Filter Types and Order

- Change ξ changes the response *type*.
  - Bessel, Butterworth, Chebyshev
  - Transfer function, Matlab, Circuit, Multisim

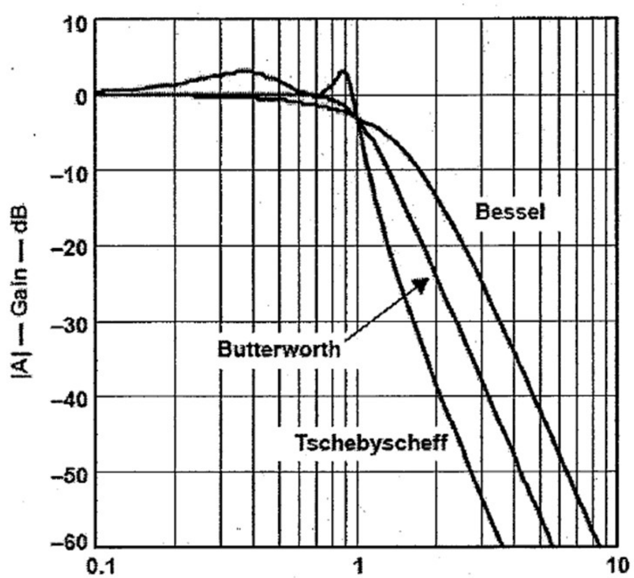
- Low Pass => High Pass
  - Transfer function, Matlab, Circuit, Multisim

Mutistages => Higher order => better roll-off

- Coefficient corrections
- Transfer function, Matlab, Circuit, Multisim

Changing the Damping

Filter Type	Damping (α)	Correction (k_{lp})
Bessel	1.732	0.785
Butterworth	1.414	1.000
3dB Chebyshev	0.766	1.390



