Integral Controller

The schematic for an op amp based Integral Controller is shown in Figure 5-31. Its transfer function is

$$\frac{\textbf{\textit{Controller out}}}{\textbf{\textit{Error}}} = \frac{k_{\mathrm{i}}}{s}$$

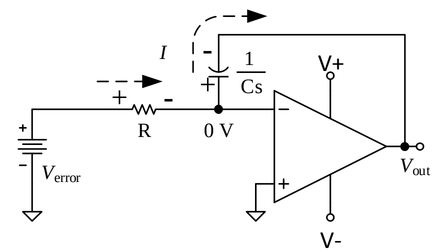

Figure 5-31 Ideal op amp integrator schematic

There is negative feedback, from the output, through the capacitor, to the op amp's inverting input pin. Because of this, the voltage at the inverting input pin is driven to virtually the same as at the noninverting input pin, ground.

That puts the entire V_{error} voltage across the input resistor R, producing the current I.

$$I = \frac{V_{\mathrm{error}}}{R}$$

The impedance into the op amp is in the Gigaohms. So, none of the current coming through the input resistor flows into the op amp. It all goes into (and out of) the capacitor, then into the op amp and down to V-.

Notice the polarity of the voltage across the capacitor. In order for the current to be pulled through the capacitor, the output voltage of the op amp must go negative.

$$V_{\mathrm{C}} = I \times \frac{1}{\mathrm{Cs}}$$

$$V_{\mathrm{out}} = -V_{\mathrm{C}} = -\frac{I}{\mathrm{Cs}}$$

This current is created by V_{error} across the input resistor, R

$$V_{\mathrm{out}} = -\frac{I}{\mathrm{Cs}} = -\frac{\frac{V_{\mathrm{error}}}{R}}{\mathrm{Cs}}$$

$$V_{\mathrm{out}} = -\frac{V_{\mathrm{error}}}{R\mathrm{Cs}}$$

$$\frac{V_{\mathrm{out}}}{V_{\mathrm{error}}} = -\frac{1}{\mathrm{RC}s}$$

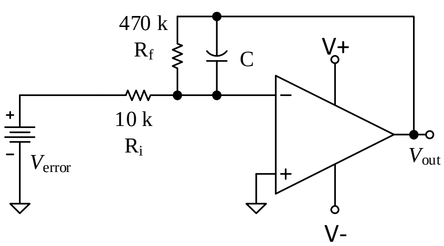

Figure 5-32 Practical op amp based integrator

Let

$$\boldsymbol{k}_{\mathbf{i}} = \frac{\mathbf{1}}{\boldsymbol{RC}}$$

$$\frac{V_{\text{out}}}{V_{\text{error}}} = -\frac{k_{\text{i}}}{s}$$

Just as with the proportional controller, the output of the op amp integrator will have to be followed by an inverter or inverting summer to correct the polarity.

Built as shown in Figure 5-31, when power is applied, the output of the op amp will slowly ramp to the power supply, and become stuck there. The capacitor is an open to DC. So there is no negative feedback for DC. All *practical* op amps have a small DC input voltage, (V_{ios}) and a small input DC current (I_{bias}). These errors are multiplied by the open loop gain of the op amp (>10^5), producing an output voltage, causing current to flow through and to charge the capacitor, producing more input voltage, producing more output voltage, charging the capacitor further, … .

This is addressed by adding R_f across the capacitor. Make

$$R_f >> R_i$$

DC the gain becomes

$$A_{\text{DC}} = -\frac{R_f}{R_i} = -47$$

Even with a few millivolts of input error, the output error is still considerably less than a volt.

Current created by the input, V_{error} through R_i, goes through the capacitor rather than the *large* R_f, so R_f does not impact the circuit's transfer function.

Figure 5-33 is the schematic for a Proportional-Integral Controller.

