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Auxetic behavior from rotating squares

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Auxetic materials exhibit the very unusual properties of becoming wider when stretched and narrower when squashed [1], that is they have negative Poisson's ratios. Apart from the pure scientific interest of having materials showing such an unconventional property, a negative Poisson's ratio gives a material several other beneficial effects such as an increased shear stiffness, an increased plane strain fracture toughness and an increased indentation resistance. These properties make auxetics superior to conventional materials for many practical applications [1, 2].

In recent years several auxetics have been fabricated by modifying the microstructure of existing materials, including foams [2] and microporous polymers [3]. A number of molecular auxetics have also been proposed [4–9] one example being -cristobalite [7]. The auxetic behavior in these materials can be explained in terms of their geometry and deformation mechanism. Thus, the hunt for new auxetic materials is frequently approached through searching for geometric features which may give such behavior [9, 10].

In this letter we present a new mechanism to achieve a negative Poisson's ratio. This is based on an arrangement involving rigid squares connected together at their vertices by hinges as illustrated in Fig. 1. This may be viewed as a two dimensional arrangement of squares or as a projection of a particular plane of a three dimensional structure. This latter type of geometry is commonly found in inorganic crystalline materials [8, 9, 11].

Referring to Fig. 1, for squares of side length "l" at an angle to each other, the dimensions of the unit cell in the Ox_i directions are given by:

$$X_1 = X_2 = 2l\left[\cos\left(\frac{}{2}\right) + \sin\left(\frac{}{2}\right)\right] \qquad (1)$$

In general, the Poisson's ratio is not constant and varies with strain and the initial geometry parameters. Hence we have a strain dependent Poisson's function for loading in the Ox_i direction defined by:

$$_{ij} = -\frac{\mathrm{d}\varepsilon_j}{\mathrm{d}\varepsilon_i}, \quad i, j = 1, 2 \qquad (2)$$

where $\mathrm{d}\varepsilon_i$ is an incrementally small strain in the Ox_i direction which is given in terms of the unit cell dimensions by:

$$\mathrm{d}\varepsilon_i = \frac{\mathrm{d}X_i}{X_i}, \quad i = 1, 2 \qquad (3)$$

where $\mathrm{d}X_i$ is an incrementally small change in the unit cell dimension X_i due to the applied load.

Thus, assuming the squares do not deform upon loading, referring to Equation 1, the incremental strains are given by:

$$\mathrm{d}\varepsilon_i = \frac{1}{X_i}\frac{\mathrm{d}X_i}{\mathrm{d}}\,\mathrm{d}\ , \quad i = 1, 2 \qquad (4)$$

where:

$$\frac{\mathrm{d}X_i}{\mathrm{d}} = l\left[\sin\left(\frac{}{2}\right) - \cos\left(\frac{}{2}\right)\right], \quad i = 1, 2 \qquad (5)$$

Hence from Equations 1–4, the Poisson's functions are given by:

$$_{12} = \ _{21} = -1 \qquad (6)$$

If we assume that the stiffness in the structure is due to the stiffness of the hinges, defined by a stiffness constant K_{h}, then the Young's moduli of the structure E_i may be derived through a conservation of energy approach. In a continuum, the strain energy due to an incrementally small strain $\mathrm{d}\varepsilon_i$ in the Ox_i direction is given by:

$$U = \frac{1}{2}E_i(\mathrm{d}\varepsilon_i)^2 \qquad (7)$$

This strain will result in a change in the angles . The work done per unit cell corresponding to this change in

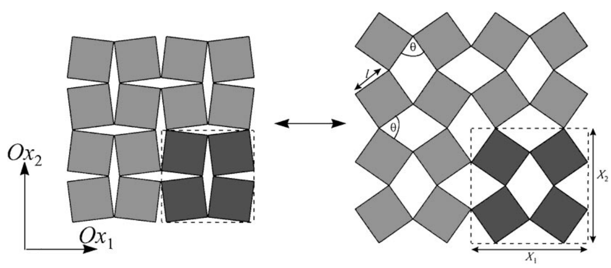

Figure 1 The geometry of the auxetic "rotating squares" structure.

