2

Advance Analysis Techniques

Introduction

When faced with a *huge* network, it is very tempting to panic. "Oh no! I have *no* idea where to start, or what to do! I can't do this. Shoot!" In reality, you *do* know how to solve the problem. You are just being overwhelmed by its magnitude.

Objectives

Upon completion of this chapter, you will be able to analyze linear passive networks with multiple loops and multiple AC and DC sources using:

- Impedance Combination.
- Superposition.
- Mesh.

2.1 Impedance Combination
Fundamental Technique

One solution to a complicated network, with a single sourse, is to combine and reduce, combine and reduce, combine and reduce the circuit until it becomes a simple series or parallel circuit. Then, work your way back out, with Ohm's law, Kirchhoff's laws, and the voltage divider law. Determine the effect the source has on each layer as you reintroduce the complexity by undoing the combinations you created to simplify the problem.

The trick in all of this combining is the documentation. Only the most gifted can keep all of the simplifications and combinations in their heads. It is critical to develop a consistent way of naming the intermediate combinations of components, and to redraw the circuit *every* time you combine elements, changing its topology.

Once you have developed a clear, workable plan, it is then just a matter of plugging in the numbers, step-by-step, combination-by-combination, working your way through the simplified schematics out to the full schematic.

High-frequency Cabling

When interconnecting pieces of electronic equipment, it is critical that the signals be passed accurately. As the signal's frequency exceeds 1 MHz, you must consider the effects of the signal generator's output resistance, the cable's *RLC* model, and the load's resistance, capacitance, and inductance. A circuit that appears as a simple generator driving a load resistor becomes much more complex as you consider the high-frequency effects. Figure 2-1 shows both the simple and the first approximation of the high-frequency model for the interconnections.
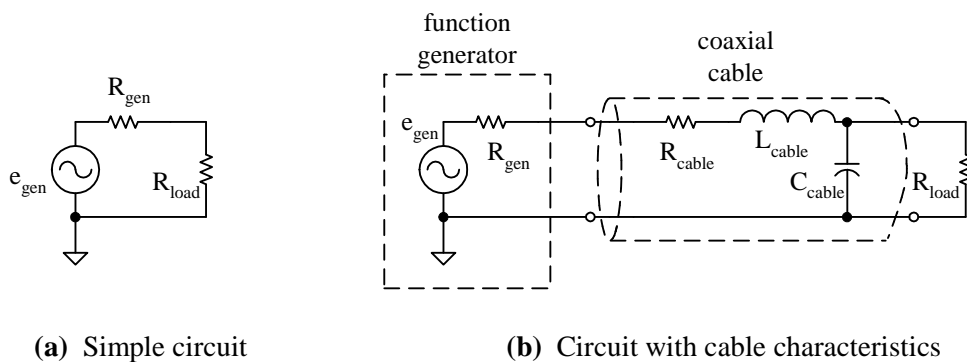
(a) Simple circuit   (b) Circuit with cable characteristics

Figure 2-1 First approximation of a high-frequency connection

Specifications for high-frequency cables include:

- Resistance per unit length.
- Capacitance per unit length.
- Characteristic impedance, Z_o.

The characteristic impedance of the cable plays a significant role in accurately transmitting high-frequency signals. Most of the details are best left to a complete text on rf electronics. For our purposes, the characteristic impedance can be used to determine the cable's inductance.

$$Z_o = \sqrt{\frac{L_{cable}}{C_{cable}}}$$

$$L_{cable} = C_{cable} \times Z_o^{\,2}$$

Terminated in its characteristic impedance (50 Ω in Example 2-1) a cable provides a well behaved, critically damped, low-pass performance. (There is much more about these parameters in later chapters). However, as soon as the termination no longer matches the cable's characteristic impedance, the resulting *RLC* filter may become underdamped. The voltage delivered to the load can even be *greater* than that provided by the function generator.

The circuit becomes even more complicated when you begin to use hook-up wire to connect from the cable's BNC connector to the input of your circuit. The inductance of wire is given as

$$L \approx 2 \times 10^{-7} \frac{H}{m} l \left[2.303 \log_{10} \left(\frac{4l}{d} \right) - 1 \right]$$

where l = length of the wire.
    d = diameter of the wire.

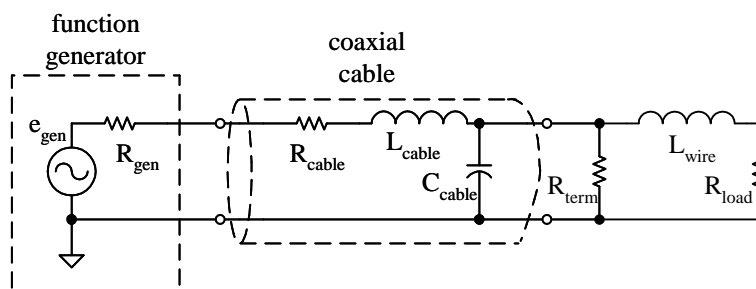At high frequencies, this inductance adds another rung to the circuit diagram ladder, and another set of calculations

Figure 2-2 Schematic for Example 2-1

Example 2-1

The following specifications are given for a coaxial cable:

$R = 275 \ \Omega/1000$ ft, $Z_o = 50 \ \Omega$, $C = 24.2$ pF/ft

Use a a 6-foot-long cable, add 50 Ω terminator, followed by a 12-inch piece of wire leading to a 100 Ω load.