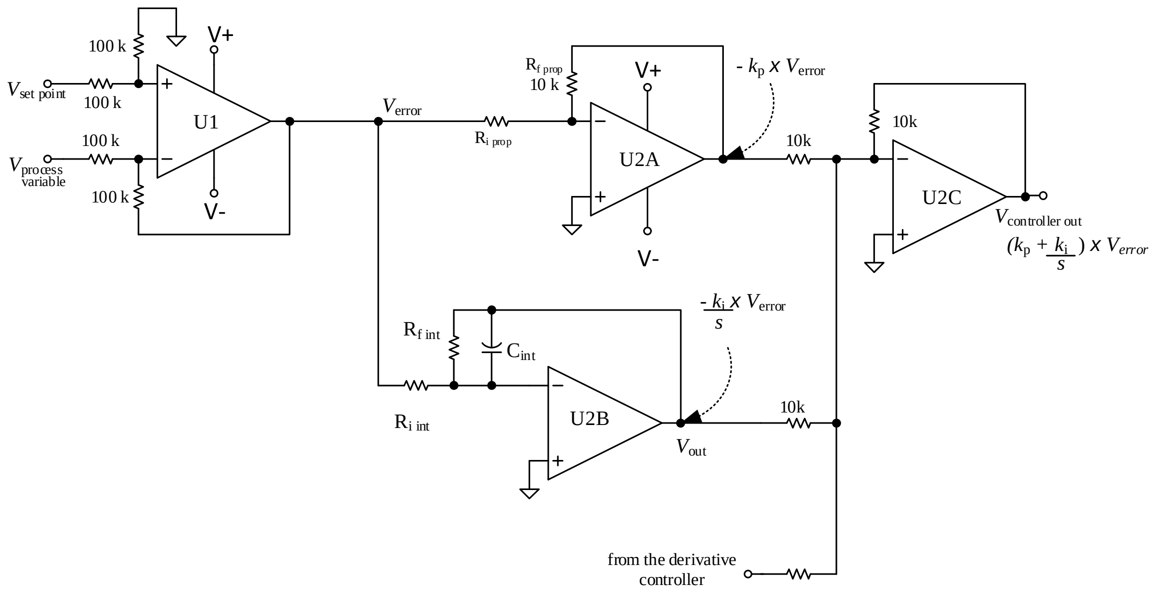

Figure 5-33 Op amp based Proportional-Integral Controller

Example 5-6

Design and verify the operation of a Proportional-Integral control system from previous examples to provide a critically damped response to a step from 227 RPM ($V_{\text{set point}}$ = 1.34 V) to 1030 RPM ($V_{\text{set point}}$ = 5 V). This is a repeat of Example 5-4 using an op amp based Proportional-Integral Controller.

m_{motor} = 206 RPM/V
m_{tach} = 0.005 V/RPM
τ = 0.362 sec
ξ = 1 critically damped

For the PI controller

$$k_p = \frac{1}{m}$$

$$k_i = \frac{1}{m\tau\xi^2}$$

Solutions

a. Parameter calculations

With the tachometer between the motor's RPM and the error amp,

$$m = 206 \times 0.005 = 1.03$$

$$k_p = \frac{1}{m} = \frac{1}{1.03} = 0.97$$

$$\frac{\mathrm{R}_{\mathrm{f\ prop}}}{\mathrm{R}_{\mathrm{i\ prop}}} = k_{\mathrm{p}} \qquad \mathrm{R}_{\mathrm{i\ prop}} = \frac{10\ \mathrm{k}\Omega}{0.97} = 10.3\ \mathrm{k}\Omega$$

Use $\mathrm{R}_{\mathrm{f\ prop}} = 10\ \mathrm{k}\Omega$ and $\mathrm{R}_{\mathrm{i\ prop}} = 10\ \mathrm{k}\Omega$

$$k_{\mathrm{i}} = \frac{1}{m\tau\xi^2} = \frac{1}{1.03 \times 0.36\ \mathrm{sec} \times 1^2} = 2.7\ /\mathrm{sec}$$

$$k_i = \frac{1}{\mathrm{R}_{\mathrm{i\ int}} \times \mathrm{C}_{\mathrm{int}}}$$

Resistors in the 1 kΩ to 100 kΩ range work well with op amps, not asking for too much current, but being insensitive to small noise induced currents. To get into the right range, begin with

$$\mathrm{R}_{\mathrm{i\ int}} \sim 10\ \mathrm{k}\Omega$$

$$\mathrm{C}_{\mathrm{int}} \sim \frac{1}{10\ \mathrm{k}\Omega \times 2.7} = 37\ \mu\mathrm{F}$$

Pick $\mathrm{C}_{\mathrm{int}} = 33\ \mu\mathrm{F}$.

