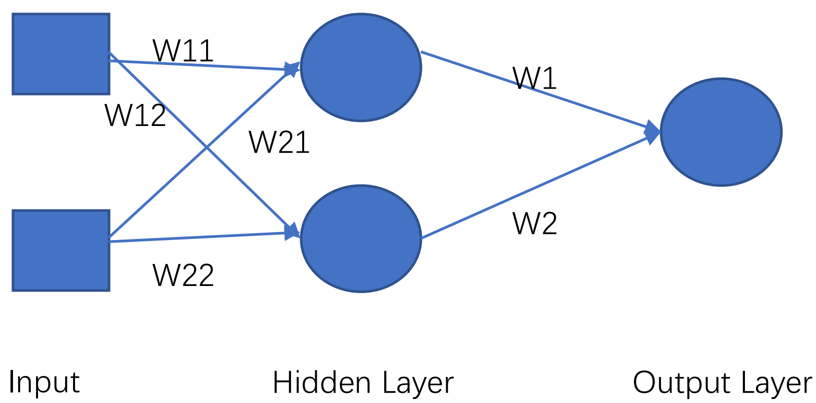
Homework 6

Backpropagation Algorithm

Question 1: Architecture Definition and Reason



In this homework, I build 2-layers neural network which contains one hidden layer and one output layer. The reason that I choose two neurons in hidden layer was I need two lines to divide training data into two different class. The sigmoid function I used in this project is shown in below:

$$y = \frac{1}{1 + \exp\left(-output\right)}$$

Every data in dataset has been training for 500 times. In each iteration, the weights and bias need to be updated.

Question 2: Plot Corresponding Decision Boundaries and Test Self-Generated Data.

Figure 1. Decision boundaries of training data

Figure 2. Self-generated dataset for training

Figure 2 shows the new generated dataset. The network has 98.5% accuracy to predict this dataset.

```
Accuracy of Prediction:  98.5 %
```

Question 3: Multiple Architecture:

Other architecture can solve this XOR problem.

If I use multi hidden layer structure, decision boundaries will be like the figure 3.

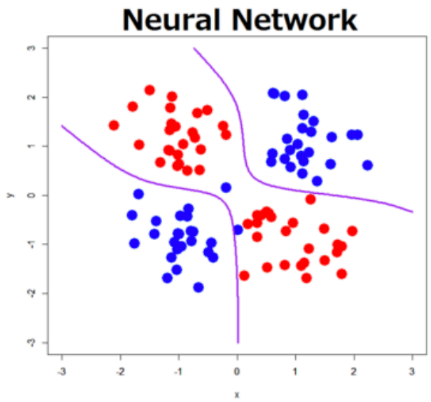


Figure 3: Decision Boundaries by implements multi-hidden layers neural Network

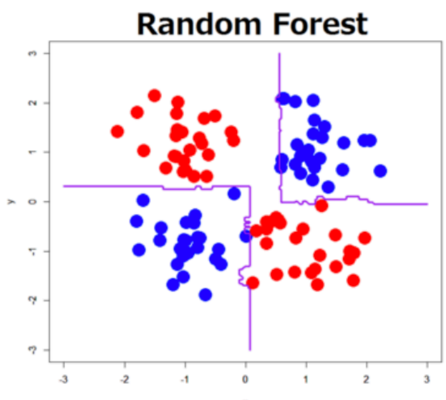


Figure 4: Decision Boundaries by implements Random Forest architecture

Or, we can use random forest architecture (Shown in Figure 4) to solve this problem. Even it seems the random forest provide more precisely decision boundaries. I would choose neural network because I think the random forest method focus too much on local dataset which might cause overfitting problem. It might have high training accuracy. On the other hand, it probably has lower predict accuracy.

