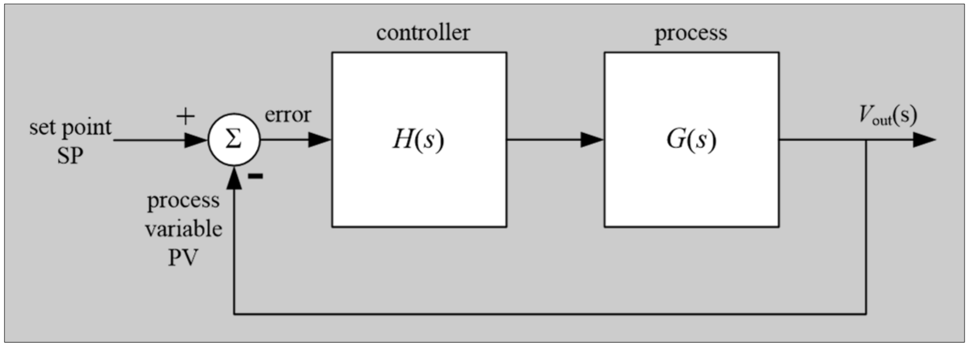
Motor with Proportional-Integral Controller

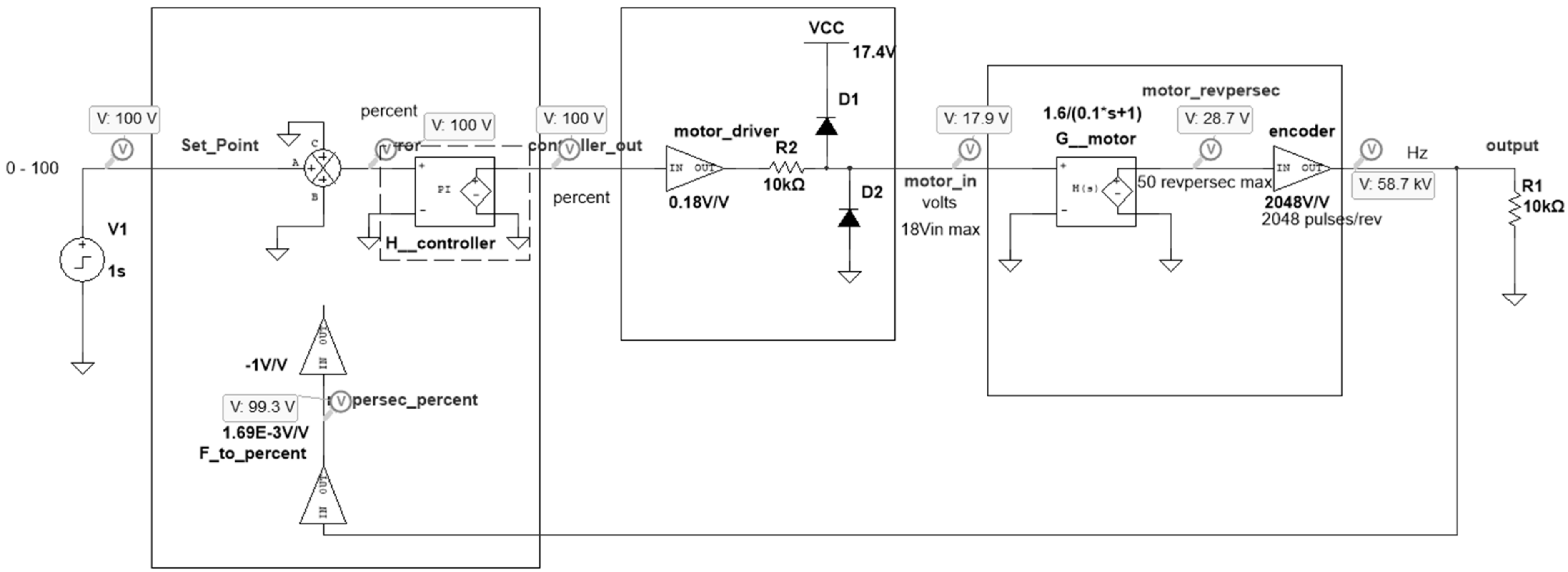


$$\frac{V_{out}}{SP} = \frac{GH}{1 + GH}$$

Integral output increases until error = 0

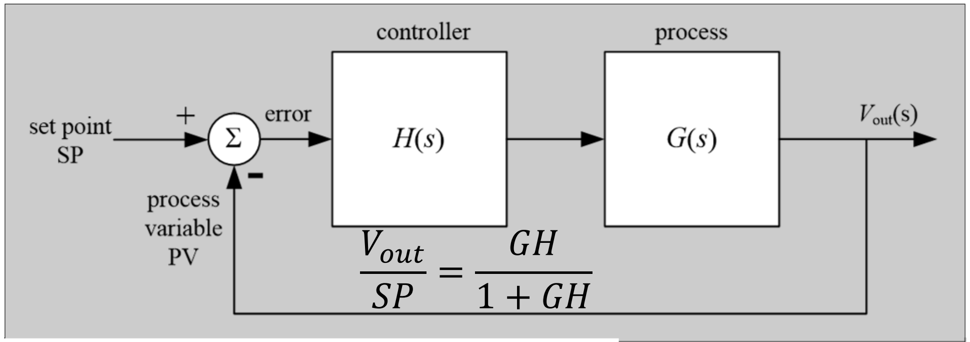
$$G = \frac{m_{\text{motor}}}{\tau_{\text{motor}} s + 1}$$

