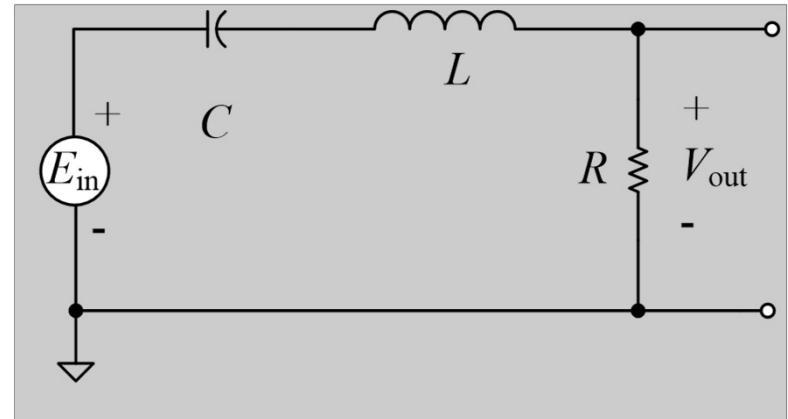


What if there is an L and a C?

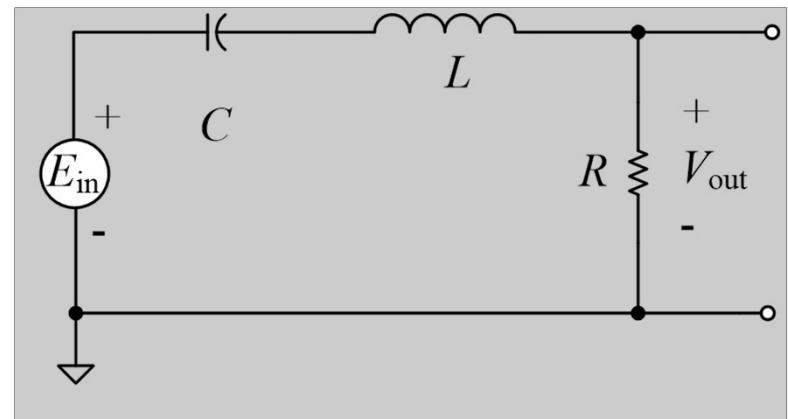
UNKNOWN

Ψ	R	C	L
$i(t)$	$\frac{v(t)}{R}$	$C \cdot \frac{dv(t)}{dt}$	$\frac{1}{L} \int_0^t v(t) dt + I_o$
$v(t)$	$R \cdot i(t)$	$\frac{1}{C} \int_0^t i(t) dt + V_o$	$L \cdot \frac{di(t)}{dt}$



UN KNOWN

Ψ	R	C	L
$i(t)$	$\frac{v(t)}{R}$	$C \cdot \frac{dv(t)}{dt}$	$\frac{1}{L} \int_0^t v(t) dt + I_o$
$v(t)$	$R \cdot i(t)$	$\frac{1}{C} \int_0^t i(t) dt + V_o$	$L \cdot \frac{di(t)}{dt}$



$$E = v_C + v_L + v_R$$

$$E = \frac{1}{C} \int_0^t idt + V_{oc} + Li' + Ri$$

$$\frac{dE}{dt} = 0 = \frac{1}{C} i + Li'' + Ri'$$

$$Li'' = -Ri' - \frac{1}{C} i$$

$$i'' = -\frac{R}{L} i' - \frac{1}{LC} i$$

$$i(0) = 0$$

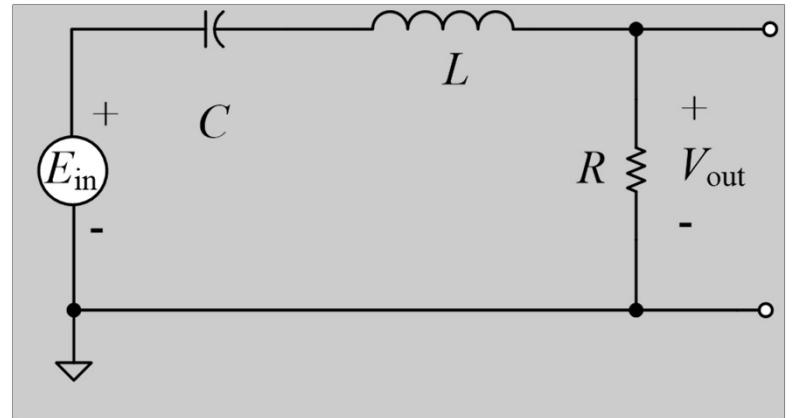
$$i'(0) = ?$$

An inductor can't change its voltage instantaneously, so at $t = 0+$ the inductor looks like an open and all the voltage appears across the L.

