1

Analog Fundamentals

Introduction

Linear electrical and electronic systems are made of resistors, capacitors, inductors, and op amps. These analog systems can be analyzed and designed with a few fundamental laws; Ohm's law, Kirchhoff's laws, and the voltage divider law. Once you understand the behavior of these few components, and can apply these laws, you will be able to handle the majority of analog systems.

Objectives

Upon completion of this chapter, you will be able to analyze and design:

- Unloaded and loaded voltage dividers.
- Op amp circuits, including voltage followers, noninverting and inverting and difference amplifiers, and the inverting summer.
- A Thevenin's equivalent circuit for a complex system.
- Simple AC circuits using phasors.

1.1 Basic Principles

Voltage Divider

Ohm's Law defines the relationship among the *voltage* (pressure), *current* (flow), and *resistance* (friction) in an electrical circuit.

Ohm's Law

$$V = I \times R \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

The units for V is **V, volts**; I is **A,amperes** or **amps** R is **Ω, ohms**

Placing two resistors end-to-end, in series, causes the total opposition to add, shown in Figure 1-1.

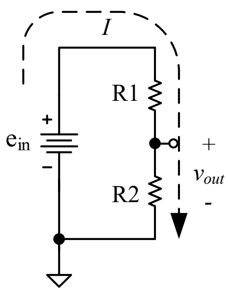


Figure 1-1 Resistors in series to form a *voltage divider*

$$R_{\text{total}} = R1 + R2$$

The voltage divider, shown in Figure 1, is used to reduce the input voltage e_{in} to a lower (more usable) level, v_{out}. The input voltage e_{in} is applied across R_{total}, producing a current I_{total}.

$$I_{\text{total}} = \frac{e_{\text{in}}}{R_{\text{total}}}$$

This current flows through R1, then R2. So , $I_{\text{R2}} = I_{\text{total}}$

$$v_{\text{out}} = v_{\text{R2}} = I_{R2} \times R2$$

$$v_{\text{out}} = I_{\text{total}} \times R2 = \frac{e_{\text{in}} \times R2}{R_{\text{total}}}$$

Voltage Divider Law

$$v_{out} = \frac{e_{\text{in}} \times R2}{R1 + R2}$$

or

$$\frac{v_{out}}{e_{in}} = \frac{R2}{R1+R2}$$

Op Amp Voltage Follower

One of the most common **linear** op amp circuits (one that uses negative feedback) is the voltage follower. Before beginning the analysis of this circuit, however, you must recall two facts about the op amp itself. These are illustrated in Figure 2.

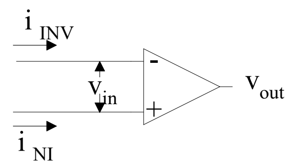


Figure 1-2 Fundamental op amp characteristics

First, since the input impedance is very high, the **signal** current that flows into either input terminal (- or +) is negligibly small.

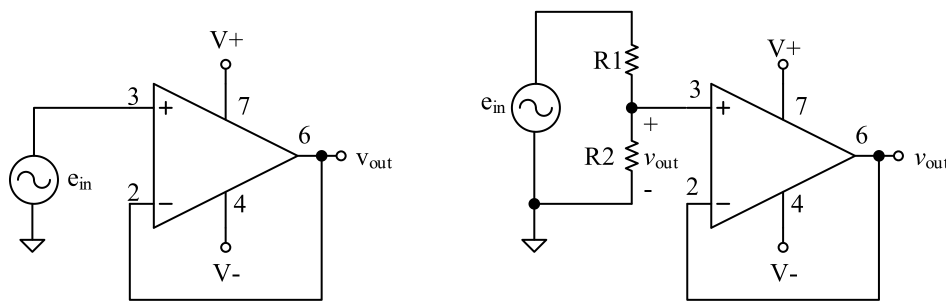
$$i_{INV} \approx i_{NI} \approx 0$$

Second, the open loop gain, v_{out}/v_{in}, is extremely large. This means that for any reasonably sized output (any output in the linear region, not $\pm V_{SAT}$), v_{in} must be negligibly small.

$$v_{in} \approx 0$$

The two inputs leads are at virtually the same potential. You certainly will never be able to measure any significant potential difference between the two inputs during linear operation.