Calculate the voltage across the 100 Ω load at 15 MHz.

Solution

a.
$$R_{cable} = 275 \frac{\Omega}{1000\,ft} \times 6\,ft = 1.65\,\Omega$$

$$C_{cable} = 24.2 \frac{pF}{ft} \times 6\,ft = 145\,pF$$

$$L_{cable} = C_{cable} \times Z_o^{\,2} = 145\,pF \times (50\,\Omega)^2 = 363\,nH$$

$$X_{L\,cable} = 2\pi \times 15\,MHz \times 363\,nF = 34.21\,\Omega$$

$$X_{C\,cable} = \frac{1}{2\pi \times 15\,MHz \times 145\,pF} = 73.17\,\Omega$$

The 12-inch piece of wire is

$$l = 12\,inches \times \frac{2.54\,cm}{inch} \times \frac{1\,m}{100\,cm} = 0.3045\,m$$

Hook-up wire has a diameter of 0.643 millimeters.

$$L_{wire} \approx 2 \times 10^{-7} \frac{H}{m} \times 0.305\,m \left[2.303 \log_{10} \left(\frac{4 \times 0.305\,m}{0.643 \times 10^{-3}\,m} \right) - 1 \right]$$

$$L_{wire} = 399.5\,nH$$

$$X_{L\,wire} = 2\pi \times 15\,MHz \times 399.5\,nH = 37.65\,\Omega$$
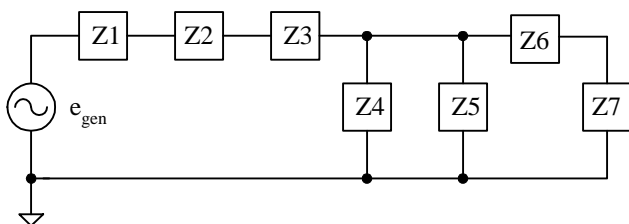
Redraw the schematic in terms of impedances, Figure 2-3.

$$\overline{Z1} = (50\,\Omega\angle 0°)$$
$$\overline{Z2} = (1.7\,\Omega\angle 0°)$$
$$\overline{Z3} = (34.21\angle 90°)$$
$$\overline{Z4} = (73.17\angle -90°)$$
$$\overline{Z5} = (50\,\Omega\angle 0°)$$
$$\overline{Z6} = (37.7\,\Omega\angle 90°)$$
$$\overline{Z7} = (100\,\Omega\angle 0°)$$

Figure 2-3 Block diagram for Example 2-1

Begin the impedance combination as far from the source as possible. Combine Z6 and Z7 to form Z_{67}. Draw the diagram to keep track of the circuit's configuration, shown in Figure 2-4.
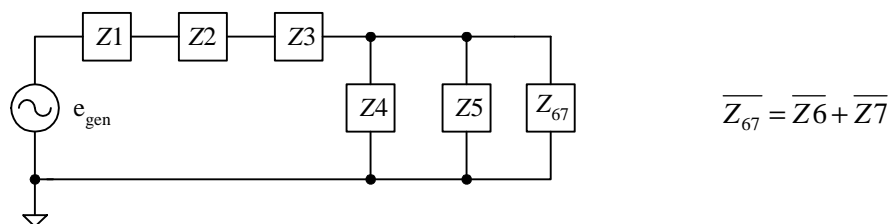

$$\overline{Z_{67}} = \overline{Z6} + \overline{Z7}$$

Figure 2-4 Impedance combination of Z6 and Z7

Impedances Z4, Z5, and Z_{67} are in parallel. They can be combined into a single impedance, Z_{4-7}. The new configuration is shown in Figure 2-5.

$$\overline{Z_{4-7}} = \left(\overline{Z4}^{-1} + \overline{Z5}^{-1} + \overline{Z_{67}}^{-1} \right)^{-1}$$
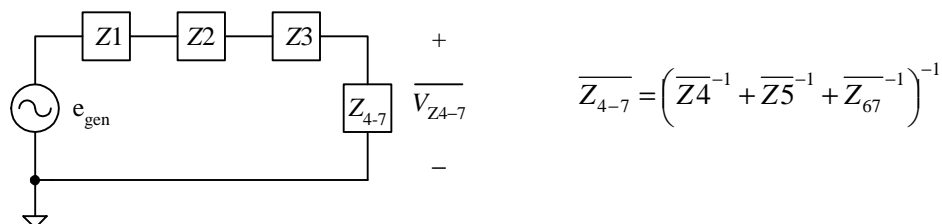
Figure 2-5 Impedance combination of Z4, Z5, and Z_{67}

This is a simple series circuit. No further impedance combinations are needed. Apply the voltage divider law to calculate $V_{Z4\text{-}7}$, the voltage across $Z_{4\text{-}7}$.