$$v_L(0) = Li'(0)$$

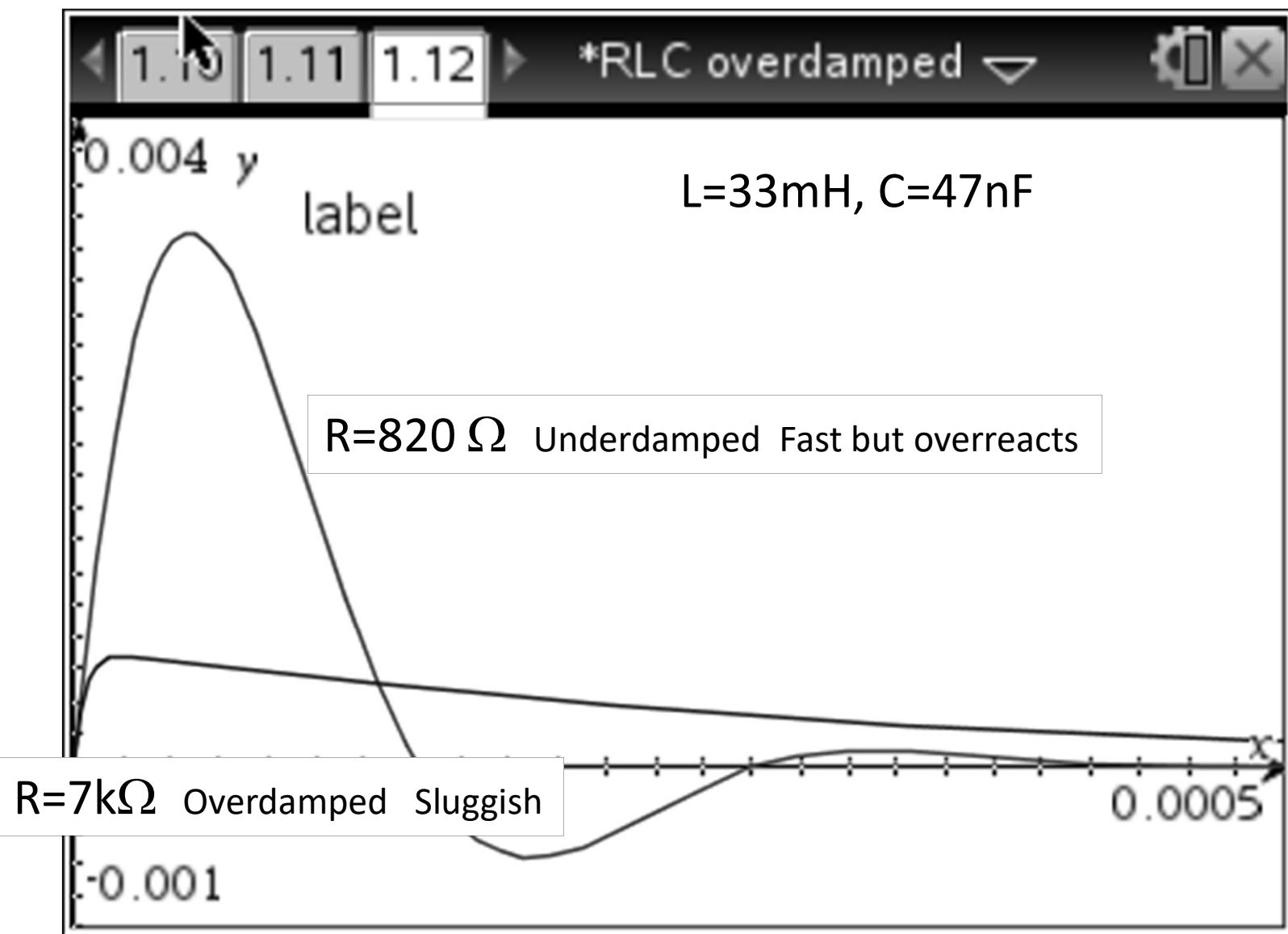
$$i'(0) = \frac{E}{L}$$

$$i'' = -\frac{R}{L}i' - \frac{1}{LC}i \quad i(0) = 0 \quad i'(0) = \frac{E}{L}$$



deSolve($i'' = -(R/L)*i' - (1/(L*C))*i$ and $i(0)=0$ and
 $i'(0)=(E/L), t, i$)

