The schematic and the AC Sweep set up are in the upper half of the figure. The AC analysis shown in the lower half indicates that A_o = 20 $\text{Log}_{10}(5)$ = 14 dB. f_{-3dB} occurs where the gain has fallen -3 dB from 14 dB to 11 dB. This happens at 2 kHz. An octave is a change from 5 kHz to 10 kHz. The gain falls -12 dB. On the phase plot, the shift is -90° at 2 kHz. This is f_o. So this is, indeed, a Butterworth filter, because $f_o = f_{-3dB.}$

6-2.3 Higher-Order Low Pass Sallen-Key Characteristics

The first-order and second-order filters, though relatively easy to build, may not provide an adequate roll-off rate. The only way to improve the roll-off rate is to increase the order of the filter as illustrated in Figure 6-7. Each increase in order produces a 20dB/decade or 6dB/octave increase in the roll off-rate.

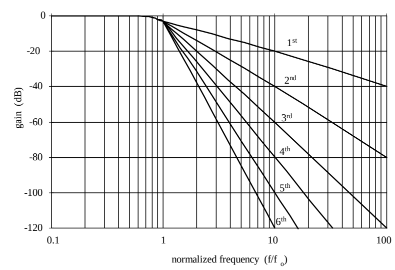


Figure 6-7 Roll-off rate comparison

Higher-order filters can be built by cascading the proper number of first- and second-order filter sections. For a fifth-order filter, this technique results in a transfer

$$\frac{A_o}{\underbrace{\left(s^2+\alpha_1 s+\omega_1^2\right)}_{\text{second-order section}}\underbrace{\left(s^2+\alpha_2 s+\omega_2^2\right)}_{\text{another second-order section}}\underbrace{\left(s+\omega_3\right)}_{\text{first-order section}}}$$

Each term in the denominator has its own damping coefficient and critical frequency. To obtain a given, well-defined response (Bessel, Butterworth, or Chebyshev), the transfer function, **as a whole,** must be solved and the appropriate coefficients determined.

$$\frac{A_o}{s^5+as^4+bs^3+cs^2+ds+e}$$

It is unreasonable to expect the α's and ω's of the cascaded filter transfer function to correlate in a simple way with the coefficients of the overall filter. You do **not** get a fifth-order, 1kHz Bessel filter by cascading two 1kHz, second-order Bessel filters and a first -order, RC passive stage.

The mathematics used to solve these higher-order polynomials is beyond the scope of this book. The results are presented in Table 6-4.

Table 6-4 Higher order damping and frequency correction factors

Filter Order	Section		Bessel	Butterworth	3dB Cheby
2	2	α	1.732	1.414	0.766
		k_{lp}	0.785	1.000	1.390
3	1	α	-	-	-
		k_{lp}	0.753	1.000	3.591
	2	α	1.447	1.000	0.326
		k_{lp}	0.687	1.000	1.172
4	2	α	1.916	1.848	0.929
		k_{lp}	0.696	1.000	2.349
	2	α	1.242	0.765	0.179
		k_{lp}	0.621	1.000	1.095
5	1	α	-	-	-
		k_{lp}	0.665	1.000	5.762
	2	α	1.775	1.618	0.468
		k_{lp}	0.641	1.000	1.670
	2	α	1.091	0.618	0.113
		k_{lp}	0.569	1.000	1.061
6	2	α	1.959	1.932	0.958
		k_{lp}	0.621	1.000	3.412
	2	α	1.636	1.414	0.289
		k_{lp}	0.590	1.000	1.408
	2	α	0.977	0.518	0.078
		k_{lp}	0.523	1.000	1.042

Example 6-4

Design a fourth-order Bessel filter with f_{-3dB} = 3 kHz.

