Automatic control project

MASTER OF SCIENCE

IN

MECHATRONICS ENGINEERING

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1 Introduction

This document presents the results of a project conducted as part of the Automatic Control course. The aim of the project was to develop a control system for effectively managing the motion of a drone.

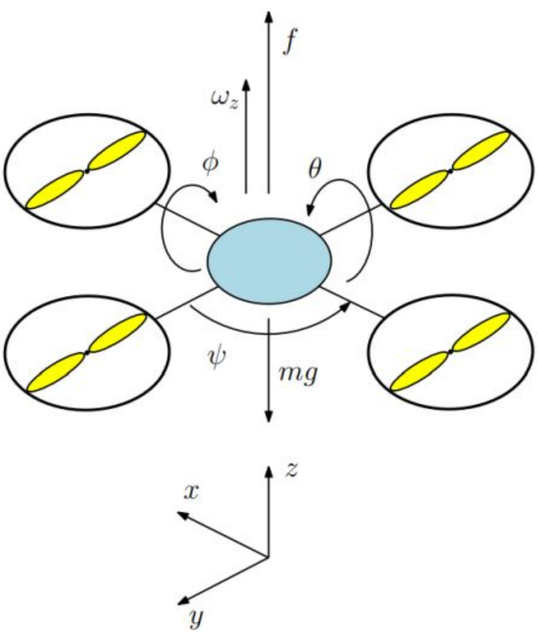
In this project, a simplified version of the drone's dynamics is used

$$\begin{cases} \dot{e}_z = v_z \\ \dot{v}_z = -g + \frac{f}{m_d} \\ \dot{e}_\psi = \omega_z \\ \dot{\omega}_z = \frac{\tau_C}{I_z} \end{cases} \tag{1}$$

The variables used in the equations represent the following quantities:

- v represents the velocity of the drone,
- ω represents the angular velocity of the drone,
- e represents the error,
- f represents the thrusting force,
- τ_C represents the control torque,
- m_d represents the mass of the drone,
- I_z represents the moment of inertia of the drone with respect to the z-axis.

These parameters are illustrated in the following picture:



2 Drone dynamics

The state space, the output and the input vector are defined as:

$$x = \begin{bmatrix} e_z \\ v_z \\ e_\psi \\ \omega_z \end{bmatrix} \quad z = \begin{bmatrix} e_z \\ e_\psi \end{bmatrix} \quad u = \begin{bmatrix} \hat{f} \\ \tau_C \end{bmatrix}$$

where e_z, v_z, e_ψ, and ω_z represent the altitude error, vertical velocity, heading error, and angular velocity around the vertical axis, respectively.

The output consists of the altitude error e_z and the heading error e_ψ.

$\hat{f}$ is calculated as the modified thrust force f minus the product of the drone's mass m_d and the gravitational acceleration g.

2.1 State Space representation

The system is:

$$\begin{cases} \dot{x} = A \cdot x + B \cdot u \\ z = C \cdot x + D \cdot u \end{cases}$$

The following matrices describe the system:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_d} & 0 \\ 0 & 0 \\ 0 & \frac{1}{I_z} \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

To determine the controllability of the system, the controllability matrix was calculated using MatLab. Subsequently, the rank of the controllability matrix was compared with the size, denoted by 'n', of the matrix A. The fact that both numbers are equal indicates that the system is controllable.

3 Full-state static feedback control

A feedback control has been implemented using the equation $u = K \cdot x$, where K is the gain matrix. The control system has a convergence rate of $\alpha = 1$, ensuring stable and efficient control. Additionally, there is a limit on the magnitude of the actuator effort, represented by $\|K\| \leq \overline{k} = 200$. After solving the control problem, the specific gain matrix K is obtained as follows:

$$K = \begin{bmatrix} -2.6734 & -1.7870 & 0 & 0 \\ 0 & 0 & -0.2833 & -0.1670 \end{bmatrix}$$

Consequently, it is feasible to calculate the real components of the eigenvalues associated with the closed-loop system $A + BK$. These real components are provided in the following vector:

$$\begin{bmatrix} -1.787 & -1.787 & -1.669 & -1.669 \end{bmatrix}$$

4 Wind perturbations environment

The assumption is made that the drone will now operate in an environment where it is exposed to wind disturbances. These disturbances are represented as an additional force and torque acting on the drone's dynamics. The perturbation vector is defined as follows: $w = [w_z \quad w_\psi]^T$.