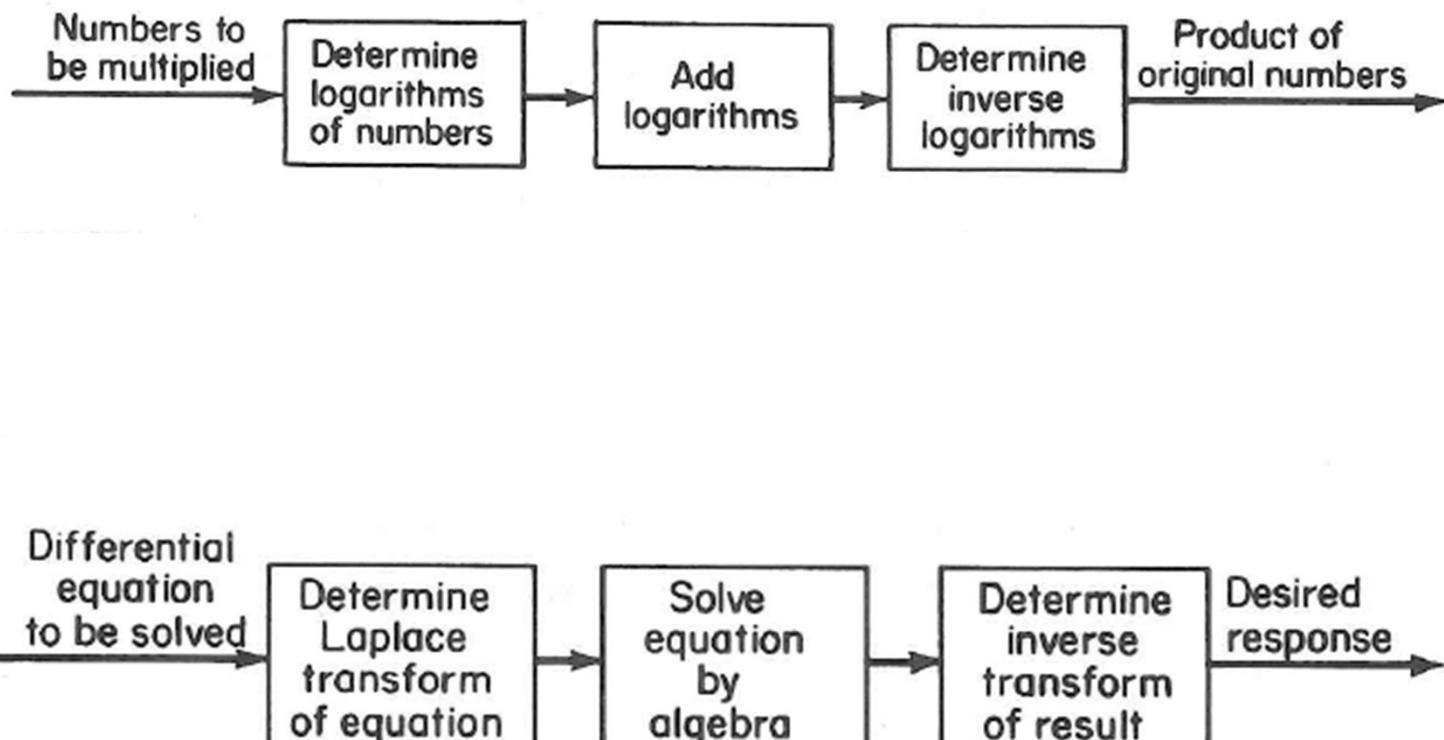
$$i = \frac{e \cdot \text{sign}(l) \cdot e^{\frac{-\sqrt{\frac{c \cdot r^2 - 4 \cdot l}{c}} \cdot t}{2 \cdot |l|}} - \frac{r \cdot t}{2 \cdot l}}{\sqrt{\frac{c \cdot r^2 - 4 \cdot l}{c}}} - \frac{e \cdot \text{sign}(l) \cdot e^{\frac{-\sqrt{\frac{c \cdot r^2 - 4 \cdot l}{c}} \cdot t}{2 \cdot |l|}} - \frac{r \cdot t}{2 \cdot l}}{\sqrt{\frac{c \cdot r^2 - 4 \cdot l}{c}}}$$