Schematics for a voltage follower are given in Figure 1-3.



a) Simple **b)** Connected to voltage divider

Figure 1-3 Voltage follower

voltage follower

***i*in = 0**

***v*out = *v*in**

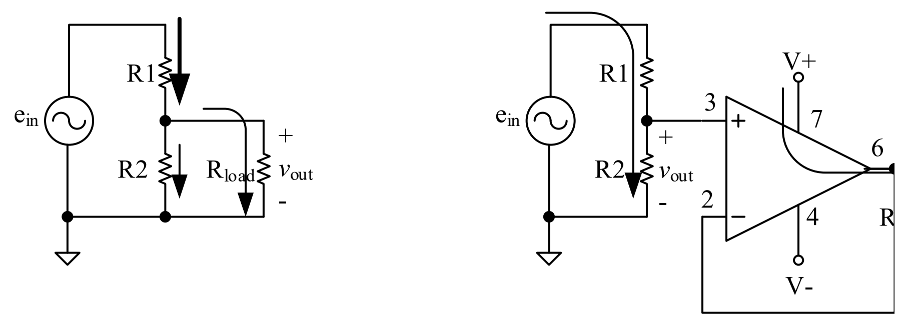
***gain* = 1**

There is no difference between the two inputs. This means that since the noninverting input is at the source potential, so is the inverting input. However, the inverting input is tied **directly** to the output. The output voltage equals the voltage at the noninverting input, which is the input voltage. So the output voltage equals the input voltage, giving a gain of 1.

Remember, no significant current flows into the input of the op amp. So, this circuit will draw *no* current from e_{in}. More importantly, if a voltage divider (Figure 1-3b) with R1 and R2 was connected between e_{in} and the op amp, *none* of I_{R2} would flow into the op amp. The voltage across R2 would *not* change. It is said that the op amp (Figure 1-3a) does not ***load down*** the voltage divider (Figure 1-3b).

The Effect of Loading and Buffering

Placing a load (a resistor or something else to use the output voltage) across the output of a voltage divider lowers that voltage. The act of taking energy out of a circuit lowers the energy available. That is *loading*. This is shown in Figure 1-4a.



a) Loaded voltage divider **b)** Op amp buffer

Figure 1-4 Loading and buffering a voltage divider

Without the load resistor, *all* of the current that flows through R1 then flows through R2, producing v_{out}. However, connecting R_{load} in parallel with R2 means that some of the current from R1 now is diverted to R_{load}. This lowers the current available to R2 and that lowers v_{out}.

However, placing a voltage follower (buffer) between the output of the voltage follower and the load prevents this loading. In Figure 1-4b, no current flows into the input of the op amp. So, all of the current from R1 flows through R2, just as it did without the load connected. The voltage follower passes v_{out} to its output, where R_{load} is connected. The current for R_{load} now comes from the power supplies, through the op amp's output pin.

Example 1-1

a. Given R1 = 5 kΩ, R2 = 10 kΩ, e_{in} = 3 V, calculate v_{out} with no load

b. Calculate the effect of connecting R_{load} = 7 kΩ.

Solution

a.

$$\boldsymbol{v}_{\mathbf{out}} = \frac{3\ \mathrm{V} \times 10\ \mathrm{k\Omega}}{5\ \mathrm{k\Omega} + 10\ \mathrm{k\Omega}} = \mathbf{2\ V}$$

b.

Remember parallel circuits? Connecting R_{load} in parallel with R2 produces a lower resistance.

$$R_{\text{parallel}} = \frac{1}{\frac{1}{R2} + \frac{1}{R_{\text{load}}}} = \frac{1}{\frac{1}{10\ \mathrm{k\Omega}} + \frac{1}{7\ \mathrm{k\Omega}}} = 4.1\ \mathrm{k\Omega}$$

$$\boldsymbol{v}_{\mathbf{out}} = \frac{3\ \mathrm{V} \times 4.11\ \mathrm{k\Omega}}{5\ \mathrm{k\Omega} + 4.11\ \mathrm{k\Omega}} = \mathbf{1.35\ V}$$

This is a 35% reduction (loading down) of v_{out}. Using the voltage follower of Figure 1-4b will prevent this error.

Noninverting Amplifier

The voltage follower uses 100% negative feedback. But, as a result, it produced an output voltage no larger than its input. To produce a larger output, you must *reduce* the amount of negative feedback. This is done by the voltage divider made of R_f and R_i in Figure 1-6.

