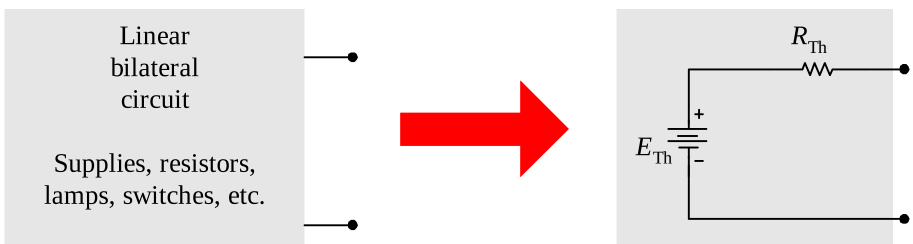
1.2 Thevenin’s Theorem

Adapted from DC/AC Circuits and Electronics by Robert Herrick

Any *linear* system can be simplified to a single voltage source in series with a single impedance. This makes figuring out how your circuit will interact with a large, complex, even unknown system *much* simpler. You only have to complete two tests or two calculations and you are done.

Figure 1-13 demonstrates this theorem. Figure 1-13(a) represents a complicated circuit and Figure 8-28(b) represents its simple Thévenin model equivalent.



(a) (b)

Figure 1-13 Thévenin model equivalent of a circuit

The model represents the actual circuit in terms of all possible load conditions. Once the model is established, it is easier to work with than an original complicated circuit.

E_{Th} by *Measurement*

So there is an electronic gismo with a pair of output terminals where the load is attached (such as a function generator). How can the Thévenin voltage E_{Th} of Figure 1-13(a) be measured? You cannot go inside the circuit and measure this voltage; you can only work at the terminals of the original circuit of Figure 1-13(a). Just *measure* the *open circuit voltage* at the terminals. Since there is no current with the unloaded circuit, the voltmeter measures the value of E_{Th}. Good voltmeters have very high internal resistances (1 MΩ or higher). If R_{Th} is a much lower resistance than this, then the voltmeter provides an accurate measurement of E_{TH}.

$$E_{TH} = V_{\text{no load}} = V_{\text{open circuit}}$$

R_{Th} by *Measurement*

To measure R_{Th} attach a load and draw a load current through R_{Th}. Then, measure two of the following: (1) the load voltage, and either (2) the load current, or the load resistance. Ohm's law then can be used to calculate R_{TH}. Look at Figure 1-14.

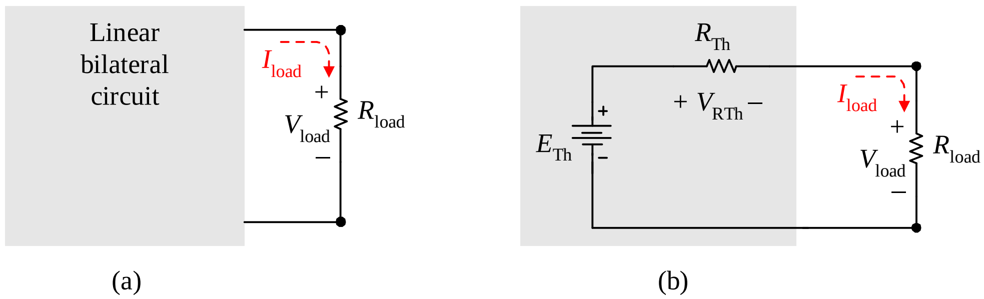


Figure 1-14 Finding R_{Th} with loaded circuit and model

Example 1-4

a. A half-bridge circuit is analyzed in the lab. The load is removed and its open circuit voltage is measured to be 10 V. A load resistance of 4 kΩ is reattached and the load voltage is measured to be 8 V. Find and draw the Thévenin model for this circuit.

b. If a 1 kΩ resistor were attached as a load instead of the 4 kΩ resistor, what would the load voltage and current be?

Solution

a. Based upon the no load measurement,

$$E_{TH} = 10\text{ V}$$

Figure 1-15(a) represents the Thévenin model for this circuit with known values: open circuit voltage, loaded circuit voltage, and load resistance.