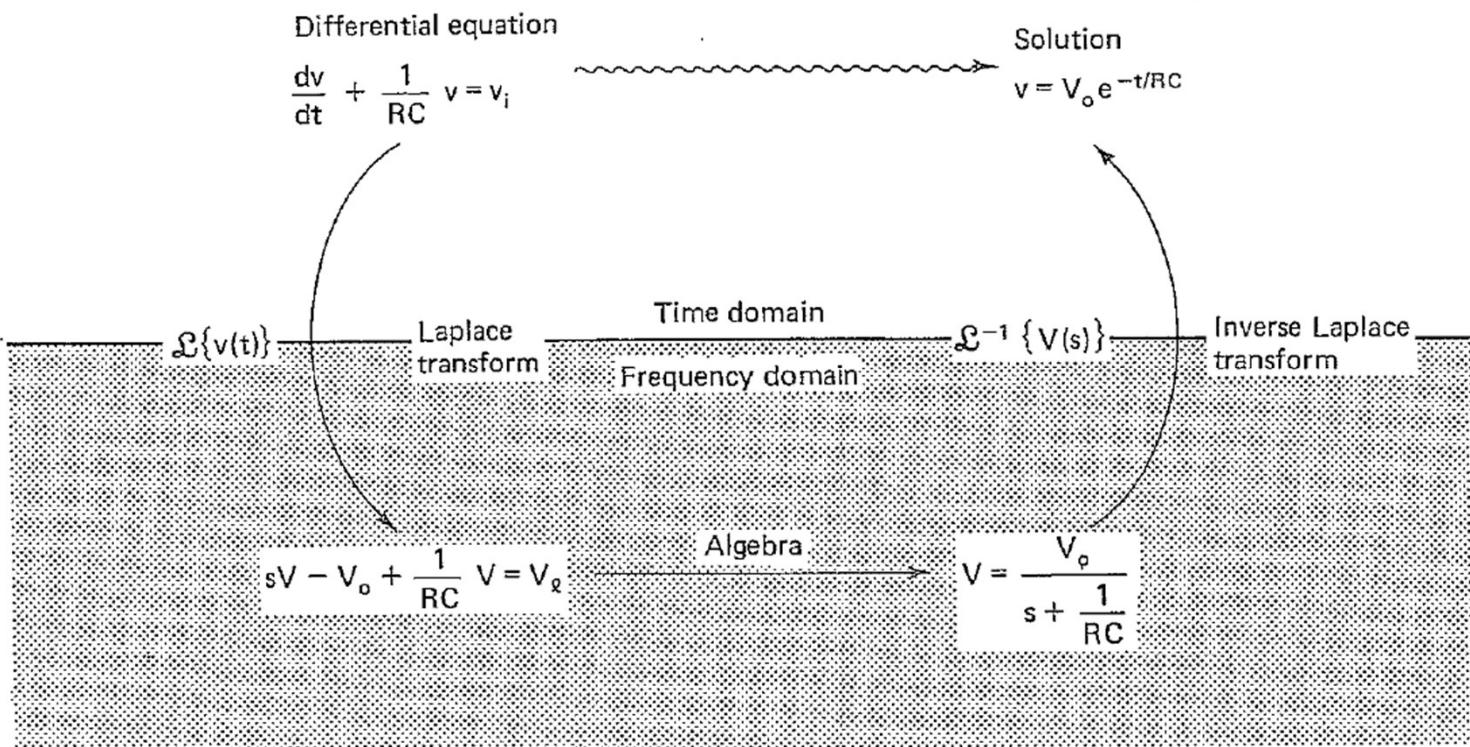
Introduction to Laplace Transforms

- Transform Concept
- Calculus => transform tables
- Transform Operations
 - Addition
 - Multiplication
 - Differentiation
 - Integration
- Roots, poles, zeros and the quadratic equation

