6

Filter Terminology

Introduction

Filters are circuits that pass certain signals while rejecting others, depending on the signals' frequencies. These circuits find wide applications in communications, sensor processing (your bathroom scale, furnace thermostat, car's engine), and industrial power distribution.

Each filter amplifies or attenuates a signal's voltage, as set by its components' impedances, like the series and parallel circuits you have already studied. But the *results* of manipulating these voltages, currents, and impedances are specified by a set of terms that are unique to filters.

These parameters all center around a plot of the gain (magnitude and phase) of a circuit on the vertical axis, versus the signal's frequency on the horizontal axis. This is the **frequency response plot**. In the first several sections of this chapter, you will learn to identify filter types and performance from that plot, then to scale its horizontal axis logarithmically and its vertical axis in decibels.

How steeply the filter transitions from passing one frequency to blocking another is defined by the **roll-off rate**, which in turn is established by the filter's **order**. The **pass band gain** is the key vertical axis specification, while **critical frequency** and the **half-power point** scale the horizontal axis. These are defined in the chapter's later sections.

Objectives

Upon completion of this chapter, you will be able to do the following:

- Identify low-pass, high-pass, and band-pass responses.
- Properly draw and logarithmically scale a frequency response plot.
- Convert between ratio and a variety of decibel parameters.
- Define and locate on a frequency response plot:

    Pass band gain, critical frequency, half-power point,

    filter order, roll-off rate

6.1 The Frequency Response Plot and Filter Types

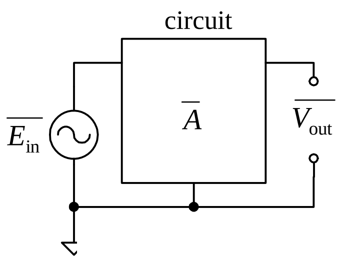


Figure 6-1
Gain definition

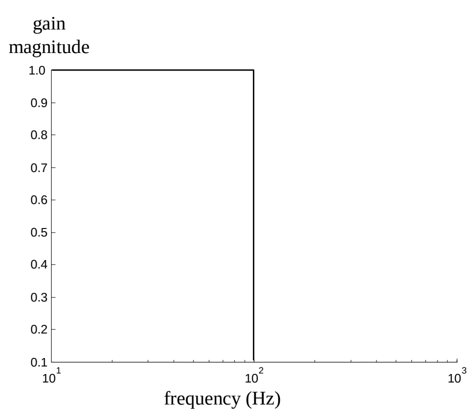
The **gain** of a circuit is the ratio of the output parameter to the input parameter. Often these parameters are voltage. Look at Figure 6-1.

$$\overline{A} = \frac{\overline{V}_{out}}{\overline{E}_{in}}$$

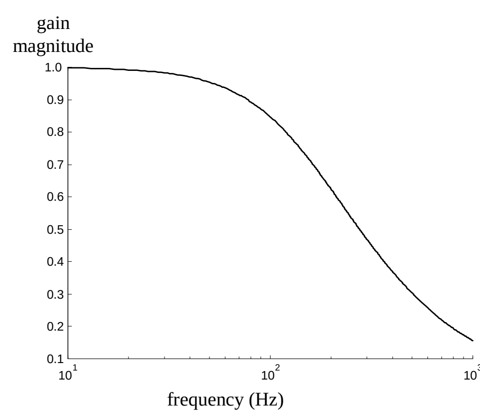
There are two important observations. First, all quantities are *phasors*. The gain indicates how much larger (amplification) or smaller (attenuation) the signal becomes as it passes through the circuit. It also shows how much the signal's phase is shifted, lead or lag. Secondly, when the parameters are input and output voltage, the magnitude of the gain is unitless, that is volts/volts.

Filters provide different gain to signals of different frequencies. Some frequency signals are passed with amplification and little phase shift, while other frequency signals may be severely attenuated with significant phase shift. It is common to illustrate this with a frequency response plot, usually with gain magnitude on the vertical axis and frequency on the horizontal. Look at Figure 6-2.

These are the responses of a low-pass filter. The ideal response,



(a) Ideal low-pass response



(b) Practical low-pass response

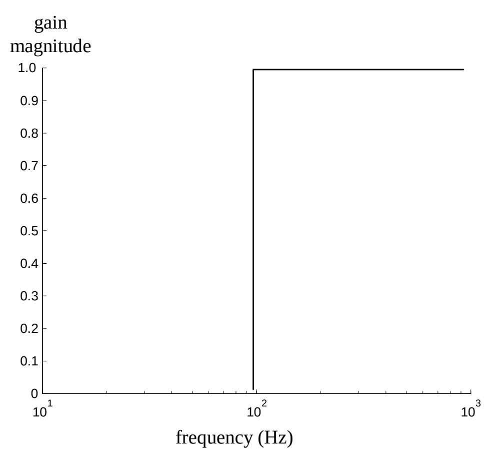
Figure 6-2 Low-pass filter frequency response

Figure 6-2 (a), indicates that signals at or below about 10^2 Hz are passed with a gain magnitude of 1.0. That is, the output is the same as the input. Above 10^2 Hz, none of the input is passed to the output.

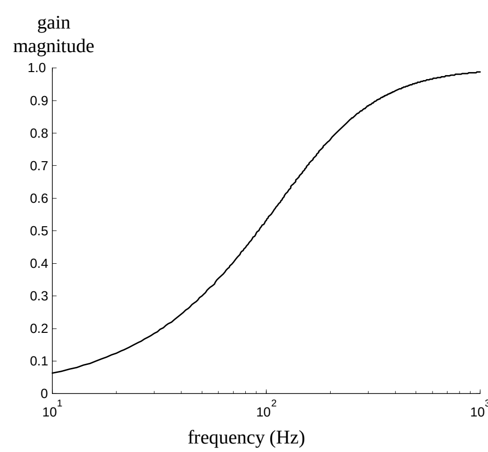
A more practical response is shown in Figure 6-2 (b). As the frequency goes up, the gain falls. Low-frequency signals are passed with little loss. But higher frequency signals are severely attenuated.

Low-pass filters are used to pass the bass signals to woofer loudspeakers, while attenuating the high pitches (to which the woofer cannot respond). They are also used extensively when measuring physical parameters, such as weight, pressure, or temperature. These signals vary at the rate of the physical world that produces them. It takes several seconds for the oil pressure or engine temperature in your car to change noticeably. However, these signals must exist next to the spark plug pulse of hundreds of volts that occurs in a few microseconds. Low-pass filters assure that interference from the high-frequency spark plugs pulses, or radio, or microprocessors are attenuated, while the slow variations from the sensors are passed on to the instrumentation and control cluster.

Figure 6-3 shows the frequency response of a high-pass filter. Low frequencies are attenuated while signals at higher frequencies are passed. This filter passes the high pitches of an audio signal to the tweeter loudspeaker, while blocking the low tones that should go only to the woofer. You may have also seen these filters as the coupler between amplifier



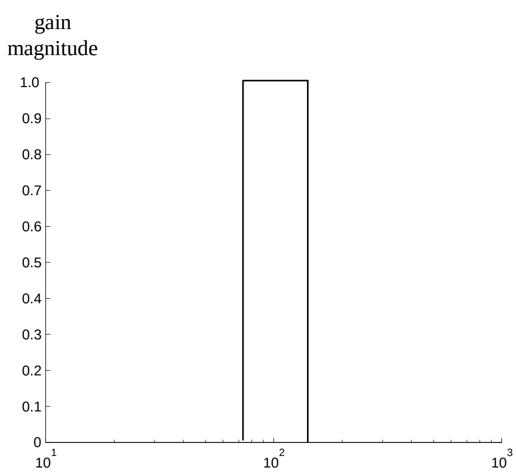
(a) Ideal high-pass response



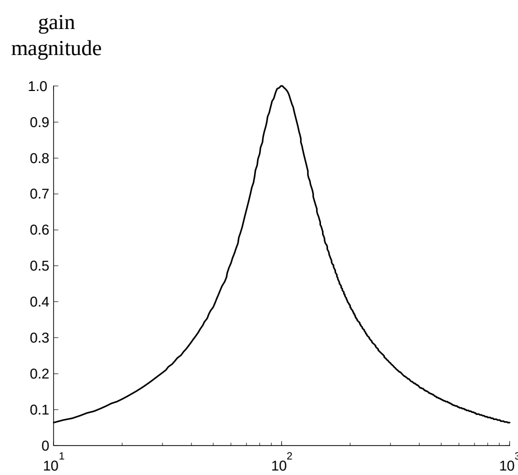
(b) Practical high-pass response

Figure 6-3 High-pass filter frequency response stages, or whenever you ac couple the input channel of an oscilloscope. The dc bias voltage (0 Hz) is blocked, while the ac signal passes.