Solution

A fourth-order filter can be built by cascading two second-order stages. From Table 6-4, the first stage must have a damping coefficient of

$$\alpha_1 = 1.916$$

$$A_{\xi 1} = 3 - 1.916 = 1.084$$

For that section, the frequency is set by

$$f_{-3\mathrm{dB}} = \frac{k_{\mathrm{lp}}}{2\pi \mathrm{RC}}$$

$$k_{\mathrm{lp}} = 0.696$$

Pick C1 = 0.01μF.

$$\mathrm{R1} = \frac{k_{\mathrm{lp}}}{2\pi f_{-3\mathrm{dB}}\mathrm{C}} = \frac{0.696}{2\pi \times 3\,\mathrm{kHz} \times 0.01\mu.} = 3.69\,\mathrm{k\Omega}$$

Build this with a 3.3 kΩ resistor in series with a 330 Ω resistor.

$$A_{\xi 1} = 1 + \frac{\mathrm{R}_{\mathrm{f1}}}{\mathrm{R}_{\mathrm{i1}}} = 1.084$$

$$\frac{\mathrm{R}_{\mathrm{f1}}}{\mathrm{R}_{\mathrm{i1}}} = 0.084 \quad \text{or} \quad \mathrm{R}_{\mathrm{f1}} = 0.084\mathrm{R}_{\mathrm{i1}}$$

Pick R_{i1} = 10 kΩ R_{f1} = 840 Ω

The second stage is handled in the same way, by using:

$$\alpha_2 = 1.242 \qquad k_{\mathrm{lp2}} = 0.621$$

This gives:

C2 = 0.01 μF   R2 = 3.3 kΩ
R_{i2} = 10 kΩ R_{f2} = 7.58 kΩ

Neither section, **alone**, exhibits a Bessel response or has the correct cut-off. However, together the proper response is produced.