This is the voltage across Z4, Z5, and Z_{67}. Figure 2-6 is a repeat of Figure 2-4, with this voltage added.
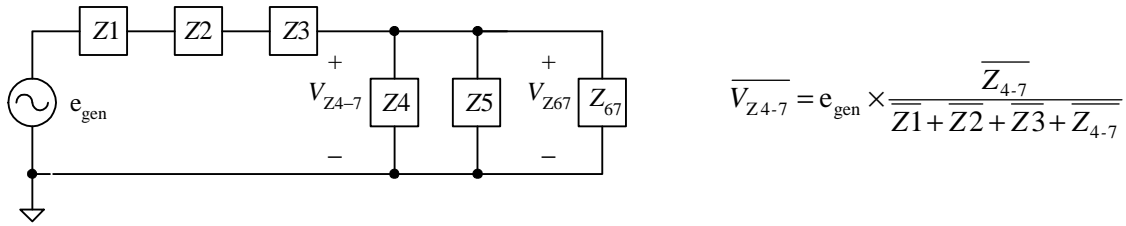

$$\overline{V_{Z4\text{-}7}} = e_{gen} \times \frac{\overline{Z_{4\text{-}7}}}{\overline{Z1} + \overline{Z2} + \overline{Z3} + \overline{Z_{4\text{-}7}}}$$

Figure 2-6 Voltage across $Z_{4\text{-}7}$

The voltage V_{Z4-7} is across the impedance Z_{67}.

$$\overline{V_{Z67}} = \overline{V_{Z4-7}}$$

This voltage produces the drop across Z7, and can be calculated by another application of the voltage divider law. This is shown in Figure 2-7.
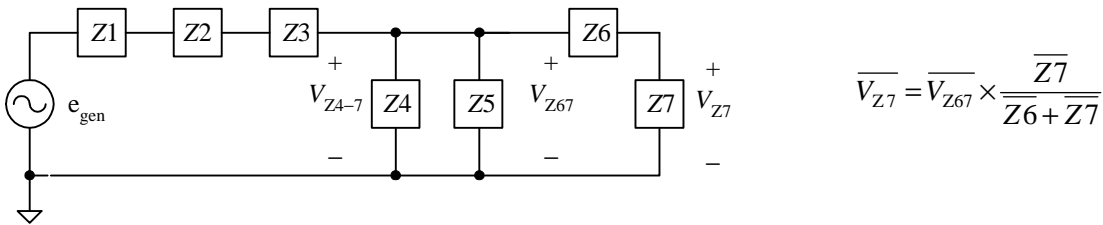

$$\overline{V_{Z7}} = \overline{V_{Z67}} \times \frac{\overline{Z7}}{\overline{Z6} + \overline{Z7}}$$

Figure 2-7 Voltage across Z7

You have a plan to calculate the voltage across $Z7$, the output. Only after the plan is completed and reviewed should you begin to insert numbers and calculate results.

$$\overline{Z1} = (50\,\Omega\angle 0°)$$

$$\overline{Z2} = (1.7\,\Omega\angle 0°)$$

$$\overline{Z3} = (34.21\angle 90°)$$

$$\overline{Z4} = (73.17\angle -90°)$$

$$\overline{Z5} = (50\,\Omega\angle 0°)$$

$$\overline{Z6} = (37.65\angle 90°)$$

$$\overline{Z7} = (100\,\Omega\angle 0°)$$

$$\overline{Z_{67}} = (37.65\,\Omega\angle 90°) + (100\,\Omega\angle 0°)$$

$$\overline{Z_{67}} = (106.9\,\Omega\angle 20.6°)$$

$$\overline{Z_{4-7}} = \left[(73.17\,\Omega\angle -90°)^{-1} + (50\,\Omega\angle 0°)^{-1} + (106.9\,\Omega\angle 20.6°)^{-1}\right]^{-1}$$

$$\overline{Z_{4-7}} = (32.71\,\Omega\angle -19.8°)$$

$$\overline{V_{Z4-7}} = \frac{(1\,V_{rms}\angle 0°)\times(32.71\,\Omega\angle -19.8°)}{(50\,\Omega\angle 0°) + (1.7\,\Omega\angle 0°) + (34.21\,\Omega\angle 90°) + (32.71\,\Omega\angle -19.8°)}$$

$$\overline{V_{Z4-7}} = (381.9\,mV_{rms}\angle -35.5°)$$

$$\overline{V_{Z7}} = \frac{(381.9\,mV_{rms}\angle -35.5°)\times(100\,\Omega\angle 0°)}{(37.65\,\Omega\angle 90°) + (100\,\Omega\angle 0°)}$$