Properly configured, the low-pass and high-pass filters combine to form the band-pass filter shown in Figure 6-4. Signals below or above the selected band are rejected. Only those within the chosen band are passed with little attenuation. The band-pass filter used in a communications receiver is very sharp, selecting one signal at 100 MHz, while rejecting another adjacent station only 0.3 MHz away. However, the mid-range audio band-pass filter may pass signals from 200 Hz to 2 kHz.



(a) Ideal band-pass response



(b) Practical band-pass response

Figure 6-4 Bandpass filter frequency response

6.2 The Frequency Response Plot's Horizontal Axis

Look carefully at the horizontal axis of Figure 6-4. The scale is *not* linear. It is logarithmic. Equal intervals are assigned to decade increases in frequency. On a linear scale the divisions would have gone up by integers, 1, 2, 3. However, on this logarithmic scale, the divisions are decades, 10^1, 10^2, 10^3; that is 10, 100, 1000.

Scaling the horizontal axis of a frequency response plot logarithmically allows a wide range of signals to be displayed without giving the higher tones a disproportionately large part of the axis.

Logarithmically scaled plots have three key characteristics. Look at Figure 6-5. First, equal distances advance the frequency by a factor of 10. This is called a **decade**. In Figure 6-5, the major divisions go from 10, to 100, to 1000, to 10,000, to 100,000. Secondly, there is no zero. That is because the log 0 is undefined. (There is no exponent of 10 that will result in 0.) If you move a major division further to the left the frequency drops by 0.1, from 10 to 1, then to 0.1, then to 0.01, and so on.

Finally, the minor divisions are *not* linearly placed. They bunch-up as you get closer to the next decade. You will see some graphs that only have every other minor division. Between 10 and 100 there may be only 4 ticks. These are at 20, 40, 60, and 80, even though the spacing may seem unusual.

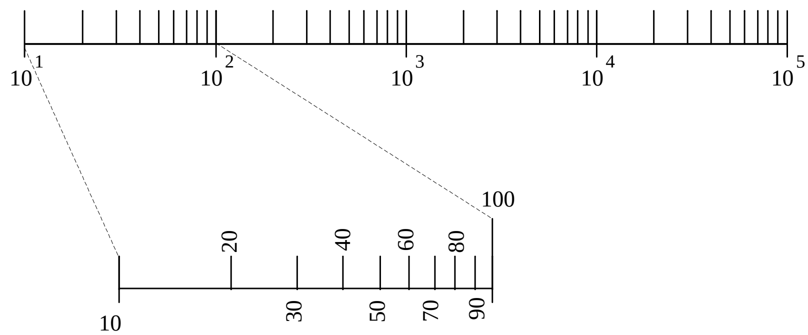


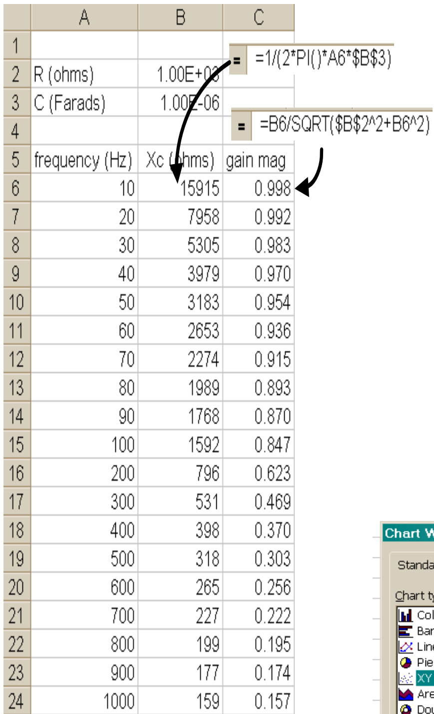
Figure 6-5 Logarithmic horizontal scale

Example 6-1

The low-pass filter frequency response shown in Figure 6-2 was generated from the set of equations : $R = 1\text{k}\Omega \qquad C = 1\mu\text{F}$

$$X_C = \frac{1}{2\pi f C} \qquad |A| = \frac{X_C}{\sqrt{R^2 + X_C^{\,2}}}$$

Plot this gain magnitude versus frequency, from 10 Hz to 1 kHz, with the horizontal axis scaled logarithmically. Use a spreadsheet and MATLAB.



	A	B	C
1			
2	R (ohms)	1.00E+03	
3	C (Farads)	1.00E-06	
4			
5	frequency (Hz)	Xc (ohms)	gain mag
6	10	15915	0.998
7	20	7958	0.992
8	30	5305	0.983
9	40	3979	0.970
10	50	3183	0.954
11	60	2653	0.936
12	70	2274	0.915
13	80	1989	0.893
14	90	1768	0.870
15	100	1592	0.847
16	200	796	0.623
17	300	531	0.469
18	400	398	0.370
19	500	318	0.303
20	600	265	0.256
21	700	227	0.222
22	800	199	0.195
23	900	177	0.174
24	1000	159	0.157

Solution

The spreadsheet is shown in Figure 6-6. The frequency cells increase in steps of 10 to 100, then in steps of 100 to 1000.

The values for the resistor and for the capacitor are placed in their own cells at the top of the sheet. This way, if you want to see the effect of changing the resistor or capacitor, you change one cell, and the sheet and graph are automatically updated.

The capacitive reactance is calculated in the second column. The equation is shown in the insert. Once this cell is correctly calculated, just pull it down to fill the rest of the column.

The last column calculates the gain magnitude. The formula is also shown in the insert. The resulting calculation is then pulled down to fill the third column. The series of steps needed to create the logarithmic plot are shown in Figure 6-7.

Figure 6-6 Spreadsheet for Example 6-1

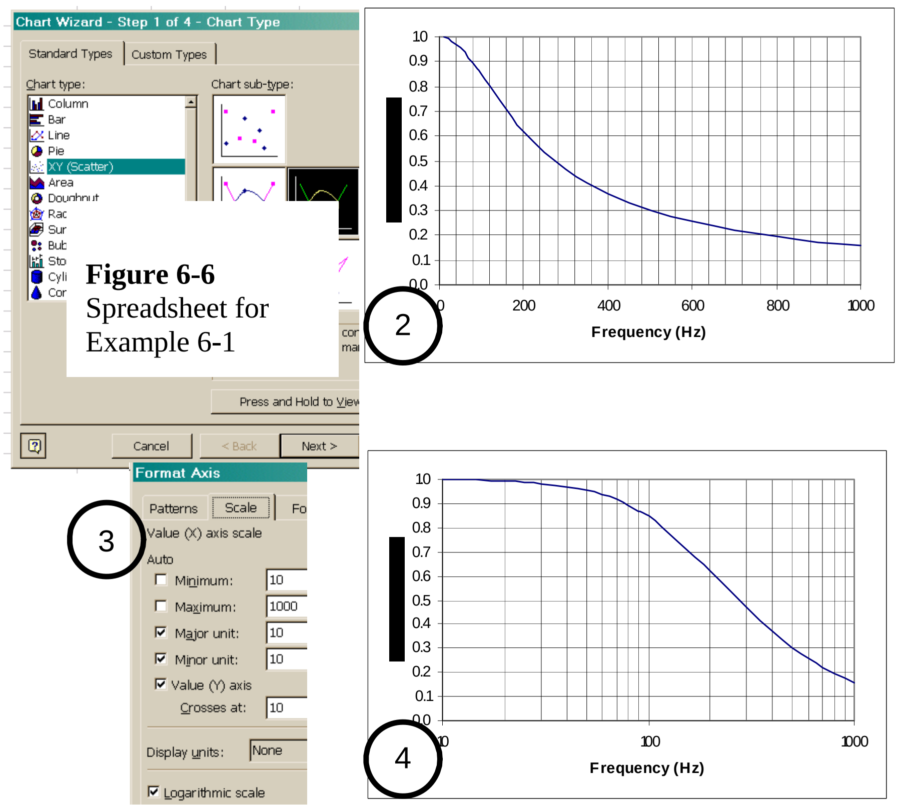
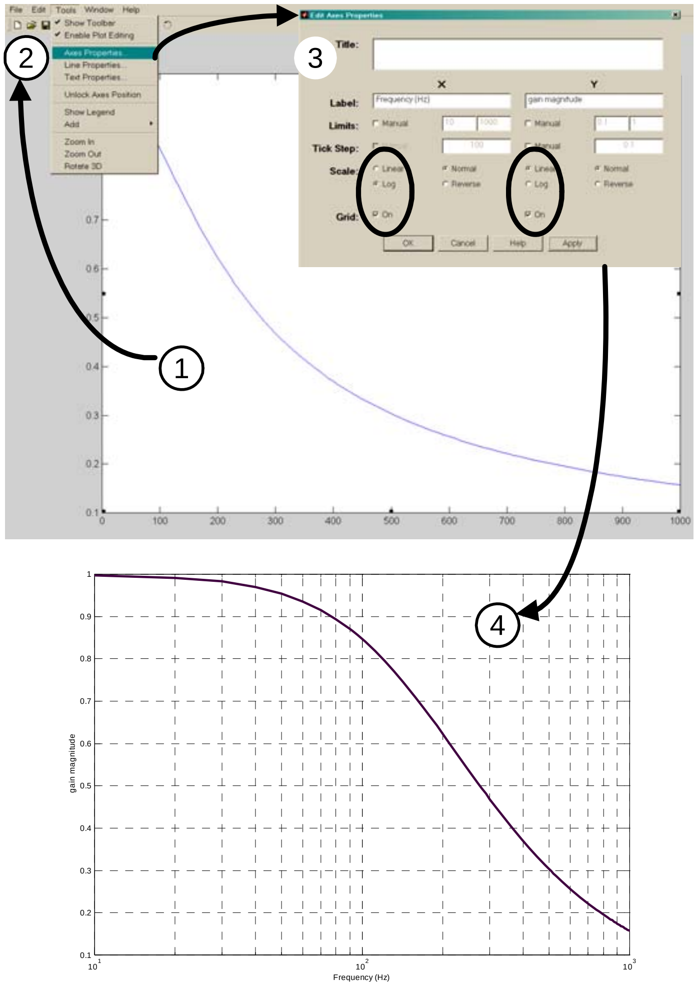


