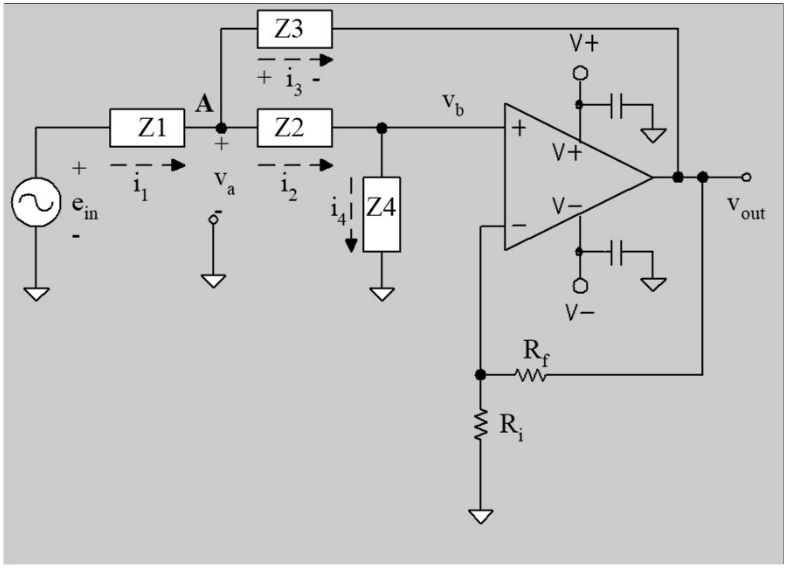
Second Order LP Butterworth Filters

- Transfer Function $G = \frac{A_o \omega_0{}^2}{s^2 + 2\xi \omega_o s + \omega_0{}^2}$
- Characteristics
  - ___________________________
  - ξ= __________
  - Damping _____________
  - f_o (-90 deg shift) _____ f_{-3dB}
  - _____dB/decade or _____ dB/octave roll off
- MATLAB

$v_{out} = A_o v_b$

Second Order L



$$v_{out} = A_o v_b$$

$$A_o = 1 + \frac{R_f}{R_i}$$

Purdue University

So $$v_b = \frac{v_{out}}{A_o}$$

Assuming that no current flows into the op amp,

$$i_4 = \frac{v_b}{Z4}$$

$$i_2 = i_4 = \frac{v_b}{Z4}$$

At node A, $$v_a = i_4(Z2 + Z4)$$

Combining these yields $$v_a = \frac{v_b}{Z4}(Z2 + Z4)$$

Current into the filter, i_1, is the difference in potential across Z1 divided by Z1.

$$i_1 = \frac{e_{in} - v_a}{Z1} = \frac{e_{in}}{Z1} - \frac{v_a}{Z1}$$

Substitute for v_a. $$i_1 = \frac{e_{in}}{Z1} - \frac{v_b(Z2 + Z4)}{Z1Z4}$$

The current through the feedback impedance, i_3, can be calculated by summing the currents at node A.

$$i_3 = i_1 - i_2$$

Combine the equations for i_1 and i_2.

$$i_3 = \frac{e_{in}}{Z1} - \frac{v_b(Z2 + Z4)}{Z1Z4} - \frac{v_b}{Z4}$$

Summing the loop from node A, Z3, and the output yields

$$v_a - i_3 Z3 - v_{out} = 0$$

$$v_{out} = v_a - i_3 Z3$$

Substitute for i_3.