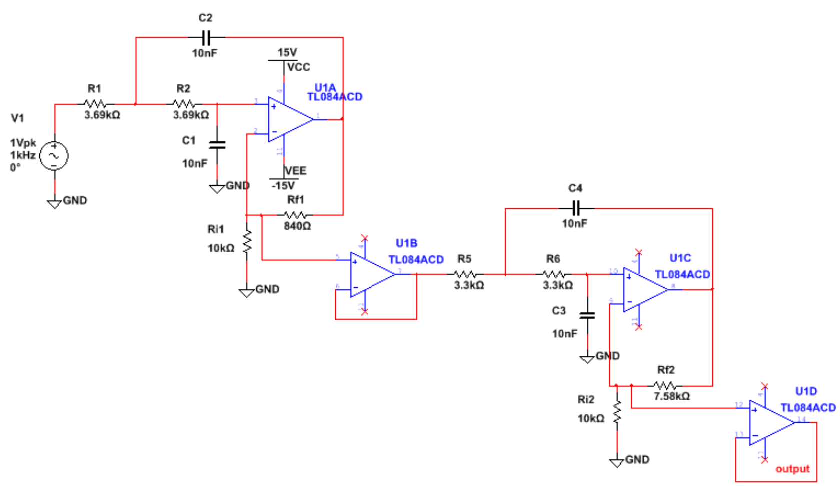


Figure 6-15 Fourth order Bessel Sallen-Key low pass filter

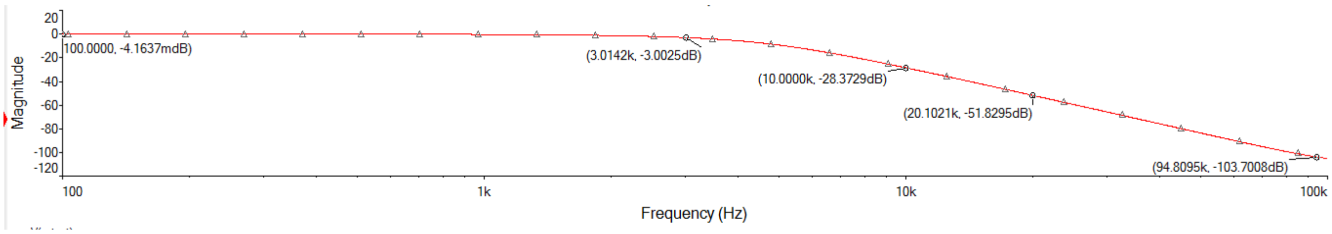


Figure 6-16 Frequency response for the circuit in Figure 6-15

The simulation schematic and overall Bode plot are given in Figures 6-15 and 6-16. The frequency response is smooth and well behaved, suggesting a Bessel (or Butterworth) response. The pass band gain from the simulation matches the manual calculations, as does $f_{-3\text{dB}}$. The roll-off rate is -23.4 dB/octave, a close match to the -24 dB/octave predicted (6dB/octave/order; fourth order).

When cascading filter sections to produce higher-order filters, be sure to use the correct damping and filter correction factor from Table 6-4. Use the lowest-order filter that meets the given specifications. Damping coefficients (and therefore filter stability) become quite small as you increase the order. Use a Butterworth filter if possible. Go to Bessel if you need better transient response. But this gives poorer **initial** roll-off. Use the Chebyshev filter if initial roll-off of the Butterworth is not adequate. However, transient response and pass band flatness suffer.

6-2.4 High Pass Sallen-Key Characteristics

The complement of the low pass filter is the high pass active filter. It is formed by exchanging the place of the resistors and capacitors in the frequency determining section of the filter. This is shown in Figure 6-17. Compare it carefully with Figure 6-11, the schematic of the second-order, low pass active filter. The general second order active filter is shown in Figure 6-10. Its transfer function was derived as

$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{A_\xi Z3Z4}{Z1Z2 + Z2Z3 + Z3Z4 + Z1Z3 + Z1Z4(1 - A_\xi)}$$

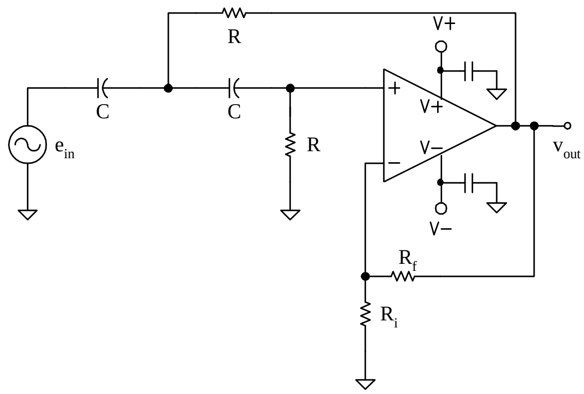


Figure 6-17 Sallen-Key, equal component, second-order, high pass, active filter

For the Sallen-Key, equal component, second order, high pass filter of Figure 9-18,

$$Z1 = \frac{1}{\mathrm{C}s} \qquad Z2 = \frac{1}{\mathrm{C}s} \qquad \mathrm{Z3} = \mathrm{R} \qquad \mathrm{Z4} = \mathrm{R}$$

Substituting these into the general transfer function gives:

$$\frac{V_{\mathrm{out}}}{E_{\mathrm{in}}} = \frac{A_\xi \mathrm{R}^2}{\frac{1}{\mathrm{C}^2 s^2} + \frac{\mathrm{R}}{\mathrm{C}s} + \mathrm{R}^2 + \frac{\mathrm{R}}{\mathrm{C}s} + \frac{\mathrm{R}}{\mathrm{C}s}\left(1 - A_\xi\right)}$$

$$= \frac{A_\xi \mathrm{R}^2\mathrm{C}^2 s^2}{1 + \mathrm{RC}s + \mathrm{R}^2\mathrm{C}^2 s^2 + \mathrm{RC}s + \mathrm{RC}s\left(1 - A_\xi\right)}$$

$$= \frac{A_\xi \mathrm{R}^2\mathrm{C}^2 s^2}{\mathrm{R}^2\mathrm{C}^2 s^2 + \mathrm{RC}\left(3 - A_\xi\right)s + 1}$$

$$\frac{V_{\mathrm{out}}}{E_{\mathrm{in}}} = \frac{A_\xi s^2}{s^2 + \frac{\left(3 - A_\xi\right)}{\mathrm{RC}} s + \frac{1}{\mathrm{R}^2\mathrm{C}^2}}$$