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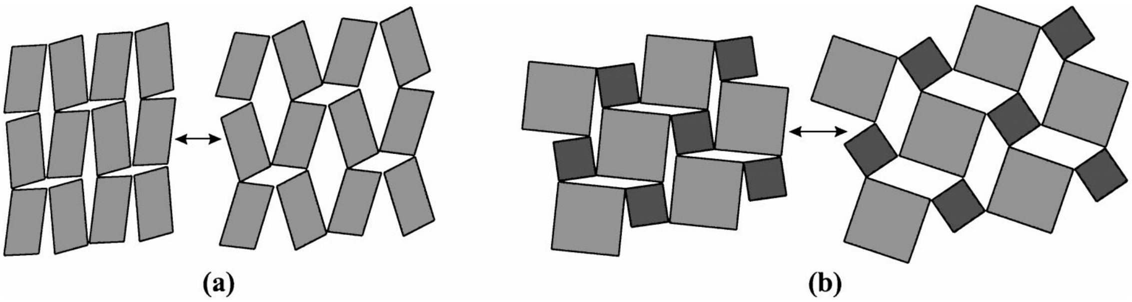

Figure 2 Variations on the "rotating squares" geometry: (a) a more general rotating parallelogram structure and (b) a "rotating squares" structure built from different sized squares.

by "d " is given by:

$$W = N\left[\frac{1}{2}K_{\mathrm{h}}(\mathrm{d}\ \ )^2\right] \tag{8}$$

where N is the number of hinges corresponding to one unit cell. Since each unit cell contains four squares, each square contains four vertices, and two vertices correspond to one hinge, then in this case, $N = 8$.

From the principle of conservation of energy Equations 7 and 8 are related through:

$$U = \frac{1}{V}W \tag{9}$$

where V is the volume of the unit cell. Assuming a unit thickness in the third dimension is this volume is given by:

$$V = X_1 X_2 1 = X_i^2 \tag{10}$$

Thus by substituting Equations 4, 7, 8, and 10 into Equation 9 we obtain:

$$\frac{1}{2}E_i\left(\frac{1}{X_i}\frac{\mathrm{d}X_i}{\mathrm{d}\ \ }\mathrm{d}\ \ \right)^2 = \frac{1}{X_i^2}\,8\left[\frac{1}{2}K_{\mathrm{h}}(\mathrm{d}\ \ )^2\right] \tag{11}$$

which simplifies to:

$$E_i = \left(\frac{\mathrm{d}X_i}{\mathrm{d}\ \ }\right)^2 = 8K_{\mathrm{h}}\left(\frac{\mathrm{d}X_i}{\mathrm{d}\ \ }\right)^2 \tag{12}$$

i.e.:

$$E_1 = E_2 = K_{\mathrm{h}}\frac{8}{l^2}\frac{1}{[1 - \sin(\ \ )]} \tag{13}$$

Since the unit cell in this structure has to be rectangular, then the compliance matrix for this structure is given by:

$$S = \begin{pmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{E_1} & \frac{-\ \ {}_{21}}{E_2} & 0 \\ \frac{-\ \ {}_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{14}$$

which from Equations 6 and 13 is clearly symmetric. This means that in the idealized scenario presented here, the shear modulus is infinite.

The off-axis mechanical properties obtained from the standard transformation equations [12] show that this idealized system is isotropic, which means that the Poisson's ratio has a constant value of -1 irrespective of the direction of loading. In practice, one would assume that the shear modulus would assume a finite value, which could reduce the symmetry to square symmetric [9].

We note that this type of behavior may also be achieved from the more general case involving hinging parallelograms or in networks built with different sized squares as illustrated in Fig. 2. An equivalent auxetic rotating triangles structure (see Fig. 3) is also possible [9].

The geometry modelled here is commonly found as a projection of a plane in inorganic crystalline materials involving octahedrally co-ordinated atoms [11]. It is also found in several zeolite crystals for which negative Poisson's ratios have been predicted through force-field based molecular modelling [8, 9]. The scenario presented here that the squares are perfectly rigid is clearly highly idealistic, and in real materials we would expect deformation of the squares to occur in parallel with the rotations. The determining factor whether a material with a geometry as in Fig. 1 is auxetic or not would depend on which of the two deformation mechanisms dominates. Nevertheless, given the many examples of crystalline materials having such a geometry, we envisage that auxetic behavior in crystals through this mechanism is highly probable and we hope that this letter encourages the mechanical testing of such crystals.

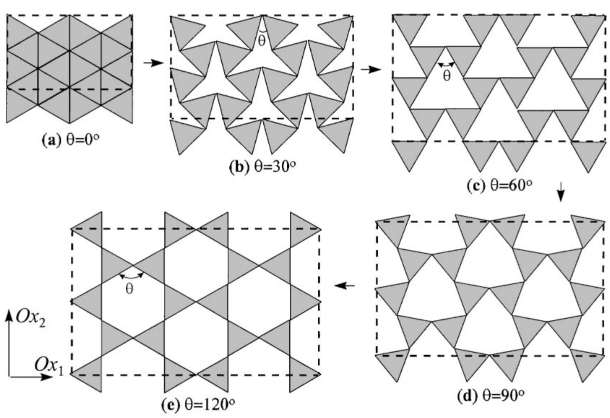

Figure 3 The idealized "rotating triangles" geometry: another structure which retains its aspect ratio on loading in a uniaxial direction (Poisson's ratio $= -1$).

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