ECET 337
Exam 2A
Spring 2025

Solutions
Name

Put your calculator in the *Exact* mode.

1. Given the time domain equation:

$$f(t) = 18 * t$$

a. [5%] Write the *integral definition* of it's Laplace transform.

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt \Big|_{s>0} = \int_0^\infty 18 \times t \times e^{-s \times t}\, dt \Big|_{s>0}$$

b. [15%] Complete the integral operation, either manually or using your calculator to find $F(s)$. If you compete the integration manually, show *all* appropriate steps. If you use your calculator, write below the *exact* information entered.

Do *not* bother to convert the answer to standard form.

$$\frac{18}{s^2}$$

Put your calculator in the *Approximate* mode.

c. [10%] Using the Laplace Transform tables, also find $f(t)$ for $F(s) = \frac{18}{s(s+5)(s+9)}$

$$f(t) = \frac{A}{s(s+a)(s+b)} \Rightarrow \frac{A}{ab}\left(1 + \frac{ae^{-bt} - be^{-at}}{b-a}\right)$$

Table line number = 20b

$A = 18$
$a = 5$
$b = 9$

$$f(t) = 0.4\left(1 + \frac{5e^{-9t} - 9e^{-5t}}{4}\right)$$

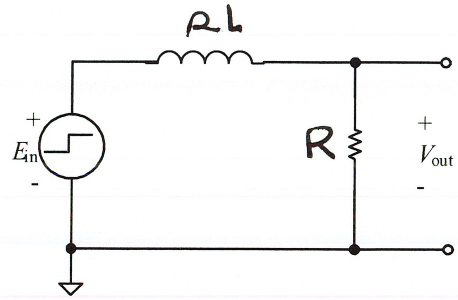


Figure 1 Schematic for Problem 2

2. [10%]**a.** *Derive* the equation for the *transfer function* in terms of *s* for the circuit above, with *symbols*, L, R, & *s*, not values.

$$\frac{R}{sL+R} \quad \frac{1/R}{1/R} = \frac{1}{\frac{L}{R}s+1} \qquad \tau = \frac{L}{R}$$

Put the answer in the *standard* form found in the Tables.

$$\frac{V_{out}}{E_{in}} = \frac{1}{\tau s+1}$$

[4%] **b.** Write the Laplace equation for $E_{in} = 18\ V$, a ***step***.

$E_{in}(s) = \frac{18}{s}$

either

[4%] **c.** Write the Laplace equation for the output voltage in response to the step.

$V_{out}(s) = \frac{18}{s}\frac{1}{(\tau s+1)} = \frac{18}{s(\tau s+1)}$

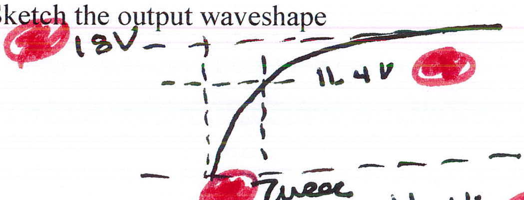
[4%] **d.** Determine the *time domain* equation for the output with symbols.

$V_{out}(t) = 18\left(1-e^{-t/\tau}\right)$ Table line # 17a

[8%] **e.** For R = 4.7 kΩ and L = 33 mH, calculate

Peak amplitude = 18V time constant = $\frac{L}{R} = \frac{33mH}{4.7k\Omega} = 7.02\mu sec$

Sketch the output waveshape



shape 4

V_{out} after one time constant = 11.4V

$18(1-e^{-1})$

$A\omega_n^2$

3. [6%] **a.** Given the transfer function: $\dfrac{12\times10^{9}}{s^2+65\times10^3 s+3\times10^9}$ for a set of R, L, and C

Calculate: $\omega_n =$ 54.8 K (2) $A_o =$ 4 (2) $\xi =$ 0.6 (2)

$\omega^2 = 3\times10^9$

$\omega = \sqrt{3\times10^9}$

$A = 12\times10^9 / 3\times10^9$

$A = 4$

$2\xi\omega_n = 65\times10^3$

$\xi = \dfrac{65\times10^3}{2\times 54.8K}$

b. [6%] If A_o, only, decreases, what happens to the following responses?
Answer: Increase, no change, decrease

