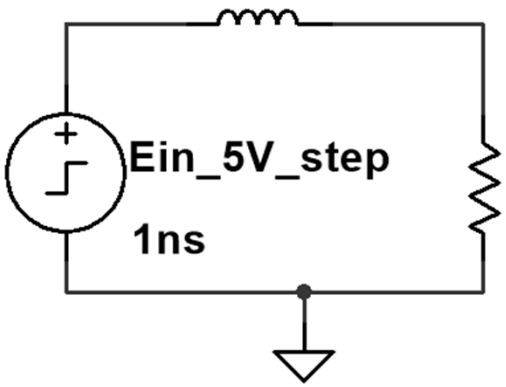


TABLE A-1 *(continued)*

No.	$F(S)$	$f(t)$	Comments
7.	$\frac{As}{s^2+\omega^2}$	$A\cos\omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s)-f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s)-sf(0)-\frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t)\,dt$	
13a.	$\frac{A}{\tau s+1}$	$\frac{A}{\tau}e^{-t/\tau}$	Free response of first-order system
13b.	$\frac{A}{s+a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{A}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$	Free response of second-order system ($\zeta>1$)
14b.	$\frac{A}{(s+a)(s+b)}$	$\frac{A}{b-a}(e^{-at}-e^{-bt})$	
15a.	$\frac{A}{(\tau s+1)^2}$	$\frac{At}{\tau^2}e^{-t/\tau}$	Free response of second-order system ($\zeta=1$)
15b.	$\frac{A}{(s+a)^2}$	Ate^{-at}	
16.	$\frac{A\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t)$	Second-order system, free response ($\zeta<1$)
17a.	$\frac{A}{s(\tau s+1)}$	$A(1-e^{-t/\tau})$	First-order system response to a step input
17b.	$\frac{A}{s(s+a)}$	$\frac{A}{a}(1-e^{-at})$	
18a.	$\frac{A}{s^2(\tau s+1)}$	$A\tau\left(e^{-t/\tau}+\frac{t}{\tau}-1\right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s+a)}$	$\frac{A}{a^2}(e^{-at}+at-1)$	

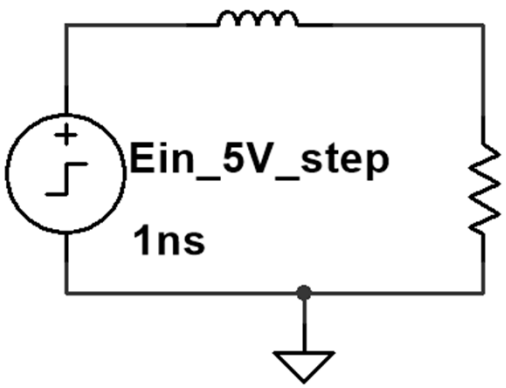
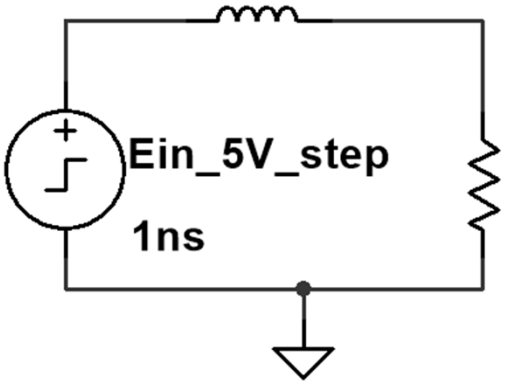


TABLE A-1 *(continued)*

No.	$F(S)$	$f(t)$	Comments
7.	$\frac{As}{s^2+\omega^2}$	$A\cos\omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s)-f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s)-sf(0)-\frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t)\,dt$	
13a.	$\frac{A}{\tau s+1}$	$\frac{A}{\tau}e^{-t/\tau}$	Free response of first-order system
13b.	$\frac{A}{s+a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{A}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$	Free response of second-order system ($\zeta>1$)
14b.	$\frac{A}{(s+a)(s+b)}$	$\frac{A}{b-a}(e^{-at}-e^{-bt})$	
15a.	$\frac{A}{(\tau s+1)^2}$	$\frac{At}{\tau^2}e^{-t/\tau}$	Free response of second-order system ($\zeta=1$)
15b.	$\frac{A}{(s+a)^2}$	Ate^{-at}	
16.	$\frac{A\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t)$	Second-order system, free response ($\zeta<1$)
17a.	$\frac{A}{s(\tau s+1)}$	$A(1-e^{-t/\tau})$	First-order system response to a step input
17b.	$\frac{A}{s(s+a)}$	$\frac{A}{a}(1-e^{-at})$	
18a.	$\frac{A}{s^2(\tau s+1)}$	$A\tau\left(e^{-t/\tau}+\frac{t}{\tau}-1\right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s+a)}$	$\frac{A}{a^2}(e^{-at}+at-1)$	