Resistors in Parallel

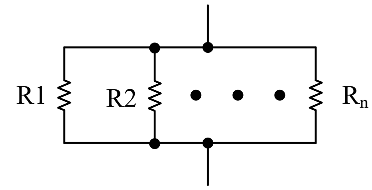


Figure 1-5 Resistors in parallel

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{\mathrm{R1}} + \frac{1}{\mathrm{R2}} + \cdots + \frac{1}{\mathrm{R_n}}$$

$$R_{\text{parallel}} = \frac{1}{\frac{1}{\mathrm{R1}} + \frac{1}{\mathrm{R2}} + \cdots + \frac{1}{\mathrm{R_n}}}$$

or

$$R_{\text{parallel}} = (\mathrm{R1}^{-1} + \mathrm{R2}^{-1} + \cdots + \mathrm{R_n^{-1}})^{-1}$$

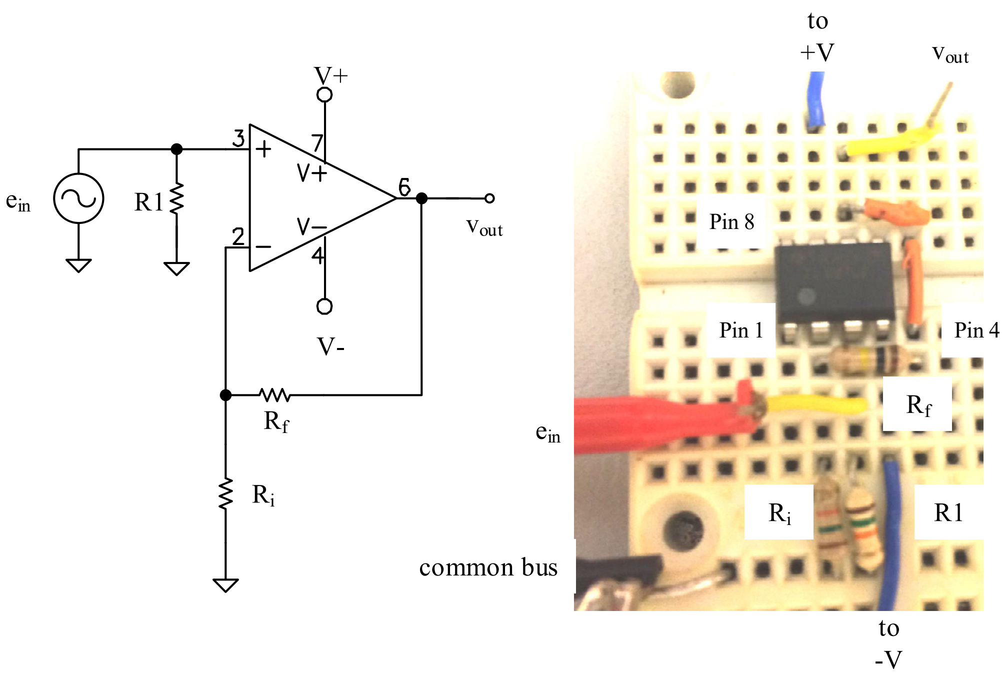


Figure 1-6 Noninverting amplifier

The gain, A_v is

Noninverting Amplifier Gain

$$A_{\mathrm{v}} = \frac{v_{\mathrm{out}}}{e_{\mathrm{in}}} = 1 + \frac{R_{\mathrm{f}}}{R_{\mathrm{i}}}$$

The voltage gain can be derived by recalling that there is virtually no difference between the two inputs of the op amp (during linear operation with negative feedback).

$$v_{Ri} = e_{in}$$

$$i_{R_i} = \frac{v_{R_i}}{R_i} = \frac{e_{in}}{R_i}$$

Since no significant current flows into the inputs of an op amp, the current through R_i is also the current through R_f.

$$i_{R_i} = i_{R_f}$$

The voltage dropped by this current across R_f is

$$v_{R_f} = i_{R_f} \times R_f$$

$$= \frac{e_{in}}{R_i} \times R_f$$

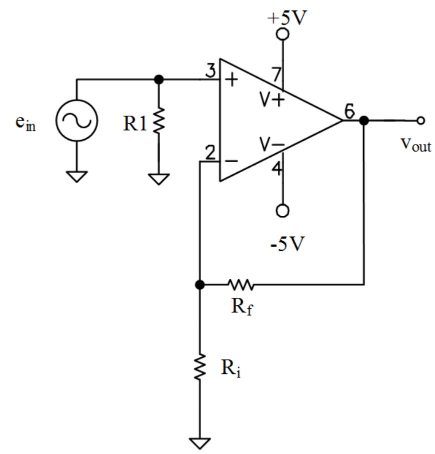


Figure 1-6 Noninverting amplifier

The output voltage is the voltage across R_i plus the voltage across R_f.