Figure 6-7 A spread-sheet-based logarithmically scaled plot




```
» f=(10:10:1e3);
» R=1e3;
» C=1e-6;
»
» Xc=1./(2*pi*f*C);
» gain=Xc./sqrt(R.^2+Xc.^2);
»
» plot(f,gain)
```

1. **When the plot command is executed, the linear plot is created in a separate window.**

2. **From the top menu bar, select <u>Tools</u>.**

   Then select <u>Axis Properties</u>.

3. **In the Axis Properties dialog box,**

   enter appropriate labels
   check the X axis Scale Log button
   check the Grid On box

4. **The *X* axis is scaled logarithmically**

Figure 6-8 MATLAB-based logarithmically scaled plot

Practice: Plot the frequency response for the following filter with 10 Hz $\le f \le$ 1 kHz. Use both a spreadsheet and MATLAB.

$$R = 1\ \text{k}\Omega, \qquad C = 1\ \mu\text{F}, \qquad |A| = \frac{R}{\sqrt{R^2 + X_C{}^2}}$$

Answers: See Figure 6-3.

6.3 The Frequency Response Plot's Vertical Axis

The frequency response plot's horizontal axis is scaled logarithmically to display a wide range of frequencies, and to accommodate the way we hear. For these same reasons, special units scale the *vertical* axis too.

There has been considerable study into the relationship between the amplitude of the signal sent to a telephone's loudspeaker and the perceived volume. As with many other human senses, it is *logarithmic*, and depends on the *power* sent to the loudspeaker, not its voltage. For that reason, the **bel** (in honor of Alexander Graham Bell) was defined.

$$bel = B = \log \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}$$

Notice that the bel is a measure of power *gain*. It is the log of a ratio.

It is common for a circuit to produce a power gain that is between 0.2 and 5. Over this region of operation, the resulting gain in bels is a fraction. Especially before the arrival of pocket calculators, this was inconvenient. So, the decibel became the traditional unit.

$$dB = 10 \log \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}$$

Key points you should notice about the decibel include:

- A decade (factor of 10) increase or decrease in the ratio produces a linear increase or decrease in dB.
- A ratio gain of 1 means the input and output power are the same. The dB gain is 0. Nothing is added or subtracted from the gain.
- Attenuation, where the output power is smaller than the input power, is expressed as a *negative* dB gain.
- Increasing the gain by 100 adds 20 dB, while attenuating the gain by $\frac{1}{100}$ subtracts 20 dB (i.e., –20 dB).
- Doubling the output power is expressed as a 3 dB gain, while cutting it in half results in –3 dB. You will see this particular point on the frequency response plot repeatedly.

Voltage provided at the input and delivered to the load is much easier to measure than power. Gain in dB can be expressed in terms of input and output *voltage*.

$$dB = 10\log\frac{P_{out}}{P_{in}} \qquad P_{out} = \frac{(V_{load})^2}{R_{load}} \qquad P_{in} = \frac{(E_{in})^2}{R_{in}}$$

$$dB = 10\log\frac{\dfrac{(V_{load})^2}{R_{load}}}{\dfrac{(E_{in})^2}{R_{in}}}$$

$$dB = 10\log\left(\frac{(V_{load})^2}{R_{load}} \times \frac{R_{in}}{(E_{in})^2}\right)$$

$$dB = 10\log\left(\left(\frac{V_{load}}{E_{in}}\right)^2 \times \frac{R_{in}}{R_{load}}\right)$$

In balanced systems, $R_{in} = R_{load}$. Communications systems impedances are forced to 50 Ω or 75 Ω, while the original telephone and audio systems set all impedances to 600 Ω. In these two cases the resistances cancel, and the ratio becomes considerably simpler. Even for most amplifiers and filters, it is common to ignore the ratio of the resistances.

$$dB = 10\log\left(\left(\frac{V_{load}}{E_{in}}\right)^2\right)$$

$$\log(x^2) = 2\log x$$

$$dB = 20\log\left(\frac{V_{load}}{E_{in}}\right)$$

dB gain in terms of voltages

Example 6-2

Complete the following calculations.

a. Given: P_{in} = 100 mW, P_{out} = 25 W
Calculate: gain dB.

b. Given: V_{out} = 50 mV$_{rms}$, gain = –45 dB

Calculate: E_{in}.

Solution

a.

$$dB = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

$$dB = 10\log\left(\frac{25\,\text{W}}{100\,\text{mW}}\right) = 23.98\,\text{dB}$$

b. Voltage calculation requires that the equation be rearranged.

$$dB = 20\log\left(\frac{V_{out}}{E_{in}}\right)$$

$$\frac{dB}{20} = \log\left(\frac{V_{out}}{E_{in}}\right)$$

$$10^{\frac{dB}{20}} = \frac{V_{out}}{E_{in}}$$

$$E_{in} = \frac{V_{out}}{10^{\frac{dB}{20}}} = \frac{50\,\text{mV}_{rms}}{10^{\frac{-45\,\text{dB}}{20}}} = 8.89\,\text{V}_{rms}$$

The proper function is

log10

```
» dB=20*log10(50e-3/8.89)
dB =
     -44.999
```

Practice:	**a.** E_{in} = 3 V_{rms}, V_{out} = 62 mV_{rms}	Calculate the dB gain.
	b. E_{in} = 1.5 V_{rms}, dB = –28 dB	Calculate V_{out}.
Answers:	**a.** dB = –33.69 dB	**b.** V_{out} = 59.72 mV_{rms}.