$$H = k_p + \frac{k_i}{s}$$

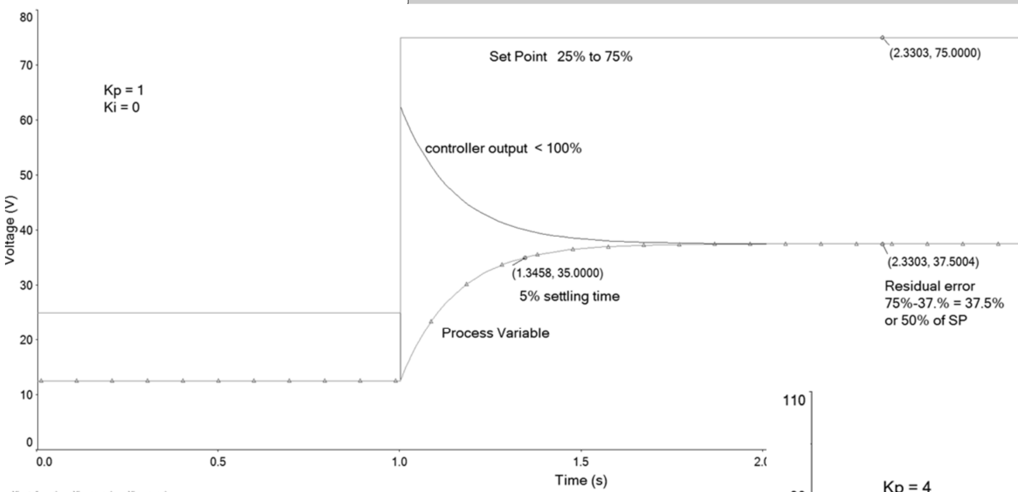


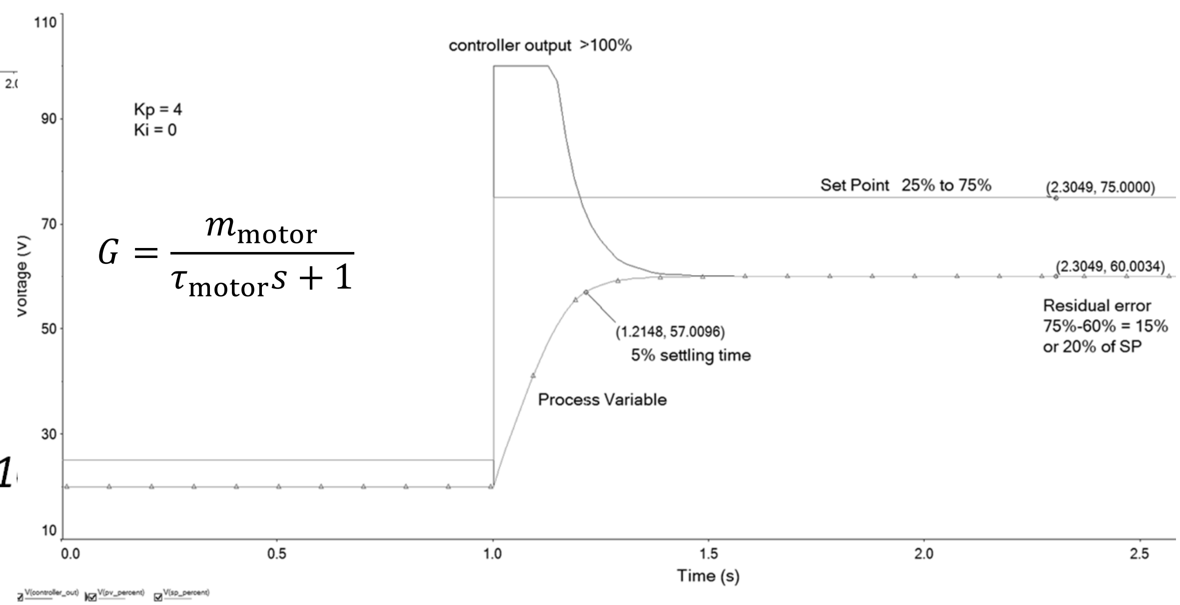
New motor, Spring 2025

Motor with Proportional Controller



$$\frac{V_{out}}{SP} = \frac{GH}{1+GH}$$





$$G = \frac{m_{\text{motor}}}{\tau_{\text{motor}} s + 1}$$

Integral Control

- Output __________ until __________.
- ___________ of CO proportional to _________.
- ___ CO = ________ CO $= k_{\mathrm{I}} * \frac{ER}{s}$
- $\mathrm{H} = \frac{CO}{ER} =$ ________

PI Motor Speed Control

$$H_{\mathrm{prop}} = k_{\mathrm{p}} \qquad H_{\mathrm{integral}} = \frac{k_{\mathrm{i}}}{s}$$

$$H_{\mathrm{pi}} = k_{\mathrm{p}} + \frac{k_{\mathrm{i}}}{s} = \frac{k_{\mathrm{p}}s + k_{\mathrm{i}}}{s}$$

$$G = \frac{m}{\tau s + 1}$$

$$\frac{PV}{SP} = \frac{GH}{1 + GH}$$

$$\frac{PV}{SP} = \frac{\dfrac{m}{\tau s + 1} \times \dfrac{k_{\mathrm{p}}s + k_{\mathrm{i}}}{s}}{1 + \dfrac{m}{\tau s + 1} \times \dfrac{k_{\mathrm{p}}s + k_{\mathrm{i}}}{s}}$$