To counteract these perturbations, two integral terms are incorporated into the controller, introducing two additional state variables:

$$\begin{cases} \dot{\sigma z} = e_z \\ \dot{\sigma \psi} = e_\psi \end{cases}$$

4.1 Dynamic with perturbations

The new dynamics are described by the following system:

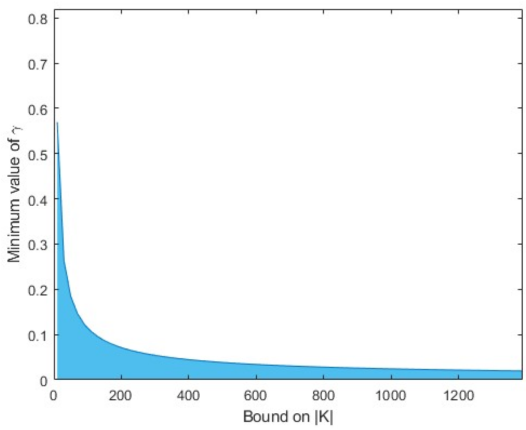
$$\begin{cases} \dot{x} = A \cdot x + B \cdot u + E \cdot w \\ z = C \cdot x + D \cdot u + F \cdot w \end{cases}$$

Now, the state space is defined as: $x = [\sigma_z \quad e_z \quad v_z \quad \sigma_\psi \quad e_\psi \quad \omega_z]^T$, with the same output vector. Now, the matrices are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_d} & 0 \\ 0 & 0 \\ 0 & \frac{1}{I_z} \end{bmatrix} C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ \frac{1}{m_d} & 0 \\ 0 & 0 \\ 0 & \frac{1}{I_z} \end{bmatrix} F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4.2 Optimality curve

We will now formulate the control problem considering the $\mathcal{L}_2$ gain. Our objective is to find the optimal curve while imposing a constraint on the value of $\bar{k}$, which is bounded within the interval $[10, 1400]$. With this constraint, the resulting optimality curve is as follows:



Furthermore, it is necessary to verify the feasibility of the problem considering the following requirements: $\gamma < 0.1$, $\alpha = 1$, and $||K|| \leq 200$. Upon running the control problem, it is determined to be infeasible.

4.3 Colleague solution

We are now required to verify if the control problem requirements are satisfied using $\overline{k} = 1200$. Upon running the Linear Matrix Inequality (LMI) again, it is observed that the problem is feasible, and the following parameters are obtained:

- The gain matrix found is:

$$K = \begin{bmatrix} -122.8601 & -119.1662 & -3.9170 & 0 & 0 & 0 \\ 0 & 0 & 0 & -31.7029 & -29.5815 & -0.9167 \end{bmatrix}$$

- Its norm is:

$$||K|| = 171.2031$$

- The real part of the closed-loop eigenvalues are:

$$\begin{bmatrix} -1.0631 & -3.3854 & -3.3854 & -8.6136 & -8.6136 & -1.1074 \end{bmatrix}$$

- And γ is:

$$\gamma = 0.0960$$

All the requirements are satisfied.

4.4 System simulation

Now, the system is simulated for a duration of 15 seconds, with the following initial conditions:

$$x(0) = \begin{bmatrix} 0 \\ 1 \\ 0.1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

with constant disturbances:

$$w_z = A_z = 2, \quad w_\psi = A_\psi = 0.2$$

Based on the simulation results obtained using the `ode45()` function, it is evident that the controller design has been successful in achieving the desired stabilization of the errors e_z and e_ψ. The errors progressively converge towards zero as the simulation progresses, indicating the effectiveness of the controller in minimizing these errors. The convergence behavior of the errors can be clearly observed in the accompanying diagram, which demonstrates the steady reduction of the errors over time. This confirms that the designed controller effectively guides the system towards the desired state, ensuring accurate and precise control of the drone's motion.