The decibel is a measure of *gain*, the log of the *ratio* of output compared to input. The dB gain tells you the output level only if you know the level of the input. However, it is convenient to express level in dB. To do this, a reference (input) must be specified.

dBV: **voltage level**
1 V_{rms} reference

$$dBV = 20\log\left(\frac{V}{1\,\text{V}_{rms}}\right)$$

Often audio systems describe signal level in dBu.

dBu: **voltage level**
0.775 V_{rms} reference

$$dBu = 20\log\left(\frac{V}{0.775\,\text{V}_{rms}}\right)$$

This reference level, 0.775 V_{rms}, was established because that is the voltage necessary to deliver 1 mW to a 600 Ω load.

$$P = \frac{(V_{rms})^2}{R}$$

$$P = \frac{(0.775\,\mathrm{V_{rms}})^2}{600\,\Omega} = 1.00\,\mathrm{mW}$$

When expressing power levels, rather than rms voltage, dB can be used as well. The dBm compares the measured power to 1 mW into a 600 Ω load. This is usually used for signal processing equipment.

$$dBm = 10\log\left(\frac{P}{1\,\mathrm{mW}}\right)\bigg|_{R=600\,\Omega}$$

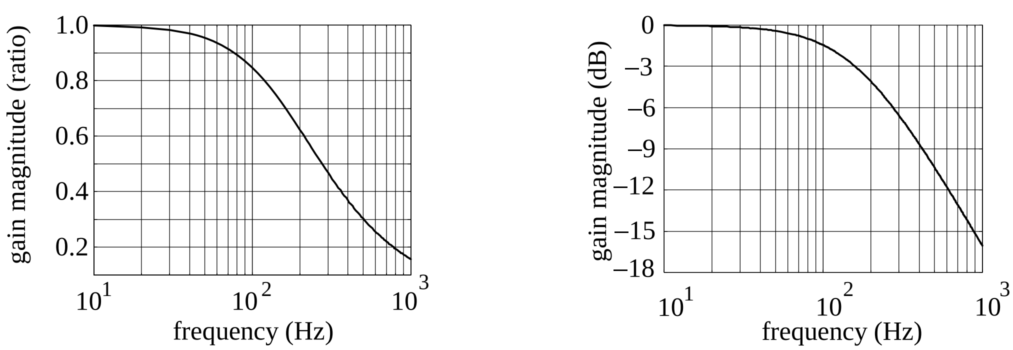
dBm: **power level 1 mW reference**

The dBW uses 1 W as the reference, more appropriate for amplifiers.

$$dBW = 10\log\left(\frac{P}{1\mathrm{W}}\right)$$

dBW: **power level 1 W reference**

The frequency response plots shown in the previous figures have scaled their vertical axis in *ratio* as shown again in Figure 6-9 (a). It is the standard, however, to scale the vertical axis of a frequency response plot in *dB*. This is shown in Figure 6-9 (b).



(a) Y axis: ratio (b) Y axis: decibels

Figure 6-9 Frequency response plots: vertical axis comparison

The **pass band gain**, $\mathbf{A_o}$, is the largest gain at the extreme of the filter's operation. For a low-pass filter, the pass band gain is determined at dc. For a high-pass filter, it is measured at the circuit's highest operating frequency. The band-pass filter peaks somewhere in the middle frequencies. That is where A_o is measured. Figure 6-10 illustrates these.

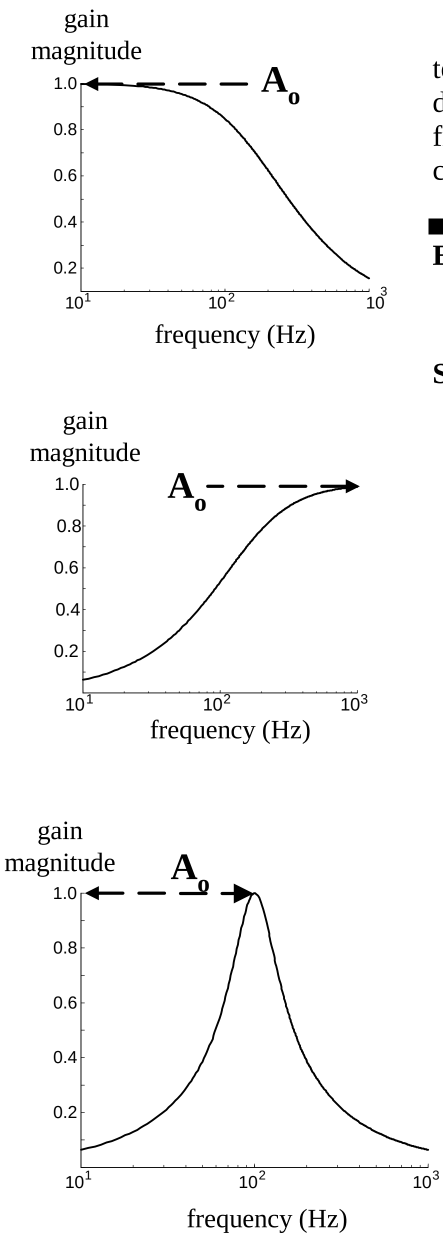


Figure 6-10
Pass band gain for several filters

Example 6-3

Use simulation software to plot the frequency response of the filter in Figure 6-11 and to determine its pass band gain.

Solution

Once you have drawn the schematic, next verify that the dc bias voltages are correct. The dc voltages on the noninverting input pin (pin 12), the inverting input pin (pin 13) and the output pin (pin 14) should all be 2.5 V_{dc}.

When the bias is correct, verify that the waveform at the output is correct. At 500 Hz, the output should be a sine wave with a 2.5 mV_p amplitude, riding on a 2.5 V_{dc} level.

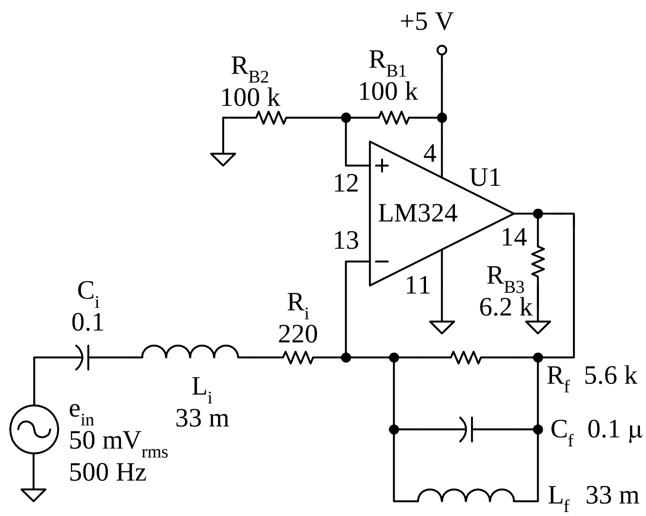


Figure 6-11 Filter schematic for Example 6-3

The frequency response simulation is shown in Figure 6-12.

Look carefully at the **Bode Plotter's** connections, **F** and **I** blocks

The cursor may be moved across the display. The gain magnitude and frequency at the intersection of the cursor and the frequency response plot are indicated in the lower right window of the **Bode Plotter**. In this example, A_o = 28.1 dB at 2.75 kHz.

Practice: Determine A_o if R_i = 120 kΩ, and $L_i = L_f$ = 470 mH.