$$\frac{PV}{SP} = \frac{\dfrac{m\left(k_{\mathrm{p}}s + k_{\mathrm{i}}\right)}{s(\tau s + 1)}}{\dfrac{s(\tau s + 1) + m\left(k_{\mathrm{p}}s + k_{\mathrm{i}}\right)}{s(\tau s + 1)}}$$

$$\frac{PV}{SP} = \frac{m\left(k_{\mathrm{p}}s + k_{\mathrm{i}}\right)}{s(\tau s + 1) + m\left(k_{\mathrm{p}}s + k_{\mathrm{i}}\right)}$$

$$\frac{PV}{SP} = \frac{mk_{\mathrm{p}}s + mk_{\mathrm{i}}}{\tau s^2 + s + mk_{\mathrm{p}}s + mk_{\mathrm{i}}}$$

$$\frac{PV}{SP} = \frac{\dfrac{mk_{\mathrm{p}}}{\tau}s + \dfrac{mk_{\mathrm{i}}}{\tau}}{s^2 + \dfrac{1}{\tau}s + \dfrac{mk_{\mathrm{p}}}{\tau}s + \dfrac{mk_{\mathrm{i}}}{\tau}}$$

$$\frac{PV}{SP} = \frac{\frac{mk_{\mathrm{p}}}{\tau} s + \frac{mk_{\mathrm{i}}}{\tau}}{s^2 + \frac{1 + mk_{\mathrm{p}}}{\tau} s + \frac{mk_{\mathrm{i}}}{\tau}}$$

$$\frac{\boldsymbol{A\omega_n^2}}{\boldsymbol{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

