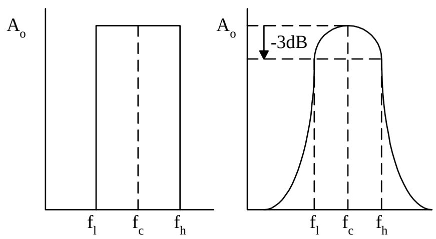
6.3 The Band pass Active Filter



(a) Ideal response (b) Actual response

Figure 6-9 Band pass filter response

The band pass filter was introduced in Section 6-1.2. Its frequency responses, both ideal and actual, are given in Figure 6-9.

The parameters of importance in a band pass filter are the high and low frequency cut-offs (f_h and f_l), the bandwidth (Δf), the center frequency (f_c), the center frequency gain (A_o), and the selectivity (Q). The bandwidth is the distance between the high frequency and low frequency -3dB points.

$$\Delta f = f_h - f_l$$

The center frequency is at the **geometric** mean of these cut-off frequencies. It is also the critical frequency.

$$f_o = f_c = \sqrt{f_h f_l}$$

On a **log** plot, this puts f_c halfway between f_h and f_l. This is **not** true on a linear graph. You must be careful. With a low frequency cut-off of 100 Hz, and a high frequency cut-off of 3k Hz, the center frequency is at 548 Hz, not 1.55 kHz.

The selectivity, Q, is the ratio of the center frequency to the bandwidth.

$$Q = \frac{f_c}{\Delta f}$$

As such, it gives a measure of the relative narrowness of the filter. For a particular center frequency, the higher the Q, the narrower the bandwidth, and therefore the more selective the filter. In addition, Q gives a more usable number than bandwidth alone. A bandwidth of 1 kHz is very broad for an audio filter with a center of 500 Hz and Q of 0.5. But that same 1 kHz bandwidth is very selective if the center frequency is 100 kHz producing a Q of 100.

Conversely, knowing the circuit's Q and center frequency, you can calculate the upper and the lower cut-off frequencies.

$$f_l = f_c\left(\sqrt{\frac{1}{4Q^2}+1} - \frac{1}{2Q}\right)$$

$$f_h = f_c\left(\sqrt{\frac{1}{4Q^2}+1} + \frac{1}{2Q}\right)$$

6-3.1 Single Op Amp Band Pass Filter

The transfer function of a second-order band pass filter is

Band pass filter

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{A_{\text{o}}\alpha\omega_{\text{o}}s}{s^2 + \alpha\omega_{\text{o}}s + \omega_{\text{o}}^{\;2}}$$

where

A_{o} = the gain at the center frequency
α = the damping coefficient
ω_{o} = the critical frequency = the center frequency

$$\frac{A_{\text{o}}\omega_{\text{o}}^2}{s^2 + \alpha\omega_{\text{o}}s + \omega_{\text{o}}^2}$$

Low pass filter

It has the same denominator as the second-order low pass filter and second-order high pass filter. The numerator of the band pass filter contains the first power of s. The numerator of the low pass filter's transfer function has no s term, while the numerator of the high pass filter has s^2.

$$\frac{A_o s^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

High pass filter

To obtain a plot of gain versus frequency, first normalize the transfer function by setting $\omega_{\text{o}} = 1$, just as was done for the low pass and the high pass filters. Next multiply the numerator and the denominator by the complex conjugate of the denominator. Then simplify. This allows you to separate the real and the imaginary parts of the equation. The gain's magnitude, then, is

$$\left|\overline{\text{G}}\right| = \sqrt{\text{Real}^2 + \text{Imaginary}^2}$$

$$\left|\overline{G}\right| = \frac{A_{\text{o}}\alpha\omega}{\sqrt{\omega^4 + \left(\alpha^2 - 2\right)\omega^2 + 1}}$$

The Q, defined by center frequency and bandwidth, is set by the damping.

***Q* and damp (α) relationship**

$$Q = \frac{1}{\alpha}$$

Figure 6-20 is a plot of the frequency response (gain magnitude) of a second-order band pass filter. The horizontal axis and the vertical axis have been normalized for convenience. To convert to a particular center frequency, f_c, multiply the horizontal axis by f_c. Add A_o (in dB) to the vertical axis to scale it for a particular A_o (dB).

There are two points to notice. The higher the Q, the sharper (and more selective) the filter. This corresponds to lowering the damping. However, this difference appears only between approximately $0.5f_c$ and $2f_c$. Below $0.5f_c$ and above $2f_c$, the filters all roll off at 20 dB/decade (6 dB/octave) independent of the Q. This is because this is a second-order filter (two reactive components). One reactive component (inductor or capacitor) causes 20 dB/decade roll-off at low frequencies. The other governs high frequencies. A sharper roll off away from the center frequency calls for a higher order, using multiple op amps.