Transfer function

$$G = \frac{A_o \omega_0{}^2}{s^2 + 2\xi\omega_o s + \omega_0{}^2}$$

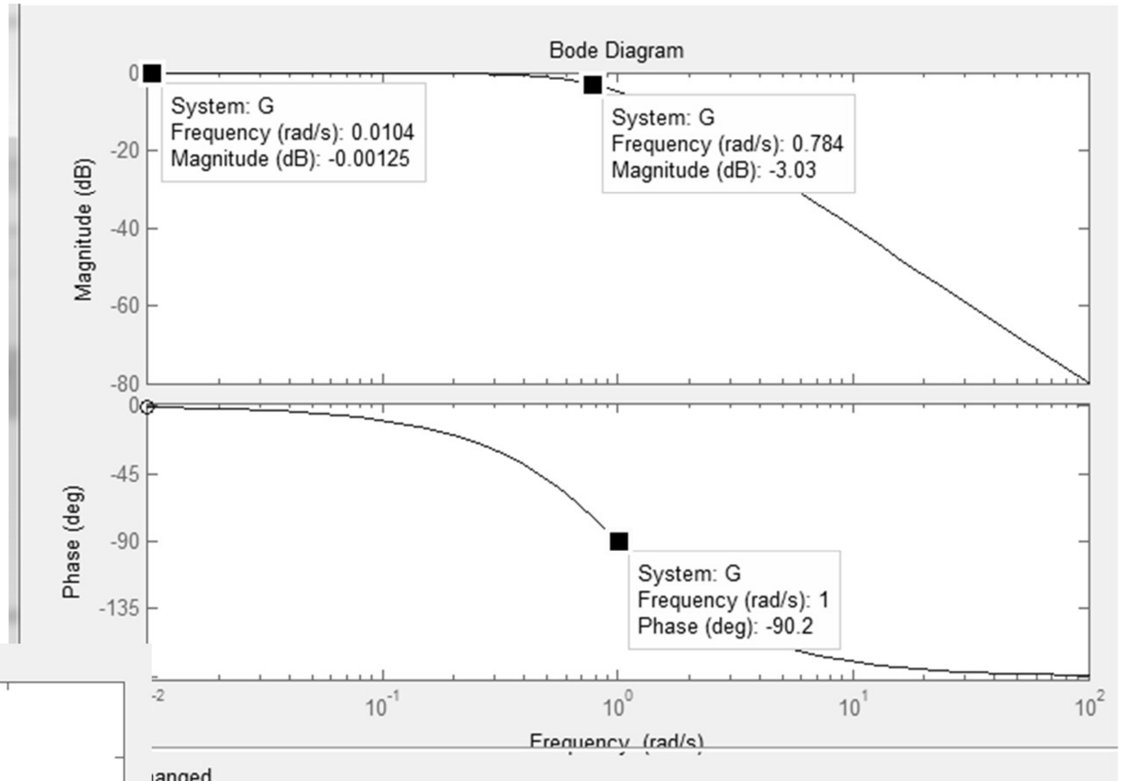
Bessel Characteristics

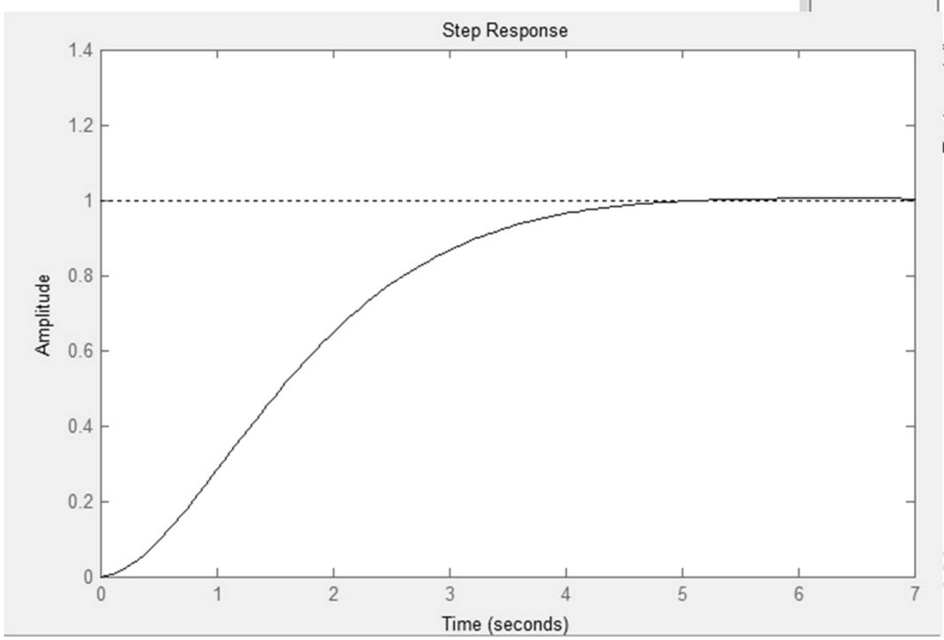
- Flat in the pass band:
- Slower *early* roll-off
- $\xi = 0.866$
  - Nearly critically damped
  - Practically *no* overshoot
  - Good for filtering digital signals