$$\frac{PV}{SP} = \frac{\frac{mk_{\mathrm{p}}}{\tau} s}{s^2 + \frac{1 + mk_{\mathrm{p}}}{\tau} s + \frac{mk_{\mathrm{i}}}{\tau}} + \frac{\frac{mk_{\mathrm{i}}}{\tau}}{s^2 + \frac{1 + mk_{\mathrm{p}}}{\tau} s + \frac{mk_{\mathrm{i}}}{\tau}}$$

$$\omega_{\mathrm{n}}^2 = \frac{mk_{\mathrm{i}}}{\tau} \qquad \omega_{\mathrm{n}} = \sqrt{\frac{mk_{\mathrm{i}}}{\tau}}$$

$$A\omega_{\mathrm{n}}^2 = \frac{mk_{\mathrm{i}}}{\tau} \qquad A = 1$$

$$\frac{PV}{SP} = \frac{\frac{mk_{\mathrm{p}}}{\tau}s + \frac{mk_{\mathrm{i}}}{\tau}}{s^2 + \frac{1 + mk_{\mathrm{p}}}{\tau}s + \frac{mk_{\mathrm{i}}}{\tau}}$$

$$\boldsymbol{\frac{A\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

$$2\xi\omega_{\mathrm{n}} = \frac{1 + mk_{\mathrm{p}}}{\tau}$$

$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\omega_{\mathrm{n}}\tau}$$

$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\tau\sqrt{\frac{mk_{\mathrm{i}}}{\tau}}}$$

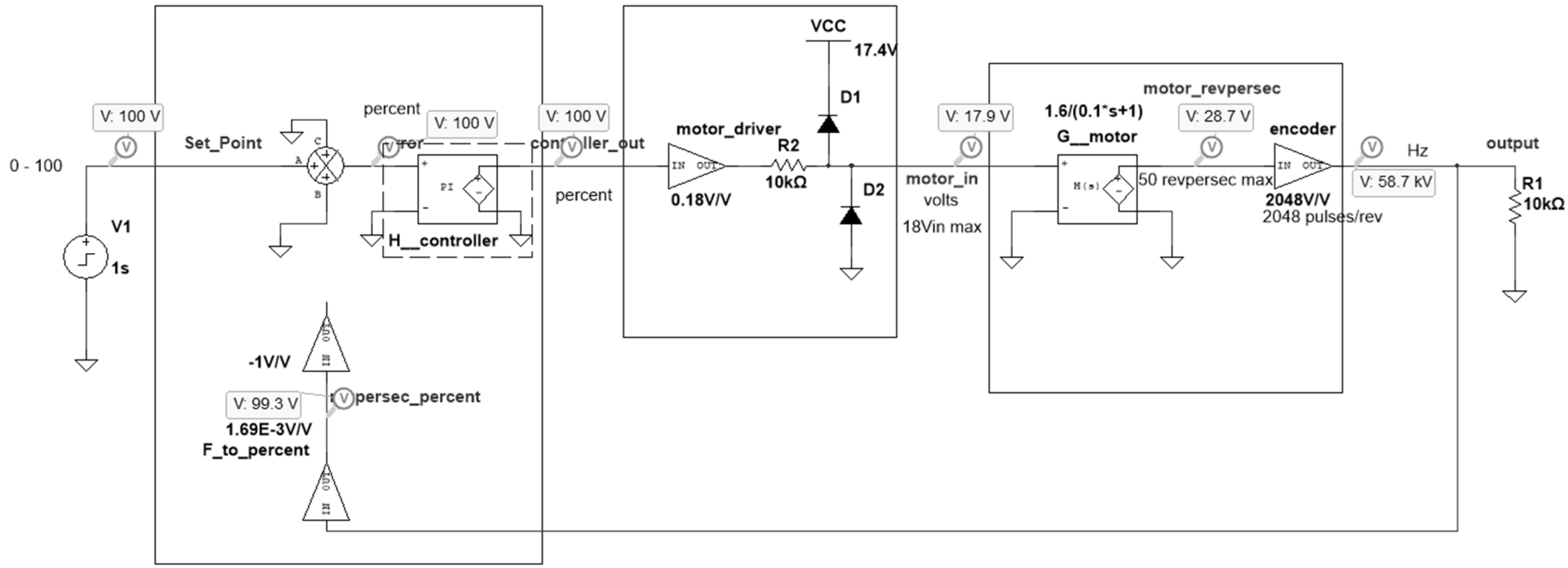
$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\sqrt{mk_{\mathrm{i}}\tau}}$$

