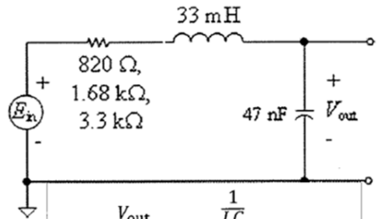
2

Laplace Second Order – Series RLC



$$\frac{V_{out}}{E_{in}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Quadratic equation *roots*

$$s^2 + 2\zeta\omega_n s + \omega_n{}^2 = 0$$

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

A = 1 $\quad B = 2\zeta\omega_n \quad C = \omega_n{}^2$

Real
Overdamped

Two REAL ROOTS

$\sqrt{B^2-4AC}$ is real

$B^2 > 4AC$

$\frac{R^2}{L^2} > \frac{4}{LC}$

$R^2 > 4\frac{L}{C}$

$\boxed{R > 2\sqrt{\frac{L}{C}}}$

Equal
Critically damped

Two EQUAL Roots

$\sqrt{B^2-4AC} = 0$

$B^2 = 4AC$

$\frac{R^2}{L^2} = \frac{4}{LC}$

$R^2 = 4\frac{L}{C}$

$\boxed{R = 2\sqrt{\frac{L}{C}}}$

Complex
Under damped

Two Complex Roots

$\sqrt{B^2-4AC}$ is imaginary

$B^2 < 4AC$

$\frac{R^2}{L^2} < \frac{4}{LC}$

$R^2 < 4\frac{L}{C}$

$\boxed{R < 2\sqrt{\frac{L}{C}}}$

(3)

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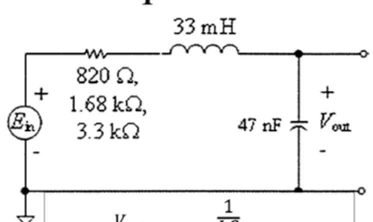

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Real
Overdamped

$B^2 > 4AC$

$(2\zeta\omega_n)^2 > 4\omega_n{}^2$

$4\zeta\omega_n{}^2 > 4\omega_n{}^2$

$\zeta > 1$

Equal
Critically damped

$\zeta = 1$

Complex
Under damped

$\zeta < 1$

$2\zeta\omega_n = \frac{R}{L}$

$\zeta = \frac{1}{2}\frac{1}{\omega_n}\frac{R}{L}$

$\zeta = \frac{1}{2}\frac{1}{\frac{1}{\sqrt{LC}}}\frac{R}{L}$

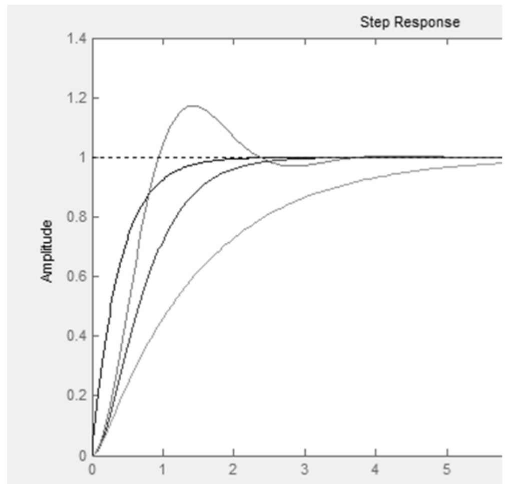
$\zeta = \frac{1}{2}\sqrt{LC}\,\frac{R}{L}$

$\boxed{\zeta = \frac{1}{2}R\sqrt{\frac{C}{L}}}$

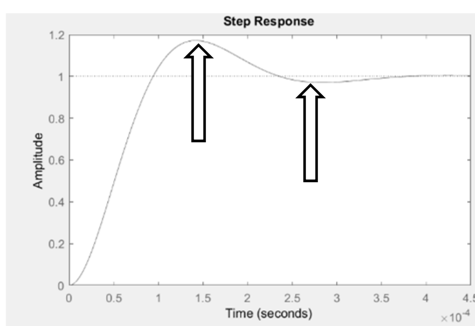
RLC responses to a step

22. $$\frac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \qquad A\left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t - \psi\right)\right]$$

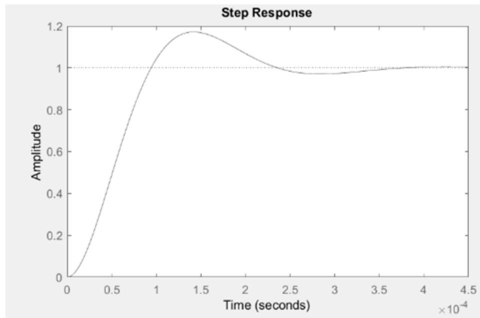
- clc
- clear
- s=tf('s')
-
-
- L=33e-3;
- C=47e-9;
-
- R1=820;
- R2=1.68e3;
- R3=3.3e3;
-
- G=1/(R1*C*s+1)
- G1=(1/(L*C))/(s*s+(R1/L)*s+1/(L*C))
- G2=(1/(L*C))/(s*s+(R2/L)*s+1/(L*C))
- G3=(1/(L*C))/(s*s+(R3/L)*s+1/(L*C))
- ltiview(G,G1,G2,G3)



22. $$\frac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \qquad A\left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t - \psi\right)\right]$$



22. $\dfrac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ $\qquad$ $A\left[1 + \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t - \psi\right)\right]$



$\zeta = 0$?