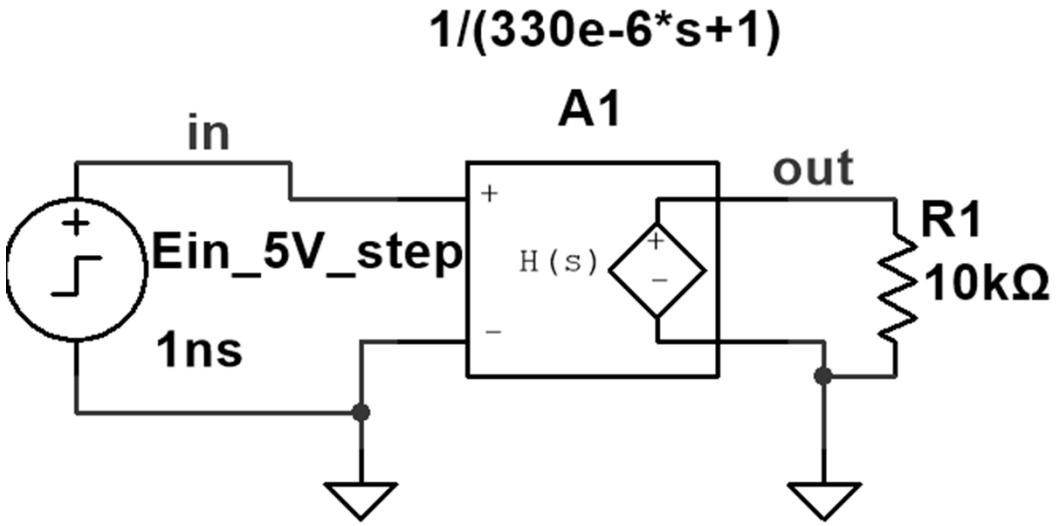


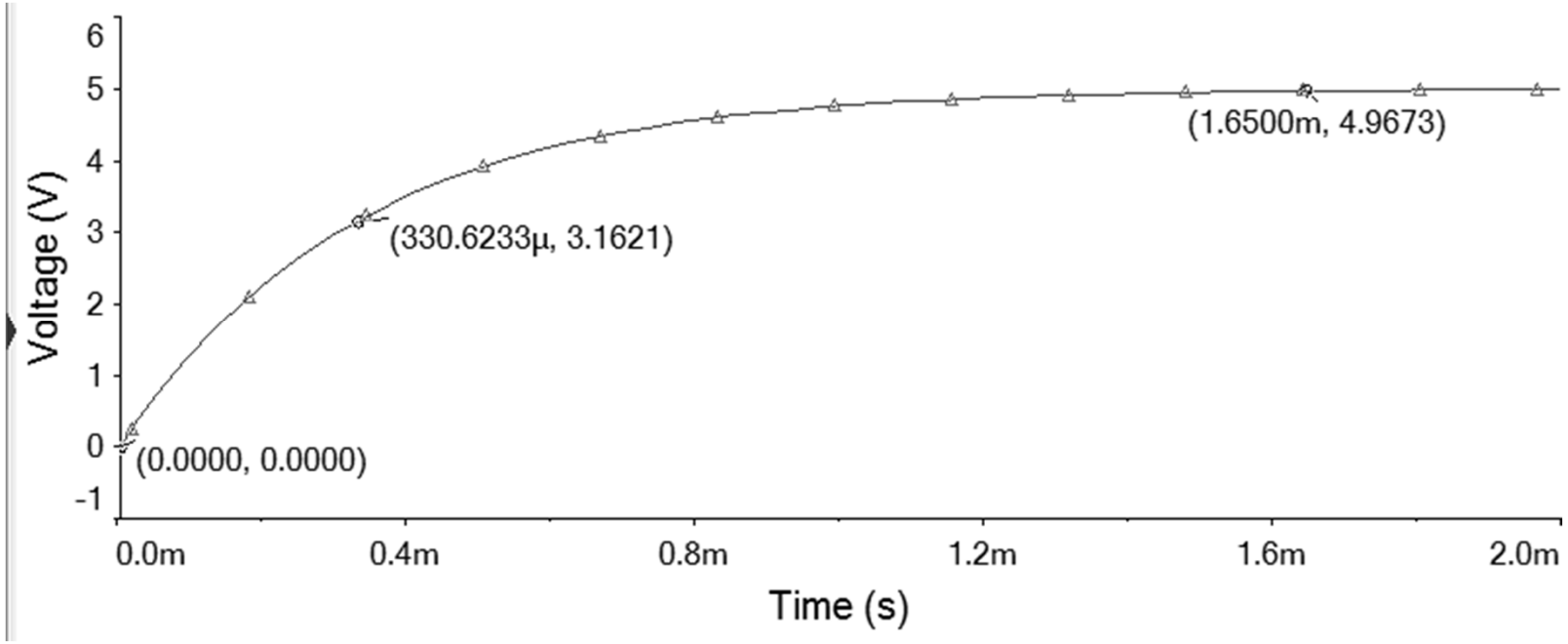
LR Circuit – Response to 5 V step
R=100 Ω, L=33 mH