$$\Rightarrow \quad \boldsymbol{k_{\mathrm{p}} = \frac{1}{m}}$$

$$\xi = \frac{1 + m\frac{1}{m}}{2\sqrt{mk_{\mathrm{i}}\tau}} = \frac{1 + 1}{2\sqrt{mk_{\mathrm{i}}\tau}}$$

$$\xi = \frac{1}{\sqrt{mk_{\mathrm{i}}\tau}}$$

$$\Rightarrow \quad \boldsymbol{k_{\mathrm{i}} = \frac{1}{m\tau\xi^2}}$$



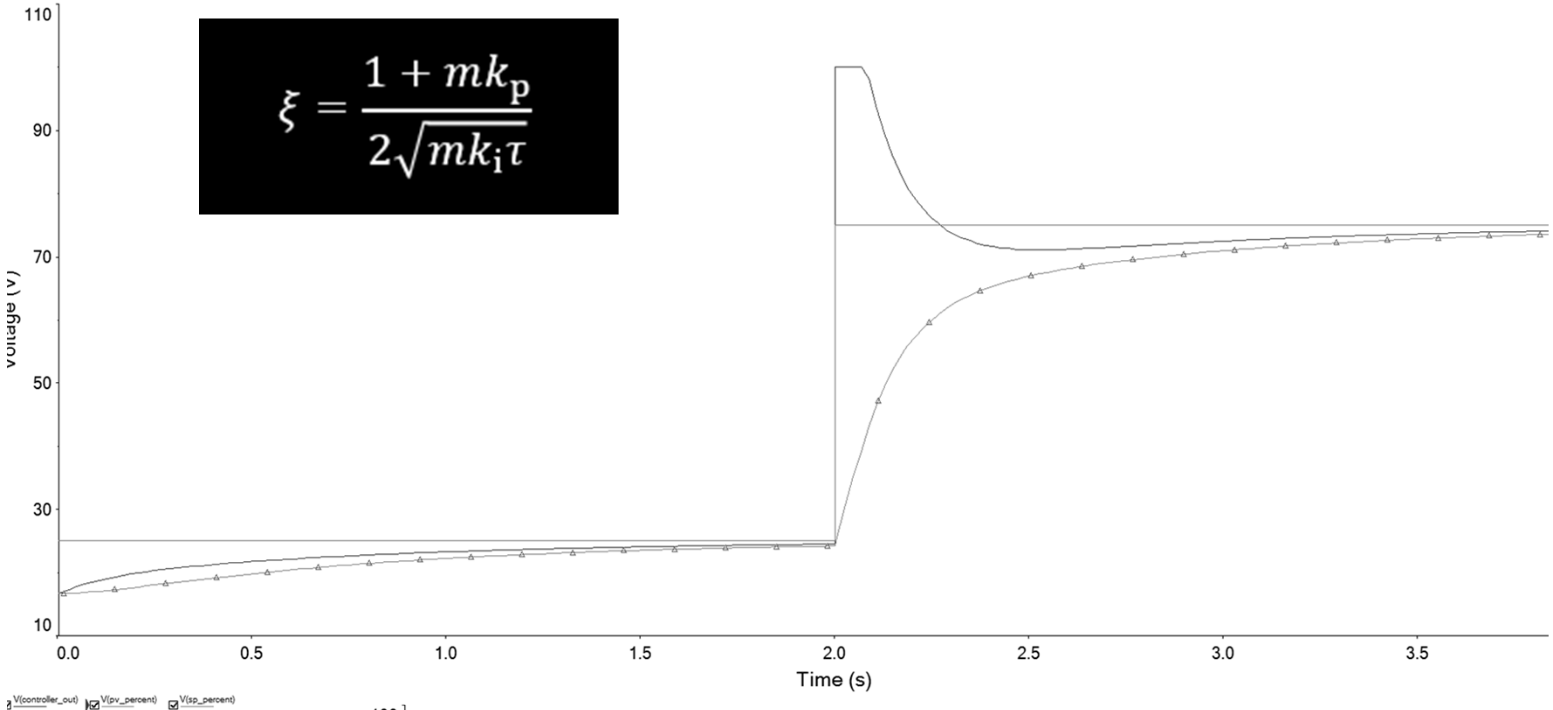
New motor, Spring 2025

$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\sqrt{mk_{\mathrm{i}}\tau}}$$