$$\overline{V_{Z7}} = (357.4\,mV_{rms}\angle -56.1°)$$

The simulation result is shown in Figure 2-8 and agrees with the calculation above. When using MultiSIM, remember that the generator voltage is displayed in V_p. The panel meters normally display dc. You must adjust them to indicate V_{rms}.
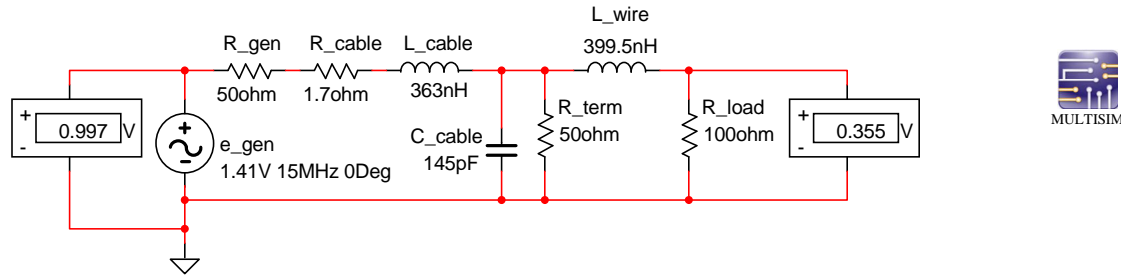

Figure 2-8 Simulation of Example 2-1

The x10 Oscilloscope Probe

A BNC cable presents considerable capacitance between the signal, on its center conductor, and common, on its shield. Combined with the other parasitic capacitances around an amplifier, the cable often makes the amplifier break into oscillations. That is, the amplifier is working fine until you connect a cable to its output to measure it. Then it breaks into oscillations. Remove the cable and the oscillations stop. The very act of trying to measure the amplifier's performance destroys that performance.

Instead of connecting to the amplifier's output directly with a BNC cable, use a ×10 oscilloscope probe. A 9 MΩ resistor is added at the tip of the probe, and considerable effort is made to lower the cable's inductance and capacitance. A simplified schematic is shown in Figure 2-9.
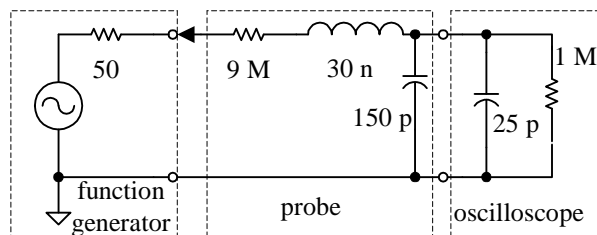

Figure 2-9 Simple ×10 oscilloscope probe

At first glance, this seems reasonable. The 9 MΩ resistor separates the cable's capacitance from the circuit, eliminating oscillations. At dc, the inductor is a short and the capacitors are opens. This just leaves a voltage divider of 9 MΩ and 1 MΩ, producing a ×10 attenuation.

However, the probe's resistance along with the cable's inductance and capacitance form a second order *RLC* low-pass filter. The probe's performance may be seriously degraded.

Example 2-2

Determine, by simulation, the output amplitude of the circuit in Figure 2-9 at DC and at 2 kHz.
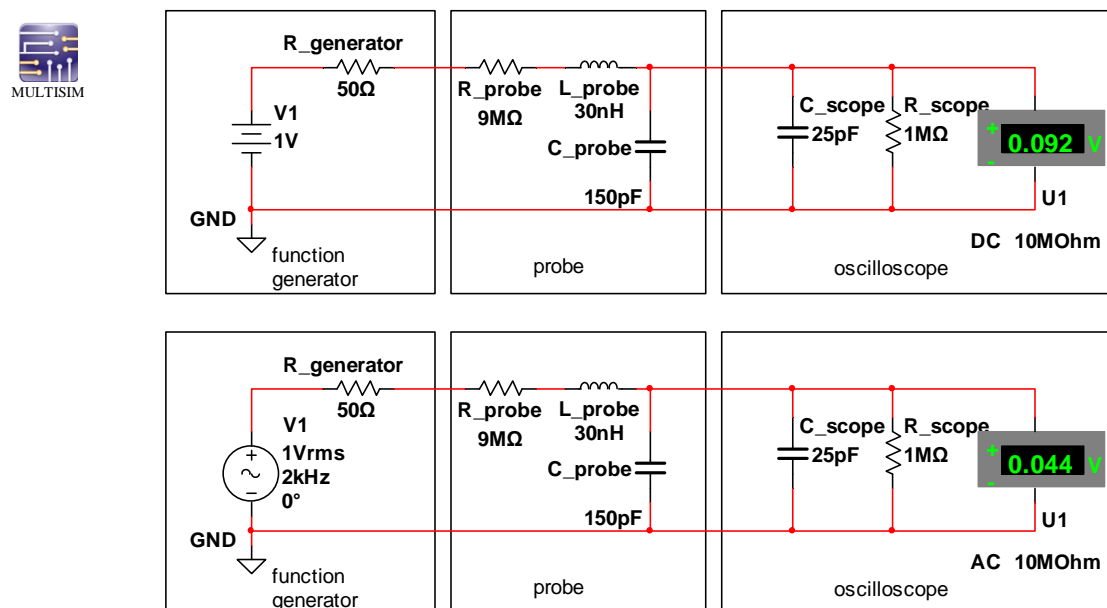
Solution

Figure 2-10 DC and AC responses of simple ×10 probe

At DC, the inductor looks like a short, and the capacitors look like opens, properly dividing the 1 V_{DC} down to nearly 0.1 V_{DC}. The 50 Ω R_generator accounts for the error.