Question 4: Train by 8 points of each class

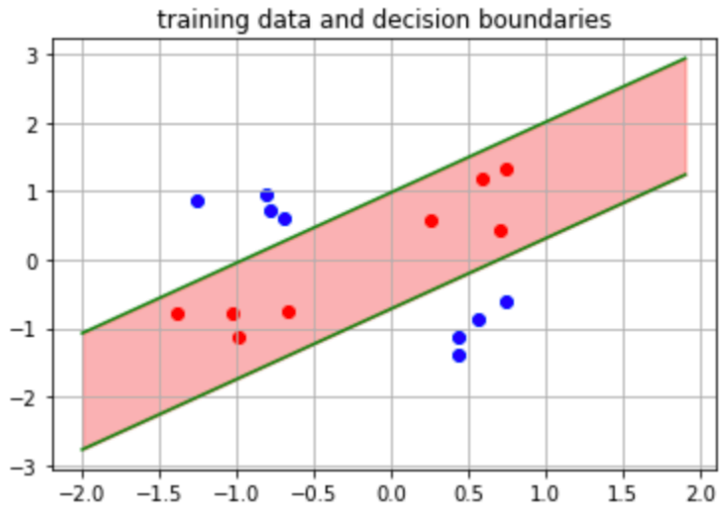


Figure 5: Decision boundaries

Yes, the decision boundaries seem optimal. So, the generalization ability of my network seems perfect with such few points.

```
[[0.258140455863934,0.567315287097103,0],
 [0.734317095363312,1.3195912881375,0],
 [0.592050478464416,1.19667090632313,0],
 [0.699169771902131,0.433616111743741,0],
[-0.985097960238142, -1.12082573650756, 0],
[-1.38177934789019, -0.778890498143647, 0],
[-1.02250556227963, -0.793451426629458, 0],
[-0.674450149157024, -0.764088469196094, 0],
[0.426882921196544, -1.39040516540311, 1],
 [0.745479567524074, -0.610730018707631, 1],
 [0.427053593889095, -1.14155143108763, 1],
 [0.564108502431285, -0.86010860171292, 1],
[-1.26109374697199, 0.860117010560185, 1],
 [-0.780648589258112, 0.728503411389706, 1],
 [-0.805034057569262, 0.963331827863979, 1],
 [-0.694074012697457, 0.59385595699395, 1]]
```

Figure 6: Chosen 16 points dataset for training

Question 5: Choose solutions for hand-select parameters

As we can see the decision boundaries from the Figure 1, few points of class 0, which are supposed to covered by red area, are divided into class 1. In order to enlarge the red area to cover more dataset, I can adjust the slope and bias to achieve that. The decision functions are: $\begin{cases} y1 = 1.09578x1 + 0.85105 \\ y2 = 1.09578x2 - 0.85105 \end{cases}$.

Figure 7. Adjusted decision boundaries

Figure 1. Decision boundaries of training data

After hand-select the slope, the adjusted decision boundaries is shown in figure 7. And the decision functions are: $\begin{cases} y1 = 0.89578x1 + 0.85105 \\ y2 = 0.89578x2 - 0.85105 \end{cases}$.

Compare Figure 1 and Figure 7, we can see there are less class 0 points out of boundaries. Actually, this is just small change from the solutions learned from MLP.

Question 6: How to improve generalization accuracy?

There are many methods to improve the accuracy. Increase hidden layer can find more features of training data which can improve the performance of this neural network. More iterations can also improve the predictive accuracy of this network. Besides, increase number of data can greatly help us to learning a better network.