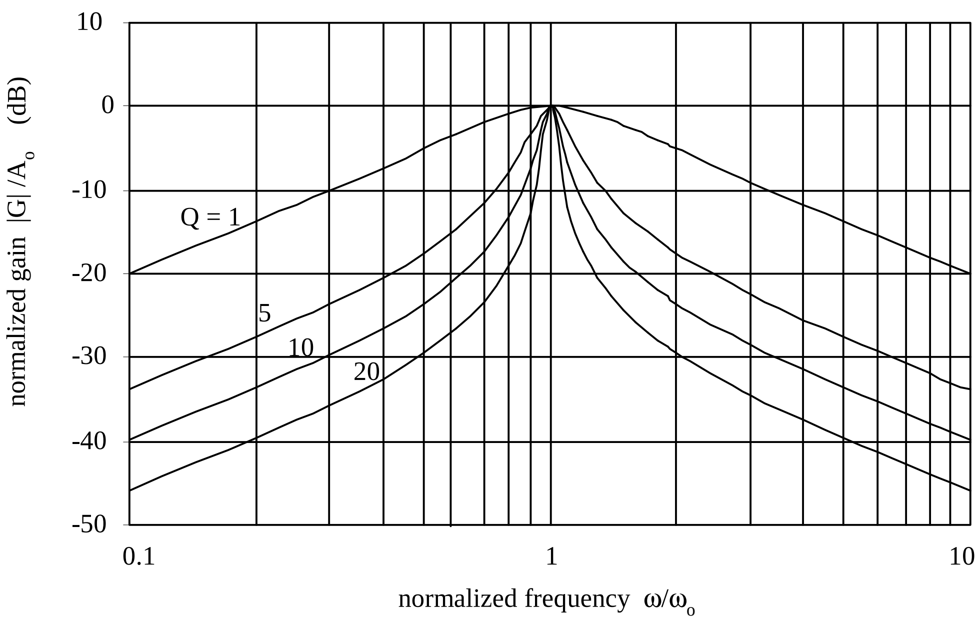


Figure 6-20 Single op amp band pass filter response (gain and frequency both normalized)

The circuit in Figure 6-21 can be analyzed mathematically to determine how component values affect each of these parameters. The capacitors are equal to each other. As with the analysis of the other op amp circuits, because of high open loop gain and negative feedback, there is no significant difference in potential between the op amp's inverting and noninverting inputs. Also, no signal current flows into either op amp input.

Therefore, I_3 flows through both the capacitor and R3. The left end of R3 is at virtual ground. This means that all of V_{out} is dropped across R3 by I_3.

$$I_3 = -\frac{V_{\text{out}}}{\text{R3}}$$

The voltage at node A with respect to ground is the voltage dropped across the capacitor by I_3.

$$V_{\text{A}} = \frac{I_3}{\text{C}s}$$

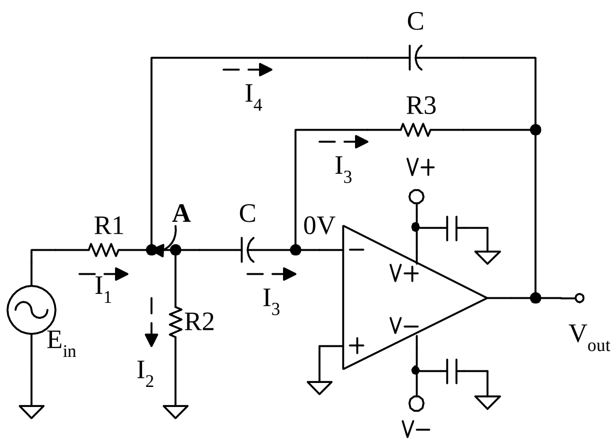


Figure 6-21 Single op amp band pass filter

$$V_{\text{A}} = -\frac{V_{\text{out}}}{\text{R3C}s}$$

The current through R2 is produced by V_{A}.