Second-order physical systems have been studied extensively for many years. Mechanical, hydraulic, and chemical, as well as electrical, second-order systems behave similarly. The transfer function for one group of such second-order systems is

$$\frac{A_o s^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

where A_o = the gain
ω_o = the critical frequency
α = the damping coefficient

Compare the general second-order transfer function to that for the Sallen-Key, equal component, second-order, high pass filter.

$$A_o = A_\xi = 1 + \frac{R_f}{R_i}$$

$$\omega_o = \frac{1}{RC} \qquad f_o = \frac{1}{2\pi RC}$$

$$\alpha = 3 - A_\xi$$

This is **identical** to the relationships developed for the Sallen-Key, equal component, second-order, low pass filter.

Normalizing the transfer function sets $\omega_o = 1$. It then becomes

$$\frac{A_\xi s^2}{s^2 + \alpha s + 1}$$

To determine the gain and phase relationships in the frequency domain, substitute $s = j\omega$ into the transfer function.

$$\frac{V_{out}}{E_{in}} = \frac{-A_\xi\omega^2}{-\omega^2 + j\alpha\omega + 1}$$

$$= \frac{-A_\xi\omega^2}{(1-\omega^2) + j\alpha\omega} \times \frac{(1-\omega^2) - j\alpha\omega}{(1-\omega^2) - j\alpha\omega}$$

$$= \frac{-A_\xi\omega^2(1-\omega^2) + jA_\xi\alpha\omega^3}{(1-\omega^2)^2 + \alpha^2\omega^2}$$

$$\text{Real} = \frac{-A_\xi \omega^2 \left(1-\omega^2\right)}{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}$$

$$\text{Imaginary} = \frac{A_\xi \alpha \omega^3}{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}$$

$$\left|\overline{\mathrm{G}}\right| = \sqrt{\text{Real}^2 + \text{Imaginary}^2}$$

$$= \frac{\sqrt{A_\xi^2 \omega^4 \left(1-\omega^2\right)^2 + A_\xi^2 \alpha^2 \omega^6}}{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}$$

$$= \frac{A_\xi \omega^2 \sqrt{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}}{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}$$

$$\left|\overline{G}\right| = \frac{A_\xi \omega^2}{\sqrt{\left(1-\omega^2\right)^2 + \alpha^2\omega^2}}$$

For the high pass filter, the magnitude is **directly** proportional to the square of the frequency. An increase in frequency causes an increase in the gain. This is plotted in Figure 6-18. Compare these curves to the three for the low pass filter in Figure 6-13. The curves have just been mirrored around the $\omega = 1$ axis.

Also, look *closely* at the two phase shift plots. At the critical frequency ($\omega = 1$), the low pass filters all shift the phase by -90°. Low pass filters *lag*. But the high pass filters all shift the phase by +90°. High pass filters *lead*.