- $f_{-3dB} = k_{lp} * f_o = 0.785 * f_o$ *early*

MATLAB

Bessel - MATLAB

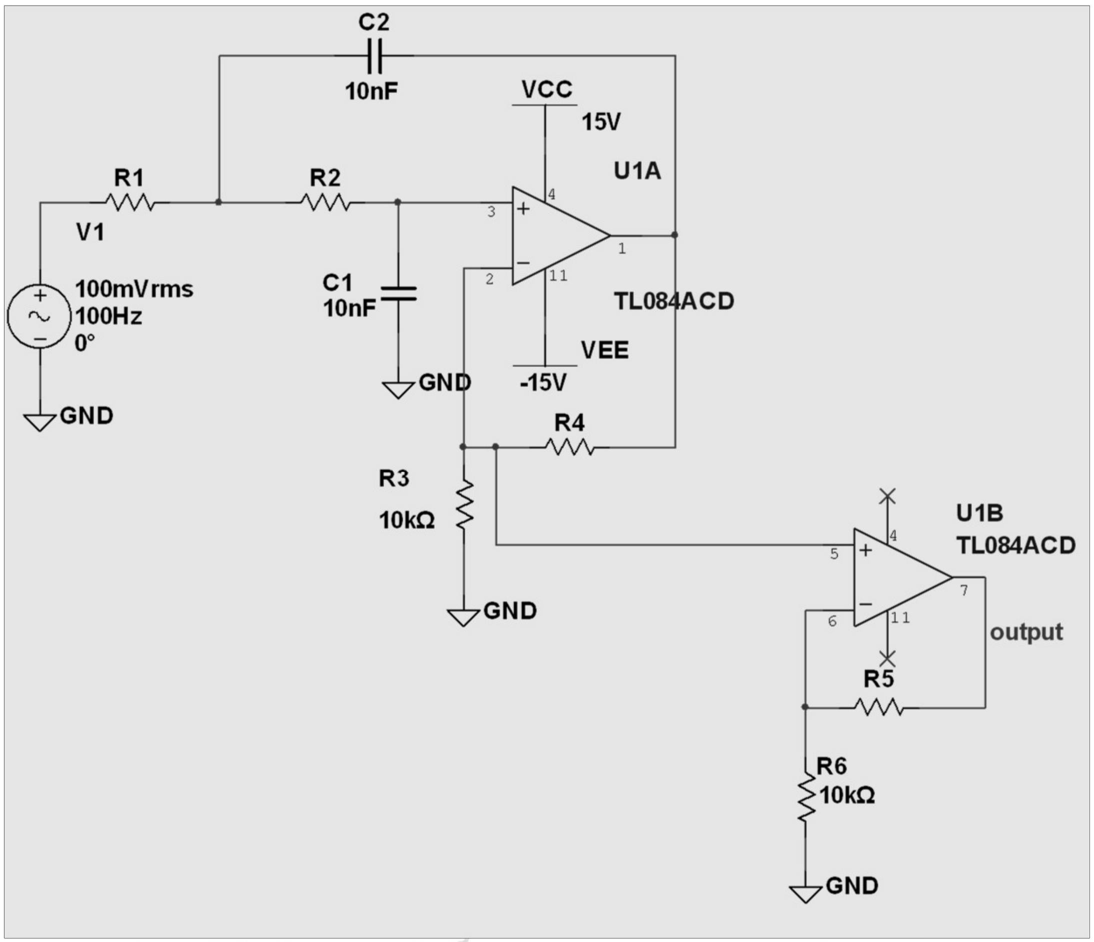
```
clear
format short G
s=tf('s')

Ao=1;
wo=1;
zeta=0.866;

G=Ao*wo^2/(s^2+2*zeta*wo*s+wo^2)

opts = bodeoptions('cstprefs');
opts.FreqUnits = 'Hz';
opts.grid = 'on';
opts.PhaseWrapping = 'on';
opts.MagLowerLimMode = 'manual';
opts.MagLowerLim = -90;

bodeplot(G,{1e-1,1e1},opts);
```