$$v_{out} = v_{R_i} + v_{R_f}$$

$$= e_{in} + \frac{e_{in}}{R_i} R_f$$

$$= e_{in}\left(1 + \frac{R_f}{R_i}\right)$$

Since the gain is the output voltage divided by the input voltage,

$$A_v = \frac{v_{out}}{e_{in}} = 1 + \frac{R_f}{R_i}$$

Noninverting Amp input Impedance

with R1:
Z_{in} = R1

without R1:
Z_{in} = GΩ

The resistor R1 has two functions. First it is an input pull-down resistor, providing a path from ground for the nA of *bias* current that must flow into the input of the op amp for it to work correctly. If the input is left open, then this tiny amount of current (that is usually ignored) cannot properly bias the input transistors into their linear region. They turn off, sending the op amp's output to the rail, i.e. the maximum positive or negative voltage available from the supplies.

Secondly, the impedance that e_{in} sees looking into the noninverting terminal of the op amp is in the GΩ. Though this is so large as to often be considered an open, it does vary from IC to IC, and with operating conditions. Resistor R1 sets that impedance to a defined, stable, predictable value.

Inverting Amplifier

The circuit shown in Figure 1-7 is an *inverting* amplifier. It is inverting because e_{in} is, eventually, connected to the inverting input rather than the noninverting input (as it is in the noninverting amplifier in Figure 1-6). *Negative* feedback is needed to assure linear operation, an amplifier. In Figure 1-7, this is provided from the output, through R_f to the inverting input. That is just how negative feedback is provided for the noninverting amplifier in Figure 1-6 too.

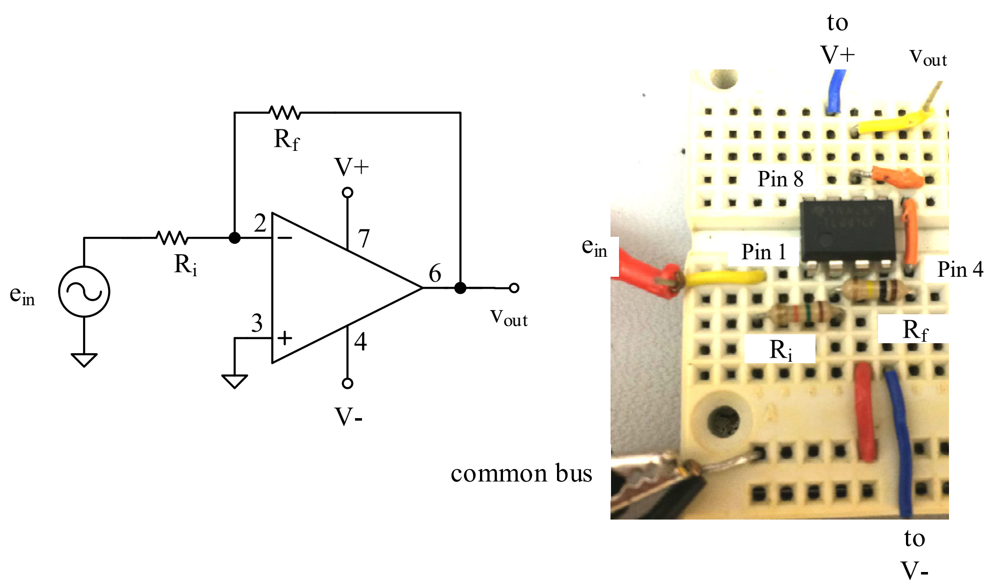


Figure 1-7 Inverting amplifier

The inverting input is at

virtual ground.

This negative feedback drives the output voltage to whatever potential is necessary to assure that there is no significant difference between the inverting input and the noninverting input. Since the noninverting input is connected directly to ground, the inverting input voltage is also at (nearly, *virtually*) ground. This is significantly different from the noninverting amplifier, and dictates its operation.

Inverting Amp input Impedance

$Z_{in} = R_i$

This means that between the input source, e_{in}, and ground (or virtual ground) is R_i. That resistor sets the input impedance. This is a significant consideration when analyzing or designing an inverting amplifier with an op amp.

