

Figure 3-6 Exponential rise with amplitude (A), and time constant (τ) defined

$$v(t) = A \left(1 - e^{\frac{-t}{\tau}} \right)$$

$$\frac{d}{dt} \left[A \left(1 - e^{\frac{-t}{\tau}} \right) \right] = \frac{A}{\tau} e^{\frac{-t}{\tau}}$$

$$\int_0^t \left[A \left(1 - e^{\frac{-t}{\tau}} \right) \right] dt = A \left[t - \tau \left(1 - e^{\frac{-t}{\tau}} \right) \right]$$

$$\frac{d}{dt} \left(A \cdot \left(1 - e^{\frac{-t}{\tau}} \right) \right) = \frac{A \cdot e^{\frac{-t}{\tau}}}{\tau}$$

$$\int_0^t \left(A \cdot \left(1 - e^{\frac{-t}{\tau}} \right) \right) dt = A \cdot \left((t - \tau) \cdot e^{\frac{-t}{\tau}} + \tau \right) \cdot e^{\frac{-t}{\tau}}$$

Figure 3-7 TI-Nspire derivative and integral of an exponential rise

$$v(t) = Ae^{\frac{-t}{\tau}}$$

$$\frac{d}{dt}\left(Ae^{\frac{-t}{\tau}}\right) = \frac{-A}{\tau}e^{\frac{-t}{\tau}}$$

$$\int_0^t \left(Ae^{\frac{-t}{\tau}}\right) dt = A\tau \left(1 - e^{\frac{-t}{\tau}}\right)$$

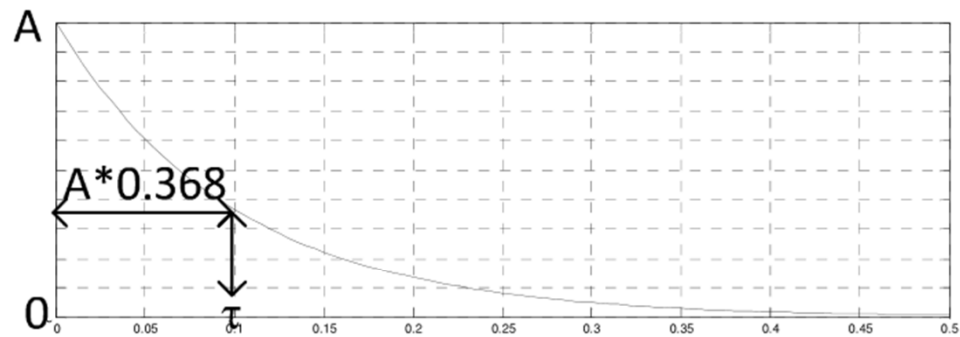


Figure 3-8 Exponential fall with amplitude (A), and time constant (τ) defined

$$\frac{d}{dt} \left(a \cdot e^{\frac{-t}{\tau}} \right) = \frac{-a \cdot e^{\frac{-t}{\tau}}}{\tau}$$

$$\int_0^t \left(a \cdot e^{\frac{-t}{\tau}} \right) dt = a \cdot \tau \cdot \left(e^{\frac{t}{\tau}} - 1 \right) \cdot e^{\frac{-t}{\tau}}$$

Figure 3-9 TI-Nspire derivative and integral of an exponential fall or spike

Ohm's Law for Capacitors

Resistors

$$i = \frac{v}{R}$$

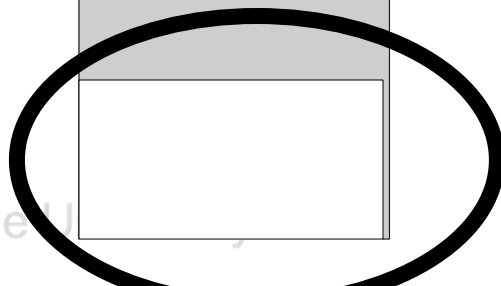
$$i = \frac{dQ}{dt}$$

$$C = \frac{Q}{V}$$

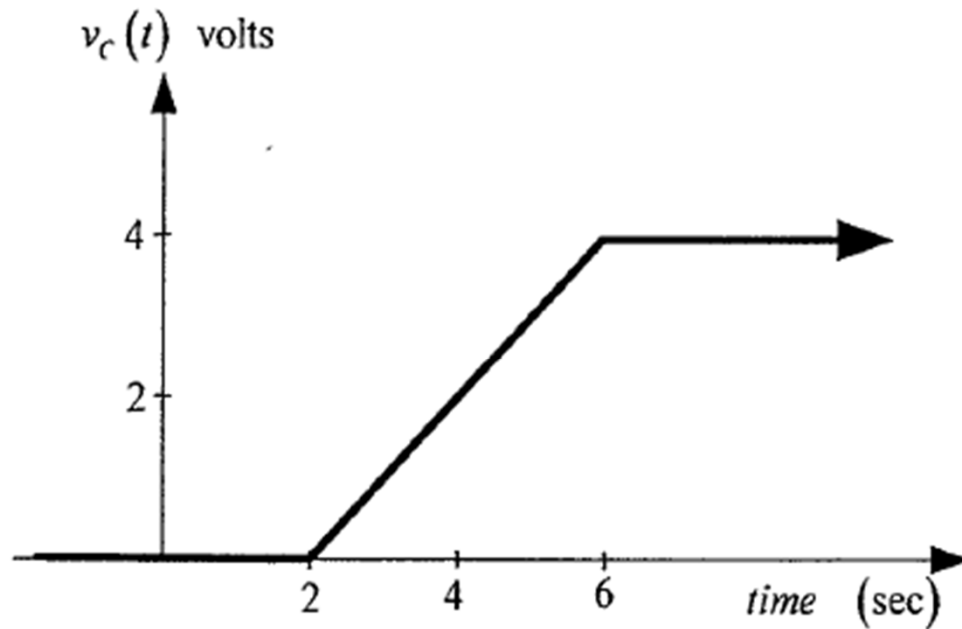
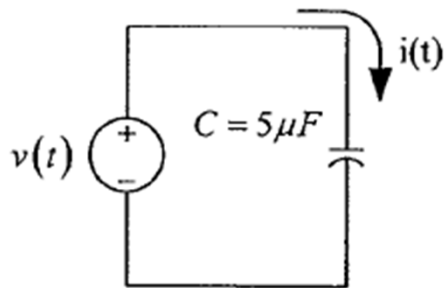
$$V = \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{1}{C} \frac{dQ}{dt}$$