Answer: A_o = –26.6 dB at 724 Hz

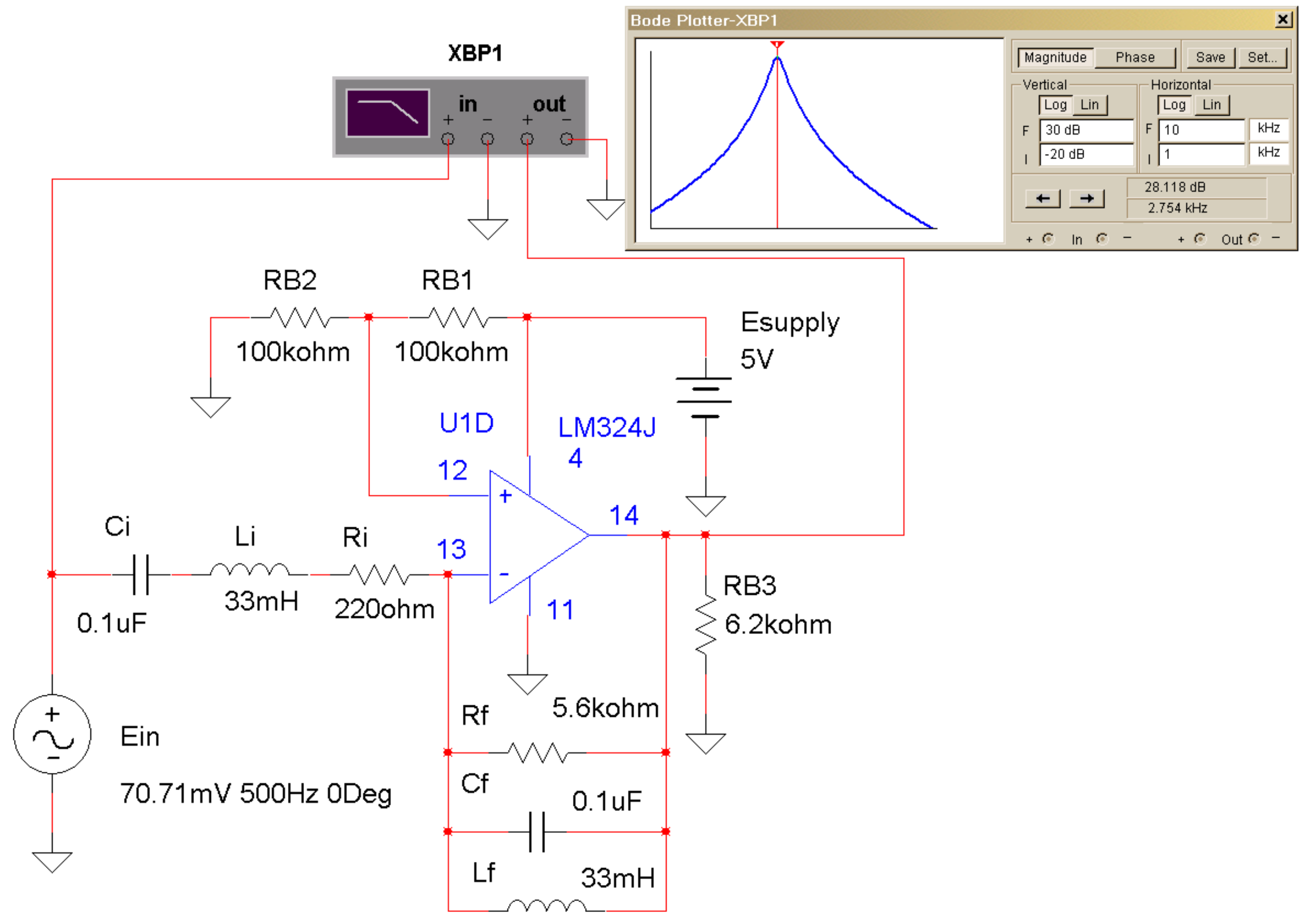


Figure 6-12 Frequency response simulation for Example 6-3

6.4 Roll-off Rate and Filter Order

You can build a filter with a resistor and a single capacitor or a single inductor. The circuit is simple and performs all of the functions of a filter. Adding capacitors and/or inductors increases the filter's complexity and cost. These added energy storage elements may also significantly improve the filter's performance. Look at Figure 6-13.

The number of inductors and/or capacitors in the circuit sets the filter's order. Each of these energy storage elements produces an increase of the derivative order needed to describe the circuit's behavior.

$$v_{\mathrm{L}} = L\frac{di}{dt} \quad \text{and} \quad i_C = C\frac{dv}{dt}$$

A circuit with resistors and two capacitors or two inductors or one capacitor and one inductor is described by a differential equation containing a second order derivative.

$$\frac{d^2}{dt^2}$$

A resistor, two capacitors and an inductor form a *third* order circuit. Its equation contains a third order derivative.

$$\frac{d^3}{dt^3}$$

Fortunately, the behavior for the more useful circuits has been fully investigated and the results tabulated. The circuits and the frequency responses in Figure 6-13 are for **Butterworth** filters. The design of both low- and high-pass, higher order filters using the Butterworth tables are covered in the following chapters.

The pass band gain and the frequency characteristics have been set to be equal. This makes comparisons easier. The major difference, then, among the various order filters is how steeply the gain falls as the frequency increases. This is called the **roll-off rate**. The higher the filter's order, the higher the roll-off rate. Since the purpose of a filter is to pass certain frequency signals and reject others, the higher the order, the more ideal the performance. Remember, however, that this improved performance is bought with increased circuit complexity and expense. Proper behavior often requires nonstandard component values, making the hardware even more difficult to actually build. It is usually a good idea to select the lowest order filter that works.

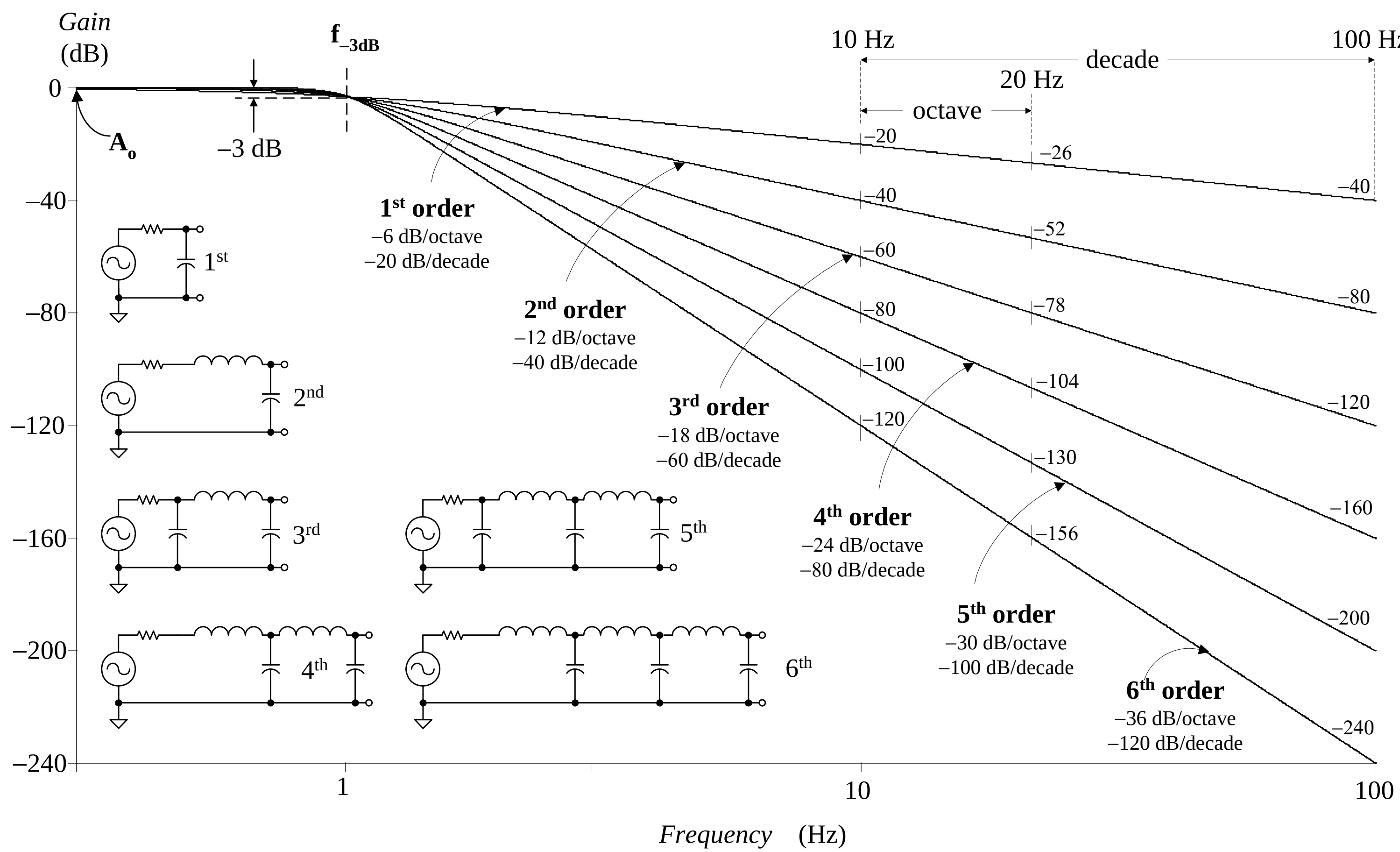


Figure 6-13 Frequency responses of first through sixth order low-pass filters

Decade:
x 10 or x 1/10

Octave:
x 2 or x 1/2

Roll-off rate is a slope, expressing how rapidly the gain *changes* with a change in frequency. The gain is measured in dB. The change in frequency is measured in either decades or octaves. A **decade** change in frequency is an increase or decrease by a **factor of ten**. In Figure 6-13, a decade change is shown between 10 Hz and 100 Hz. An **octave** change is a change by **two**. In Figure 6-13, the frequency changes an octave between 10 Hz and 20 Hz. Audio filters usually have their roll-off rates defined in dB/octave because musical notes are also arranged in octaves. This allows finer resolution. There are approximately 10 octaves between 20 Hz and 20 kHz, but only three decades.

Roll-off rate depends on the filter's order.

The higher the order, the steeper the roll-off is. When the gain first begins to fall, it falls *nonlinearly*. Within a couple of octaves, the slope becomes *linear*. The roll-off becomes constant, depending on order, at

–20 dB/decade/order   –6 dB/octave/order

The polarity is negative since the gain falls as the frequency increases. For high-pass filters the slope is positive.

6.5 Cut-off Frequencies

The key gain parameter (vertical axis) is the pass band gain, A_o. This tells you the filter's maximum gain.