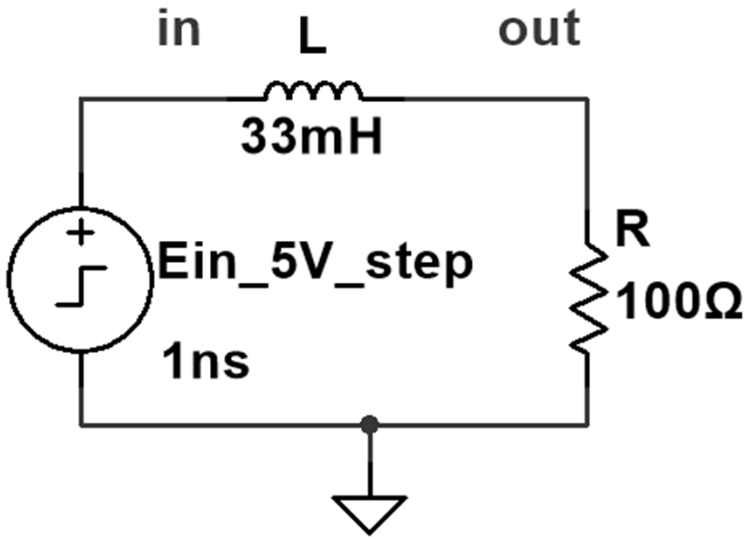
$V_{\text{out}}(s) =$

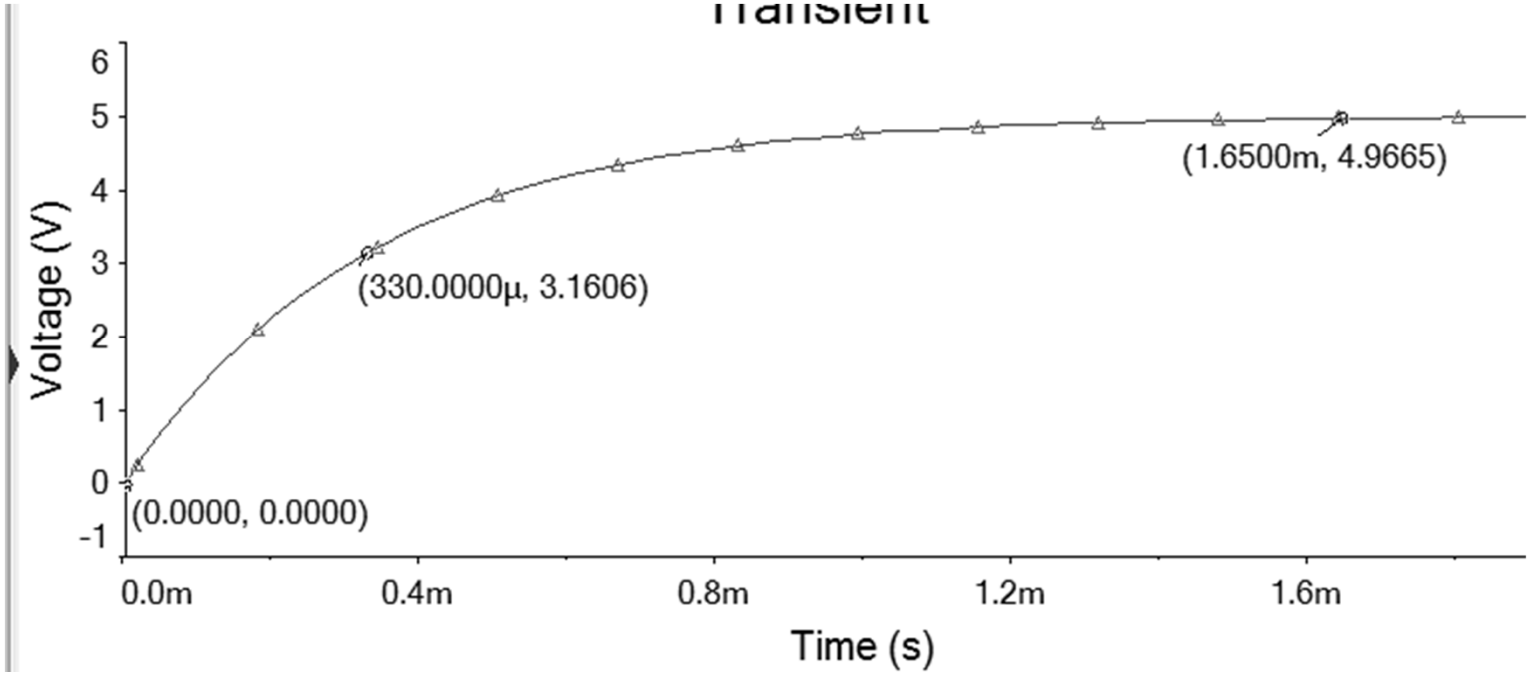
$V_{\text{out}}(t) =$

```
clc
clear

s=tf('s')
R=
L=
tau=
Ein=


G=
Vout=

ltiview(Vout)
```


1/(330e-6*s+1)
A1
in
out
Ein_5V_step
1ns
H(s)
R1
10kΩ


Voltage (V)
Time (s)
(0.0000, 0.0000)
(330.6233µ, 3.1621)
(1.6500m, 4.9673)
6
5
4
3
2
1
0
-1
0.0m
0.4m
0.8m
1.2m
1.6m
2.0m


in
L
out
33mH
Ein_5V_step
1ns
R
100Ω


Transient
Voltage (V)
6
5
4
3
2
1
0
-1
(1.6500m, 4.9665)
(330.0000µ, 3.1606)
(0.0000, 0.0000)
0.0m
0.4m
0.8m
1.2m
1.6m
Time (s)

Transform Definition => calculus => tables!

TABLE A-1 LAPLACE TRANSFORMS

No.	$F(S)$	$f(t)$	Comments
1.	1	$\delta(t)$	Unit impulse
2.	$\frac{A}{s}$	$A(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$	Step
3.	$\frac{1}{s}$	$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$	Unit step
4.	$\frac{A}{s^2}$	At	Ramp
5.	$\frac{2A}{s^3}$	At^2	Parabola
6.	$\frac{A\omega}{s^2+\omega^2}$	$A \sin \omega t$	Sine

TABLE A-1 *(continued)*

No.	$F(S)$	$f(t)$	Comments