$$\frac{dV}{dt} = \frac{1}{C} i$$



$$i = C * dv/dt$$



$$i = C * dv/dt$$

The voltage across $5 \mu\text{F}$ capacitor is given by the following equation:

$$v(t) = 4(1 - e^{-10t}) \quad \text{for } t > 0$$

Determine an expression for the current $i(t)$.

$$i = C * dv/dt$$

$$v = ?$$

$$v(t) = \frac{1}{C} \int_0^t i(t) dt + V_0$$

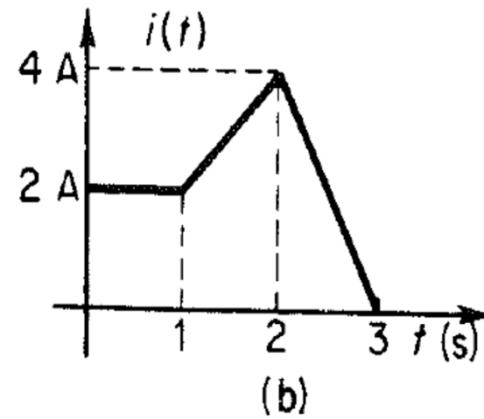
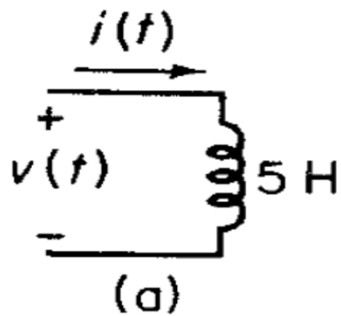
$$i(t) = (1.5)e^{-t/(1 \times 10^{-4})}$$

$$V_0 = -10V$$

Inductor Analysis in the Time Domain

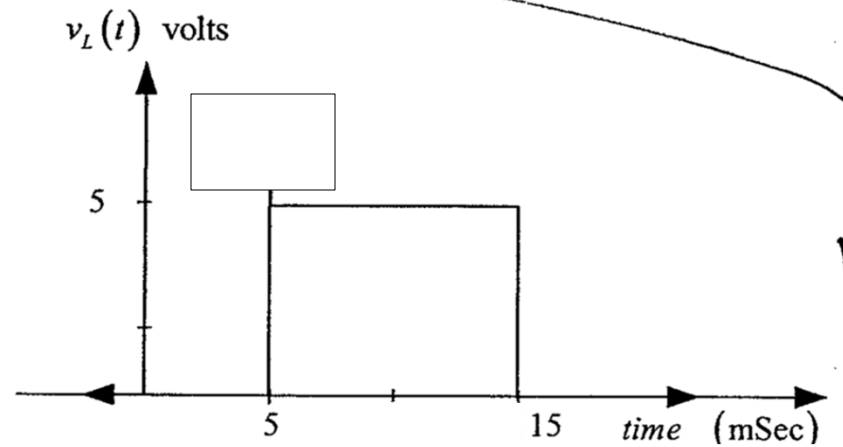
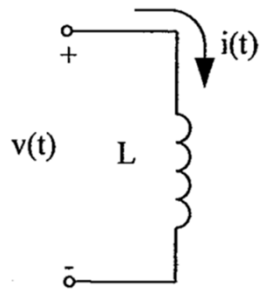
- $v = L \frac{di}{dt}$
- $i_L \Rightarrow v$
Graphical
Calculus
- $v_L \Rightarrow I$
Graphical
Calculus
- Summary

$$v = L * di/dt$$



$$i(t) = \frac{1}{L} \int_0^t v(t) dt + I_0$$

Graphical



$$L = 100 \text{ mH}$$

Example 3-6

Given a voltage ramp across an 80 mH inductor with:

$$I_o = 0 \text{ A (ramp starts at 0 and goes up)}$$

$$v_{\max} = 5 \text{ V}$$

$$f = 100 \text{ Hz}$$

determine the equation for the current through the inductor and plot it.

Summary – Circuit laws

UN KNOWN

\Downarrow	R	C	L
$i(t)$	$\frac{v(t)}{R}$	$C \cdot \frac{dv(t)}{dt}$	$\frac{1}{L} \int_0^t v(t) dt + I_o$
$v(t)$	$R \cdot i(t)$	$\frac{1}{C} \int_0^t i(t) dt + V_o$	$L \cdot \frac{di(t)}{dt}$