This is a standard value. *But* most capacitors of that size are electrolytic, polarized. If the output of the integral controller will *always* be one polarity, be sure to install the capacitor correctly. However, if the integral controller may go both positive and negative, then $\mathrm{C}_{\mathrm{int}}$ must be nonpolarized.

$$\mathrm{R}_{\mathrm{i\ int}} = \frac{1}{33\ \mu\mathrm{F} \times 2.7} = 11.2\ \mathrm{k}\Omega$$

Pick $\mathrm{R}_{\mathrm{i\ int}} = 11\ \mathrm{k}\Omega$.

b. Circuit test

Before connecting the controller to the motor, firing up the system and *hoping* everything works, it is wise to test the controller alone. Testing the error amplifier, then the proportional controller, was done with fixed voltages. However, the integral controller is going to *change* its output as the capacitor charges or discharges. So, apply a step to the *Set Point* input and a constant voltage as the *Process Variable*. The simulation schematic is in Figure 5-34, and the results in Figure 5-35.

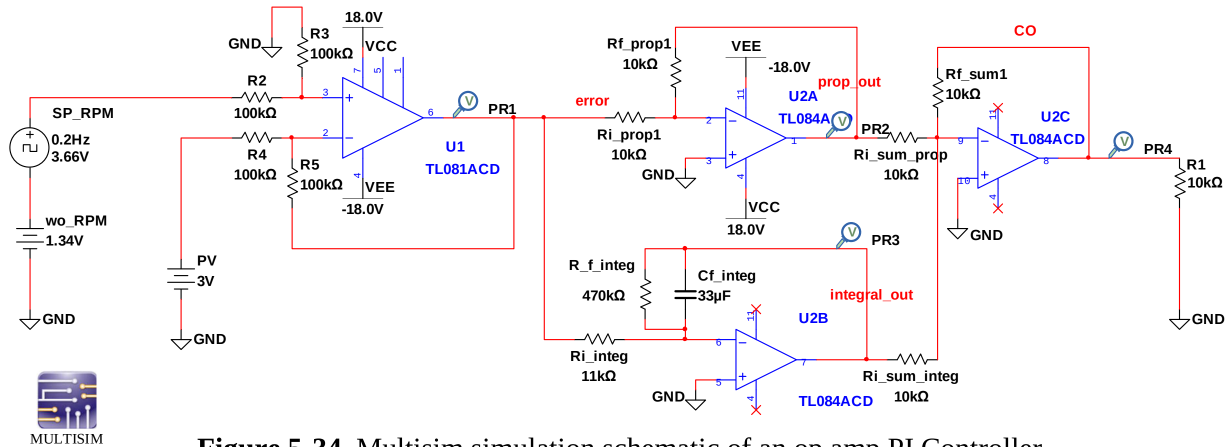

Figure 5-34 Multisim simulation schematic of an op amp PI Controller

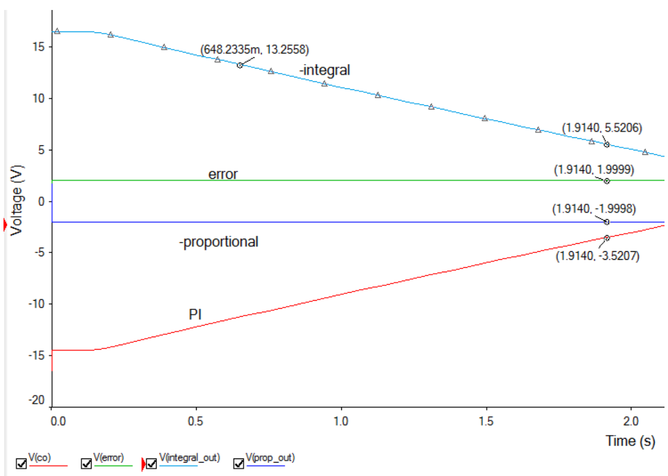

Figure 5-35 Simulation transient response

With $V_{\text{set point}} = 1.34\text{V}+3.66\text{V} = 5\text{V}$
and $V_{\text{process variable}} = 3\text{V}$
$V_{\text{error}} = 5\text{ V} - 3\text{ V} = 2\text{ V}$