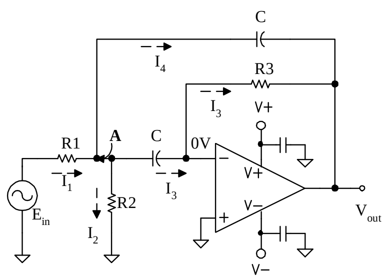


Figure 6-21 Single op amp band pass filter

$$I_2 = \frac{V_A}{R2} = -\frac{V_{out}}{R2R3Cs}$$

The current through the upper feedback capacitor, I_4, is

$$I_4 = \frac{V_A - V_{out}}{\frac{1}{Cs}} = (V_A - V_{out})Cs$$

After some algebraic manipulation, this becomes

$$I_4 = -\frac{V_{out}(R3Cs + 1)}{R3}$$

The input current, I_1, is

$$I_1 = \frac{E_{in} - V_A}{R1}$$

$$I_1 = \frac{E_{in}}{R1} + \frac{V_{out}}{R1R3Cs}$$

The current entering node A (I_1) must equal the current leaving the node (I_2, I_3, and I_4).

$$I_1 = I_2 + I_3 + I_4$$

Combining all of these equations yields

$$\frac{E_{in}}{R1} + \frac{V_{out}}{R1R3Cs} = -\frac{V_{out}}{R2R3Cs} - \frac{V_{out}}{R3} - \frac{V_{out}(R3Cs + 1)}{R3}$$

With diligence, this can be simplified to

$$\frac{V_{out}}{E_{in}} = -\frac{R2R3Cs}{(R1R2R3C^2)s^2 + (2R1R2C)s + (R1+R2)}$$

or
$$\frac{V_{\text{out}}}{E_{in}} = -\frac{\frac{1}{\text{R1C}} s}{s^2 + \frac{2}{\text{R3C}} s + \frac{\text{R1} + \text{R2}}{\text{R1R2R3C}^2}}$$

Now, compare this to the general transfer function for a band pass filter.

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{A_{\text{o}} \alpha \omega_{\text{o}} s}{s^2 + \alpha \omega_{\text{o}} s + \omega_{\text{o}}^2}$$

$$\omega_{\text{o}}^2 = \frac{\text{R1} + \text{R2}}{\text{R1 R2 R3C}^2}$$

$$f_{\text{o}} = f_{\text{c}} = \frac{1}{2\pi} \sqrt{\frac{\text{R1} + \text{R2}}{\text{R1 R2 R3C}^2}}$$

Center frequency

So all components combine to set the center frequency. This is a bit inconvenient.

$$\alpha \omega_{\text{o}} = \frac{2}{\text{R3 C}}$$

Equating the numerators of the two equations, gives

$$A_{\text{o}} \alpha \omega_{\text{o}} = -\frac{1}{\text{R1 C}}$$

Dividing these two equations gives

$$A_{\mathbf{o}} = -\frac{\text{R3}}{2\text{R1}}$$

Pass band gain

This shows that at the center frequency, the gain is set by the ratio of the feedback resistor and the input resistor, similar to a simple inverting amplifier.

$$\alpha\omega_o = \frac{2}{\mathrm{R3\,C}}$$

$$\alpha = \frac{2}{\mathrm{R3}\,\omega_o \mathrm{C}}$$

$$Q = \frac{1}{\alpha} = \frac{\mathrm{R3}\,\omega_o\,\mathrm{C}}{2}$$

$$= \frac{2\pi\, f_o\, \mathrm{R3\,C}}{2}$$

$$Q = \frac{f_o}{\Delta f}$$

$$\frac{f_o}{\Delta f} = \frac{2\pi\, f_o\, \mathrm{R3\,C}}{2}$$

Bandwidth

$$\Delta f = \frac{2}{2\pi\,\mathrm{R3\,C}}$$

The bandwidth is set **solely** by R3 and C. Tuning R1 or R2 does not alter the bandwidth.

The three highlighted equations allow you to analyze a given single op amp active filter to determine its center frequency, pass band gain, and bandwidth. Then Q, low frequency cut-off, and high frequency cut-off can also be calculated.

For design, this same group of equations must be manipulated a bit and applied in a specific order as illustrated in Example 6-6.

Example 6-6

Design a second-order band pass active filter to meet the following specifications:

$f_l = 3$ kHz $f_h = 3.5$ kHz $A_o = -5$ (ratio)

Solution

The center frequency is

$$f_{\mathrm{c}} = \sqrt{f_{\mathrm{l}} f_{\mathrm{h}}}$$

$$f_{\mathrm{c}} = \sqrt{3\,\mathrm{kHz} \times 3.5\,\mathrm{kHz}} = 3240\,\mathrm{Hz}$$

The bandwidth is

$$\Delta f = f_{\mathrm{h}} - f_{\mathrm{l}}$$

$$\Delta f = 3.5\,\mathrm{kHz} - 3\,\mathrm{kHz} = 500\,\mathrm{Hz}$$

$$Q = \frac{f_{\mathrm{c}}}{\Delta f}$$

$$Q = \frac{3240\,\mathrm{Hz}}{500\,\mathrm{Hz}} = 6.5$$

This is within the limits set for a single op amp band pass filter.