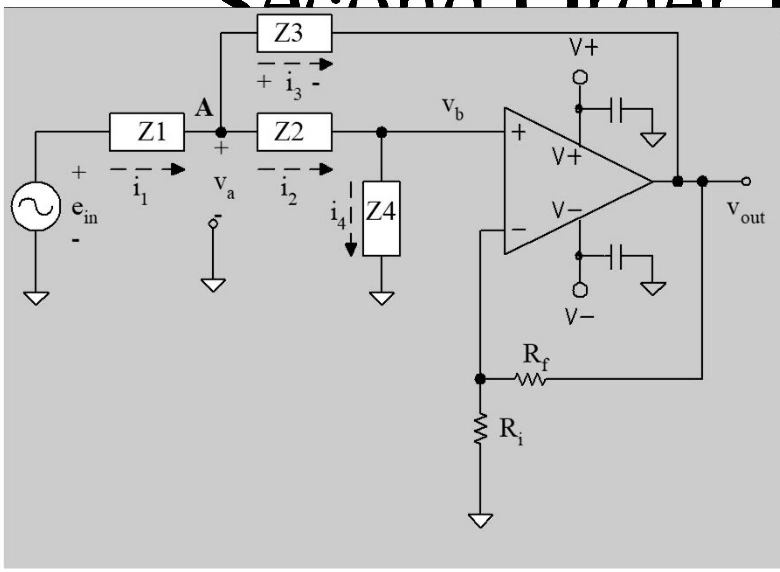
$$v_{out} = \frac{v_b}{Z4}(Z2 + Z4) - \left[\frac{e_{in}}{Z1} - \frac{v_b(Z2 + Z4)}{Z1Z4} - \frac{v_b}{Z4}\right]Z3$$

Combine this with the initial relationship for the input to output voltages of the op amp.

$v_{out} = A_o v_b$

Second Order LP Butterworth Filters

$$v_{out} = \frac{v_b}{Z4}(Z2+Z4) - \left[\frac{e_{in}}{Z1} - \frac{v_b(Z2+Z4)}{Z1Z4} - \frac{v_b}{Z4}\right]Z3$$



$$\frac{v_{out}}{e_{in}} = \frac{A_o Z3Z4}{Z1Z2 + Z2Z3 + Z3Z4 + Z1Z3 + Z1Z4(1-A_o)}$$

$$\boxed{A_{int}} = 1 + \frac{R_f}{R_i}$$

$$\frac{V_{out}}{E_{in}} = \frac{\frac{A_o}{C^2s^2}}{R^2 + \frac{R}{Cs} + \frac{1}{C^2s^2} + \frac{R}{Cs} + \frac{R}{Cs}(1-A_o)}$$

$$= \frac{\frac{A_o}{C^2s^2}}{\frac{R^2C^2s^2 + RCs + 1 + RCs + RCs(1-A_o)}{C^2s^2}}$$

$$= \frac{A_o}{R^2C^2s^2 + 2RCs + RCs(1-A_o) + 1}$$

$$\frac{V_{out}}{E_{in}} = \frac{A_o}{R^2C^2s^2 + RC[2+(1-A_o)]s + 1}$$

$$\omega_o^2 = \frac{1}{R^2C^2}$$

$$\omega_o = \frac{1}{RC}$$

The quadratic in the denominator is more easily solved if the coefficient of s^2 is 1. Dividing numerator and denominator by R^2C^2, you obtain

$$\frac{V_{out}}{E_{in}} = \frac{\frac{A_o}{R^2C^2}}{s^2 + \left(\frac{3-A_o}{RC}\right)s + \frac{1}{R^2C^2}}$$

$$f_o = \frac{1}{2\pi RC}$$

Second-order systems have been studied extensively. Mechanical and chemical as well as electrical second-order systems behave similarly. One transfer function is

$$\alpha\omega_o = \frac{3-A_o}{RC}$$

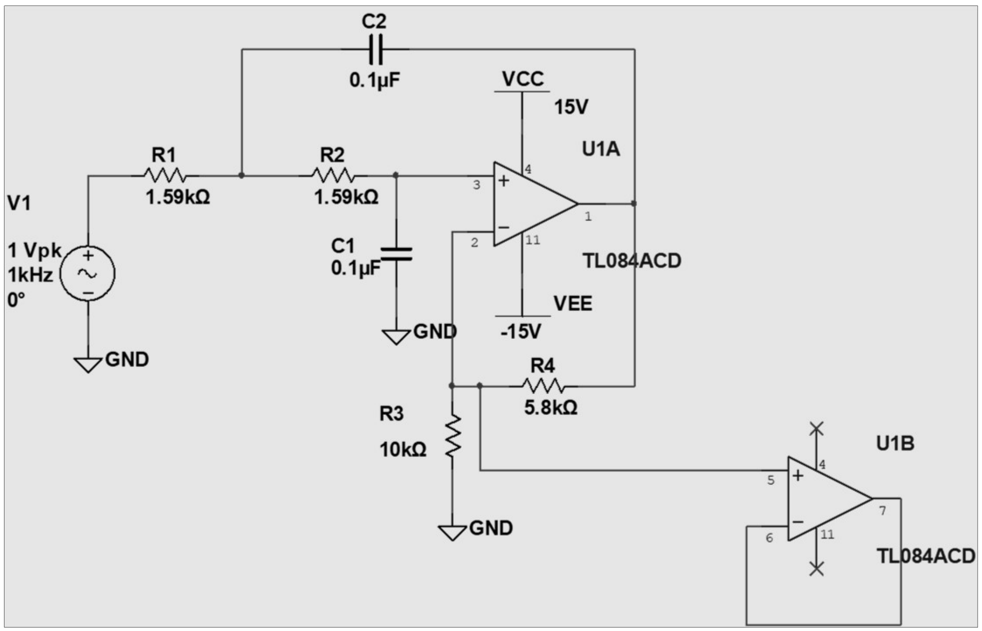
$$\frac{A_o\omega_o^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