Bessel - Sallen Key Implementation

$$G = \frac{A_o \omega_0{}^2}{s^2 + 2\xi\omega_o s + \omega_0{}^2}$$



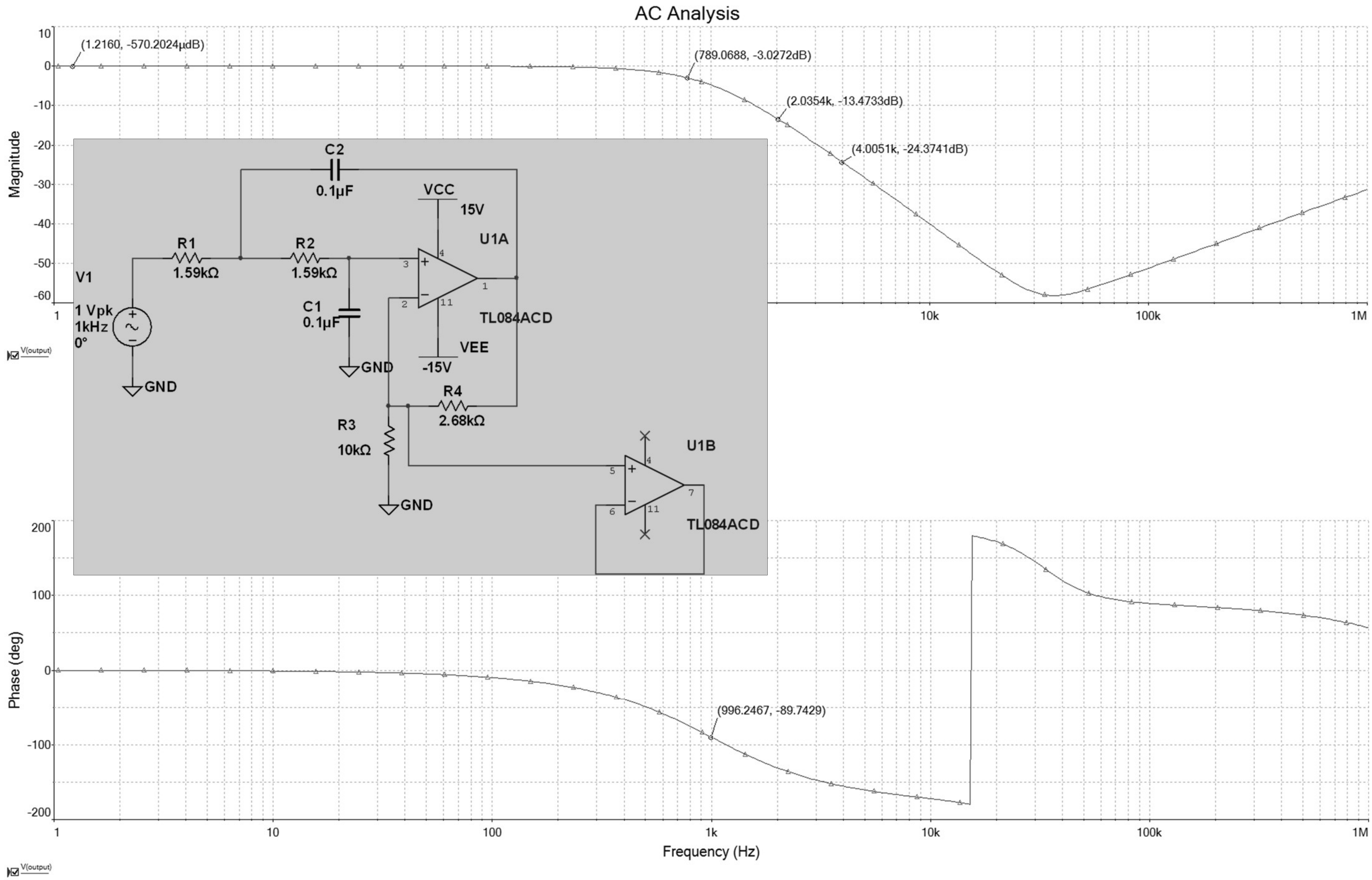
$f_o = 1/(2\pi RC)$

$\alpha = 2\xi = 2*0.866 = 1.732$
$= 3 - A_{int}$
$A_{int} = 1 + R_f/R_i$
$A_{int} = 3 - 1.732$
$= 1.268$