Laplace Transform Concept



Laplace Transform Concept



Transform Definition => calculus

$$\mathcal{L}\{f(t)\} = F(s)$$

$$f(t) = e^{-at}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

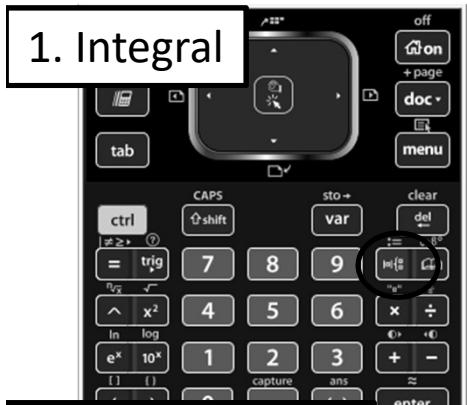
$$f(t) = \int_{a-i\infty}^{a+i\infty} F(s) e^{st} dt$$

Transform Definition => calculus => TI-nspire

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

1. Integral

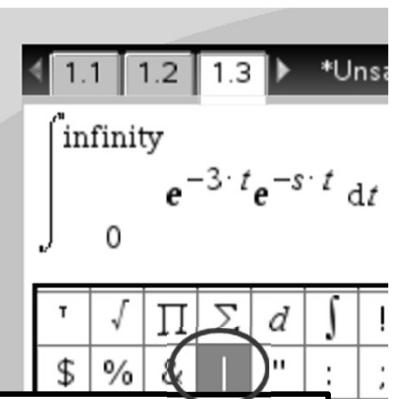


2. Exponential
exp() or button

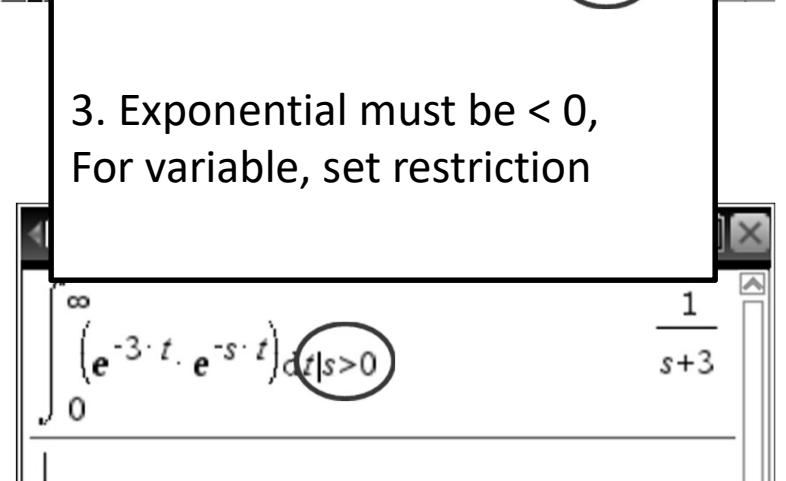
Must be < 0,
not variable



$$f(t) = e^{-3t}$$



3. Exponential must be < 0,
For variable, set restriction



Transform Definition => calculus

$$\mathcal{L}\{f(t)\} = F(s)$$

$$f(t) = \sin(\omega t)$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

Transform Operations

$$\mathcal{L}[Kf(t)] = K\mathcal{L}[f(t)] = KF(s)$$

$$\mathcal{L}[f_1(t) + f_2(t) + \cdots + f_n(t)] = \mathcal{L}[f_1(t)] + \mathcal{L}[f_2(t)] + \cdots + \mathcal{L}[f_n(t)]$$

$$\mathcal{L}[f_1(t)f_2(t)] \neq F_1(s)F_2(s)$$

Transform Definition => calculus => tables!

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$f(t) = \int_{a-i\infty}^{a+i\infty} F(s) e^{st} dt$$

TABLE 5-1
Common Laplace transform pairs

$f(t)$	$F(s) = \mathcal{L}[f(t)]$	
$\delta(t)$	1	(T-1)
1 or $u(t)$	$\frac{1}{s}$	(T-2)
t	$\frac{1}{s^2}$	(T-3)
$e^{-\alpha t}$	$\frac{1}{s + \alpha}$	(T-4)
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	(T-5)
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	(T-6)
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s + \alpha)^2 + \omega^2}$ ^a	(T-7)
$e^{-\alpha t} \cos \omega t$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$ ^a	(T-8)
t^n	$\frac{n!}{s^{n+1}}$	(T-9)
$e^{-\alpha t} t^n$	$\frac{n!}{(s + \alpha)^{n+1}}$	(T-10)