When plotting the gain *magnitude*, the key *frequency* parameter (horizontal axis) is the **half-power point frequency**, $\boldsymbol{f_{-3dB}}$. The purpose of a filter is to pass power to the load from signals of certain frequencies, and to block power to the load for signals of other frequencies. At the half-power point frequency, the power passed to the load is one-half of the power passed to the load at the filter's maximum gain (A_o).

$$P_{\text{half-power}} = \frac{1}{2} P_{\text{max}}$$

$$\frac{P_{\text{half-power}}}{P_{\text{max}}} = \frac{1}{2}$$

$$dB_{\text{half-power}} = 10\log\left(\frac{P_{\text{half-power}}}{P_{\text{max}}}\right) = 10\log\left(\frac{1}{2}\right) = -3\,\text{dB}$$

At the half-power point, the power delivered to the load has fallen –3 dB below the power delivered when the filter is passing its maximum output.

In terms of voltage gain,

$$\frac{\left(V_{\text{half-power}}\right)^2}{R_{\text{load}}} = \frac{1}{2}\frac{\left(V_{\max}\right)^2}{R_{\text{load}}}$$

$$\left(V_{\text{half-power}}\right)^2 = \frac{1}{2}\left(V_{\max}\right)^2$$

$$V_{\text{half-power}} = \frac{1}{\sqrt{2}}V_{\max} = 0.707V_{\max}$$

At the half-power point, the output voltage has fallen to 0.707 of its maximum value. That's a 30% drop in voltage, producing a 50% drop in power delivered to the load.

To determine the effect on the voltage *gain* at the half-power point, divide each side of the equation by E_{in}.

At $f_{-3\text{dB}}$:

$P = 0.5\ P_{\max}$

$V = 0.707\ V_{\max}$

$A_{\text{dB}} = A_{\text{o dB}} - 3\ \text{dB}$

$$\frac{V_{\text{half-power}}}{E_{\text{in}}} = \frac{1}{\sqrt{2}} \times \frac{V_{\max}}{E_{\text{in}}}$$

$$A_{\text{half-power}} = \frac{1}{\sqrt{2}} \times A_o$$

$$A_{\text{half-power dB}} = 20\log\left(\frac{1}{\sqrt{2}} \times A_o\right)$$

The log of the product is the sum of the log of each number.

$$A_{\text{half-power dB}} = 20\log\left(A_o\right) + 20\log\left(\frac{1}{\sqrt{2}}\right)$$

$$A_{\text{half-power dB}} = A_{\text{o dB}} - 3\text{dB}$$

Example 6-4

Run a simulation for the circuit in Figure 6-14.

a. Produce a frequency response plot and determine $f_{-3\text{dB}}$.

b. Determine the rms voltage and power delivered to the resistor at 10 $f_{-3\text{dB}}$ and at $f_{-3\text{dB}}$.

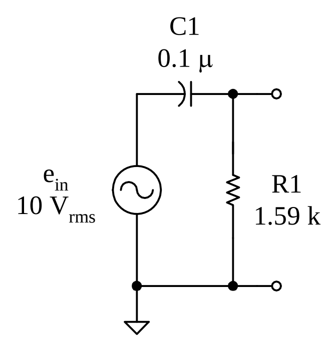


Figure 6-14
Filter for Example 6-4

Solution

The frequency response plot is given in Figure 6-15. This is a high-pass filter with A_o = 0 dB and f_{-3dB} = 1 kHz.

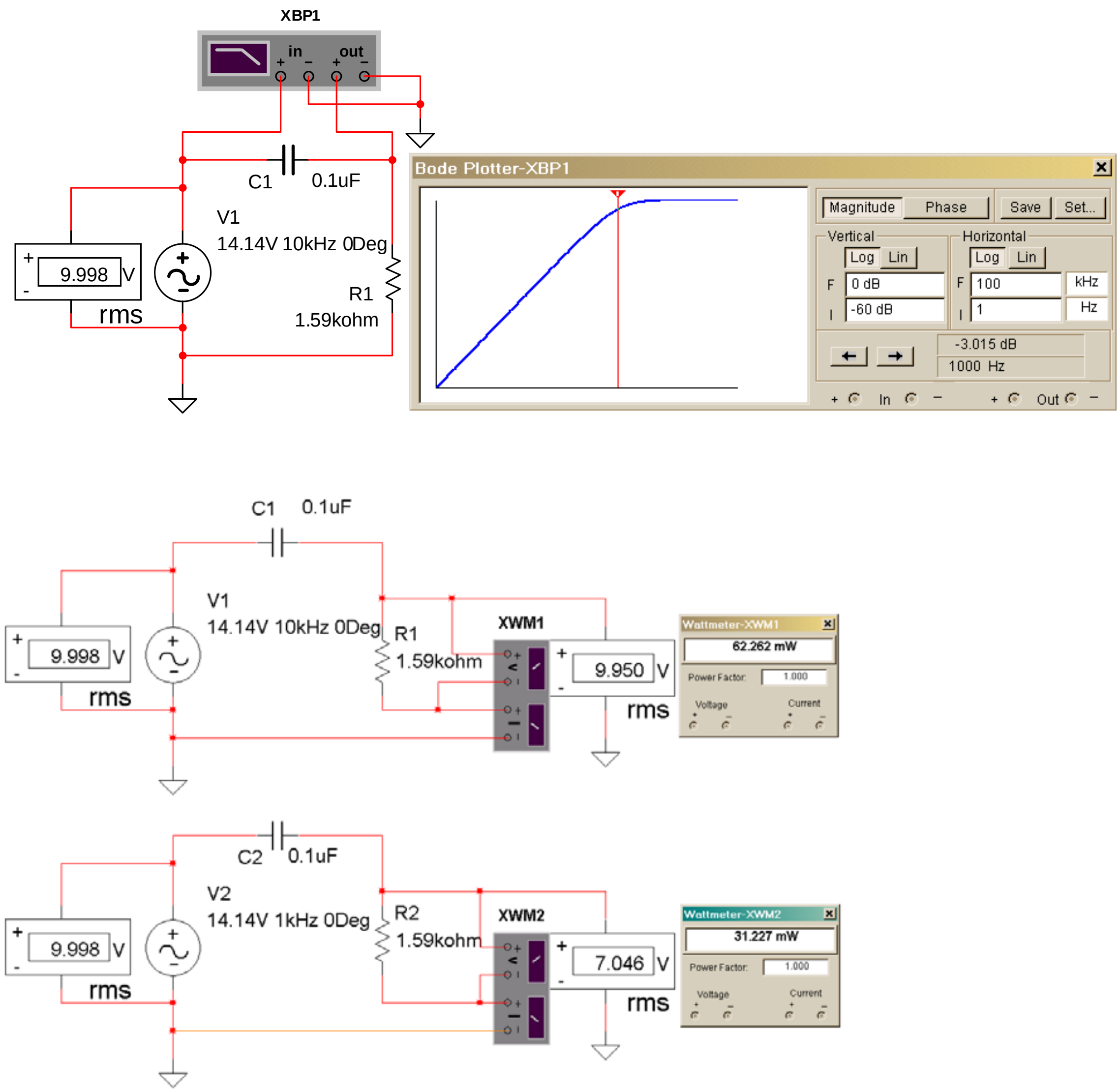


Figure 6-15 Simulation results for Example 6-4

Practice: Replace the 0.1 μF capacitor in Figure 6-14 with a 25.3 mH inductor and repeat the simulation.

Answer: This is a low-pass filter. $A_o = 0$ dB, $f_{-3dB} = 10$ kHz, $P_{R\ @\ 1kHz} = 62$ mW, $V_{R\ @\ 1\ kHz} = 10\ V_{rms}$, $P_{R\ @\ 10\ kHz} = 31$ mW, $V_{R\ @\ 10\ kHz} = 7.0\ V_{rms}$

The frequency response plots shown so far have been for gain *magnitude* versus frequency. However, a change in frequency also shifts the phase of a filter's output voltage. In audio systems, a phase shift between two loudspeakers makes the sound seem to move, without fading. The phase of the carrier signal of one form of communications system is shifted to encode the information being transmitted (rather than shifting the amplitude, AM, or the frequency, FM). When many signals of different frequencies and phases all arrive at a filter at the same time, the shape of the composite output signal strongly depends on the phase shift each of the different sine waves receives. The phase shift produced by the filter has a critical effect on the output voltage.

