Homework 3

September 19, 2017

Due: September 26, 2017, 11:59 PM

Instructions

Your homework submission must cite any references used (including articles, books, code, websites, and personal communications). All solutions must be written in your own words, and you must program the algorithms yourself. If you do work with others, you must list the people you worked with. Submit your solutions as a PDF to the E-Learning at UF (http://elearning.ufl.edu/).

Your programs must be written in either MATLAB or Python. The relevant code to the problem should be in the PDF you turn in. If a problem involves programming, then the code should be shown as part of the solution to that problem. If you solve any problems by hand just digitize that page and submit it (make sure the problem is labeled).

To complete this assignment, please make sure you download the data files provided: "training.txt", "validate.txt", "testnoisy.txt", and "test.txt".

If you have any questions address them to:

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- Sheng Zou (TA) – shengzou@ufl.edu

Material

For this assignment, you were given four files including: a training set ("training.txt"), a validation set ("validate.txt"), and a noisy test set ("testnoisy.txt"). See figure 1 to see their plots.

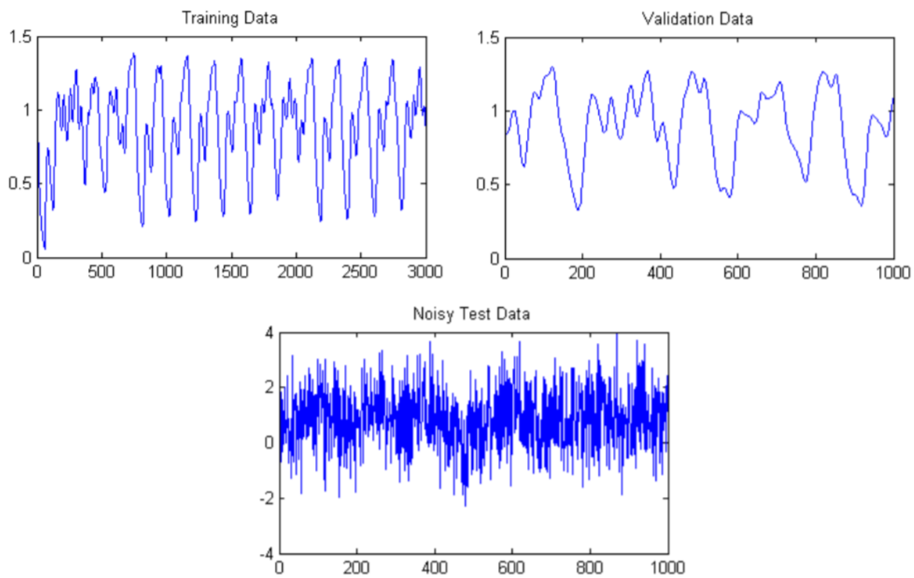

Figure 1: Three segments provided in the course website. Top left: training set segment of 3000 points ("training.txt"). Top right: validation set segment of 1000 points ("validate.txt"). And bottom middle: noisy test set segment of 1000 points ("testnoisy.txt").

Question 1 – 10 points

The least squares regression algorithm is a class of adaptive filter used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean squares of the error signal (difference between the desired and the actual signal).

Optimize an adaptive filter to predict the next point in the time series, using **least square regression** (analytic solution). Since this data is experimental, you may need to use the regularized solution

$$\mathbf{w}^* = (\mathbf{R} + \lambda \mathbf{I})^{-1} \mathbf{p}$$

Use the training data set to find your hyper-parameters; the validation data set to validate the hyper parameters (filter order M and regularization term λ) you have introduced in the solution; and the test data set to evaluate how good of approximation your algorithm gives you.

1.1 (5 points) Select the hyper-parameters for this adaptive filter, namely, **filter order** and **regularization parameter** (trained with "training.txt" and validated using "validate.txt"). The idea is to experiment several different values in training and select the one that provides the best mean square error in the validation set. Present a plot of the validation set in the 2-d space of filter order and regularization parameter.

1.2 (5 points) Quantify the performance in the MSE-sense of your algorithm using filter orders 4, 8 and 30 in the noisy data ("testnoisy.txt"). Which filter order works better for this noisy data? Explain the results.

Question 2 – 10 points

Optimize an adaptive filter to predict the next point in the time series, using **least mean squares (LMS)**

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \eta \nabla \mathbf{e}(n)$$

Use the training data set to find your hyper-parameters; the validation data set to validate the hyper parameters (filter order M and step-size η) you have introduced in the solution; and the test data set to evaluate how good of approximation your algorithm gives you.

2.1 (5 points) Select the hyper-parameters for this adaptive filter, namely, **filter order** and **step-size** (trained with "training.txt" and validated using "validate.txt"). Quantify the performance in the MSE-sense of your algorithm using filter orders 4, 8 and 30 in the noisy data ("testnoisy.txt"). Which filter order works better for this noisy data? Explain the results.

2.2 (5 points) Now, choose a fixed filter model order, e.g. $M = 4$, and change the step-size. Plot the learning curves and weight tracks for several step-sizes. What happens when the step-size is too large? What happens when the step-size is too small? What is the compromise that must be made in order to find an *optimal* solution? Explain the results.