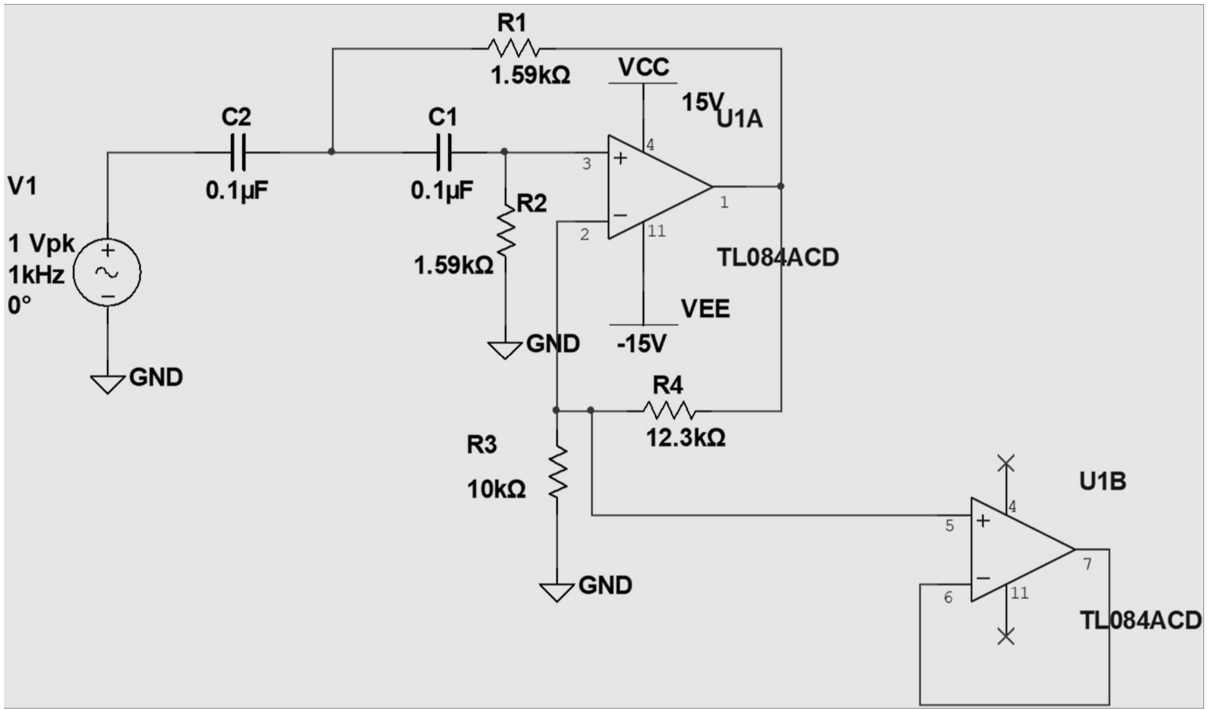
Set damping

Multisim Demo

Bessel - Sallen Key Implementation

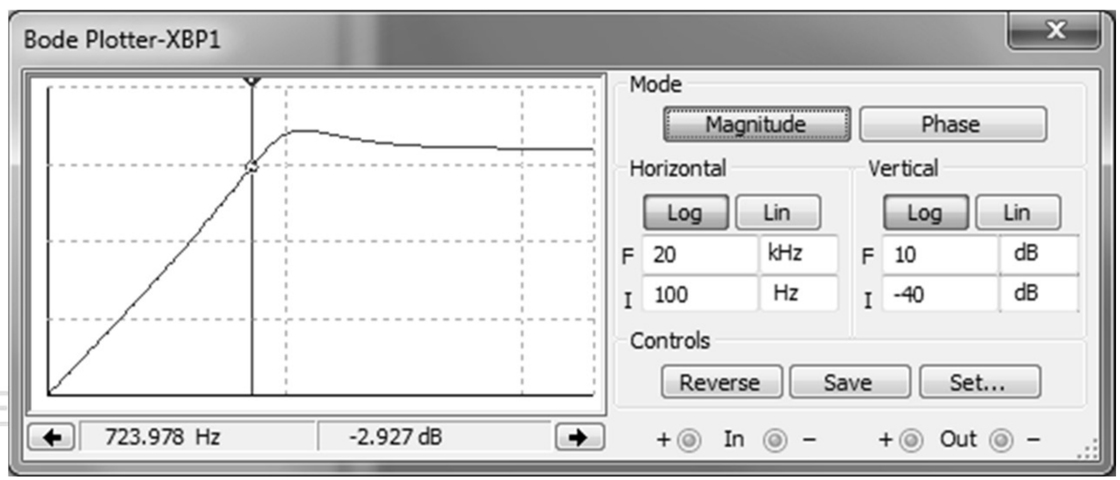


Second Order HP Chebyshev Filters



- $G = \dfrac{A_o s^2}{s^2+2\xi\omega_o s+\omega_0{}^2}$

- Multisim



Second Order HP Chebyshev Filters

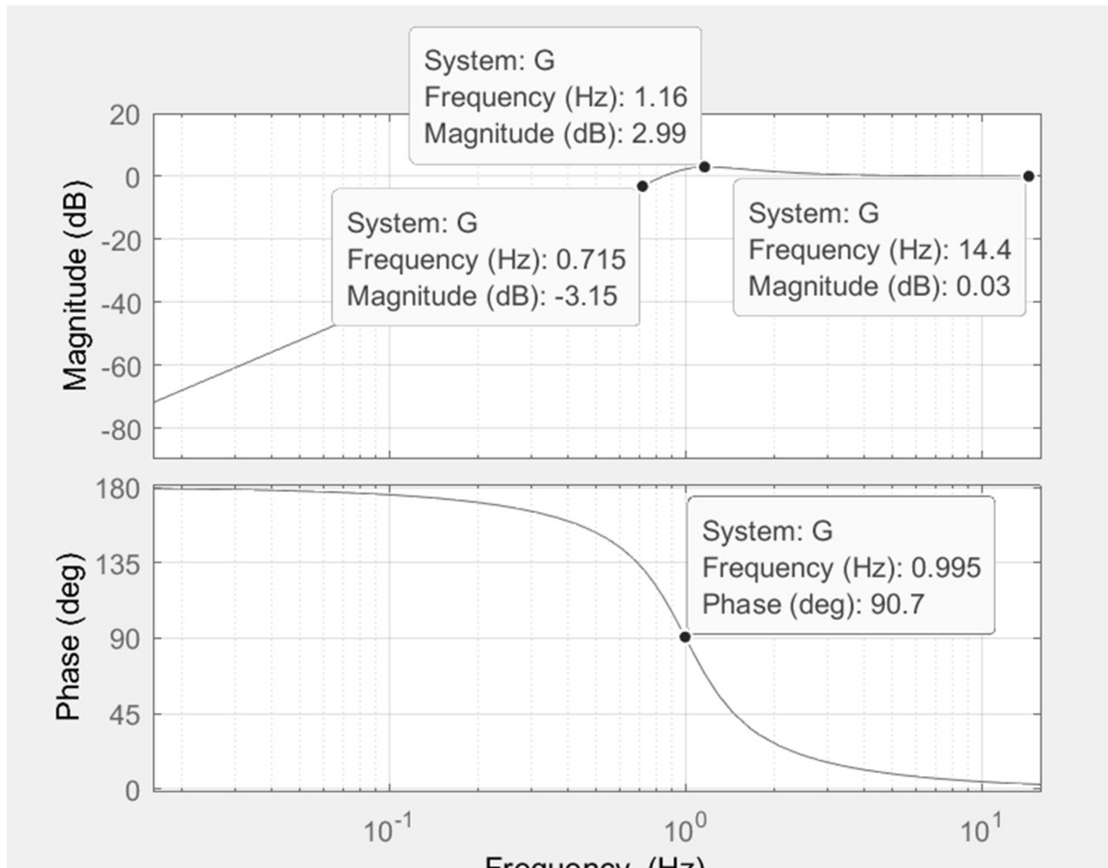
$f_{-3dBLP}=k_{lp}*f_o$

$k_{HP}=1/k_{lp}$

$f_{-3dBHP}=k_{HP}*f_o$

- $G=\frac{A_o s^2}{s^2+2\xi\omega_o s+\omega_0{}^2}$
- MALAB

```
clear
format short G
s=tf('s')

Ao=1;
fo=1;
wo1=2*pi*fo;
zeta1=0.383;


G=(Ao*s^2/(s^2+2*zeta1*wo1*s+wo1^2))

opts = bodeoptions('cstprefs');
opts.FreqUnits = 'Hz';
opts.grid = 'on';
opts.PhaseWrapping = 'on';
opts.MagLowerLimMode = 'manual';
opts.MagLowerLim = -90;

bodeplot(G,{1e-1,1e2},opts);
```



