3.1 Math with Key Waveforms

All of the contributions that electronics make to our lives are carried on a small number of different wave shapes. Once you can write the time domain equation for each, its derivative, and its integral, you can determine the effect any circuit has on that waveform, or the circuit needed to shape what you have into what you need.

A more complex waveform can be assembled from the piece-wise linear combination of this small set of standard signals. And, derivatives and integrals can then be completed, in pieces.

DC

DC is just a constant. Mathematically, it is the simplest to manipulate. The time domain equation is a constant, since the voltage does not change. That also means that the derivative is 0 (no change with respect to time). The integral of a constant is that constant times the variable *t*. As time goes on, more and more is added to the summation (the integral). The area under the line increases as the width (t) increases.

$$v(t) = A$$

$$\frac{d}{dt} A = 0$$

$$\int_0^t A\,dt = At$$

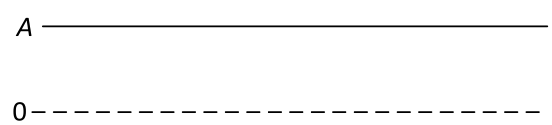


Figure 3-1 DC waveform

Sinusoid

The sine wave is probably the second most common wave form. Commercial power is produced and distributed as sinusoids. It also is a valuable test signal since it does *not* change its shape as it is processed by *linear* components. Its amplitude and phase may be altered, but the fundamental shape passes through unchanged. Look at the derivative and integral below. This makes testing circuits and detecting nonlinearity and distortion simpler. Finally, with Fourier Analysis, *any* repetitive wave shape can be built by combining different frequency and amplitude sine waves.

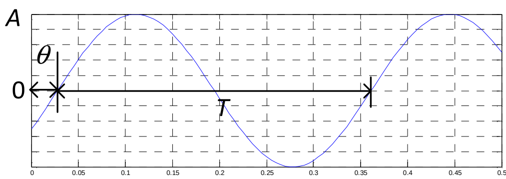


Figure 3-2 Sinusoid with amplitude (*A*), period (*T*), and phase (*θ*) defined

$$v(t) = A\sin(2\pi f t - \theta) \quad f = \frac{1}{T}$$

$$\frac{d}{dt} A\sin(2\pi f t - \theta) = A \times 2\pi f \times \cos(2\pi f t - \theta)$$

$$\int_0^t A\sin(2\pi f t - \theta)dt = \frac{-A}{2\pi f}[\cos(2\pi f t - \theta) - \cos(\theta)]$$

Basic calculus or fundamental tables give the derivative and the integral of sin (ϕ). But, what about all of the other variables? What do they do to the resulting function? The chain rule can get a little complicated to implement. The most direct way to evaluate this waveform's derivative and integral is with the TI-Nspire calculator.

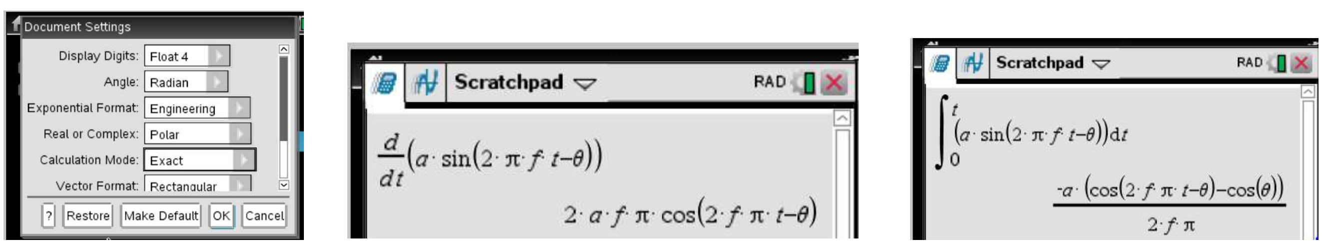


Figure 3-3 TI-Nspire derivative and integral of a general sine wave

The key screens are shown in Figure 3-3. First in the **Document settings**, select **Radians**, and **Exact**. Then, in the menu, select **4:Calculus**, then **1:Derivative**. The ***θ*** symbol is on the book key. The integral is found the same way, selecting **3:Integral** instead of **1:Derivative**.

Ramp

The ramp or triangle wave rises or falls at a constant rate, taking a set time to rise or fall one volt or one amp. The rising wave is shown in Figure 3-4. Often voltage is easier to sense than time. So many timing circuits have a ramp at their heart. Knowing how fast the ramp is rising, the circuit just waits until the voltage has risen to proscribed level. When voltage is an accurate measure of position, such as in a cathode ray tube or a robotic servo position control system, then a ramp can be used to move the controlled object at a constant speed.

$$v(t) = mt + b = \frac{A}{T}t + b$$

$$\frac{d}{dt}\left(\frac{A}{T}t + b\right) = \frac{A}{T}$$

$$\int_0^t \left(\frac{A}{T}t + b\right)dt = \frac{A}{2T}t^2 + bt$$

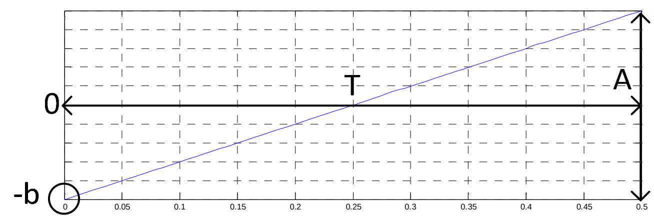


Figure 3-4 Ramp with amplitude (*A*), period (*T*) and offset (-*b*) defined

TI-nspire CX CAS

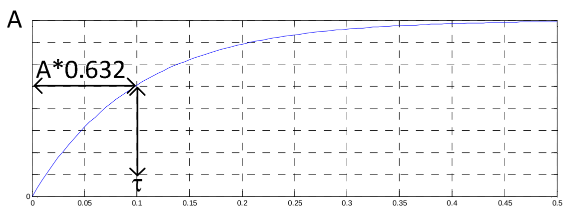
Figure 3-5 TI-Nspire derivative and integral of a ramp

The TI-Nspire calculations for the ramp are shown in Figure 3-5. This version of software does *not* recognize the difference between upper and lower case, *T* and *t*. So, the period has been represented by *p*.