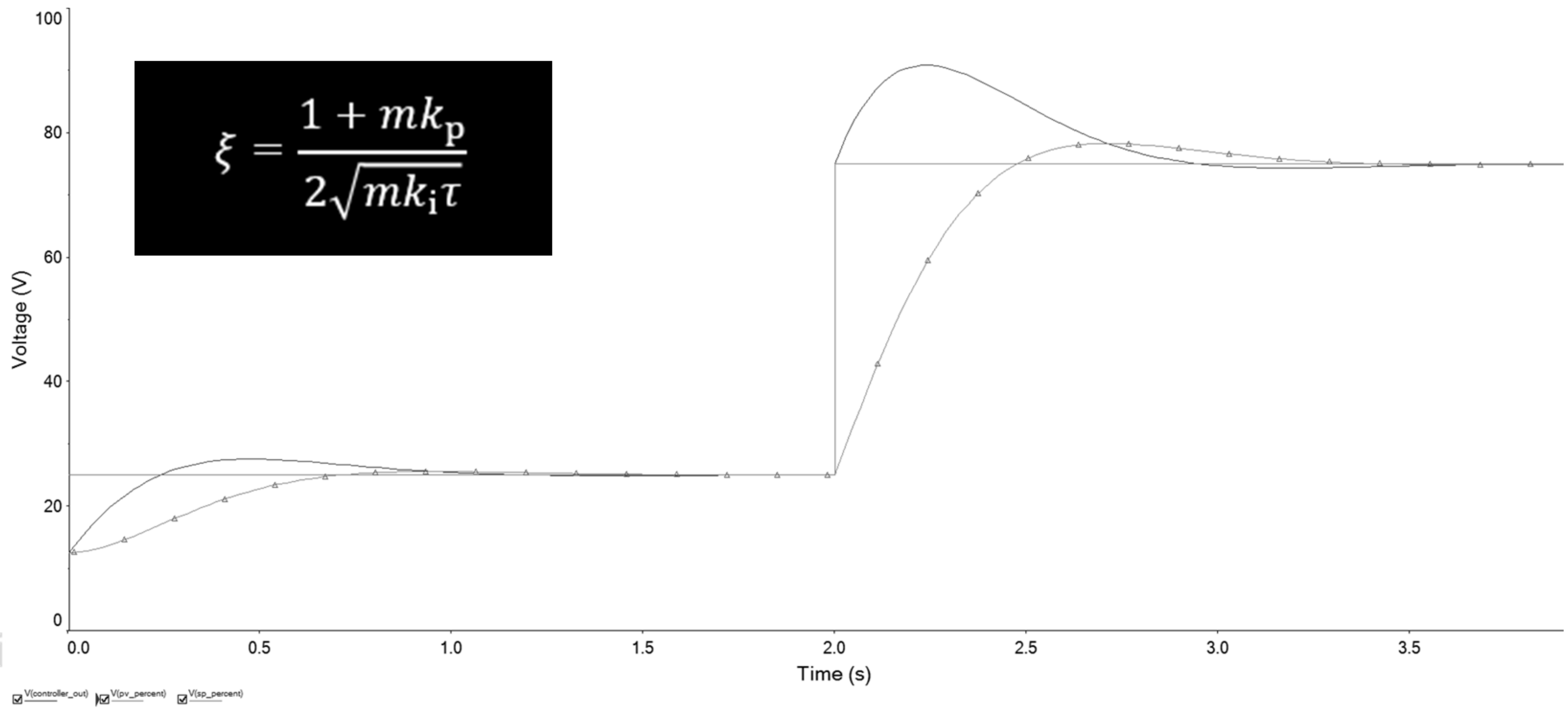
10/(13s+1)

10/(10s+1)

$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\sqrt{mk_{\mathrm{i}}\tau}}$$



$$\xi = \frac{1 + mk_{\mathrm{p}}}{2\sqrt{mk_{\mathrm{i}}\tau}}$$



Other changes

- Double k_p
  - Effects on

- Motor ages