For a first order, *RC*, low-pass filter, the phase shift is

$$\Theta_{\text{low pass}} = -90° + \arctan\left(\frac{X_C}{R}\right)$$

Configured as a first order high-pass filter the phase shift is

$$\Theta_{\text{high pass}} = \arctan\left(\frac{X_C}{R}\right)$$

Adding an inductor produces a band-pass filter with a phase shift of

$$\Theta = -\arctan\left(\frac{X_L - X_C}{R}\right)$$

These three basic filters' phase frequency responses are plotted in Figure 6-16. The MATLAB commands used are also included. The low-pass filter's phase frequency response begins near zero, and shifts more and more negative as the frequency increases, headed toward, but never reaching –90°. Low-pass filters cause the phase of the output to **lag.**

The high-pass filter is the complement of the low-pass. At high frequencies, where the filter is passing the input signal, the phase is shifted very little. As the frequency falls, and the filter begins to reject the signal, the phase is *advanced*, toward +90°. High-pass filters cause the phase of the output to **lead** the input signal.

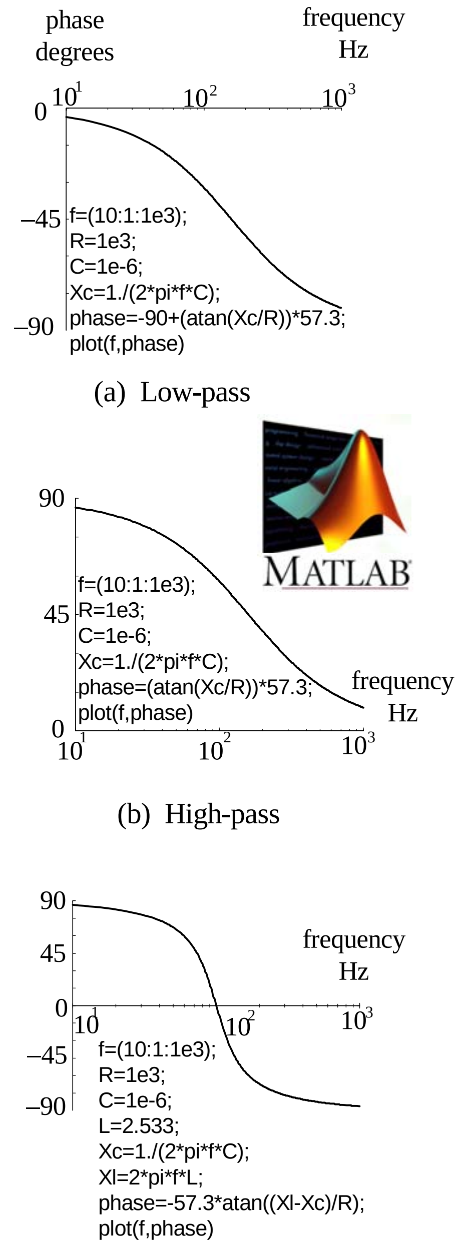


Figure 6-16 Phase shift frequency responses with MATLAB commands

For the band-pass the low frequencies are rejected. This is like a high-pass filter. The phase at low frequencies also behaves like it does in a high-pass filter. It leads, being shifted nearly +90° at very low frequencies, and dropping to 0°. The bandpass filter rejects high frequencies too, just as a low-pass filter does. The upper frequency output phase lags, starting at 0° near the center, and shifting toward –90°.

The **critical frequency**, $\boldsymbol{f_o}$ is that frequency at which the phase has been shifted ±45°/order. Low-pass filters cause the phase to lag (–45°/order). High-pass filters cause the phase to lead (+45°/order). Increasing the order of the filter increases the phase shift at f_o. Figure 6-17 is a plot of the phase shift of low-pass filters, first to sixth order.

These are the same filters whose magnitude frequency response, and schematics, are shown in Figure 6-13. For certain classes of filters these two key frequency parameters occur at the same frequency. For others, the half-power point frequency is related to the critical frequency. Remember, these two parameters define two different phenomena. At the half-power point frequency (f_{-3dB}), the power to the load has fallen by 50% and the voltage gain's *magnitude* has dropped to 0.707 A_o. At the critical frequency, the phase has been shifted some multiple of 45°.

Phas

±45
ord

Low

High

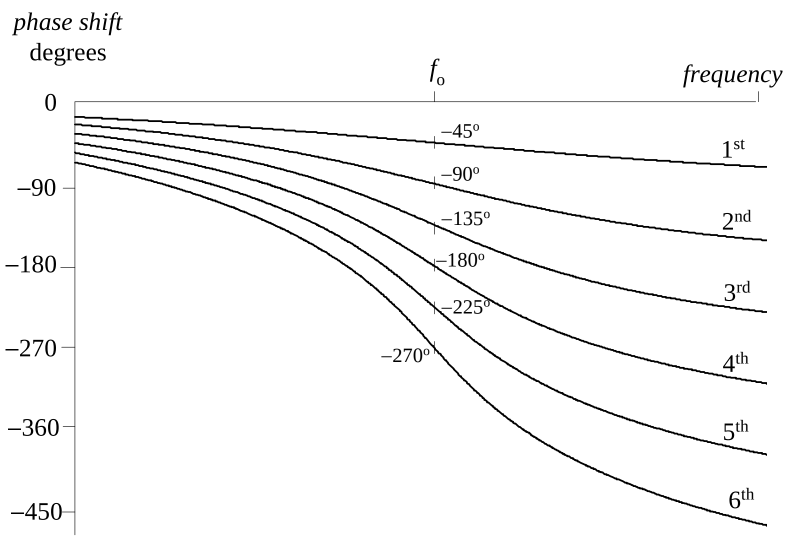


Figure 6-17 Phase shift of first through sixth order low-pass filters

Summary

Gain is the *phasor* ratio of output to input. Both magnitude and phase are calculated. Filters provide different gain for different frequency signals, allowing certain frequency signals to pass while blocking others. Low-pass filters are used to drive bass loudspeakers, and to remove high frequency. High-pass filters pass the notes to the tweeter loudspeaker, and couple signals between amplifier stages while blocking their dc bias. Band-pass filters drive the mid-range loudspeakers.

The horizontal axis of a filter's frequency response plot is scaled logarithmically. Both spreadsheets and MATLAB allow graphs to have their horizontal axis scaled logarithmically.

Gain magnitude often is expressed in decibels. The dB may indicate the *power* gain or the *voltage* gain. There are two key dB levels. A gain of 0 dB indicates that the output power or voltage equals the input power or voltage. A gain of −3 dB indicates that the output is below the input. The output power is half of the input power. The output voltage has fallen to 0.707 of the input voltage. Since dB is a measure of *gain*, it must be a ratio. However, the dB may also be used to express a level if a reference is specified, such as dBV, dBu, dBm, and dBW.

$$dB = 10 \times \log \frac{P_{out}}{P_{in}}$$

$$= 20 \times \log \frac{V_{out}}{E_{in}}$$

$$dBV = 20 \times \log \frac{V_{out}}{1V_{rms}}$$

$$dBu = 20 \times \log \frac{V_{out}}{0.775V_{rms}}$$

$$dBm = 10 \times \log \frac{P_{out}}{1mW}$$

$$dBW = 10 \times \log \frac{P_{out}}{1W}$$

A filter's order is equal to the number of inductors and capacitors in the circuit. The higher the filter's order is, the more steeply the gain magnitude rolls-off. Roll-off rate may be expressed in dB/decade or dB/octave. Low-pass filters have a negative slope, while the slope for high-pass filters is positive.

Roll-off rate:
20 dB/decade/order
6 dB/octave/order

The pass band gain, A_o, is the largest gain at the extreme of the filter's operation. For a low-pass filter, the pass band gain is determined at dc. For a high-pass filter, it is measured at the circuit's highest operating frequency. The band-pass filter's gain peaks somewhere in the middle.

The key frequency parameter on a gain magnitude plot is the half-power point frequency, f_{-3dB}. At that frequency, the dB gain has fallen 3 dB below A_o. Power to the load is 50% of what it is at the filter's maximum output. The voltage gain falls to 0.707 A_o.

The critical frequency f_o is the main frequency parameter of the gain phase frequency response. At that frequency the phase has shifted by ±45°/order. Low-pass filters lag, sending the phase negative. High-pass filters lead, producing a positive phase shift. The higher the filter's order is, the greater the phase shift at the critical frequency.

For many filters, the half-power point frequency f_{-3dB} and the critical frequency f_o occur at the same frequency. For other types of filters, these two parameters are related, but occur at different frequencies.