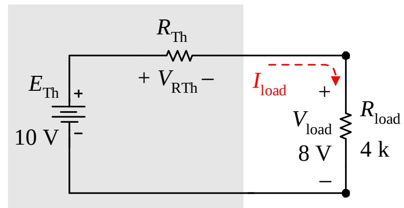


Figure 1-15(a) Model loaded with 4 kΩ load resistor

Find the voltage drop.

$$V_{RTh} = 10\,\text{V} - 8\text{ V} = 2\text{ V}$$

Next, calculate the load current I_{load}.

$$I_{RTh} = I_{load} = \frac{8\text{ V}}{4\text{ k}\Omega} = 2\text{ mA}$$

$$\mathrm{R}_{\mathbf{Th}} = \frac{2\,\mathbf{V}}{2\,\mathbf{mA}} = 1\,\mathbf{k\Omega}$$

b. Figure 1-15(b) represents the Thévenin model of the half-bridge circuit with a 1 kΩ load attached for analysis. Find the total circuit resistance of the series circuit.

$$R_{total} = 1\text{ k}\Omega + 1\text{ k}\Omega = 2\text{ k}\Omega$$

Find the series circuit current.

$$I_{load} = \frac{10\text{ V}}{2\text{ k}\Omega} = 5\text{ mA}$$

Find the load voltage.

$$V_{load} = 5\text{ mA} \times 1\text{ k}\Omega = 5\text{ V}$$

So, the original bridge circuit with a 1 kΩ load resistor instead of a 4 kΩ resistor would have a load current of 5 mA and a load voltage drop of 5 V. Notice that the details of the original circuit were not even known, but using Thevenin's Theorem its interaction with the rest of the world was calculated.

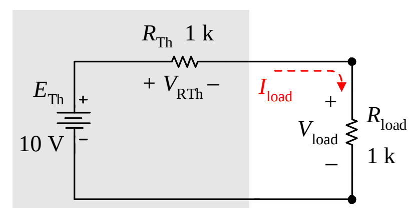


Figure 1-15(b) Model with a 1 kΩ load

Thevenin's Values by *Analysis*

The response of the RC charge circuit in Figure 1-16 is covered elsewhere. In summary, when e_{in} steps up, current begins to flow, limited by the resistor. As time goes on, charge from that current accumulates on the plates of the capacitor. Its voltage rises. Given enough time, the voltage on the capacitor matches the input voltage. At that point current stops and the capacitor continues to hold that charge (voltage) until the input drops to discharge the capacitor. How far the capacitor charges depends on the input voltage, and how quickly it can charge. That charge rate depends on both R and C. The mathematical relationships are developed elsewhere.

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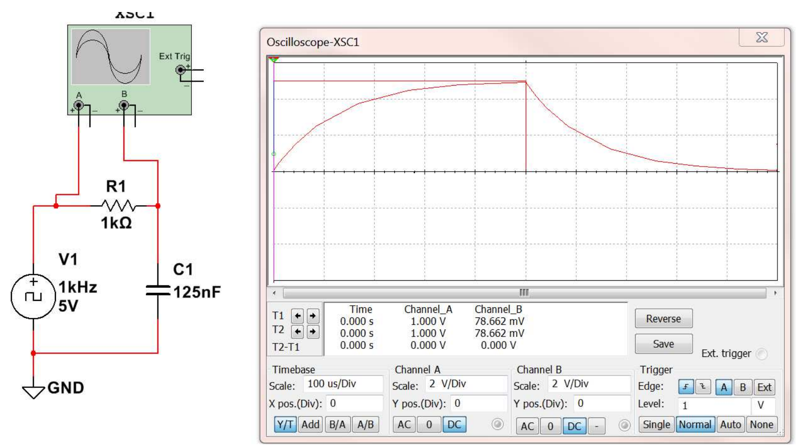
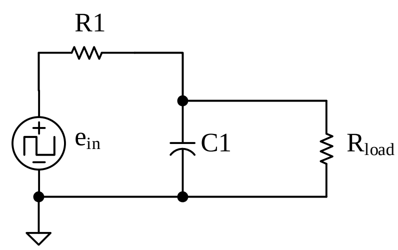