^a Complex roots

Transform Table

TABLE A-1 LAPLACE TRANSFORMS

No.	$F(s)$	$f(t)$	Comments
1.	1	$\delta(t)$	Unit impulse
2.	$\frac{A}{s}$	$A(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$	Step
3.	$\frac{1}{s}$	$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$	Unit step
4.	$\frac{A}{s^2}$	At	Ramp
5.	$\frac{2A}{s^3}$	At^2	Parabola
6.	$\frac{A\omega}{s^2 + \omega^2}$	$A \sin \omega t$	Sine

TABLE A-1 (continued)

No.	$F(s)$	$f(t)$	Comments
7.	$\frac{As}{s^2 + \omega^2}$	$A \cos \omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s) - f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t) dt$	
13a.	$\frac{A}{ts + 1}$	$\frac{A}{\tau} e^{-\nu t}$	Free response of first-order system
13b.	$\frac{A}{s + a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{A}{\tau_1 - \tau_2} (e^{-\nu_1 t} - e^{-\nu_2 t})$	Free response of second-order system ($\zeta > 1$)
14b.	$\frac{A}{(s + a)(s + b)}$	$\frac{A}{b - a} (e^{-at} - e^{-bt})$	
15a.	$\frac{A}{(\tau s + 1)^2}$	$\frac{At}{\tau^2} e^{-\nu t}$	Free response of second-order system ($\zeta = 1$)
15b.	$\frac{A}{(s + a)^2}$	Ate^{-at}	
16.	$\frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{A\omega_n e^{-\zeta\nu t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t)$	Second-order system, free response ($\zeta < 1$)
17a.	$\frac{A}{s(\tau s + 1)}$	$A(1 - e^{-\nu t})$	First-order system response to a step input
17b.	$\frac{A}{s(s + a)}$	$\frac{A}{a}(1 - e^{-\nu t})$	
18a.	$\frac{A}{s^2(\tau s + 1)}$	$A\tau \left(e^{-\nu t} + \frac{t}{\tau} - 1 \right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s + a)}$	$\frac{A}{a^2} (e^{-\nu t} + at - 1)$	

Source: Floyd E. Nixon, *Handbook of Laplace Transformation: Fundamentals, Applications, Tables and Examples*, 21e, © 1965, Referenced and adapted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

09-Class-Laplace & Inverse Laplace Transforms - Feb 8

09b-Laplace Transforms Table

No.	$F(s)$	$f(t)$	Comments
19a.	$\frac{A\omega}{(s^2 + \omega^2)(ts + 1)}$	$\frac{A\omega t}{1 + \omega^2 t^2} e^{-\nu_1 t} + \frac{A}{\sqrt{1 + \omega^2 t^2}} \sin(\omega t - \psi)$ where $\psi = \tan^{-1} \omega t$ ($0 < \psi < \pi$)	First-order system response to a sine input
19b.	$\frac{A\omega}{(s^2 + \omega^2)(s + a)}$	$\frac{A\omega e^{-at}}{\omega^2 + a^2} + \frac{A}{\sqrt{\omega^2 + a^2}} \sin(\omega t - \psi)$ where $\psi = \tan^{-1} \omega a$ ($0 < \psi < \pi$)	
20a.	$\frac{A}{s(\tau_1 s + 1)(\tau_2 s + 1)}$	$A \left(1 + \frac{\tau_1 e^{-\nu_1 t} - \tau_2 e^{-\nu_2 t}}{\tau_2 - \tau_1} \right)$	Second-order system response to a step input ($\zeta > 1$)
20b.	$\frac{A}{s(s + a)(s + b)}$	$\frac{A}{ab} \left(1 + \frac{ae^{-bt} - be^{-at}}{b - a} \right)$	
21a.	$\frac{A}{s(ts + 1)^2}$	$A \left(1 - \frac{\tau + t}{\tau} e^{-\nu t} \right)$	Second-order system response to a step input ($\zeta = 1$)
21b.	$\frac{A}{s(s + a)^2}$	$\frac{A}{a^2} [1 - (1 + at)e^{-\nu t}]$	
22.	$\frac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$A \left[1 + \frac{e^{-\zeta\nu t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t - \psi) \right]$ where $\psi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$ ($0 < \psi < \pi$)	Second-order system response to a step input ($\zeta < 1$)
23a.	$\frac{A}{s^2(\tau_1 s + 1)(\tau_2 s + 1)}$	$A \left(t - \tau_1 - \tau_2 - \frac{\tau_2^2 e^{-\nu_2 t} - \tau_1^2 e^{-\nu_1 t}}{\tau_1 - \tau_2} \right)$	Second-order system response to a ramp input ($\zeta > 1$)
23b.	$\frac{A}{s^2(s + a)(s + b)}$	$\frac{A}{ab} \left[t - \frac{a + b}{ab} - \frac{(b/a)e^{-bt} - (a/b)e^{-at}}{b - a} \right]$	Second-order system response to a ramp input ($\zeta = 1$)
24a.	$\frac{A}{s^2(\tau s + 1)^2}$	$A[t - 2\tau + (t + 2\tau)e^{-\nu t}]$	
24b.	$\frac{A}{s^2(s + a)^2}$	$\frac{A}{a^2} \left[t - \frac{2}{a} + \left(t + \frac{2}{a} \right) e^{-\nu t} \right]$	
25.	$\frac{A\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$A \left[t - \frac{2\tau}{\omega_n} + \frac{e^{-\zeta\nu t}}{\omega_n \sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t - \psi) \right]$ where $\psi = 2 \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$ ($0 < \psi < \pi$)	Second-order system response to a ramp input ($\zeta < 1$)