Code:

```
# import needed library
import numpy as np
import matplotlib.pyplot as plt
from math import exp
from random import seed
from random import random
%matplotlib inline
# input training data
data = np.loadtxt("HW6_data.txt")
label = np.zeros([400,1])
dataset = np.zeros([400,3])
dataset = data
label = data[:,2]
dataset = dataset.tolist()
for n in dataset:
	n[2]=int(n[2])
# Initialize a network
def init_network(num_inputs, num_hidden, num_outputs):
	network = list()
	hidden_layer1 = [{'weights':[random() for i in range(num_inputs + 1)]} for i in range(num_hidden)]
	network.append(hidden_layer1)
	output_layer = [{'weights':[random() for i in range(num_hidden + 1)]} for i in range(num_outputs)]
	network.append(output_layer)
	return network
# Calculate neuron activation for an input
def activate(weights, inputs):
	activation = weights[-1]
```

```
	for i in range(len(weights)-1):
		activation += weights[i] * inputs[i]
	return activation

# Transfer neuron activation
def sigmoid(activation):
	return 1.0 / (1.0 + exp(-activation))
# Forward propagate input to a network output
def forward_prop(network, row):
	inputs = row
	for layer in network:
		new_inputs = []
		for neuron in layer:
			activation = activate(neuron['weights'], inputs)
			neuron['output'] = sigmoid(activation)
			new_inputs.append(neuron['output'])
		inputs = new_inputs
	return inputs

# Calculate the derivative of an neuron output
def transfer_derivative(output):
	return output * (1.0 - output)
# Backpropagate error and store in neurons
def backward_propagate_error(network, expected):
	for i in reversed(range(len(network))):
		layer = network[i]
		errors = list()
		if i != len(network)-1:
			for j in range(len(layer)):
				error = 0.0
```

```
                for neuron in network[i + 1]:
                    error += (neuron['weights'][j] * neuron['delta'])
                errors.append(error)
        else:
            for j in range(len(layer)):
                neuron = layer[j]
                errors.append(expected[j] - neuron['output'])
        for j in range(len(layer)):
            neuron = layer[j]
            neuron['delta'] = errors[j] * transfer_derivative(neuron['output'])
# Update network weights with error
def update_weights(network, row, l_rate):
    for i in range(len(network)):
        inputs = row[:-1]
        if i != 0:
            inputs = [neuron['output'] for neuron in network[i - 1]]
        for neuron in network[i]:
            for j in range(len(inputs)):
                neuron['weights'][j] += l_rate * neuron['delta'] * inputs[j]
            neuron['weights'][-1] += l_rate * neuron['delta']

# Train a network for a fixed number of epochs
def train(network, train, l_rate, n_epoch, n_outputs):
    for epoch in range(n_epoch):
        sum_error = 0
        for row in train:
            outputs = forward_prop(network, row)
            expected = [0 for i in range(n_outputs)]
            expected[row[-1]] = 1
            #print(expected)
```

```
            sum_error += sum([(expected[i]-outputs[i])**2 for i in range(len(expected))])
            backward_propagate_error(network, expected)
            update_weights(network, row, l_rate)
def test(network,row):
    outputs = forward_prop(network,row)
    return outputs.index(max(outputs))
# Test training backprop algorithm
seed(1)
num_inputs = len(dataset[0]) - 1
num_outputs = len(set([row[-1] for row in dataset]))
print(num_outputs)
network = initialize_network(2, 2, 2)
train(network, dataset, 0.2, 500, num_outputs)
layer1=[]
for layer in network:
    layer1.append(layer)
    print(layer)
# Train a network for a fixed number of epochs
def train_network(network, train, l_rate, n_epoch, n_outputs):