1) *final* height in response to a step input decrease (3)

2) ringing NC (3)

c. [8%] If ξ, only, is cut in half, what would happen to each of the following responses?
Answer: Increase, no change, decrease

1) *final* height in response to a step input NC (4)

2) ringing increase (4)

e. [10%] Identify the type of damping that describes each of the following characteristics.
Answer ***over*** or ***critical*** or ***under***

Characteristics	damping	
1. $\xi = 1$	critical	C
2. $\xi > 1$	over	O
3. $\xi < 1$	under	U
4. fastest without overshooting	critical	C
5. overshoots	under	U
6. smallest 5% settling time	under	U
7. sluggish	over	O
8. two equal, real roots	critical	C
9. different real roots	over	O
10. two complex roots ~~two~~	under	U

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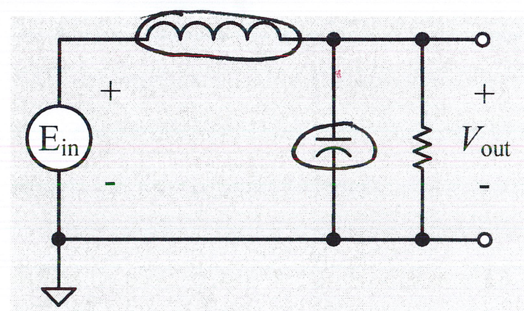
4. [10%] Answer the following for the circuit to the right.



a) Does this: (choose one)
pass DC and block pulses or
pass pulses and block DC (3)

b) Which component(s) set the resonant (critical) frequency?
(circle in the schematic) (2)

c) Does changing R affect the damping?
Yes or No (circle one) (2)

d) What is the circuit's order?
1st 2nd 3rd (circle one)

(3)

ECET 337
Exam 2B
Spring 2025

Solutions
Name

Put your calculator in the *Exact* mode.

1. Given the time domain equation:

$$f(t) = 12 * t + 3$$

a. [5%] Write the *integral definition* of it's Laplace transform. 1

$F(s) = 1 \int_0^{\infty} f(t)\, e^{-st}\, dt \Big|_{s>0}$ 1 1 1

b. [15%] Complete the integral operation, either manually or using your calculator to find *F*(s). If you compete the integration manually, show *all* appropriate steps. If you use your calculator, write below the *exact* information entered.

Do *not* bother to convert the answer to standard form.

$$\int_0^{\infty} (12*t+3) * e^{-s*t}\, dt \Big|_{s>0} =$$ 12

$$\frac{3(s+4)}{s^2}$$ 3

Put your calculator in the *Approximate* mode.

c. [10%] Using the Laplace Transform tables, also find *f*(t) for $F(s) = \frac{12}{s(s+3)(s+7)}$

f(t) = ~~0.571 s(~~ $0.571\left(1 + \frac{3e^{-7t} - 7e^{-3t}}{4}\right)$

Table line number = 20b 3

$$\frac{A}{s(s+a)(s+b)} \Rightarrow \frac{A}{ab}\left(1 + \frac{ae^{-bt} - be^{-at}}{b-a}\right)$$ 2 2

$A = 12$ 1
$a = 3$ 1
$b = 7$ 1

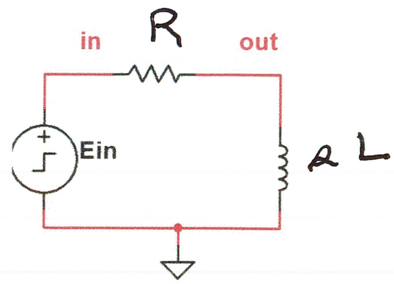


Figure 1 Schematic for Problem 2

2. [10%]**a.** *Derive* the equation for the *transfer function* in terms of *s* for the circuit above, with *symbols*, L, R, & *s*, not values.

$$\frac{V_{out}}{E_{in}} = \frac{sL}{R + sL} \cdot \frac{1/R}{1/R} = \frac{\frac{L}{R}s}{\frac{L}{R}s + 1} \qquad \tau = L/R$$

1 1 1 1 1 1 1 1 1

Put the answer in the *standard* form found in the Tables.