Higher Order

- Adjust each stage RC and A_{int} Corrections $\alpha = 2\xi = 3\text{-}A_{int}$
  $A_{int} = 1+R_f/R_i$ $f_{-3dB}= k_{LP1}f_o$ $f_{-3dB}= k_{LP2}*f_o$

Filter Order	Section		Bessel	Butterworth	3dB Cheby
2	2	α	1.732	1.414	0.766
		k_{lp}	0.785	1.000	1.390
3	1	α	-	-	-
		k_{lp}	0.753	1.000	3.591
	2	α	1.447	1.000	0.326
		k_{lp}	0.687	1.000	1.172
4	2	α	1.916	1.848	0.929
		k_{lp}	0.696	1.000	2.349
	2	α	1.242	0.765	0.179
		k_{lp}	0.621	1.000	1.095
5	1	α	-	-	-
		k_{lp}	0.665	1.000	5.762
	2	α	1.775	1.618	0.468
		k_{lp}	0.641	1.000	1.670
	2	α	1.091	0.618	0.113
		k_{lp}	0.569	1.000	1.061
6	2	α	1.959	1.932	0.958
		k_{lp}	0.621	1.000	3.412
	2	α	1.636	1.414	0.289
		k_{lp}	0.590	1.000	1.408
	2	α	0.977	0.518	0.078
		k_{lp}	0.523	1.000	1.042

3rd Order LP Bessel

Filter Order	Section		Bessel
2	2	α	1.732
		k_{lp}	0.785
3	1	α	-
		k_{lp}	0.753
	2	α	1.447
		k_{lp}	0.687

$$\omega_o = \frac{1}{\tau}$$

$$f_{3dB} = k_{lp} \times f_o \qquad f_o = \frac{f_{3dB}}{k_{LP}}$$

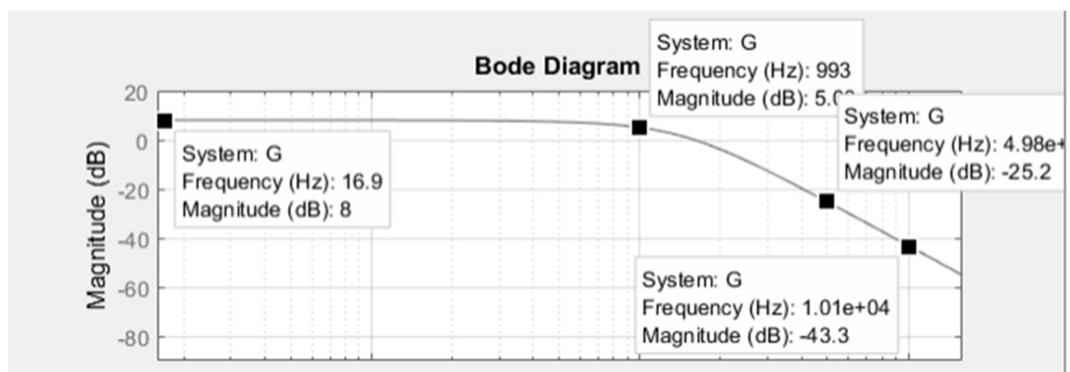
$$G = \frac{1}{\tau_1 s + 1} \times \frac{A_o \omega_{o2}}{s^2 + \alpha_2 \omega_{o2} s + \omega_{02}{}^2}$$

```
clear
format short G
s=tf('s')

AdB=8;
Ao=10^(AdB/20);
f3dB=1e3;
kLP1=0.753;
kLP2=0.687;
a2=1.447;

fo1=f3dB/kLP1;
wo1=2*pi*fo1;
tau1=1/wo1;

fo2=f3dB/kLP2;
wo2=2*pi*fo2;

G=(1/(tau1*s+1))*(Ao*wo2^2/(s^2+a2
*wo2*s+wo2^2))

opts = bodeoptions('cstprefs');
opts.FreqUnits = 'Hz';
opts.grid = 'on';
opts.PhaseWrapping = 'on';
opts.MagLowerLimMode = 'manual';
opts.MagLowerLim = -90;

bodeplot(G,{1e2,100e3},opts);
```

$$\omega_o = \frac{1}{\tau}$$

$$f_{3dB} = k_{lp} \times f_o \qquad f_o = \frac{f_{3dB}}{k_{LP}}$$

$$G = \frac{1}{\tau_1 s + 1} \times \frac{A_o \omega_{o2}}{s^2 + \alpha_2 \omega_{o2} s + \omega_{02}{}^2}$$