Figure 1-16 RC charge circuit

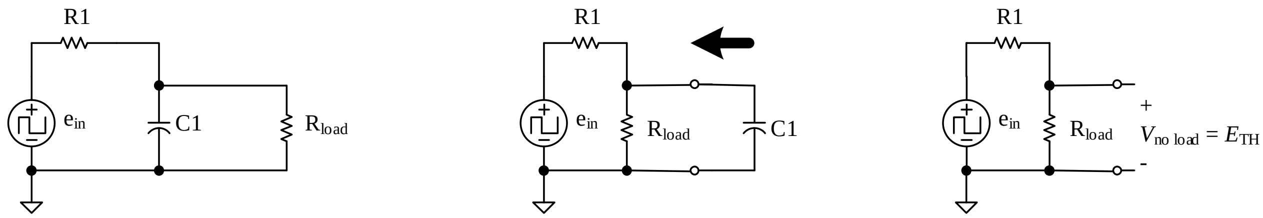
What happens when a load resistance is placed in parallel with C1, to use this ramp in a following circuit? Look at Figure 1-17.



(a) Loaded RC network

Figure 1-17 RC circuit loading

The first step is to rearrange the components, placing the one of interest to the far right. Since the charging of the capacitor is the process being considered, the capacitor has been moved in Figure 1-17b.



(a) Loaded RC network **(b)** Network seen by the capacitor **(c)** $V_{\text{no load}} = E_{\text{TH}}$

Figure 1-17 RC circuit loading

First, remember that

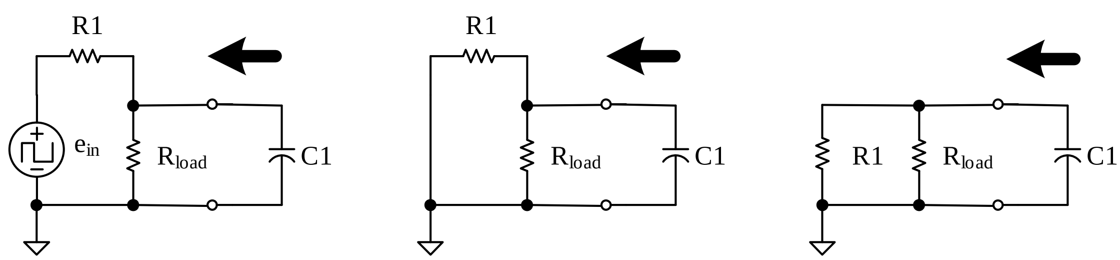
$$E_{\text{TH}} = V_{\text{no load}}$$

So, redraw the circuit with the load (the capacitor) removed, then calculate the resulting output voltage. The schematic is shown in Figure 1-17 (c). For this particular circuit, using the voltage divider law,

$$E_{\text{TH}} = e_{\text{in}} \frac{R_{\text{load}}}{R1 + R_{\text{load}}}$$

A different network provides a different relationship between e_{in} and E_{TH}.

To calculate R_{TH}, sources are replaced by their characteristic impedance. For a voltage source, e_{in}, is replaced by a short. Then, looking back into the circuit from the capacitor, determine the resistance seen. This is shown in Figure 17 (d) and (e).



(b) Network seen by the capacitor **(d)** Source replace with short **(e)** R1 & R_{load} in parallel

Figure 1-17 RC circuit loading

By inspection, for this particular network, R1 and R_{load} are in parallel, *not* series.

$$R_{\mathrm{TH}} = \frac{1}{\frac{1}{\mathrm{R1}} + \frac{1}{R_{\mathrm{load}}}}$$

A different network provides a different relationship between its components and R_{TH}.

Example 1-5

a. For the circuit in Figure 1-16, determine the Thevenin's equivalent circuit when a 2 kΩ resistor is connected as a load.

b. Verify that the model is correct by simulating both the loaded circuit and the model.