Purdu

$$\alpha = 3 - \boxed{A_{int}}$$

Sallen Key Implementation

$$G = \frac{A_o {\omega_0}^2}{s^2 + 2\xi\omega_o s + {\omega_0}^2}$$



$f_o =$ __________

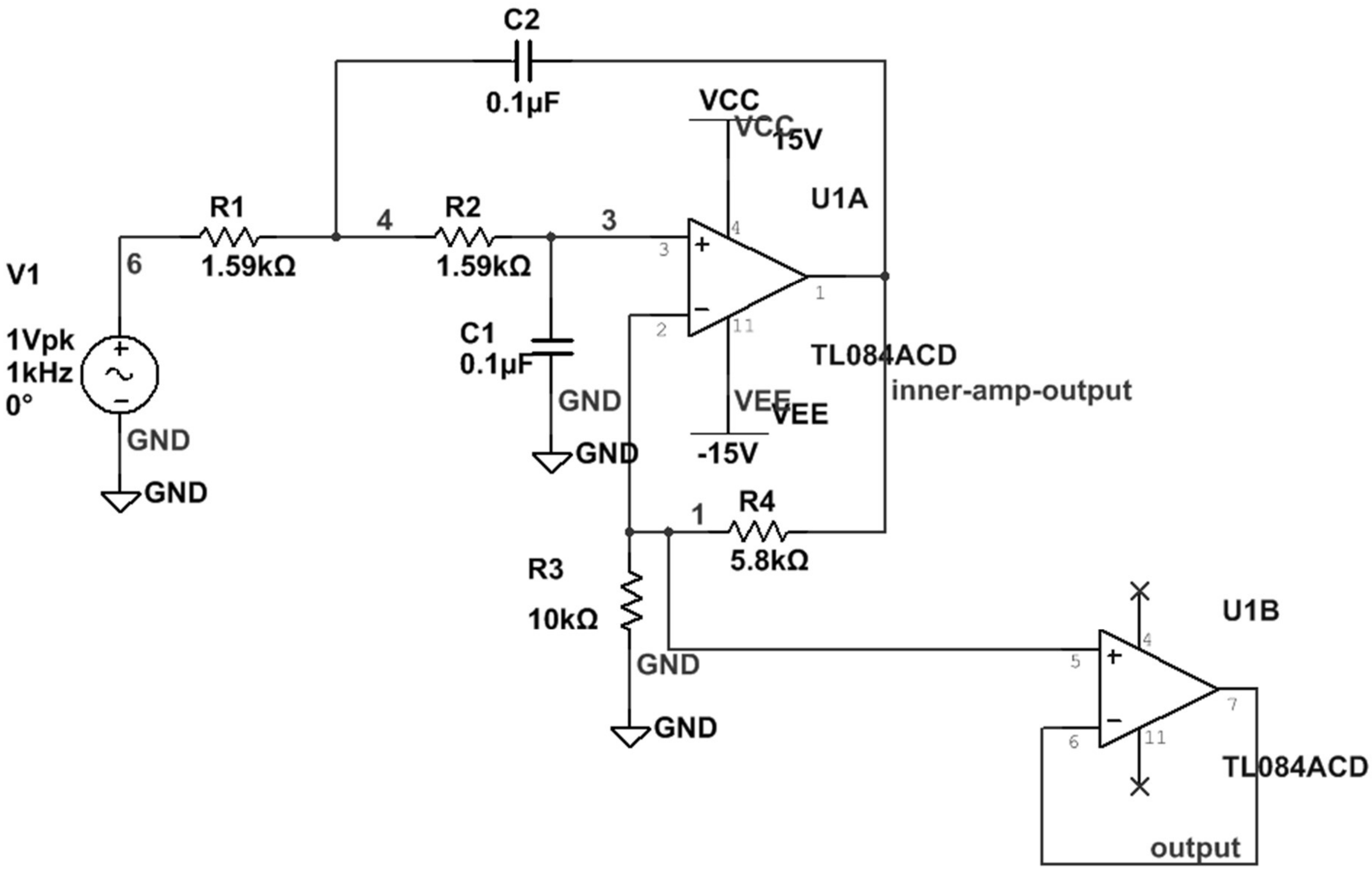
$\alpha =$ ______________________

$A_{int} =$ _____________

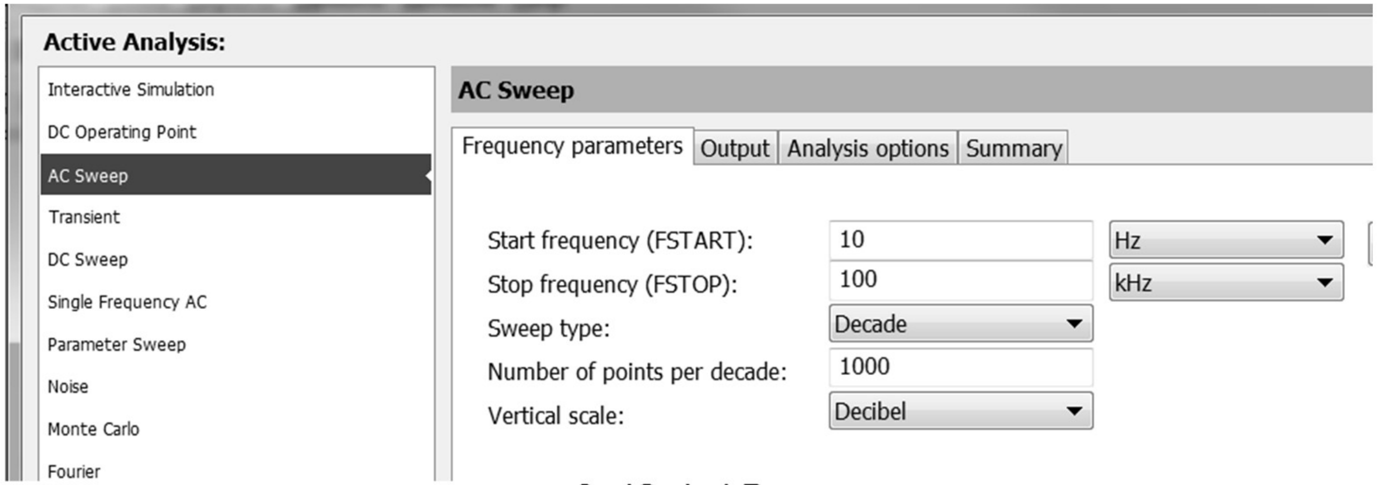
Sets _____________

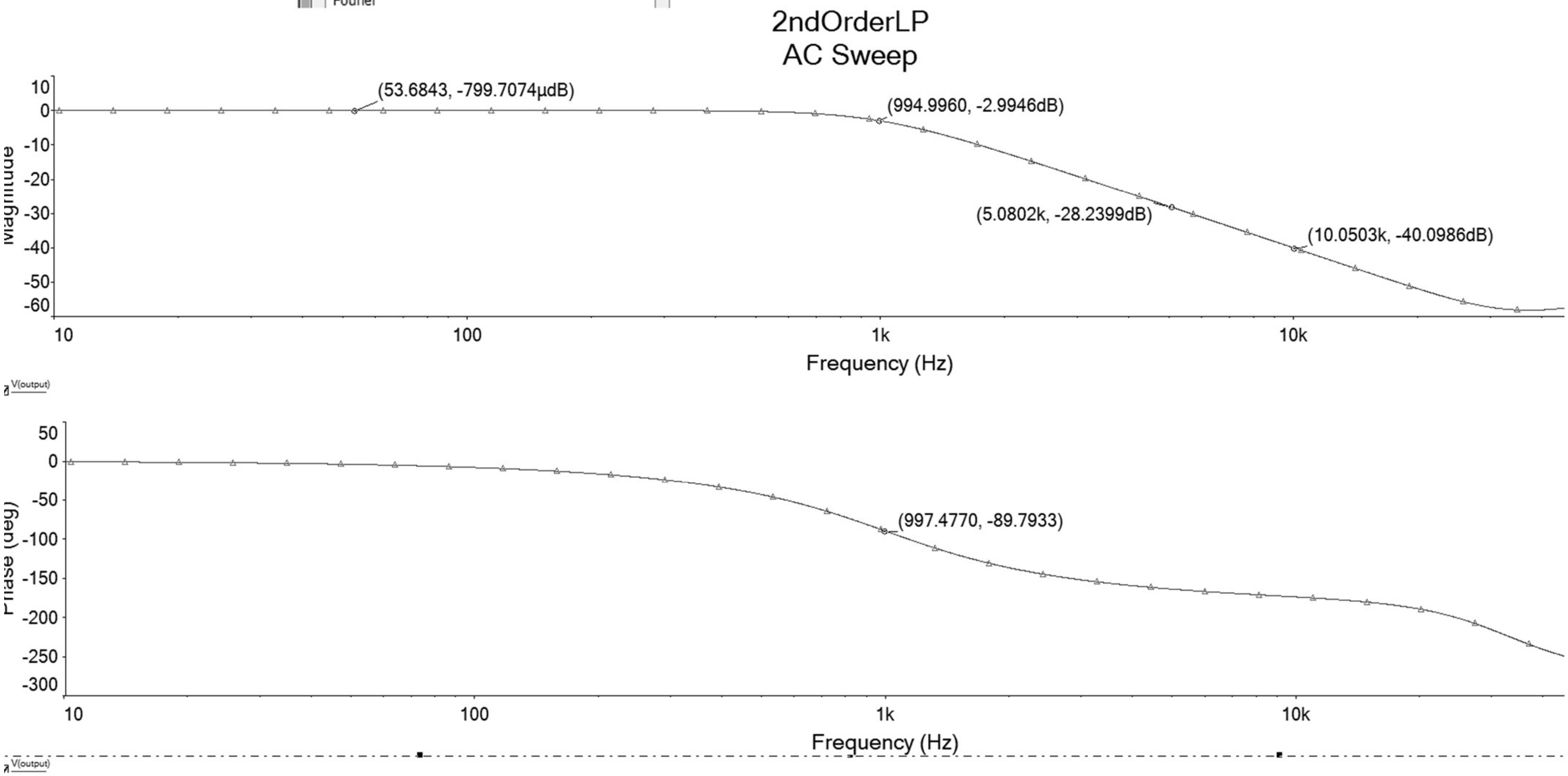
U1B ???

Multisim Demo



C2
0.1µF
VCC
VCC