But, at 2 kHz, the voltage passed to the oscilloscope's measuring electronics is 56% low. This makes the probe useful only for DC and *very* low frequencies.

The solution to this severe low-pass filter performance is the *addition* of two more capacitors. Capacitor C1 is placed in parallel with the 9 MΩ input resistor. Combined with the oscilloscope's 1 MΩ input resistor, C1 forms a high-pass filter. It boosts the high frequencies. The 9 MΩ input resistor, the cable's (150 pF) capacitance, the oscilloscope's input capacitance (25 pF), and C2 form a low-pass filter. As this low-pass filter begins to attenuate the high-frequency signals, the high-pass filter (C1 and the 1 MΩ resistor) begins to boost. When the values are properly set, the attenuation from the low-pass filter is precisely offset by the boost from the high-pass filter. The frequency response is flat out to 100 MHz or more.

For the capacitors to provide a $\times 10$ attenuation, the impedance of the three parallel capacitors must be $\frac{1}{9}$ the impedance of C1. This allows the capacitive impedance to match the attenuation provided by the resistors. Since impedance and capacitance are reciprocals, the entire parallel capacitance must be nine times larger than C1.

$$X_{C\,\text{paralllel}} = \frac{1}{2\pi f\, C_{\text{parallel}}} = \frac{1}{9}X_{C1} = \frac{1}{9(2\pi f\, C1)}$$

$$C_{\text{parallel}} = 9 \times C1$$

The exact value of the cable's capacitance and the oscilloscope's input are rarely known. So C2 is a variable capacitor built into the connector of the probe. To adjust C2 to the correct value, apply a square
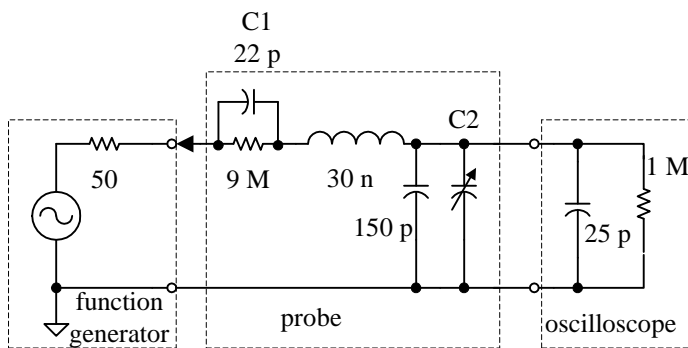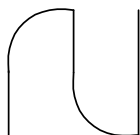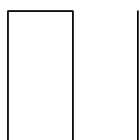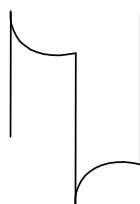

Figure 2-11 Frequency compensated $\times 10$ oscilloscope probe

(a) C2 is too large, the low-pass filter is dominant

(b) C2 is correctly adjusted

(c) C2 is too small, the high-

Figure 2-12 Oscilloscope probe adjustment

wave to the oscilloscope and probe. Adjust the capacitor in the probe's connector until the display is a clean square wave, with no rounding (too much low-pass) or overshoot (too much high-pass). See Figure 2-12.

The impedance from the parallel capacitance is now $\frac{1}{9}$ that from C1. This precisely matches the resistor divider. Since frequency affects both C1 and the parallel capacitance the same, regardless of the frequency, the attenuation is $\frac{1}{10}$.
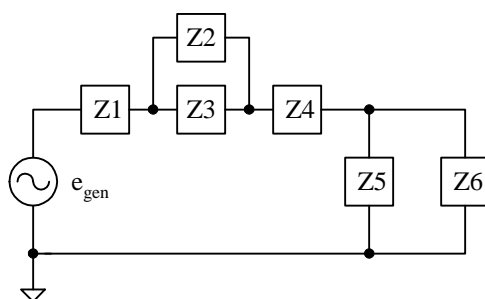

Example 2-3

    a. Calculate the correct value for C2 in the probe schematic of Figure 2-11.

    b. Use the impedance combination technique to calculate the voltage across the oscilloscope's 1 MΩ input resistor at 2 kHz.

    c. Confirm your calculation with a simulation. Also use the simulation to display the response of the oscilloscope's input voltage as the frequency sweeps from 1 Hz to 100 MHz.