Practical single op amp limit on Q

$1 < Q < 10$

To calculate component values, first pick a convenient size capacitor. Let

$C = 0.027\ \mu F$

Next, R3, which alone sets the bandwidth, must be determined.

$$\mathrm{R3} = \frac{2}{2\pi \Delta f\ \mathrm{C}}$$

$$\mathrm{R3} = \frac{2}{2\pi \times 500\,\mathrm{Hz} \times 0.027\,\mu\mathrm{F}} = 23.6\,\mathrm{k}\Omega$$

Pick R3 = 24kΩ.

The gain is set by both R1 and R3. Since you now have R3, R1 can be calculated.

$$A_o = -\frac{R3}{2R1}$$

$$R1 = -\frac{R3}{2A_o}$$

$$R1 = -\frac{24\,k\Omega}{2(-5)} = 2.4\,k\Omega$$

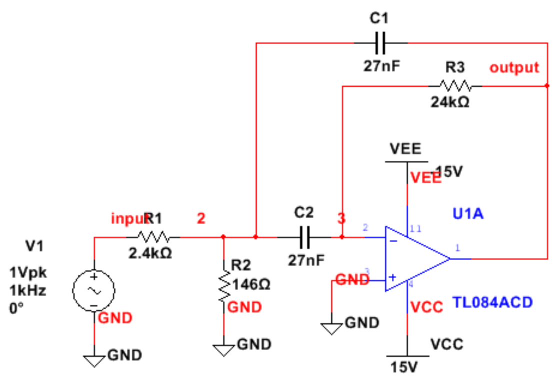
Finally, the value of R2 must be calculated to set the center frequency.

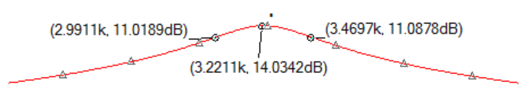
$$f_c = \frac{1}{2\pi}\sqrt{\frac{R1 + R2}{R1\ R2\ R3\ C^2}}$$

But, you need an equation for R2. After some more algebra, the result is

$$R2 = \frac{R1}{4\pi^2 R1\,R3\,f_c^2 C^2 - 1}$$

$$R2 = \frac{2.4\,k\Omega}{4\pi^2(2.4\,k\Omega)(24\,k\Omega)(3240\,Hz)^2(0.027\,\mu F)^2 - 1} = 146\,\Omega$$





20 Log (5) = 14 dB

Figure 6-22 Simulation schematic and AC Sweep

The **sequence** in adjusting these values is critical. Since bandwidth is affected only by R3, alter its value first to obtain the desired bandwidth. Gain is affected by R3 and R1, so, without changing R3 (bandwidth adjust), next tweak R1 to set the center frequency gain. Finally, the center frequency is affected by all three resistors. Without changing R1 or R3, adjust R2 to obtain the desired center frequency.

There are two component limitations to this single op amp band pass filter. Look at the equation for R2. To get a real value for R2, the denominator may not be zero or go negative. That is,

$$\mathrm{R2} = \frac{\mathrm{R1}}{4\pi^2 \mathrm{R1}\,\mathrm{R3}\, f_c^{\,2} \mathrm{C}^2 - 1}$$

$$4\pi^2 \mathrm{R1}\,\mathrm{R3}\, f_c^{\,2} \mathrm{C}^2 - 1 > 0$$

or

$$f_c > \frac{1}{2\pi\sqrt{\mathrm{R1}\,\mathrm{R3}}\,\mathrm{C}}$$

With a little more algebra, this same condition can be expressed in terms of the filter's initial specifications. This allows you to decide, at the beginning, if this configuration can be used to implement the band pass filter.

Q and A_o restriction

$$2Q^2 > \left|A_o\right|$$

If less gain is needed to assure that the inequality is met, set the filter's gain to a lower level, and add an amplifier after the filter to set the overall circuit's gain correctly.

The second component limitation is the op amp's gain bandwidth. At the center frequency, the op amp's open loop gain must be at least 20 Q^2 to insure less than 10% gain error. In terms of the op amp's gain bandwidth,

Op amp restriction

$$GBW \geq 20\, Q^2 \times f_c$$

One final word of warning: remember that $Q = 1/\alpha$. As the filter's Q is increased, to make it more and more selective, the circuit's damping is being lowered. For any $\alpha < 2$ ($\xi < 1$) the circuit is underdamped. This means that when the circuit is hit with a pulse, the output rings. It is possible that the circuit may oscillate at f_c if the damping is set too low. So choose the highest damping, the lowest Q, that works. If more rejection than this single op amp band pass filter can provide without excessive ringing is needed, then a multistage band pass filter may be called for.