7.	$\frac{As}{s^2+\omega^2}$	$A \cos \omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s) - f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t)\,dt$	
13a.	$\frac{A}{\tau s+1}$	$\frac{A}{\tau}e^{-t/\tau}$	Free response of first-order system
13b.	$\frac{A}{s+a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{A}{\tau_1-\tau_2}(e^{-t/\tau_1} - e^{-t/\tau_2})$	Free response of second-order system ($\zeta > 1$)
14b.	$\frac{A}{(s+a)(s+b)}$	$\frac{A}{b-a}(e^{-at} - e^{-bt})$	
15a.	$\frac{A}{(\tau s+1)^2}$	$\frac{At}{\tau^2}e^{-t/\tau}$	Free response of second-order system ($\zeta = 1$)
15b.	$\frac{A}{(s+a)^2}$	Ate^{-at}	
16.	$\frac{A\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n\sqrt{1-\zeta^2}\,t)$	Second-order system, free response ($\zeta < 1$)
17a.	$\frac{A}{s(\tau s+1)}$	$A(1 - e^{-t/\tau})$	First-order system response to a step input
17b.	$\frac{A}{s(s+a)}$	$\frac{A}{a}(1 - e^{-at})$	
18a.	$\frac{A}{s^2(\tau s+1)}$	$A\tau\left(e^{-t/\tau} + \frac{t}{\tau} - 1\right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s+a)}$	$\frac{A}{a^2}(e^{-at} + at - 1)$	