Solution

a.

$$\boldsymbol{E_{\mathrm{TH}}} = e_{\mathrm{in}} \frac{R_{\mathrm{load}}}{R1 + R_{\mathrm{load}}} = 5\,V \frac{2\ \mathrm{k}\Omega}{1\ \mathrm{k}\Omega + 2\ \mathrm{k}\Omega} = \mathbf{3.33\ V}$$

$$\boldsymbol{R_{\mathrm{TH}}} = \frac{1}{\frac{1}{\mathrm{R1}} + \frac{1}{R_{\mathrm{load}}}} = \frac{1}{\frac{1}{1\ \mathrm{k}\Omega} + \frac{1}{2\mathrm{k}\Omega}} = \mathbf{667\ \Omega}$$

b.

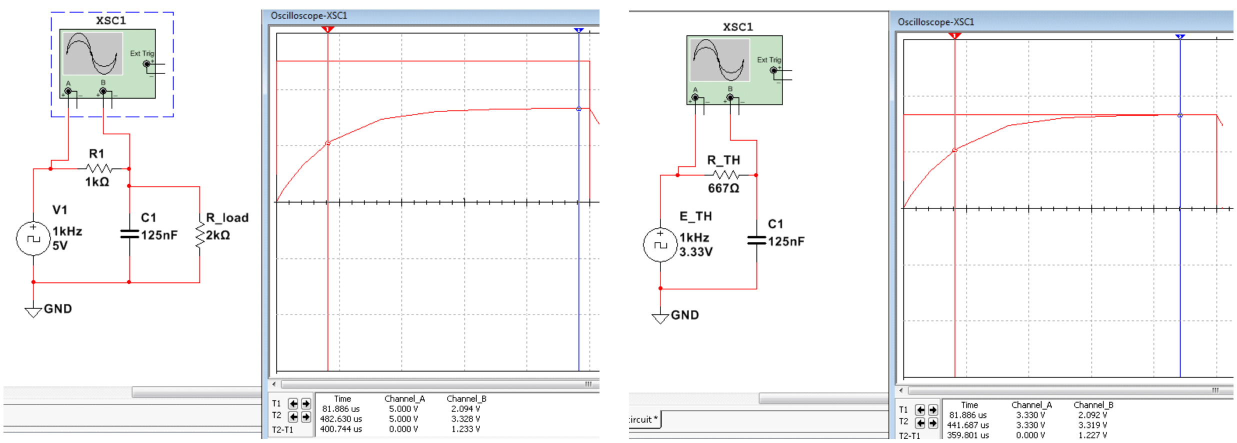


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Figure 1-18 Loaded circuit and Thevenin's model simulations

Look carefully at Figure 1-18. The loaded circuit, on the left, outputs a rising ramp, that eventually reaches 3.33 V (cursor B at T2), and takes 81 µs to reach 2.09 V in one time constant (cursor B at T1).

The Thevenin's model, on the right also reaches 3.33 V and takes the same time, 81 µs, to reach the same voltage, 2.09 V. The two circuits provide equivalent performance.

1.3 AC Phasors

Sinusoids

The sine wave is one of the most common waveshapes used in developing and testing electronic circuits. It is the only signal whose shape is not altered as it passes through linear systems; sine in=> sine out. A voltage sine wave is shown in Figure 1-19.

V_p = peak amplitude

f = frequency in Hz cycles per second

T = period in sec seconds per cycle

$$f = \frac{1}{T}$$

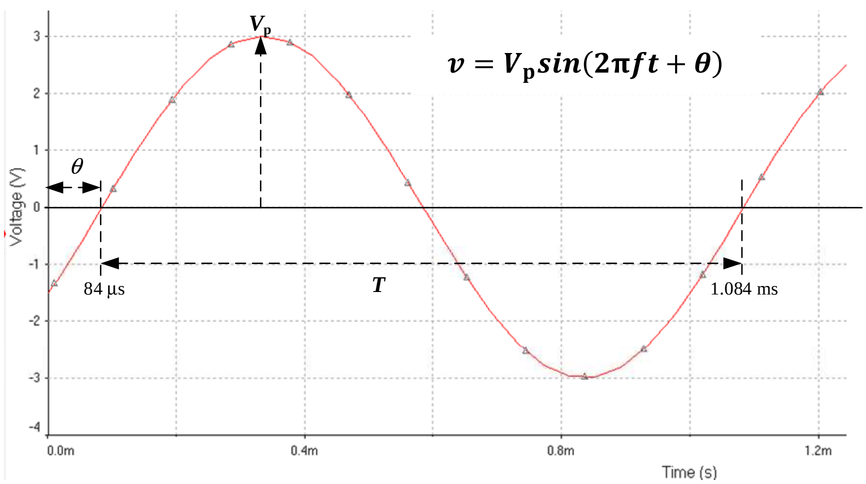


Figure 1-19 Sine wave

There are several details. The 2π are in radians. Multiplying that by *f* in cycles/second, and then by time produces an angle in *radians*. So, for the function to be properly evaluated, the phase shift, θ, must also be expressed in radians, not degrees.