Exponential Rise

Capacitance and inductance store energy in electrostatic and electromagnetic fields. As the stored energy increases, so does the voltage or current. But, the more voltage or current already present, the harder it is to force even more charge (and therefore even more voltage or current) onto the charging part. The part charges more slowly, and voltage or current rises more slowly. That is an *exponential* rise, *not* linear as in the preceding section. Look at Figure 3-6.

The time constant, τ, is defined as the length of time it takes the output to change by 63.2% (i.e. 1-e^{-1}) of its maximum possible change. In this example the output starts at 0 and will eventually reach *A*. So, its maximum possible change is *A**0.632.



$$v(t) = A\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\frac{d}{dt}\left[A\left(1 - e^{\frac{-t}{\tau}}\right)\right] = \frac{A}{\tau}e^{\frac{-t}{\tau}}$$

$$\int_0^t \left[A\left(1 - e^{\frac{-t}{\tau}}\right)\right] dt = A\left[t - \tau\left(1 - e^{\frac{-t}{\tau}}\right)\right]$$

Figure 3-6 Exponential rise with amplitude (*A*), and time constant (τ) defined

TI-nspire CX CAS

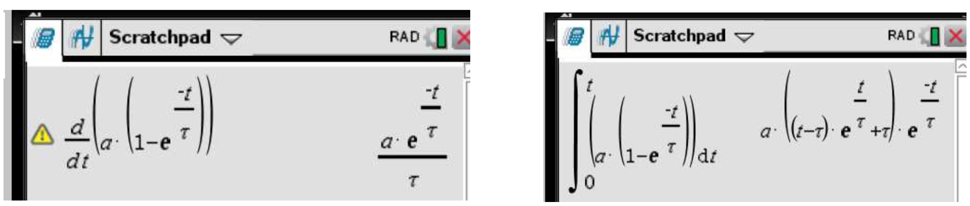


Figure 3-7 TI-Nspire derivative and integral of an exponential rise

When entering the exponential into the calculator, be sure to use the button especially for that function. Do *not* enter the letter *e* and then try to raise it to an exponent.

The integral returned by the TI-Nspire seems more complicated than the equation given beside Figure 3-6. But, a little algebra produces a match.

Exponential Spike or Fall

As capacitance or inductance *dis*charges, voltage or current also *falls* exponentially. This is shown in Figure 3-8.

Initially fully charged, energy flows rapidly off of the capacitance or inductance. Like charges oppose. So lots of charge forces charge off quickly. But as the charge falls, there is less charge left forcing charge away. So it leaves more slowly. The less there is, the more slowly it escapes.

The time constant is still the time it takes for the output to change 63.2% of its maximum change. Since it starts at *A*, a 63.2% change is a fall down to 36.8% of *A*.

$$v(t) = Ae^{\frac{-t}{\tau}}$$

$$\frac{d}{dt}\left(Ae^{\frac{-t}{\tau}}\right) = \frac{-A}{\tau}e^{\frac{-t}{\tau}}$$

$$\int_0^t \left(Ae^{\frac{-t}{\tau}}\right) dt = A\tau\left(1 - e^{\frac{-t}{\tau}}\right)$$

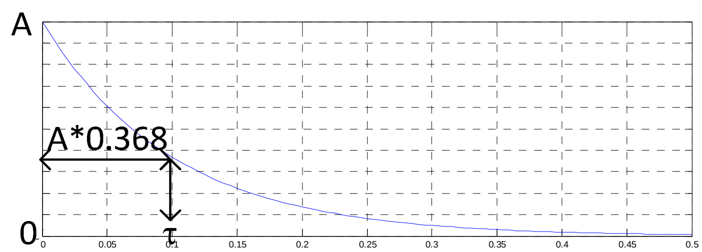


Figure 3-8 Exponential fall with amplitude (*A*), and time constant (τ) defined

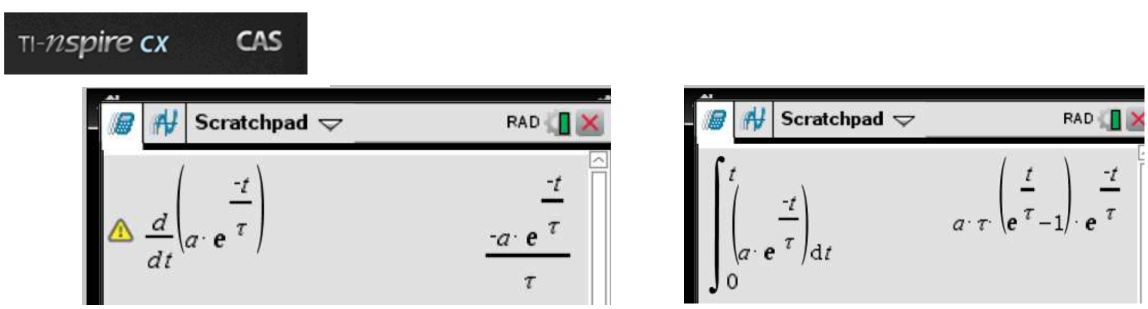


Figure 3-9 TI-Nspire derivative and integral of an exponential fall or spike

As with the exponential rise, it takes a little algebra to change the answer given by the TI-Nspire into a more convenient form.