$$\frac{V_{out}}{E_{in}} = \frac{\frac{L}{R}s}{\frac{L}{R}s + 1} = \frac{\tau s}{\tau s + 1}$$ 1

[4%] **b.** Write the Laplace equation for E_{in} = 12 *V* , a *step*. 2

$E_{in}(s) = \frac{12}{s}$ 2

[4%] **c.** Write the Laplace equation for the output voltage in response to the step. 2

$V_{out}(s) = \frac{12}{s}\frac{\tau s}{\tau s + 1} = \frac{12\tau}{(\tau s + 1)}$ 2

[4%] **d.** Determine the *time domain* equation for the output with symbols.

$V_{out}(t) = 12e^{-t/\tau}$ 2 2 Table line # 13a 1

$$\frac{A}{\tau s + 1} \Rightarrow \frac{A}{\tau} e^{-t/\tau}$$

$$A = 12\tau$$

$$v_{out} = \frac{12\tau}{\tau} e^{-t/\tau} = 12e^{-t/\tau}$$

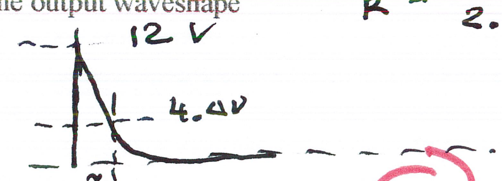
[8%] **e.** For R = 2.2 kΩ and L = 56 mH, calculate 1

Peak amplitude = 12V 1 time constant = 25.5μs

$$\frac{L}{R} = \frac{56\,mH}{2.2\,k\Omega}$$

Sketch the output waveshape

shape 4



V_{out} after one time constant = 4.4V 2

$12V(e^{-1}) = 4.4V$

3. [6%] **a.** Given the transfer function: $\frac{15\times10^9 X}{s^2+56\times10^3 s+8\times10^9}$ for a set of R, L, and C

Calculate: $\omega_n =$ 89.4 k 2 $A_o =$ 1.88 2 $\xi =$ 0.31 2

$\omega_n^2 = 8\times10^9$ $A_o \omega_n^2 = 15\times10^9$ $2\xi\omega_n = 56\times10^3$

$\omega_n = 89.4k$ $A_o = 15\times10^9 / 8\times10^9$ $\xi = \frac{56\times10^3}{2 * 89.4k}$

b. [6%] If A_o, only, increases, what happens to the following responses?
Answer: Increase, no change, decrease

1) ringing NC 3
2) *final* height in response to a step increase 3

c. [8%] If ξ, only, is increased by 3, what would happen to each of the following responses?
Answer: Increase, no change, decrease

1) ringing decreases 4
2) *final* height in response to a step NC 4

e. [10%] Identify the type of damping that describes each of the following characteristics.
Answer ***over*** or ***critical*** or ***under***

10

Characteristics	damping
1. $\xi > 1$	O
2. $\xi = 1$	C
3. $\xi < 1$	U
4. sluggish	O
5. fastest with no overshoots	C
6. smallest 5% settling time	U
7. overshoots	U
8. different, real roots	O
9. two complex roots	U
10. equal, real roots	C

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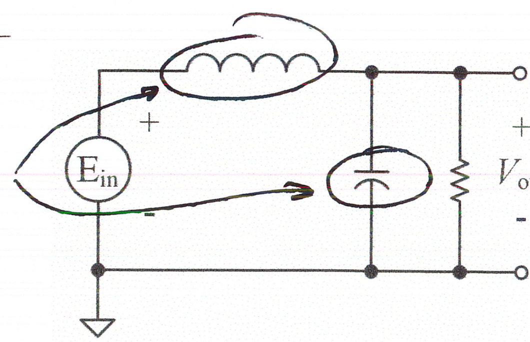
4. [10%] Answer the following for the circuit to the right.



a) What is the circuit's order?
1^{st} 2^{nd} 3^{rd} (circle one) 3

b) Which component(s) set the resonant (critical) frequency?
(circle in the schematic) 2

c) Does changing R affect the damping?
Yes or No (circle one) 2

d) Does this: (choose one)
pass pulses and block DC or
pass DC and block pulses 3