15V
U1A
R1
4
R2
3
V1
6
1.59kΩ
1.59kΩ
3
4
1
2
11
1Vpk
1kHz
0°
C1
0.1µF
TL084ACD
inner-amp-output
GND
VEE
VEE
GND
GND
-15V
GND
1
R4
5.8kΩ
R3
10kΩ
U1B
4
5
GND
7
6
11
GND
TL084ACD
output


Active Analysis:
Interactive Simulation
DC Operating Point
AC Sweep
Transient
DC Sweep
Single Frequency AC
Parameter Sweep
Noise
Monte Carlo
Fourier
AC Sweep
Frequency parameters
Output
Analysis options
Summary
Start frequency (FSTART): 10 Hz
Stop frequency (FSTOP): 100 kHz
Sweep type: Decade
Number of points per decade: 1000
Vertical scale: Decibel


2ndOrderLP
AC Sweep
(53.6843, -799.7074µdB)
(994.9960, -2.9946dB)
(5.0802k, -28.2399dB)
(10.0503k, -40.0986dB)
10
0
-10
-20
-30
-40
-50
-60
10
100
1k
10k
Frequency (Hz)
V(output)
(997.4770, -89.7933)
50
0
-50
-100
-150
-200
-250
-300
10
100
1k
10k
Frequency (Hz)
V(output)