The output of the proportional controller should be

$$-2\text{V} \times \frac{10\text{k}\Omega}{10\text{k}\Omega} = -2\text{V}$$

The current going into the integral controller is

$$\frac{2\text{V}}{11\text{ k}\Omega} = 182\mu\text{A}$$

This goes to charge the capacitor.

$$I = \text{C}\frac{dv}{dt} \qquad \frac{dv}{dt} = \frac{I}{\text{C}} = \frac{182\ \mu\text{A}}{33\ \mu\text{F}} = 5.5\frac{\text{V}}{\text{sec}}$$

The capacitor ramps down from 13.3 V to 5.5V from 0.64 sec to 1.91 sec

$$\frac{dv}{dt} = \frac{13.2\text{ V} - 5.5\text{ V}}{0.65\text{ sec} - 1.91\text{ sec}} = -6.1\ \frac{\text{V}}{\text{sec}}$$

This is close. The small error is caused by $R_{f\ int}$

The output of each controller sums and is inverted to

-(-2 V +5.5 V) = - 3.5 V

The Proportional Integral Controller is working correctly alone.

Figure 5-36 is the schematic for the full motor speed control system. The op amp Proportional-Integral Controller replaces the mathematical block from Figure 5-18.

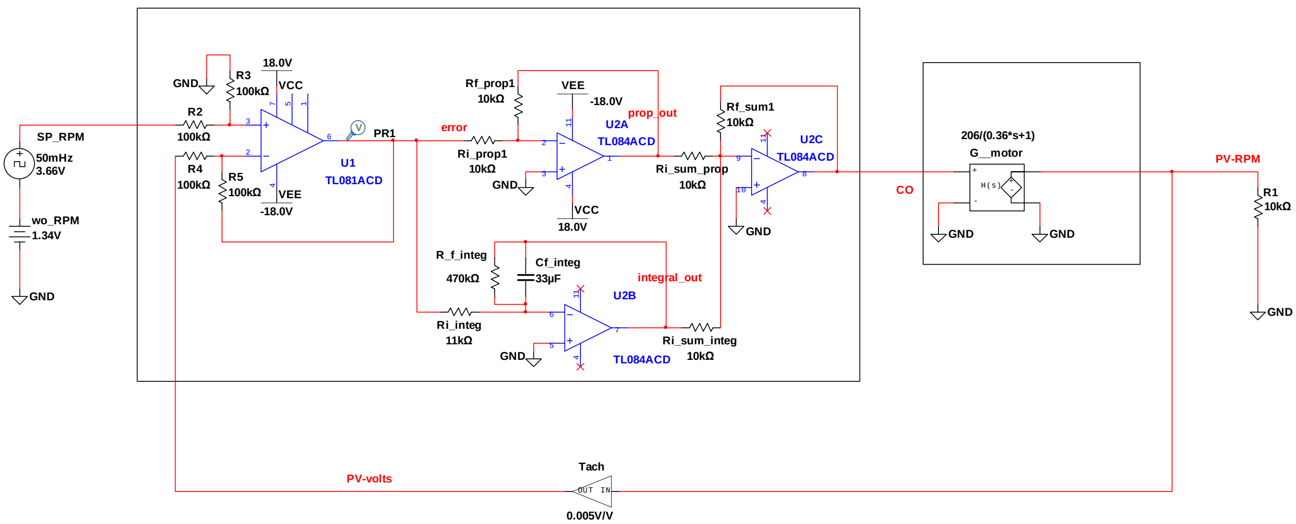

Figure 5-36 Simulation schematic for an op amp Proportional-Integral Controller

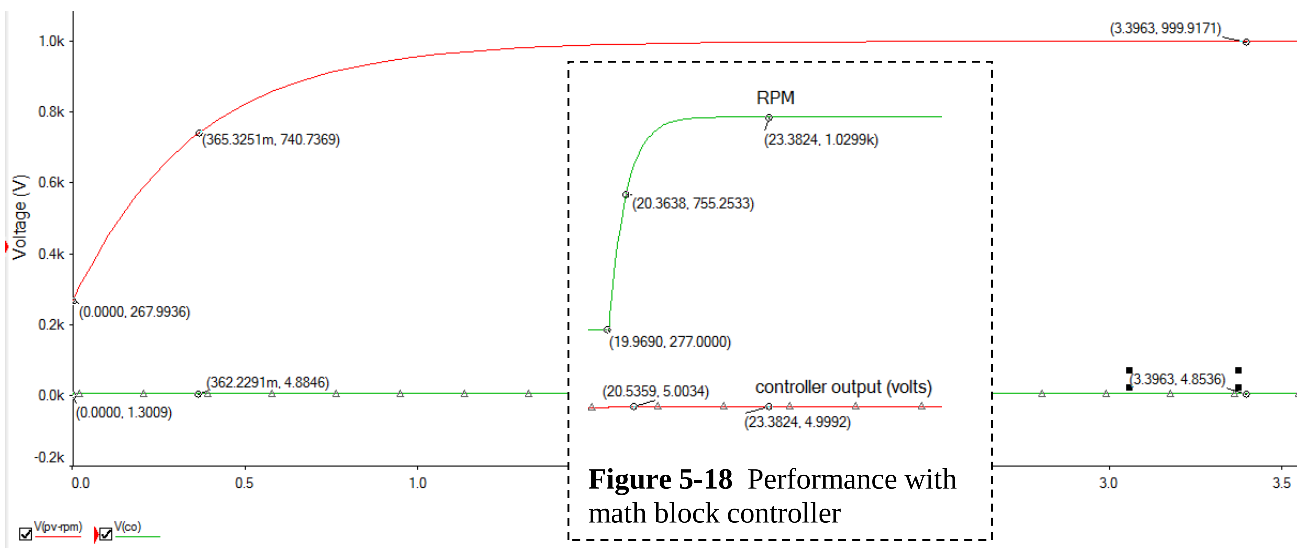

Figure 5-37 Motor control system with an op amp Proportional-Integral Controller