Solution

    a.
$$C_{parallel} = 9 \times 22\,pF = 198\,pF$$

$$C2 = 198\,pF - 150\,pF - 25\,pF = 23\,pF$$

    b. The block diagram is given in Figure 14-16.

$$\overline{Z1} = (50\,\Omega\angle 0°)$$
$$\overline{Z2} = (X_{C1}\angle - 90°)$$
$$\overline{Z3} = (9\,M\Omega\angle 0°)$$
$$\overline{Z4} = (X_{L\,cable}\angle 90°)$$
$$\overline{Z5} = (X_{C\,parallel}\angle - 90°)$$
$$\overline{Z6} = (1\,M\Omega\angle 0°)$$

Figure 2-13 Block diagram for Example 2-3

Begin combining impedances at the far right.   Since there are two sets of parallel components, simplify them both in the same step. The resulting diagram in Figure 2-14.

Again, the circuit has simplified into a series circuit. Apply the voltage divider law.

$$\overline{V_{\text{oscilloscope}}} = e_{\text{gen}} \times \frac{\overline{Z_{56}}}{\overline{Z1} + \overline{Z_{23}} + \overline{Z4} + \overline{Z_{56}}}$$

Now, follow your plan, inserting the numbers and completing the calculations. The three reactances are:

$$X_{\text{C1}} = \frac{1}{2\pi \times 2\,\text{kHz} \times 22\,\text{pF}} = 3.617\,\text{M}\Omega$$

$$X_{\text{L cable}} = 2\pi \times 2\text{kHz} \times 30\,\text{nH} = 377\,\mu\Omega \approx 0\,\Omega$$

$$X_{\text{C parallel}} = \frac{1}{2\pi \times 2\,\text{kHz} \times 198\,\text{pF}} = 401.9\,\text{k}\Omega$$

The impedances are:

$$\overline{Z1} = (50\,\Omega\angle 0°)$$

$$\overline{Z2} = (3.617\,\text{M}\Omega\angle -90°)$$

$$\overline{Z3} = (9\,\text{M}\Omega\angle 0°)$$

$$\overline{Z4} = (0\,\Omega\angle 90°)$$

$$\overline{Z5} = (401.9\,\text{k}\Omega\angle -90°)$$

$$\overline{Z6} = (1\,\text{M}\Omega\angle 0°)$$

$$\overline{Z_{56}} = \left[(401.9\,\text{k}\Omega\angle -90°)^{-1} + (1\,\text{M}\Omega\angle 0°)^{-1}\right]^{-1} = (372.9\,\text{k}\Omega\angle -68.1°)$$

$$\overline{Z_{23}} = \left[(3.617\,\text{M}\Omega\angle -90°)^{-1} + (9\,\text{M}\Omega\angle 0°)^{-1}\right]^{-1} = (3.356\,\text{M}\Omega\angle -68.1°)$$

$$\overline{V_{\text{oscilloscope}}} = \frac{(1\,\text{V}_{\text{rms}}\angle 0°) \times (372.9\,\text{k}\Omega\angle -68.1°)}{(50\,\Omega\angle 0°) + (3.356\,\text{M}\Omega\angle -68.1°) + (372.9\,\text{k}\Omega\angle -68.1°)}$$

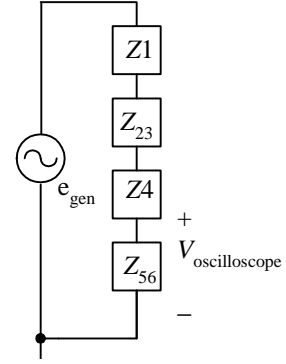$$\overline{V_{\text{oscilloscope}}} = (100.00\,\text{mV}_{\text{rms}}\angle 0°)$$

Figure 2-14
Simplified block diagram for Example 14-4

$$\overline{Z_{56}} = \left(\overline{Z5}^{-1} + \overline{Z6}^{-1}\right)^{-1}$$

$$\overline{Z_{23}} = \left(\overline{Z2}^{-1} + \overline{Z3}^{-1}\right)^{-1}$$

The compensated oscilloscope probe has reduced the voltage by $\frac{1}{10}$.

c.   The simulation is given in Figure 2-15, and confirms the manual calculations.

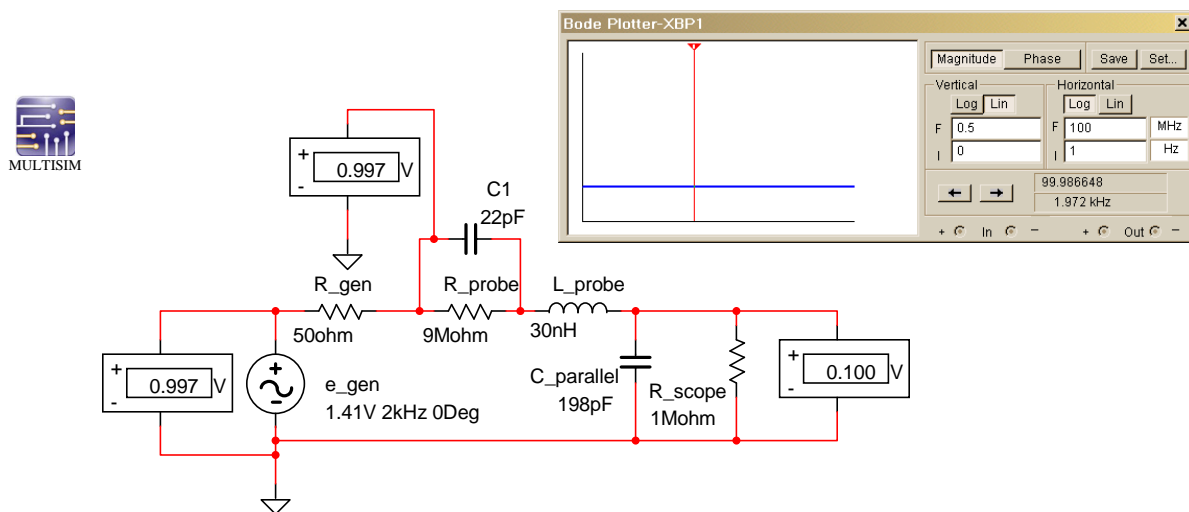The compensated oscilloscope probe's frequency response is flat.