    for epoch in range(n_epoch):
        sum_error = 0
        for row in train:
            outputs = forward_propagate(network, row)
            expected = [0 for i in range(n_outputs)]
            expected[row[-1]] = 1
            #print(expected)
            sum_error += sum([(expected[i]-outputs[i])**2 for i in range(len(expected))])
            backward_propagate_error(network, expected)
```

```
                update_weights(network, row, l_rate)
# plot the boundary lines
# plot training data
for k in range(0,200):
    plt.scatter(dataset[k][0],dataset[k][1],c='red')
for l in range(200,400):
    plt.scatter(dataset[l][0],dataset[l][1],c='blue')
# plot boundary

w2 = list(layer1[0][1].values())
w21 = w2[0][0]
w22 = w2[0][1]
b2 = w2[0][2]
x = np.arange(-2,2,0.1)
y1 = x*(-w21/w22-0.2)+ b2/w12
y2 = x*(-w21/w22-0.2)- b2/w22
plt.title('training data and decision boundaries')
plt.grid()
plt.plot(x,y1,c='green')
plt.plot(x,y2,c='green')
plt.fill_between(x, y2, y1, y2 > y1, color='#ff0000', alpha=0.3)
plt.show()
print('y=',-w21/w22-0.2,'x+',b2/w12)
print('y=',-w21/w22-0.2,'x-',b2/w12)
#generate data
mu_x = [1,-1]
delta_x = 0.25
mu_y = [1,-1]
delta_y = 0.25
z=np.zeros([200,3])
```

```
n=0
plt.title('Self-Generated Testing Data')
for i in range(2):
    for j in range(2):
        x = mu_x[i]+delta_x*np.random.randn(50)
        y = mu_y[j]+delta_y*np.random.randn(50)
        if mu_x[i]*mu_y[j] == 1:
            plt.scatter(x,y,c = 'red')
            z[n:n+50,0] = x
            z[n:n+50,1] = y
        if mu_x[i]*mu_y[j] == -1:
            plt.scatter(x,y,c='blue')
            z[n:n+50,0] = x
            z[n:n+50,1] = y
            z[n:n+50,2] = z[n:n+50,2]+1
        n=n+50
# test generated data
correct = 0
wrong = 0
for row in z:
    determine = test(network,row)
    if determine == row[-1]:
        correct = correct+1
    else:
        wrong = wrong+1
print('Accuracy of Prediction: ',100*correct/(wrong+correct),'%')
# choose 16 points for training
p16 = np.zeros([16,3])
p16 = [[0.258140455863934,0.567315287097103,0],
       [0.734317095363312,1.3195912881375,0],
```

```
      [0.592050478464416,1.19667090632313,0],
      [0.699169771902131,0.433616111743741,0],
     [-0.985097960238142, -1.12082573650756, 0],
     [-1.38177934789019, -0.778890498143647, 0],
     [-1.02250556227963, -0.793451426629458, 0],
     [-0.674450149157024, -0.764088469196094, 0],
     [0.426882921196544, -1.39040516540311, 1],
      [0.745479567524074, -0.610730018707631, 1],
      [0.427053593889095, -1.14155143108763, 1],
      [0.564108502431285, -0.86010860171292, 1],
     [-1.26109374697199, 0.860117010560185, 1],
      [-0.780648589258112, 0.728503411389706, 1],
      [-0.805034057569262, 0.963331827863979, 1],
      [-0.694074012697457, 0.59385595699395, 1]]
seed(1)
num_inputs = len(p16[0]) - 1
num_outputs = len(set([row[-1] for row in p16]))
print(num_outputs)
network = initialize_network(2, 2, 2)
train(network, p16, 0.2, 500, num_outputs)
layer1=[]
for layer in network:
    layer1.append(layer)
    print(layer)

# plot the boundary lines
# plot training data
for k in range(0,8):
    plt.scatter(p16[k][0],p16[k][1],c='red')
for l in range(8,16):
```

```
    plt.scatter(p16[l][0],p16[l][1],c='blue')
# plot boundary

w2 = list(layer1[0][1].values())
w21 = w2[0][0]
w22 = w2[0][1]
b2 = w2[0][2]
x = np.arange(-2,2,0.1)
y1 = x*(-w21/w22)+ b2/w12
y2 = x*(-w21/w22)- b2/w22
plt.title('training data and decision boundaries')
plt.grid()
plt.plot(x,y1,c='green')
plt.plot(x,y2,c='green')
plt.fill_between(x, y2, y1, y2 > y1, color='#ff0000', alpha=0.3)
plt.show()
```