22. $\dfrac{A\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ $\qquad$ $A\left[1 + \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t - \psi\right)\right]$

$\zeta = 0$?

6. $\dfrac{A\omega}{s^2 + \omega^2}$ $\qquad$ $A \sin \omega t$

6. The 5% settling time is the time from when the input step occurs until the output *settles* to within 5% of its final level. Look at Figure 1.

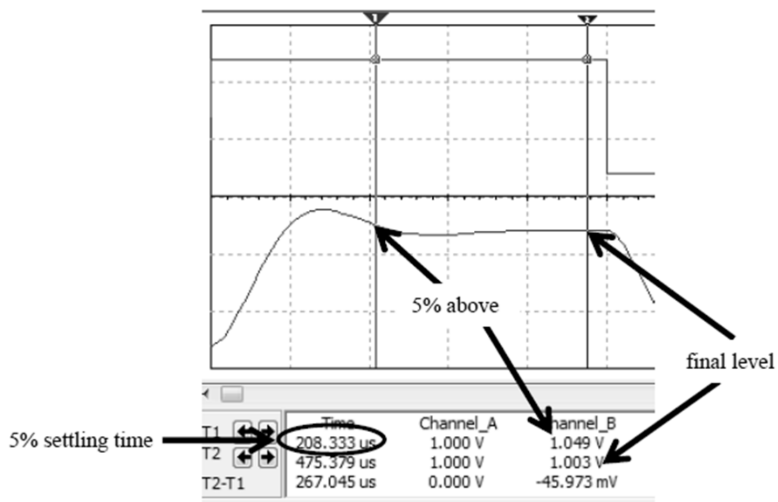
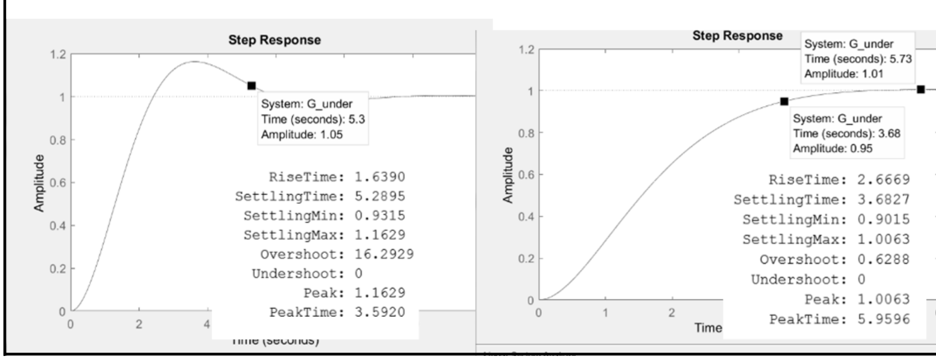


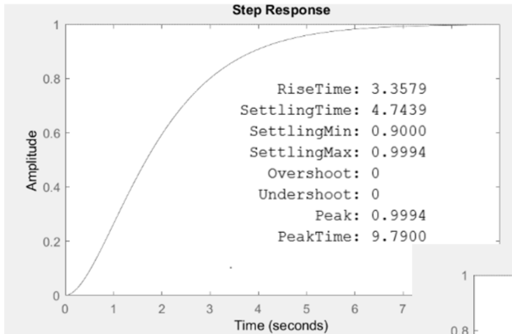
Figure 1 5% settling time measurement

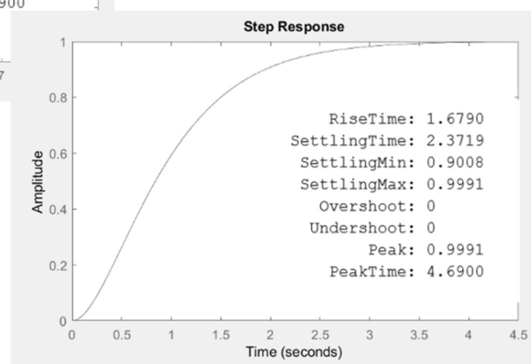
Effect of ξ

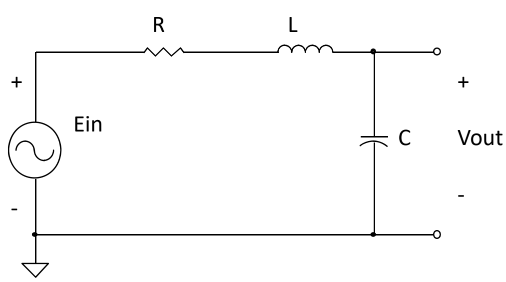
```
clc
clear
s=tf('s')
A=1;
z=0.5;
w=1;
G_under=(A*w^2)/(s^2+2*z*w*s+w^2)
ltiview(G_under)

S=stepinfo(G_under,'SettlingTimeThreshold', 0.05)
```