Figure 2-15 Simulation results for Example 2-3

2.2 Superposition

All repetitive wave shapes can be constructed from dc and the proper amplitude, frequency, and phase sine waves. The superposition theorem indicates that the composite input wave shape is just the addition of these signals. The effect that each input signal has on the output can be calculated *independently*. Finally, the composite output wave is the sum of these output signals.

Although this may sound trivial, even obvious, its implication is enormous. Regardless of the wave shape, all you need to be able to do is to figure out how the circuit responds to dc and handle the ac phasor analysis, several times. Then, add up the results.

(a) Complete circuit

Basic Principles

So far, all of the circuits have been driven from a single source. But often, several signals are applied to a circuit simultaneously. To handle these circuits, the superposition theorem states that:

> When multiple *independent* sources drive a linear network, you can determine the effect of all of the sources acting together by calculating the effect of each source individually, and then adding the results.

To turn off a source, replace it with its characteristic impedance. The impedance of an ideal voltage source is 0 Ω, a short, while the impedance of an ideal current source is an open. The output impedance of an op amp may practically be considered to be 0 Ω. Most function generators have an output impedance of 50 Ω, as does most rf circuitry.

Figure 2-16(a) shows a full schematic with three sources. The sine wave from a function generator is *RC* coupled into the circuit where it is combined with the current from a sensor, i_{sensor}, and delivered to a load. There is also a biasing supply, E_{supply}, to consider.

The effect of the function generator alone is shown in Figure 2-16(b). The current source has been replaced with an open and the bias voltage supply with a short. The circuit is much simpler: R1 is in parallel with R2 and the series combination of R3 and R_{load}.
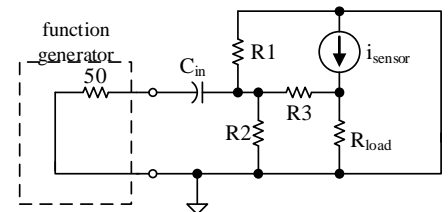
In Figure 11-1(c) the current source is considered. Both of the voltage sources have been replaced by shorts. However, the function generator's 50 Ω output impedance is still in the circuit. The current source is in parallel with the load. R1 continues to parallel R2. The generator's 50 Ω now is in series with the capacitor.
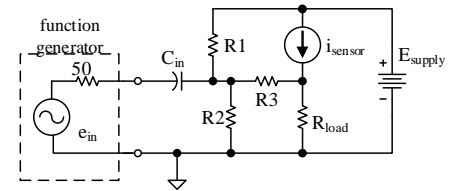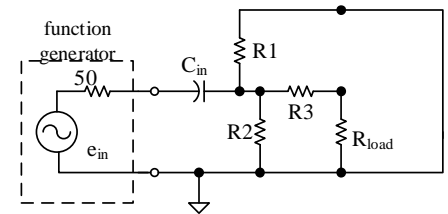
(b) Circuit with only e_{in}

(c) Circuit with only i_{sensor}

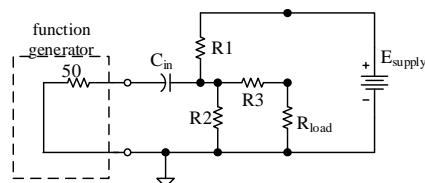**Figure 2-16** Turning sources off when using superposition

(d) Circuit with only E_{supply}

Figure 2-16 Turning sources off when using superposition

Figure 2-16 (d) shows the effect of the bias voltage supply. This time, R1 is in series with a branch containing three resistors. Since the capacitor is an open to dc, the branch containing R1 and C_{in} is ignored.

Each of these circuits results in a configuration that is quite different from the original, and from each other. The results from each are added together to produce the overall solution.

When applying superposition, you may turn *off* only the **independent** sources. Independent sources are those whose values do not change as currents or voltages elsewhere in the circuit change. All three of the sources in Figure 2-16 are independent.