Transform Table

TABLE A-1 LAPLACE TRANSFORMS

No.	$F(s)$	$f(t)$	Comments
1.	1	$\delta(t)$	Unit impulse
2.	$\frac{A}{s}$	$A(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$	Step
3.	$\frac{1}{s}$	$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$	Unit step
4.	$\frac{A}{s^2}$	At	Ramp
5.	$\frac{2A}{s^3}$	At^2	Parabola
6.	$\frac{A\omega}{s^2 + \omega^2}$	$A \sin \omega t$	Sine

TABLE A-1 (continued)			
No.	$F(s)$	$f(t)$	Comments
7.	$\frac{As}{s^2 + \omega^2}$	$A \cos \omega t$	Cosine
8.	$aF(s)$	$af(t)$	
9.	$\frac{n!}{s^{n+1}}$	t^n	
10.	$sF(s) - f(0)$	$\frac{df(t)}{dt}$	
11.	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$	$\frac{d^2f(t)}{dt^2}$	
12.	$\frac{F(s)}{s}$	$\int f(t) dt$	
13a.	$\frac{A}{ts + 1}$	$\frac{A}{t} e^{-\psi t}$	Free response of first-order system
13b.	$\frac{A}{s + a}$	Ae^{-at}	
14a.	$\frac{A}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{A}{\tau_1 - \tau_2} (e^{-\psi \tau_1} - e^{-\psi \tau_2})$	Free response of second-order system ($\zeta > 1$)
14b.	$\frac{A}{(s + a)(s + b)}$	$\frac{A}{b - a} (e^{-at} - e^{-bt})$	
15a.	$\frac{A}{(ts + 1)^2}$	$\frac{At}{t^2} e^{-\psi t}$	Free response of second-order system ($\zeta = 1$)
15b.	$\frac{A}{(s + a)^2}$	$At e^{-at}$	
16.	$\frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t)$	Second-order system, free response ($\zeta < 1$)
17a.	$\frac{A}{s(ts + 1)}$	$A(1 - e^{-\psi t})$	First-order system response to a step input
17b.	$\frac{A}{s(s + a)}$	$\frac{A}{a}(1 - e^{-\psi t})$	
18a.	$\frac{A}{s^2(ts + 1)}$	$A\tau \left(e^{-\psi t} + \frac{t}{\tau} - 1 \right)$	First-order system response to a ramp input
18b.	$\frac{A}{s^2(s + a)}$	$\frac{A}{a^2} (e^{-\psi t} + at - 1)$	

TABLE A-1 (continued)