Effect of ω_n





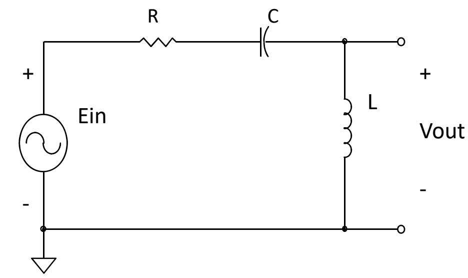


$$\frac{V_{\text{out}}}{E_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + 2\zeta\omega_n s + \omega_n{}^2 = 0$$

$\overline{G}_{f=0} = ?$

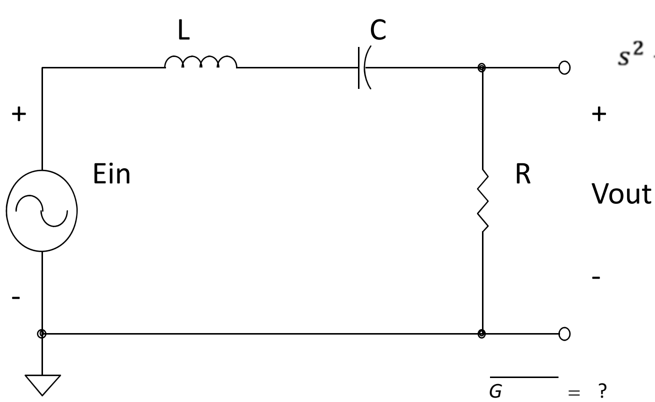
$\overline{G}_{f\to\infty} = ?$



$\overline{G_{f=0}} = ?$

$\overline{G_{f\to\infty}} = ?$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$



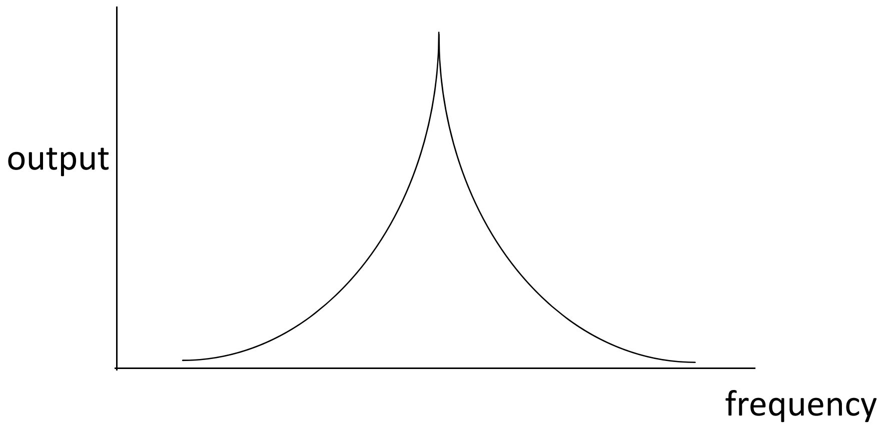
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$\overline{G_{f=0}} = ?$

$\overline{G_{f\to\infty}} = ?$

Resonance Phenomina:

Output versus Frequency



Resonance Frequency Derivation

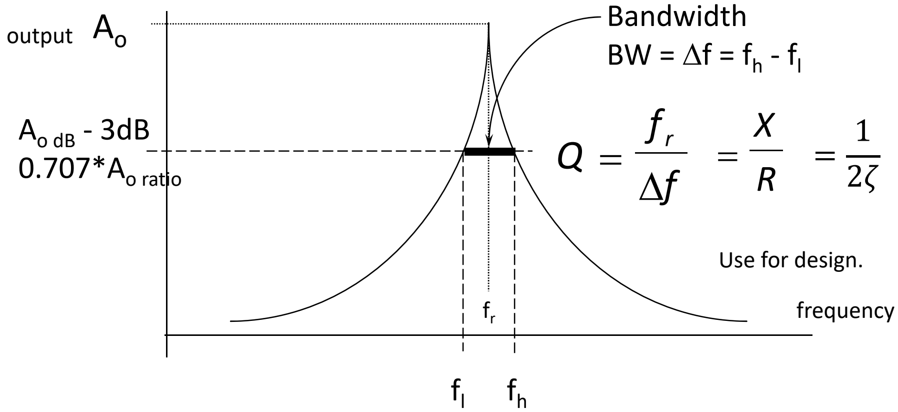
$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Selectivity



$$Q = \frac{f_r}{\Delta f} = \frac{X}{R} = \frac{1}{2\zeta}$$

Use for design.