θ = phase shift in radians

360° = 2π radians

$$\frac{\theta}{-t_{phase}} = \frac{360^o}{T}$$

The phase shift, θ, indicates when the sine wave begins, crossing ground going positive. In Figure 1-19, that happens at t_{phase} = 84 µs. This can be converted into degrees with the ratio to the right.

2.1 Impedance Combination

Fundamental Technique

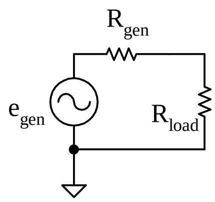
One solution to a complicated network, with a single sourse, is to combine and reduce, combine and reduce, combine and reduce the circuit until it becomes a simple series or parallel circuit. Then, work your way back out, with Ohm's law, Kirchhoff's laws, and the voltage divider law. Determine the effect the source has on each layer as you reintroduce the complexity by undoing the combinations you created to simplify the problem.

The trick in all of this combining is the documentation. Only the most gifted can keep all of the simplifications and combinations in their heads. It is critical to develop a consistent way of naming the intermediate combinations of components, and to redraw the circuit *every* time you combine elements, changing its topology.

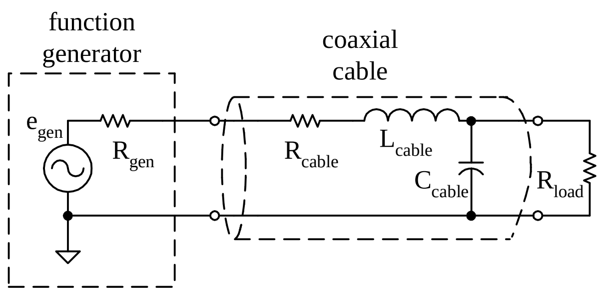
Once you have developed a clear, workable plan, it is then just a matter of plugging in the numbers, step-by-step, combination-by-combination, working your way through the simplified schematics out to the full schematic.

High-frequency Cabling

When interconnecting pieces of electronic equipment, it is critical that the signals be passed accurately. As the signal's frequency exceeds 1 MHz, you must consider the effects of the signal generator's output resistance, the cable's *RLC* model, and the load's resistance, capacitance, and inductance. A circuit that appears as a simple generator driving a load resistor becomes much more complex as you consider the high-frequency effects. Figure 2-1 shows both the simple and the first approximation of the high-frequency model for the interconnections.



(a) Simple circuit



(b) Circuit with cable characteristics

Figure 2-1 First approximation of a high-frequency connection

Specifications for high-frequency cables include:

- Resistance per unit length.
- Capacitance per unit length.
- Characteristic impedance, Z_o.

The characteristic impedance of the cable plays a significant role in accurately transmitting high-frequency signals. Most of the details are best left to a complete text on rf electronics. For our purposes, the characteristic impedance can be used to determine the cable's inductance.

$$Z_o = \sqrt{\frac{L_{cable}}{C_{cable}}}$$

$$L_{cable} = C_{cable} \times Z_o^2$$

Terminated in its characteristic impedance (50 Ω in Example 2-1) a cable provides a well behaved, critically damped, low-pass performance. (There is much more about these parameters in later chapters). However, as soon as the termination no longer matches the cable's characteristic impedance, the resulting *RLC* filter may become underdamped. The voltage delivered to the load can even be *greater* than that provided by the function generator.

The circuit becomes even more complicated when you begin to use hook-up wire to connect from the cable's BNC connector to the input of your circuit. The inductance of wire is given as

$$L \approx 2\times10^{-7}\,\frac{\text{H}}{\text{m}}\,l\left[2.303\log_{10}\left(\frac{4l}{d}\right)-1\right]$$

where l = length of the wire.
d = diameter of the wire.