The performance of the system with the math based controller is shown in the insert of Figure 5-37. The two simulations perform very similarly. Both systems reach 1000 RPM at 3.3 seconds after the set point step. There is *no* residual error in either, and no overshoot. Both rise to 750 RPM at 0.36 seconds after the step.

Derivative Controller

The schematic for an op amp based Integral Controller is shown in Figure 5-38. Its transfer function is

$$\frac{\boldsymbol{Controller\ out}}{\boldsymbol{Error}} = \boldsymbol{k_d \times s}$$

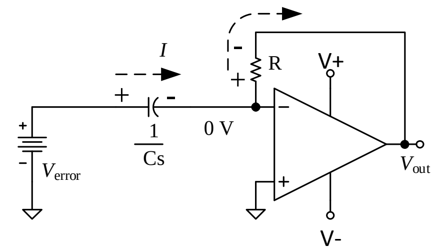

Figure 5-38 Ideal op amp differentiator schematic

There is negative feedback, from the output, through the R, to the op amp's inverting input pin. Because of this, the voltage at the inverting input pin is driven to virtually the same as at the noninverting input pin, ground.

That puts the entire V_{error} voltage across the input capacitor, producing the current I.

$$I = \frac{V_{\text{error}}}{\frac{1}{cs}} = \text{C}s \times V_{\text{error}}$$

The impedance into the op amp is in the Gigaohms. So, none of the current coming through the input resistor flows into the op amp. It all goes through the resistor, then into the op amp and down to V-.

Notice the polarity of the voltage across the resistor. In order for the current to be pulled through the resistor, the output voltage of the op amp must go negative.

$$V_{\text{R}} = I \times R$$

$$V_{\text{out}} = -V_R = -I\text{R}$$