Figure 1-8 shows the voltages and currents around the inverting amplifier. Because of negative feedback, the inverting input pin is driven to virtual ground, 0 V. This puts the entire input voltage across R_i

$$V_{\mathrm{Ri}} = e_{\mathrm{in}}$$

$$I_{\mathrm{Ri}} = \frac{V_{\mathrm{Ri}}}{R_{\mathrm{i}}} = \frac{e_{\mathrm{in}}}{R_{\mathrm{i}}}$$

The current that flows into the input pin of the op amp is so small that it can be ignored. Summing the currents at that input pin gives

$$I_{\mathrm{Ri}} = I_{\mathrm{input\ pin}} + I_{\mathrm{Rf}}$$

$$I_{\mathrm{input\ pin}} = 0$$

$$I_{\mathrm{Ri}} = I_{\mathrm{Rf}}$$

$$I_{\mathrm{Ri}} = \frac{e_{\mathrm{in}}}{R_{\mathrm{i}}} = I_{\mathrm{Rf}}$$

The voltage across R_f, V_{Rf}, is

$$V_{\mathrm{Rf}} = I_{\mathrm{Rf}} \times R_{\mathrm{f}}$$

$$V_{\mathrm{Rf}} = \frac{e_{\mathrm{in}}}{R_{\mathrm{i}}} R_{\mathrm{f}}$$

Look carefully at the direction of I_{Rf} and the polarity of the output voltage. Current can only flow left-to-right through R_f if the output voltage is negative with respect to the left end of R_f, virtual ground.

$$V_{\mathrm{out}} = -V_{\mathrm{Rf}}$$

$$V_{\mathrm{out}} = -\frac{e_{\mathrm{in}}}{R_{\mathrm{i}}} R_{\mathrm{f}} = -\frac{R_{\mathrm{f}}}{R_{\mathrm{i}}} e_{\mathrm{in}}$$

Inverting amplifier gain

$$A_{\mathrm{v}} = \frac{V_{\mathrm{out}}}{e_{\mathrm{in}}} = -\frac{R_{\mathrm{f}}}{R_i}$$

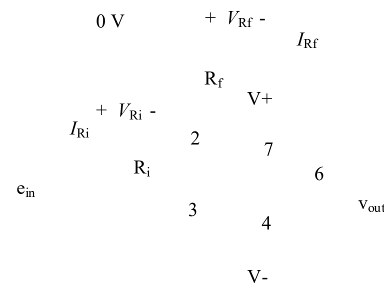


Figure 1-8 Inverting amp gain derivation

Inverting Summer

Several voltage sources can be combined by connecting each, through its own input resistor to the virtual ground of the inverting amplifier, creating an *inverting summer*. This is shown in Figure 1-8.

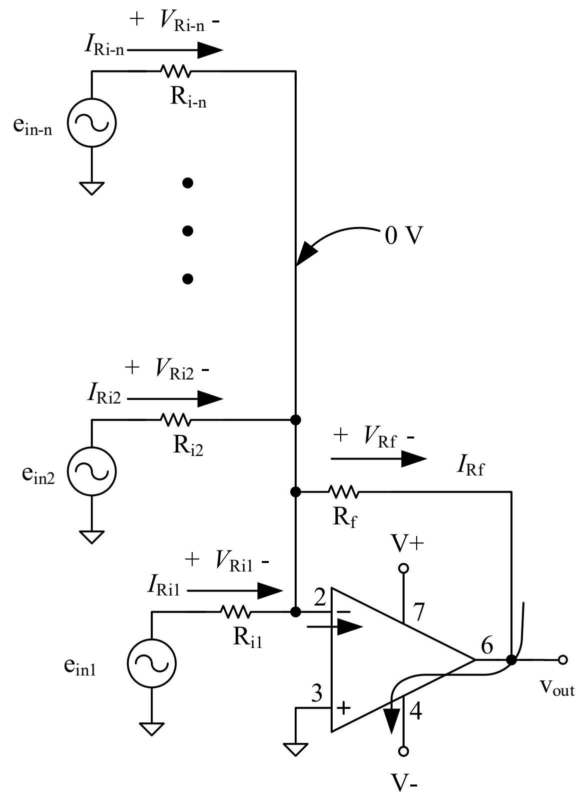


Figure 1-8 Inverting summer schematic

Each source sends its current into the virtual ground node where they combine to create the current through the feedback resistor.