Example 2-4

Superposition analysis of the circuit in Figure 2-16 produces:

$$\overline{V_{\text{load from } ein}} = (2.25\,V_{rms} \angle 0°) \text{ @ 1kHZ}$$

$$\overline{V_{\text{load from } isensor}} = (0.75\ V_{rms} \angle 0°) \text{ @ 3kHZ}$$

$$V_{\text{load from Esupply}} = 2.5\ V_{dc}$$

a. Write the equation for the composite voltage across the load.

b. Plot two cycles of the voltage across the load.

Solution

a. First, each of the phasors must be written in their time domain form.

$$v(t) = V_p \sin(\omega t + \theta)$$

Where $\quad V_p = V_{rms}\sqrt{2}$

$$\omega = 2\pi f$$

Then, they must be added together.

$$\begin{aligned} v_{load} &= [2.25\,V_{rms} \times 1.414\sin(2\pi \times 1\text{kHz} \times t)] \\ &+ [0.75\,V_{rms} \times 1.414\sin(2\pi \times 3\text{kHz} \times t)] \\ &+ 2.5\,V_{dc} \end{aligned}$$

b. The plot of this composite wave is shown in Figure 2-17. It looks much more like a square wave than a sine. Certainly just adding $2.25\ V_{rms} + 0.75\ V_{rms} + 2.5\ V_{dc} = 5.5$ V gives an entirely incorrect result.

v_{load}

```
t=(10e-6:10e-6:2e-3);
v_in=(2.25*1.414)*sin(2*pi*1e3*t);
v_sensor=(0.75*1.414)*sin(2*pi*3e3*t);
V_dc=2.5;
v_load=v_in+v_sensor+V_dc;

plot(t,v_load)
```

time (ms)

c.

$f1(x)=3.18E0 \cdot sin(2 \cdot \pi \cdot 1.E3 \cdot x)+1.06E0 \cdot sin(2 \cdot$
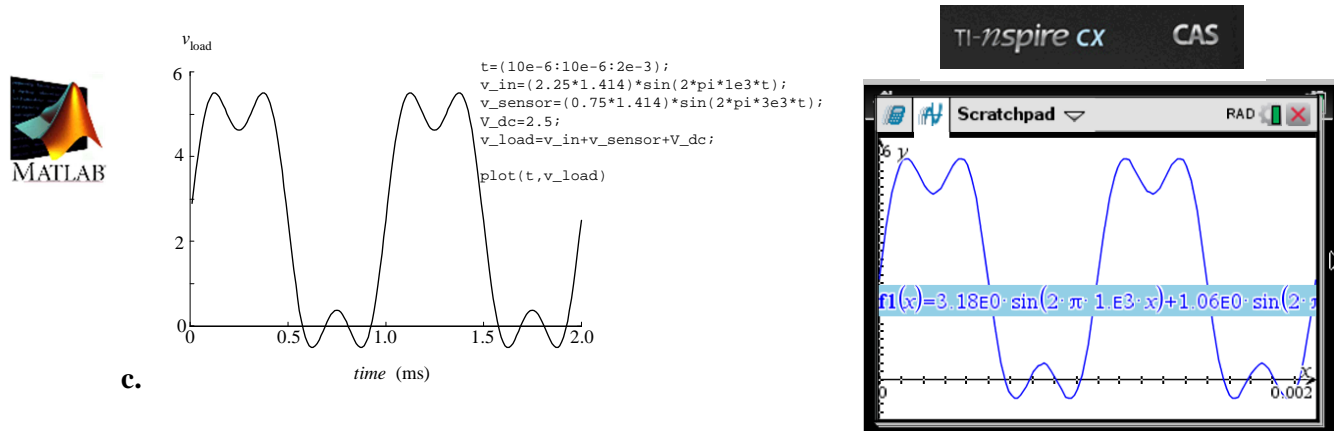
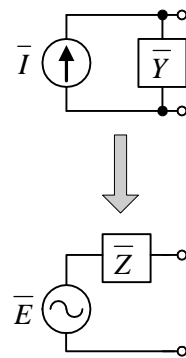**Figure 2-17** Plot and MATLAB commands for Example 2-4

2.3 Mesh Analysis

Impedance combination requires *many* steps to determine the effect of a *single* source. If the circuit has multiple sources, these steps must be repeated for each source, and the results added. At best, this is *very* tedious, and equally error prone.

Mesh analysis uses Kirchhoff's voltage law, and allows you to write a set of simultaneous equations that fully describe the circuit *by inspection*. You can then use your scientific calculations or MATLAB to solve this set of equations. That's it! There are just two steps, one of which you can complete just by looking at the circuit, and the other entering numbers into your calculator. Mesh analysis is performed on circuits that contain voltage sources. Three-phase electrical power systems commonly require mesh analysis.

Mesh analysis is based on Kirchhoff's voltage law. It requires that all sources be *voltage* sources. So, the first step is to convert any current sources to voltage sources. This conversion is shown in Figure 2-17.

$$\overline{Z} = \frac{1}{\overline{Y}} \qquad\qquad \overline{E} = \overline{I} \times \overline{Z}$$

Figure 2-17
Source Conversion