At high frequencies, this inductance adds another rung to the circuit diagram ladder, and another set of calculations

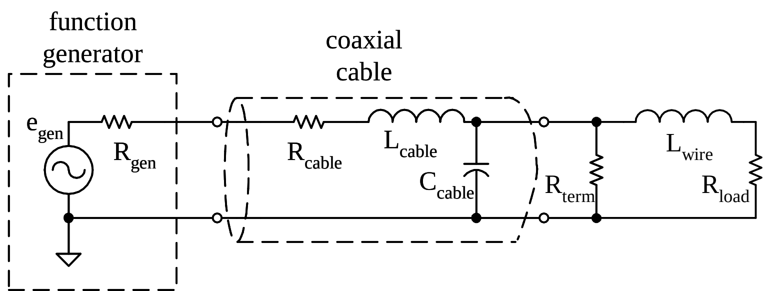


Figure 2-2 Schematic for Example 2-1

Example 2-1

The following specifications are given for a coaxial cable:

R = 275 Ω/1000 ft, Z_o = 50 Ω, C = 24.2 pF/ft

Use a a 6-foot-long cable, add 50 Ω terminator, followed by a 12-inch piece of wire leading to a 100 Ω load.

Calculate the voltage across the 100 Ω load at 15 MHz.

Solution

a.

$$R_{cable} = 275\frac{\Omega}{1000\,ft}\times 6\,ft = 1.65\,\Omega$$

$$C_{cable} = 24.2\frac{pF}{ft}\times 6\,ft = 145\,pF$$

$$L_{cable} = C_{cable}\times Z_o^{\ 2} = 145\,pF\times(50\,\Omega)^2 = 363\,nH$$

$$X_{L\,cable} = 2\pi\times 15\,MHz\times 363\,nF = 34.21\,\Omega$$

$$X_{C\,cable} = \frac{1}{2\pi\times 15\,MHz\times 145\,pF} = 73.17\,\Omega$$

The 12-inch piece of wire is

$$l = 12\,inches\times\frac{2.54\,cm}{inch}\times\frac{1\,m}{100\,cm} = 0.3045\,m$$

Hook-up wire has a diameter of 0.643 millimeters.

$$L_{wire} \approx 2\times 10^{-7}\,\frac{H}{m}\times 0.305\,m\left[2.303\log_{10}\left(\frac{4\times 0.305\,m}{0.643\times 10^{-3}\,m}\right)-1\right]$$

$$L_{wire} = 399.5\,nH$$

$$X_{L\,wire} = 2\pi\times 15\,MHz\times 399.5\,nH = 37.65\,\Omega$$

Redraw the schematic in terms of impedances, Figure 2-3.