TABLE A-1 *(continued)*

No.	$F(S)$	$f(t)$	Comments
19a.	$\frac{A\omega}{(s^2+\omega^2)(\tau s+1)}$	$\frac{A\omega\tau}{1+\omega^2\tau^2}e^{-t/\tau} + \frac{A}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t - \psi)$ where $\psi = \tan^{-1}\omega\tau \quad (0 < \psi < \pi)$	First-order system response to a sine input
19b.	$\frac{A\omega}{(s^2+\omega^2)(s+a)}$	$\frac{A\omega e^{-at}}{\omega^2+a^2} + \frac{A}{\sqrt{\omega^2+a^2}}\sin(\omega t - \psi)$ where $\psi = \tan^{-1}\omega/a \quad (0 < \psi < \pi)$	
20a.	$\frac{A}{s(\tau_1 s+1)(\tau_2 s+1)}$	$A\left(1 + \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_2 - \tau_1}\right)$	Second-order system response to a step input ($\zeta > 1$)
20b.	$\frac{A}{s(s+a)(s+b)}$	$\frac{A}{ab}\left(1 + \frac{ae^{-bt} - be^{-at}}{b-a}\right)$	
21a.	$\frac{A}{s(\tau s+1)^2}$	$A\left(1 - \frac{\tau+t}{\tau}e^{-t/\tau}\right)$	Second-order system response to a step input ($\zeta = 1$)
21b.	$\frac{A}{s(s+a)^2}$	$\frac{A}{a^2}[1 - (1+at)e^{-at}]$	
22.	$\frac{A\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$A\left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t - \psi)\right]$ where $\psi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} \quad (0 < \psi < \pi)$	Second-order system response to a step input ($\zeta < 1$)
23a.	$\frac{A}{s^2(\tau_1 s+1)(\tau_2 s+1)}$	$A\left(t - \tau_1 - \tau_2 - \frac{\tau_1^2 e^{-t/\tau_1} - \tau_2^2 e^{-t/\tau_2}}{\tau_1 - \tau_2}\right)$	Second-order system response to a ramp input ($\zeta > 1$)
23b.	$\frac{A}{s^2(s+a)(s+b)}$	$\frac{A}{ab}\left[t - \frac{a+b}{ab} - \frac{(b/a)e^{-bt} - (a/b)e^{-at}}{b-a}\right]$	
24a.	$\frac{A}{s^2(\tau s+1)^2}$	$A[t - 2\tau + (t+2\tau)e^{-t/\tau}]$	Second-order system response to a ramp input ($\zeta = 1$)
24b.	$\frac{A}{s^2(s+a)^2}$	$\frac{A}{a^2}\left[t - \frac{2}{a} + \left(t + \frac{2}{a}\right)e^{-at}\right]$	
25.	$\frac{A\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$A\left[t - \frac{2\zeta}{\omega_n} + \frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t - \psi)\right]$ where $\psi = 2\tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} \quad (0 < \psi < \pi)$	Second-order system response to a ramp input ($\zeta < 1$)

Source: Floyd E. Nixon, *Handbook of Laplace Transformation: Fundamentals, Applications, Tables and Examples*, 21e, © 1965, Referenced and adapted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Transform Operations

$$f'(t) \qquad sF(s) - f(0)$$

$$\int_0^t f(t)\,dt \qquad \frac{F(s)}{s}$$

UNKNOWN

↓	R	C	L
$i(t)$	$\frac{v(t)}{R}$	$C \cdot \frac{dv(t)}{dt}$	$\frac{1}{L}\int_0^t v(t)dt + I_o$
$v(t)$	$R \cdot i(t)$	$\frac{1}{C}\int_0^t i(t)dt + V_o$	$L \cdot \frac{di(t)}{dt}$