Since the gain magnitude plots are rotated around the normalized axis, compared to the low pass plots, the correction factors of Table 9-3 must be changed for the high pass filter.

$$k_{\text{hp}} = \frac{1}{k_{\text{lp}}}$$

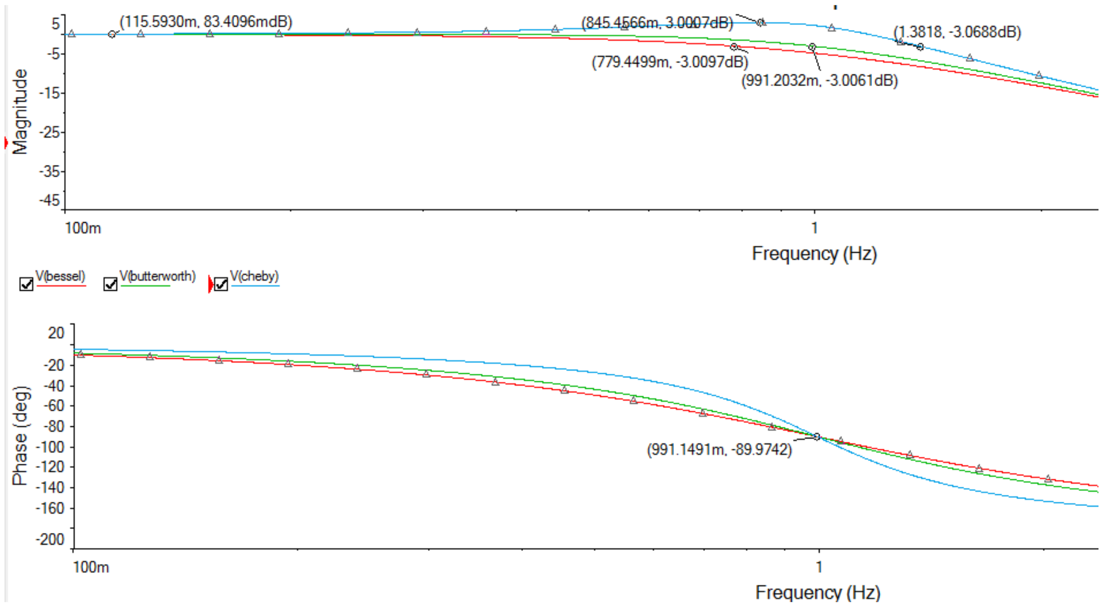


Fig 6-13 Normalized low pass filter responses

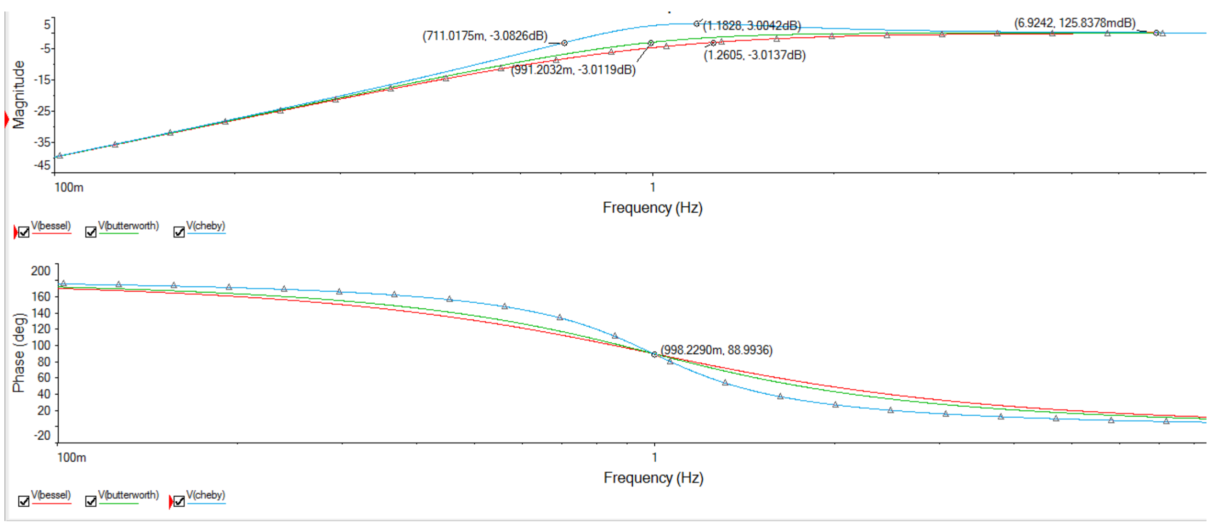


Fig 6-18 Normalized high pass filter responses

Example 6-5

For a high pass filter with the components listed below, calculate f_{-3dB}, the filter type, and A_o.