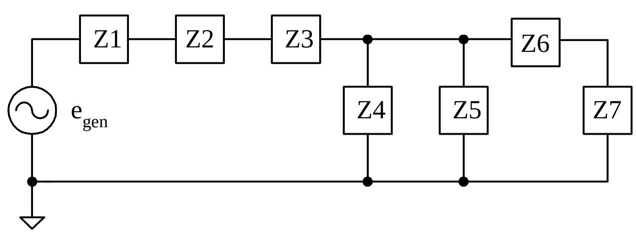


Figure 2-3 Block diagram for Example 2-1

$$\overline{Z1} = (50\,\Omega\angle 0°)$$

$$\overline{Z2} = (1.7\,\Omega\angle 0°)$$

$$\overline{Z3} = (34.21\angle 90°)$$

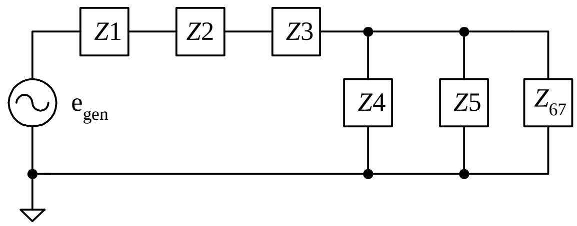
$$\overline{Z4} = (73.17\angle -90°)$$

$$\overline{Z5} = (50\,\Omega\angle 0°)$$

$$\overline{Z6} = (37.7\,\Omega\angle 90°)$$

$$\overline{Z7} = (100\,\Omega\angle 0°)$$

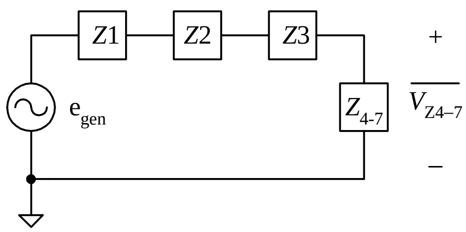
Begin the impedance combination as far from the source as possible. Combine Z6 and Z7 to form Z_{67}. Draw the diagram to keep track of the circuit's configuration, shown in Figure 2-4.



$$\overline{Z_{67}} = \overline{Z6} + \overline{Z7}$$

Figure 2-4 Impedance combination of $Z6$ and $Z7$

Impedances $Z4$, $Z5$, and Z_{67} are in parallel. They can be combined into a single impedance, Z_{4-7}. The new configuration is shown in Figure 2-5.

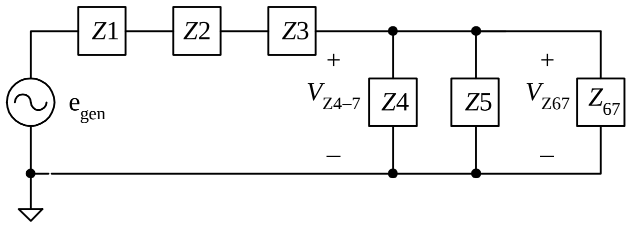


$$\overline{Z_{4-7}} = \left(\overline{Z4}^{-1} + \overline{Z5}^{-1} + \overline{Z_{67}}^{-1}\right)^{-1}$$

Figure 2-5 Impedance combination of $Z4$, $Z5$, and Z_{67}

This is a simple series circuit. No further impedance combinations are needed. Apply the voltage divider law to calculate $V_{Z4\text{-}7}$, the voltage across $Z_{4\text{-}7}$.

This is the voltage across $Z4$, $Z5$, and Z_{67}. Figure 2-6 is a repeat of Figure 2-4, with this voltage added.



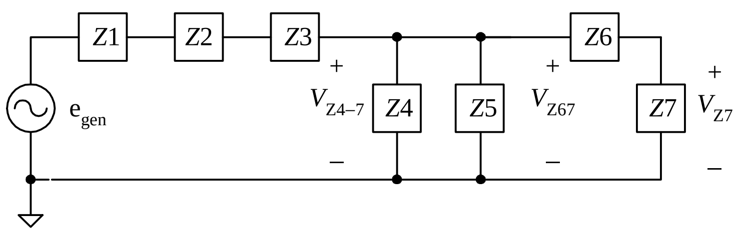
$$\overline{V_{Z4\text{-}7}} = e_{gen} \times \frac{\overline{Z_{4\text{-}7}}}{\overline{Z1} + \overline{Z2} + \overline{Z3} + \overline{Z_{4\text{-}7}}}$$

Figure 2-6 Voltage across $Z_{4\text{-}7}$

The voltage V_{Z4-7} is across the impedance Z_{67}.

$$\overline{V_{Z67}} = \overline{V_{Z4-7}}$$

This voltage produces the drop across Z7, and can be calculated by another application of the voltage divider law. This is shown in Figure 2-7.



$$\overline{V_{Z7}} = \overline{V_{Z67}} \times \frac{\overline{Z7}}{\overline{Z6} + \overline{Z7}}$$

Figure 2-7 Voltage across $Z7$

You have a plan to calculate the voltage across $Z7$, the output. Only after the plan is completed and reviewed should you begin to insert numbers and calculate results.