No.	$F(s)$	$f(t)$	Comments
19a.	$\frac{A\omega}{(s^2 + \omega^2)(ts + 1)}$	$\frac{A\omega t}{1 + \omega^2 t^2} e^{-\psi t} + \frac{A}{\sqrt{1 + \omega^2 t^2}} \sin(\omega t - \psi)$ where $\psi = \tan^{-1}\omega t$ ($0 < \psi < \pi$)	First-order system response to a sine input
19b.	$\frac{A\omega}{(s^2 + \omega^2)(s + a)}$	$\frac{A\omega e^{-at}}{\omega^2 + a^2} + \frac{A}{\sqrt{\omega^2 + a^2}} \sin(\omega t - \psi)$ where $\psi = \tan^{-1}\omega/a$ ($0 < \psi < \pi$)	
20a.	$\frac{A}{s(\tau_1 s + 1)(\tau_2 s + 1)}$	$A \left(\frac{\tau_1 e^{-\psi \tau_1} - \tau_2 e^{-\psi \tau_2}}{\tau_2 - \tau_1} \right)$	Second-order system response to a step input ($\zeta > 1$)
20b.	$\frac{A}{s(s + a)(s + b)}$	$\frac{A}{ab} \left(1 + \frac{ae^{-bt} - be^{-at}}{b - a} \right)$	
21a.	$\frac{A}{s(ts + 1)^2}$	$A \left(1 - \frac{\tau + t}{\tau} e^{-\psi t} \right)$	Second-order system response to a step input ($\zeta = 1$)
21b.	$\frac{A}{s(s + a)^2}$	$\frac{A}{a^2} [1 - (1 + at)e^{-\psi a}]$	
22.	$\frac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$A \left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t - \psi) \right]$ where $\psi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$ ($0 < \psi < \pi$)	Second-order system response to a step input ($\zeta < 1$)
23a.	$\frac{A}{s^2(\tau_1 s + 1)(\tau_2 s + 1)}$	$A \left(t - \tau_1 - \tau_2 - \frac{\tau_2^2 e^{-\psi \tau_2} - \tau_1^2 e^{-\psi \tau_1}}{\tau_1 - \tau_2} \right)$	Second-order system response to a ramp input ($\zeta > 1$)
23b.	$\frac{A}{s^2(s + a)(s + b)}$	$\frac{A}{ab} \left[t - \frac{a + b}{ab} - \frac{(b/a)e^{-bt} - (a/b)e^{-at}}{b - a} \right]$	
24a.	$\frac{A}{s^2(ts + 1)^2}$	$A[t - 2\tau + (t + 2\tau)e^{-\psi t}]$	Second-order system response to a ramp input ($\zeta = 1$)
24b.	$\frac{A}{s^2(s + a)^2}$	$\frac{A}{a^2} \left[t - \frac{2}{a} + \left(t + \frac{2}{a} \right) e^{-\psi t} \right]$	
25.	$\frac{A\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$A \left[t - \frac{2\zeta}{\omega_n} + \frac{e^{-\zeta\omega_n t}}{\omega_n \sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t - \psi) \right]$ where $\psi = 2 \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$ ($0 < \psi < \pi$)	Second-order system response to a ramp input ($\zeta < 1$)

Source: Floyd E. Nixon, *Handbook of Laplace Transformation: Fundamentals, Applications, Tables and Examples*, 21e, © 1965, Referenced and adapted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Transform Operations

UN KNOWN

Ψ	R	C	L
$i(t)$	$\frac{v(t)}{R}$	$C \cdot \frac{dv(t)}{dt}$	$\frac{1}{L} \int_0^t v(t) dt + I_o$
$v(t)$	$R \cdot i(t)$	$\frac{1}{C} \int_0^t i(t) dt + V_o$	$L \cdot \frac{di(t)}{dt}$

$$\begin{aligned} & f'(t) \\ & \int_0^t f(t) dt \end{aligned} \quad \frac{sF(s) - f(0)}{\frac{F(s)}{s}}$$

unknown

Ψ	R	C	L
I	$\frac{V}{R}$	$C \cdot sV$	$\frac{V}{sL}$
V	$R + I$	$\frac{I}{sC}$	$L \cdot sI$

Transform Ohm's Law

$$V = I \times Z_s$$

unknown

∇	R	C	L
I	$\frac{V}{R}$	$C \cdot s V$	$\frac{V}{s L}$
V	$R + I$	$\frac{I}{s C}$	$L \cdot s I$

Transform Ohm's Law

unknown

I	R	C	L
I	$\frac{V}{R}$	$C \cdot sV$	$\frac{V}{sL}$
V	$R + I$	$\frac{I}{sC}$	$L \cdot sI$
Z_s	R	$\frac{1}{sC}$	sL