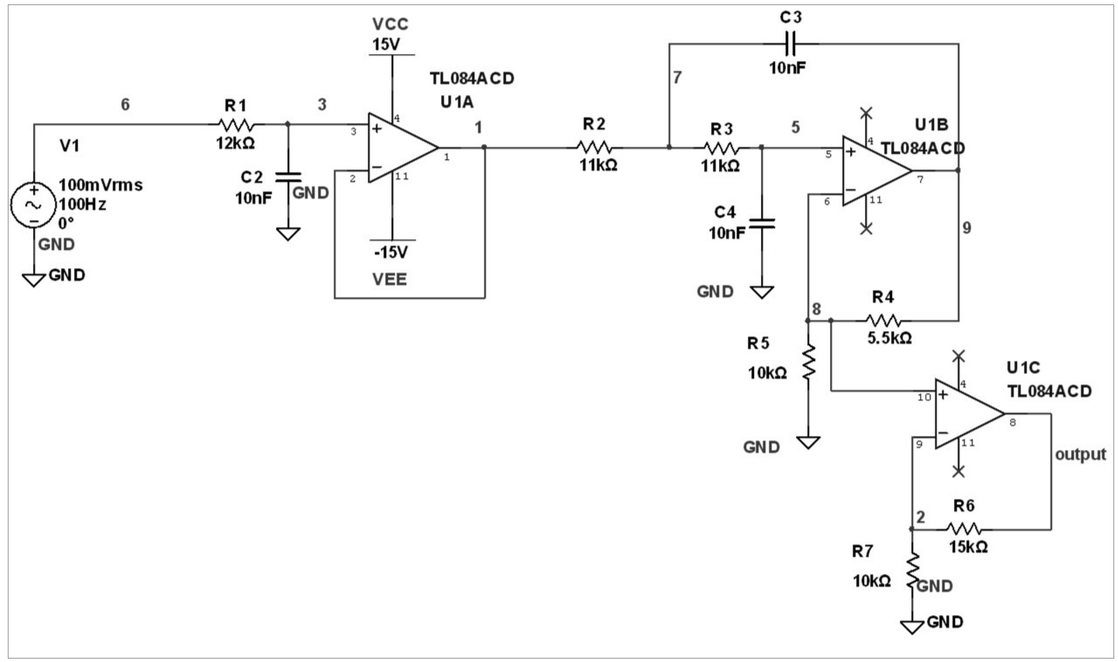


Sallen-Key Configuration

$$f_{3dB} = k_{lp} \times f_o = \frac{k_{LP1}}{2\pi R_1 C}$$

$$R_1 = \frac{k_{LP1}}{2\pi f_{3dB} \quad C}$$

$$R_2 = R_3 = \frac{k_{LP2}}{2\pi f_{3dB} \quad C}$$



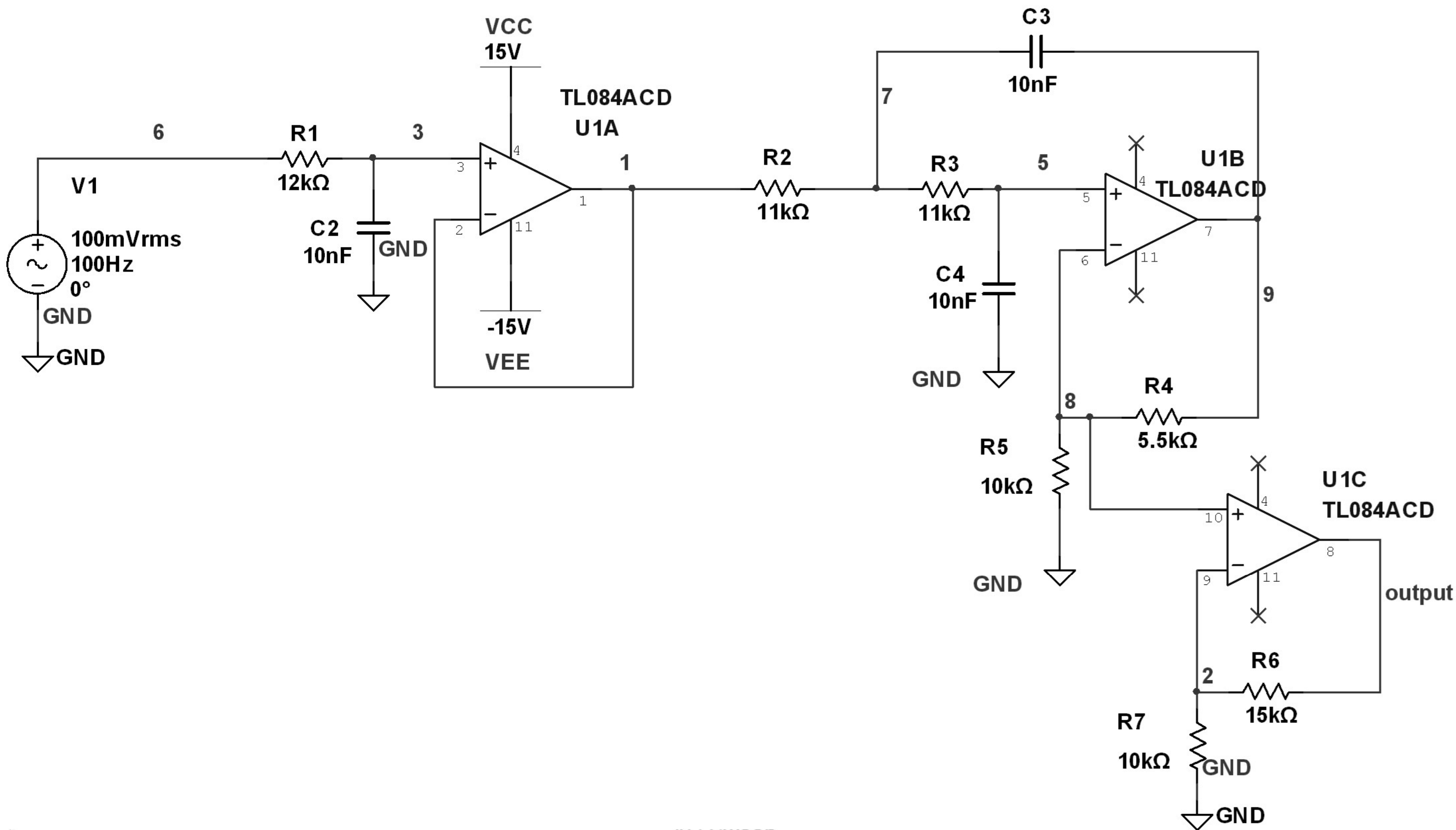
$$A_{int} = 3 - \alpha = 3 - 1.447 = 1.553$$

$$A_o = 2.5 = 1 + \frac{R_6}{7}$$

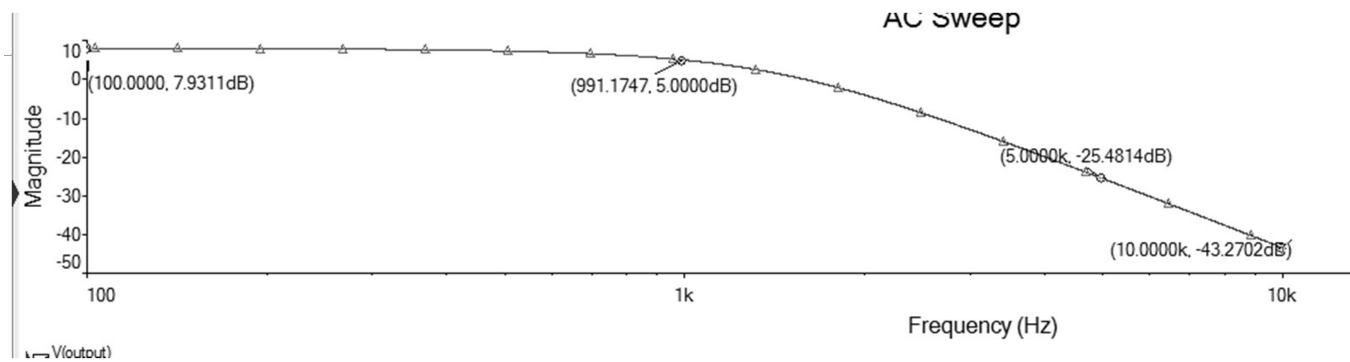
$$A_{int} = 1.553 = 1 + \frac{R_4}{R_5}$$

$R_7 = 10\ k\Omega \qquad R_6 = 15\ k\Omega$

$R_4 = 10\ k\Omega \qquad R_5 = 5.5\ k\Omega$



VCC
15V
TL084ACD
U1A
6
R1
3
12kΩ
V1
100mVrms
100Hz
0°
GND
GND
C2
10nF
GND
-15V
VEE
1
R2
11kΩ
C3
10nF
7
R3
11kΩ
5
U1B
TL084ACD
C4
10nF
GND
9
8
R4
5.5kΩ
R5
10kΩ
GND
U1C
TL084ACD
output
R6
15kΩ
2
R7
10kΩ
GND
GND



AC Sweep
(100.0000, 7.9311dB)
(991.1747, 5.0000dB)
(5.0000k, -25.4814dB)
(10.0000k, -43.2702dB)
Magnitude
Frequency (Hz)
V(output)