$$\overline{Z1} = (50\,\Omega\angle 0^\circ)$$
$$\overline{Z2} = (1.7\,\Omega\angle 0^\circ)$$
$$\overline{Z3} = (34.21\angle 90^\circ)$$
$$\overline{Z4} = (73.17\angle -90^\circ)$$
$$\overline{Z5} = (50\,\Omega\angle 0^\circ)$$
$$\overline{Z6} = (37.65\angle 90^\circ)$$
$$\overline{Z7} = (100\,\Omega\angle 0^\circ)$$

$$\overline{Z_{67}} = (37.65\Omega\angle 90^\circ) + (100\,\Omega\angle 0^\circ)$$
$$\overline{Z_{67}} = (106.9\,\Omega\angle 20.6^\circ)$$

$$\overline{Z_{4-7}} = \left[(73.17\,\Omega\angle -90^\circ)^{-1} + (50\,\Omega\angle 0^\circ)^{-1} + (106.9\,\Omega\angle 20.6^\circ)^{-1}\right]^{-1}$$
$$\overline{Z_{4-7}} = (32.71\Omega\angle -19.8^\circ)$$

$$\overline{V_{Z4-7}} = \frac{(1\,\mathrm{V_{rms}}\angle 0^\circ)\times(32.71\Omega\angle -19.8^\circ)}{(50\Omega\angle 0^\circ) + (1.7\,\Omega\angle 0^\circ) + (34.21\Omega\angle 90^\circ) + (32.71\Omega\angle -19.8^\circ)}$$
$$\overline{V_{Z4-7}} = (381.9\,\mathrm{mV_{rms}}\angle -35.5^\circ)$$

$$\overline{V_{Z7}} = \frac{(381.9\,\mathrm{mV_{rms}}\angle -35.5^\circ)\times(100\,\Omega\angle 0^\circ)}{(37.65\Omega\angle 90^\circ) + (100\,\Omega\angle 0^\circ)}$$
$$\overline{V_{Z7}} = (357.4\,\mathrm{mV_{rms}}\angle -56.1^\circ)$$

The simulation result is shown in Figure 2-8 and agrees with the calculation above. When using MultiSIM, remember that the generator voltage is displayed in V_p. The panel meters normally display dc. You must adjust them to indicate V_{rms}.

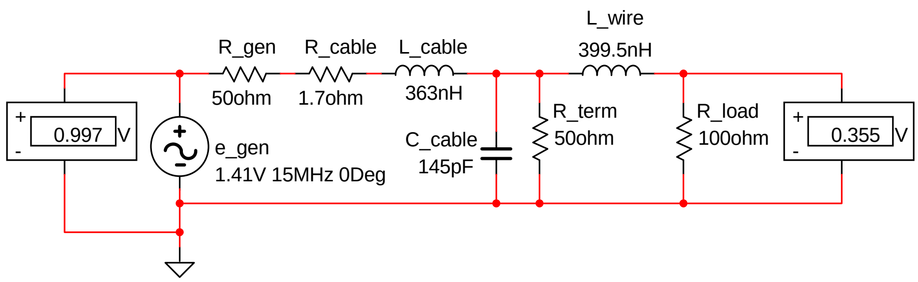


Figure 2-8 Simulation of Example 2-1

The x10 Oscilloscope Probe

A BNC cable presents considerable capacitance between the signal, on its center conductor, and common, on its shield. Combined with the other parasitic capacitances around an amplifier, the cable often makes the amplifier break into oscillations. That is, the amplifier is working fine until you connect a cable to its output to measure it. Then it breaks into oscillations. Remove the cable and the oscillations stop. The very act of trying to measure the amplifier's performance destroys that performance.

Instead of connecting to the amplifier's output directly with a BNC cable, use a ×10 oscilloscope probe. A 9 MΩ resistor is added at the tip of the probe, and considerable effort is made to lower the cable's inductance and capacitance. A simplified schematic is shown in Figure 2-9.

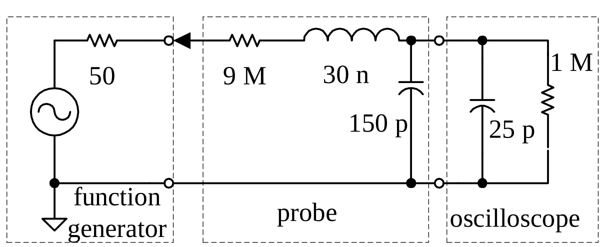


Figure 2-9 Simple ×10 oscilloscope probe