R = 10 kΩ C = 0.1µF $R_{f\xi}$ = 12.3 kΩ $R_{i\xi}$ = 10 kΩ
R_{fo} = 30 kΩ R_{io} = 10 kΩ

Solution

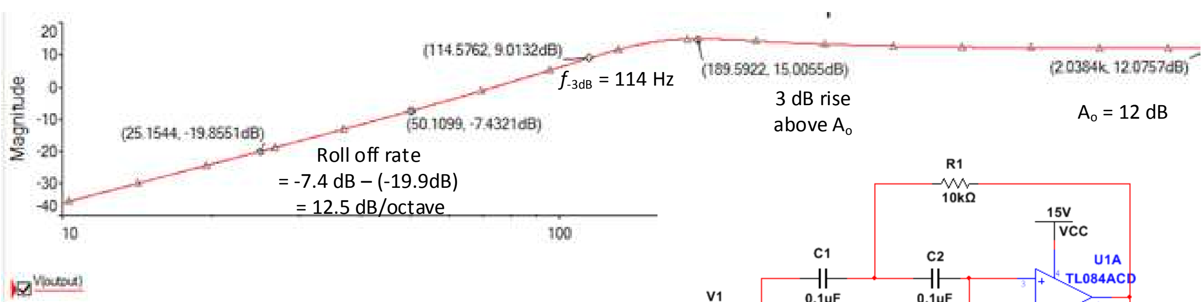
$$A_\xi = 1 + \frac{R_{f\xi}}{R_{i\xi}} = 1 + \frac{12.3\,\text{k}\Omega}{10\,\text{k}\Omega} = 2.23$$

$$\alpha = 3 - A_\xi = 3 - 2.23 = 0.77 \quad \text{This is a Chebyshev filter.}$$

$$k_{hp} = \frac{1}{k_{lp}} = \frac{1}{1.39} = 0.719$$

$$f_{-3dB} = \frac{k_{hp}}{2\pi RC} = \frac{0.719}{2\pi \times 10\,\text{k}\Omega \times 0.1\,\mu\text{F}} = 114\,\text{Hz}$$

$$A_o = 1 + \frac{R_{fo}}{R_{io}} = 1 + \frac{30\,\text{k}\Omega}{10\,\text{k}\Omega} = 4 \;\Rightarrow 12\,\text{dB}$$



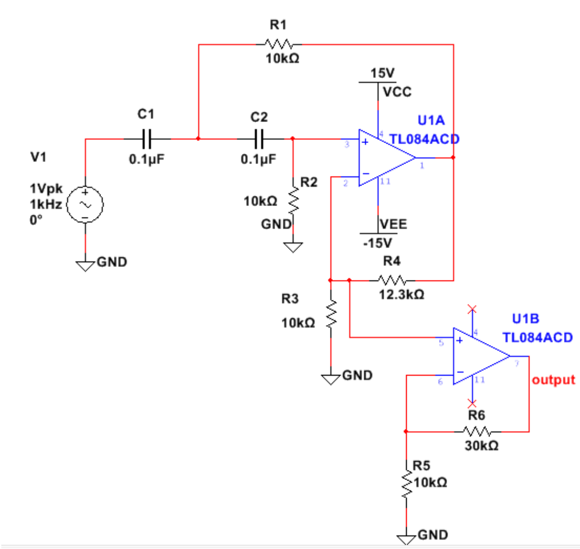


Figure 6-19 2nd order high pass Chebyshev filter

The higher-order calculation that you saw for the low pass filter can be applied equally well to high pass filters.