$$I_{\mathrm{Ri1}} = \frac{V_{\mathrm{Ri1}}}{R_{\mathrm{i1}}} = \frac{e_{\mathrm{in1}}}{R_{\mathrm{i1}}} \qquad I_{\mathrm{Ri2}} = \frac{V_{\mathrm{Ri2}}}{R_{\mathrm{i2}}} = \frac{e_{\mathrm{in2}}}{R_{\mathrm{i2}}} \qquad I_{\mathrm{Ri-n}} = \frac{V_{\mathrm{Ri-n}}}{R_{\mathrm{i-n}}} = \frac{e_{\mathrm{in-n}}}{R_{\mathrm{i-n}}}$$

$$I_{R\mathrm{f}} = I_{\mathrm{R1}} + I_{\mathrm{R2}} + \cdots + I_{\mathrm{R-n}}$$

$$I_{\mathrm{Rf}} = \frac{e_{in1}}{R_{i1}} + \frac{e_{in2}}{R_{i2}} + \cdots + \frac{e_{\mathrm{in-n}}}{R_{\mathrm{i-n}}}$$

That current produces the output voltage.

$$V_{\mathrm{out}} = -I_{\mathrm{Rf}} \times R_{\mathrm{f}}$$

$$V_{\mathrm{out}} = -\left(\frac{e_{in1}}{R_{i1}} + \frac{e_{in2}}{R_{i2}} + \cdots + \frac{e_{\mathrm{in-n}}}{R_{\mathrm{i-n}}}\right) R_{\mathrm{f}}$$

$$V_{\mathrm{out}} = -\left(\frac{R_{\mathrm{f}}}{R_{i1}} e_{in1} + \frac{R_{\mathrm{f}}}{R_{i2}} e_{in2} + \cdots + \frac{R_{\mathrm{f}}}{R_{\mathrm{i-n}}} e_{\mathrm{in-n}}\right)$$

Inverting summer output equation

Each input source has its own gain, set by its input resistor and by the feedback resistor. Changing $R_{\mathrm{i-n}}$ changes the contribution that source has on the mix of signals. Changing R_{f} changes the overall gain without altering the proportion that each separate input contributes.

Remember, each input resistor is connected to virtual ground. So it sets the input impedance that its source experiences, and the current its source must provide.

Example 1-2

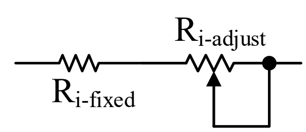
Design an audio mixer to combine a voice, a guitar, and a drum. Each input must have an:

a. impedance of at least 10 kΩ.

b. adjustable gain (mix) 0.5 to 50.

Solution

Refer to Figure 1-8. Make each R_i with a fixed resistor in series with a rheostat.



$R_{\text{i-fixed}}$ = 10 kΩ

Even when the rheostat is set to 0 Ω, the input impedance is 10 kΩ.

Each signal's gain is set by

$$A_{\mathrm{v-n}} = \frac{V_{\mathrm{out}}}{e_{\mathrm{in-n}}} = -\frac{R_{\mathrm{f}}}{R_{\mathrm{i-n}}}$$

The smaller $R_{i\text{-}n}$, the larger the gain. So when the rheostat is set to 0 Ω the gain will be at its maximum, 50.

$$\boldsymbol{R_f} = A_{v-n} \times R_{i-n} = 50 \times 10\ \text{k}\Omega = \mathbf{500\ k\Omega}$$

The smallest gain comes when $R_{i\text{-}n}$ is at its largest.

$$A_{v-n} = \frac{V_{out}}{e_{in-n}} = -\frac{R_f}{R_{i-n}} = \frac{R_f}{R_{i-fixed} + R_{i-adjust}}$$

$$R_{i-fixed} + R_{i-adjust} = \frac{R_f}{A_{v-n\ min}}$$

$$R_{i-adjust} = \frac{R_f}{A_{v-n\ min}} - R_{i-fixed}$$

$$R_{i-adjust} = \frac{500\ \text{k}\Omega}{0.5} - 10\ \text{k}\Omega$$

$$R_{i-adjust} = 990\ \text{k}\Omega$$

Select $\mathbf{R_{i\text{-}adjust}} = \mathbf{1\ M\Omega}$.

Difference Ampifier: Fundamentals

The inverting amplifier can be combined with the noninverting amplifier to create a circuit with several very useful characteristics. Its schematic is shown in Figure 1-9.

Proper operation requires that

$$R_{i1} = R_{i2} = R_i$$

$$R_{f1} = R_{f2} = R_f$$

This matching should be as perfect as possible, 1% or closer.

Assuming a perfect match, the output voltage is

Signals can be **amplified.**

$$v_{out} = \frac{R_f}{R_i}(e_{in1} - e_{in2})$$