Problems

The Frequency Response Plot

6-1 **a.** Plot the frequency response of the following gain magnitude between the frequencies of 1 kHz and 1 MHz. A spreadsheet or MATLAB will help to automate the calculations and plotting. See Example 6-1 if you need help.

$$|A| = \frac{8R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$R = 2.2\ \text{k}\Omega,\ \ L = 2.2\ \text{mH},\ \ C = 1\ \text{nF}$

b. Identify the *type* of filter.

c. Explain one application for this filter that is not described in the text.

6-2 Repeat the three parts of Problem 6-1 for the following.

$$|A| = \frac{3X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$R = 2.2\ \text{k}\Omega,\ \ L = 2.2\ \text{mH},\ \ C = 1\ \text{nF}$

The Horizontal Axis

6-3 Plot the gain magnitude for the circuit in Problem 6-1 with its horizontal axis scaled logarithmically.

6-4 Plot the gain magnitude for the circuit in Problem 6-2 with its horizontal axis scaled logarithmically.

The Vertical Axis

6-5 Calculate the *dB* for each of the following:

a. P_{in} = 200 W, P_{out} = 20 mW

b. V_{in} = 100 mV_{rms}, V_{out} = 70.7 mV_{rms}

6-6 Calculate the *dB* for each of the following:

a. P_{in} = 1.2 W, P_{out} = 300 m W

b. V_{in} = 28.3 V_{rms}, V_{out} = 250 mV_{rms}

6-7 A filter has a gain of –8 dB and an input of 12 mW. Calculate the power delivered to the load.

6-8 A filter has a gain of 12 dB and an input of 300 mV_{rms}. Calculate the output voltage.

6-9 What input power is required to deliver 1.4 W to a load from a filter with a gain of –24 dB?

6-10 What input voltage is required to deliver 4.6 V_{rms} to a load from a filter with a gain of 41 dB?

6-11 Convert the following:
- **a.** 50 mW to dBm **b.** 25 W to dBW
- **c.** 350 mV_{rms} to dBu **d.** 7.3V_{rms} to dBV

6-12 Convert the following:
- **a.** 230 mW to dBm **b.** 75 W to dBW
- **a.** 230 mV_{rms} to dBu **d.** 8.9V_{rms} to dBV

6-13 **a.** Plot the gain magnitude from Problem 6-1. Express the gain magnitude in dB. Scale the horizontal axis logarithmically.

b. Determine the pass band gain, A_o.

6-14 **a.** Plot the gain magnitude from Problem 6-2. Express the gain magnitude in dB. Scale the horizontal axis logarithmically.

b. Determine the pass band gain, A_o.

Roll-off Rate and Filter Order

6-15 Determine the roll-off rate for each of the following filters.
- **a.** Third order low-pass. Answer in dB/decade.
- **b.** Second order high-pass. Answer in dB/octave.
- **c.** Sixth order low-pass. Answer in dB/decade.
- **d.** Fifth order high-pass. Answer in dB/octave.

6-16 Determine the order needed for each of the following filters.
- **a.** At 3 kHz the gain is –3dB. At 6 kHz the gain is –15 dB.
- **b.** At 100 kHz the gain is –40 dB. At 10 kHz the gain is 0 dB.
- **c.** At 100 Hz the gain is –3dB. At 400 Hz the gain is –39 dB.
- **d.** At 1 MHz the gain is –90 dB. At 10 kHz the gain is +70 dB.

Cut-off Frequencies

6-17 Determine f_{-3dB} for the filter in Problem 6-1. There is both a low frequency and a high frequency half-power point frequency.

6-18 Determine f_{-3dB} for the filter in Problem 6-2. There is both a low-frequency and a high-frequency half-power point frequency.

6-19 If the vertical axis of the frequency response plot is scaled in ratio, and $A_o = 23$, where does f_{-3dB} occur?

Frequency Response Lab Exercise

A. Passive High-pass Filter

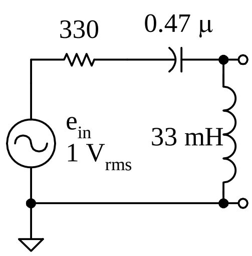


Figure 6-18
High pass filter

1. Build the circuit in Figure 6-18. Keep the leads from the signal generator and the leads to each component as short as practical.
2. Set the signal generator's amplitude to 1 V_{rms}, and its frequency to 100 Hz.
3. Verify the pass band gain (A_o) of your circuit by connecting the oscilloscope and digital multimeter between the output and common. Then, raise the frequency to 10 kHz. V_{out} should be a maximum at 10 kHz. Record this gain ratio.

A_o =

4. Lower the frequency until the output voltage has fallen to 0.707 of its value in step A3. This frequency is f_{-3dB}. Record it.

f_{-3dB}

5. Measure and record the critical frequency, f_o (+90° phase shift).

***f_o* =**

***$A_{@ 200 Hz}$* = ______ dB**

6. Lower the frequency to 200 Hz. Record the gain in dB.

***$A_{@ 100 Hz}$* = ______ dB**

7. Lower the frequency to 100 Hz. Record the gain in dB again.

***Roll-off rate* = _____dB/octave**

8. Calculate the dB/octave roll-off rate.

9. Construct a spreadsheet with columns for *frequency* (kHz), V_{out} (V_{rms}), *gain* (dB), *phase* (degrees). Provide a row for the column titles and then 19 rows for data. Enter the frequency data, one in each row (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0).

10. Create a frequency response graph of *frequency* (log scale) on the *x* axis versus *gain* (dB) on the *y* axis. Do this before you begin taking data.

11. Create a frequency response graph of *frequency* (log scale) on the *x* axis versus *phase* on the *y* axis. Do this before you begin taking data.

12. Set the input to 100 Hz, 1 V_{rms}. Measure the output rms voltage magnitude and phase. Allow the spreadsheet to calculate the dB.

13. Change the frequency to that indicated in the next row. Adjust the input voltage magnitude to 1 V_{rms}. Measure the output magnitude and phase.

14. Compete the table and the plots.

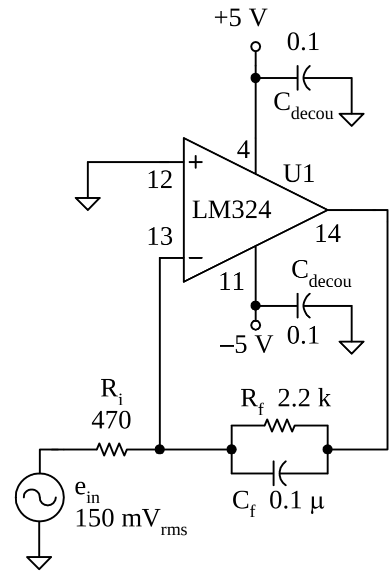


Figure 6-19
Active low-pass filter

B. Active Low-pass Filter

1. Build the circuit in Figure 6-19. Keep the leads from the signal generator and the leads to each component as short as practical.

2. Set the signal generator's amplitude to 150 mV_{rms}, and its frequency to 10 kHz.

3. Verify the pass band gain (A_o) of your circuit by connecting the oscilloscope and digital multimeter between the output and common. Then, lower the frequency to 100 Hz. The output should be a maximum at 100 Hz. Record this gain ratio.

A_o = _____

4. Raise the frequency until the output voltage has fallen to 0.707 of its value in step B3. This frequency is f_{-3dB}. Record it.

f_{-3dB} = _____

5. Measure and record the critical frequency (f_o).

f_o = _____

6. Raise the frequency to 5 kHz. Record the gain in dB.

dB

_ dB

____dB/octave

7. Raise the frequency to 10 kHz. Record the gain in dB again.

8. Calculate the dB/octave roll-off rate.

9. Construct a spreadsheet with columns for *frequency* (kHz), V_{out} (V_{rms}), *gain* (dB), *phase* (degrees). Provide a row for the column titles and then 19 rows for data. Enter the frequency data, one in each row (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0).

10. Create a frequency response graph of *frequency* (log scale) on the *x* axis versus *gain* (dB) on the *y* axis. Do this before you begin taking data.

11. Create a frequency response graph of *frequency* (log scale) on the *x* axis versus *phase* on the *y* axis. Do this before you begin taking data.

12. Set the input to 100 Hz, 150 mV_{rms}. Measure the output rms voltage magnitude and phase. Allow the spreadsheet to calculate the dB.

13. Change the frequency to that indicated in the next row. Maintain the input voltage magnitude at 150 mV_{rms}. Measure the output magnitude and phase.

14. Compete the table and the plots.